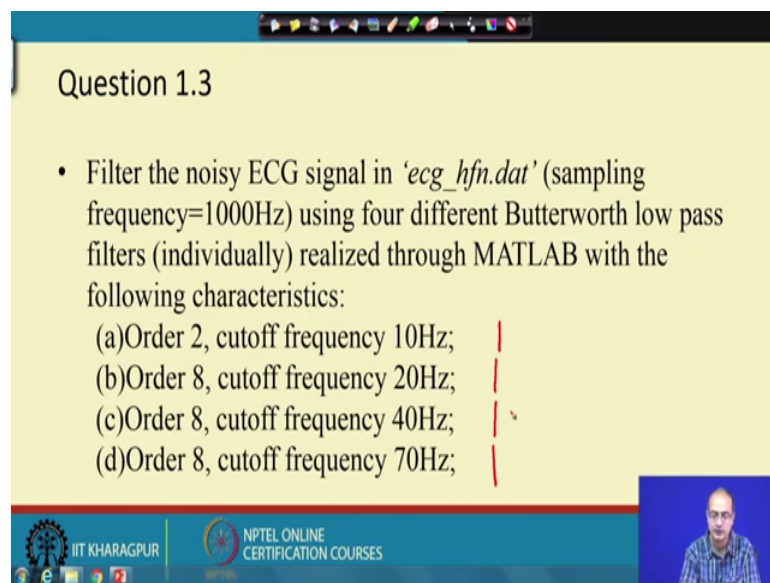


Biomedical Signal Processing
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Lecture - 49
Tutorial - I (Contd.)

So, now we are going for the third problem of the tutorial one. So, here the problem is that again we have the noisy signal that is ECG sample that 1000 hertz and we are trying with Butterworth filter. The low pass filter to actually review that high frequency noise and we have here four different options that we look at the different options of the that first is order two cut off frequency 10 hertz; that means, order is low that means, it will have a smooth transition and cut off frequency is also very low.

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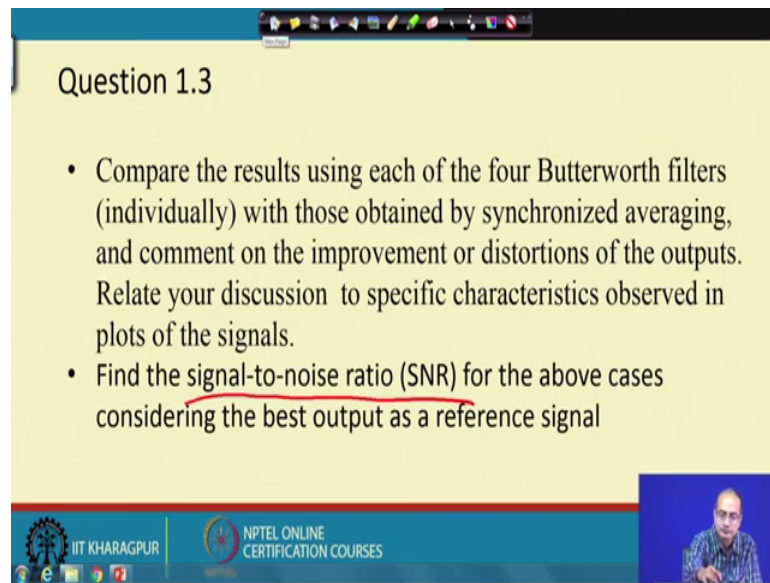
Question 1.3

- Filter the noisy ECG signal in 'ecg_hfn.dat' (sampling frequency=1000Hz) using four different Butterworth low pass filters (individually) realized through MATLAB with the following characteristics:
 - (a) Order 2, cutoff frequency 10Hz;
 - (b) Order 8, cutoff frequency 20Hz;
 - (c) Order 8, cutoff frequency 40Hz;
 - (d) Order 8, cutoff frequency 70Hz;

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Second is we have model order increase to 8 and that the cut off frequency is also increased to 20, then keeping the model order same cut off frequency increased to 40, then 70 ok. So, cut off frequency is increase means in a low pass filter that we are allowing more and more high frequency part. So, we have to see that how it helps to preserve the signal component that is ECG signal in hand and how it is successful in blocking the high frequency noise ok. So, that is the task at hand.

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Question 1.3

- Compare the results using each of the four Butterworth filters (individually) with those obtained by synchronized averaging, and comment on the improvement or distortions of the outputs. Relate your discussion to specific characteristics observed in plots of the signals.
- Find the signal-to-noise ratio (SNR) for the above cases considering the best output as a reference signal

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So, the next instruction is compare the results using each of the 4 Butterworth filters individually with those obtained by synchronous averaging the synchronized averaging, it has been taken as a gold standard in this case and for that that we would like to see that how we can actually get the output ok.

Next is we would compute the signal to noise ratio that is SNR. It compute the SNR here of the above cases considering the best output as the reference signal. So, we will take out of the 4 cases, the best one as the reference with respect to that will compute the SNR ok.

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Solution 1.3 Cont....

- Input ECG signal is available at:
http://people.ucalgary.ca/~ranga/enel563/SIGNAL_DATA_FILE/S/ecg_hfn.dat
- Sample MATLAB code to display the input ECG is available at:
http://people.ucalgary.ca/~ranga/enel563/SIGNAL_DATA_FILE/S/ecg_hfn.m

Note: Keep the input signal and the MATLAB codes in the same

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So, the first is we need to get the signal; so for that we have the signal given here and here we have got the code now again we remind you that we need to keep the input signal and the MATLAB code in the same directory . So, with that we proceed.

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Solution 1.3

- The basic Butterworth low pass filter is given by-

$$|H(j\omega)| = \frac{1}{1 + \left(\frac{j\omega}{j\omega_c}\right)^{2N}}$$

where N is the order of the filter

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So, now the first thing as a part of the solution is to look at the basic structure of the Butterworth filter ok.

So, Butterworth filter of order n here is the formula, we are getting the signal that the magnitude in the frequency domain of the signal for a nth order filter. So, that low pass

filter we get and we make use of actually MATLAB to get the help of it to compute this, the Butterworth filter and that makes the task easy for us.

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Solution 1.3

- The input Signal with High frequency noise

```
%% Load and Display Signal
% load Input ECG Signal
x = load('ecg_hfn.dat');
fs = 1000; % sampling freq
L = length(x); %Signal length
t = [1:L]/fs;
figure,
plot(t,x);
```

The slide features a plot of an ECG signal with high-frequency noise. The x-axis is labeled 'Time (sec)' and the y-axis is labeled 'V'. The plot shows a regular ECG waveform with sharp, high-frequency spikes. To the right of the plot is a block of MATLAB code with several lines underlined in red. A red circle highlights the line `t = [1:L]/fs;`, and two red arrows point to the `figure,` and `plot(t,x);` lines. The slide footer includes the IIT KHARAGPUR logo and the text 'NPTEL ONLINE CERTIFICATION COURSES'.

So, first we need to look at the input signal with high frequency noise. So, we know now by now we know the game that what we have do first we need to load the signal in a variable here that variable is x, then we need to note down that frequency of sampling fs, then take the length of the signal that mean how many samples are there in that signal and with respect to that we can actually calculate the time axis ok.

How do we do that we compute the number of samples. So, it will look like a ram the index is increasing and multiply with the one by frequency that is sampling frequency one by sampling frequency will give us the sampling interval and then we create a pen by this figure command and then issue the command for plot . So, we get this plot here with sufficient amount of high frequency noise. So, this is the signal we need to clean with the Butterworth filter or low pass Butterworth filter.

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Solution 1.3

- Magnitude and phase response of Butterworth filter

```
%% Design Butterworth low pass filter
fs = 1000; % Sampling freq
fc = 10; %for cutoff freq of 10Hz
n = 2; % order of filter is 2
[b a] = butter(n,fc/(fs/2),'low');
% plot magnitude and phase spectrum
figure;
freqz(b,a,L,fs);
Note: 'low' indicates Low Pass Filter
```

So, let us proceed towards it. So, first let us see how the magnitude and the phase response of the Butterworth filter looks like ok. So, we have the advantage of MATLAB, it is already implemented. So, to use that we need to keep few things in mind that we need to we need the sampling frequency that is one variable. Next is the cut off frequency, the first choice was that cut off frequency should be set to 10 hertz. So, we set f_c the cut off frequency variable as 10 and the model order should be 2. So, we have taken a variable n to provide the model order of the Butterworth filter.

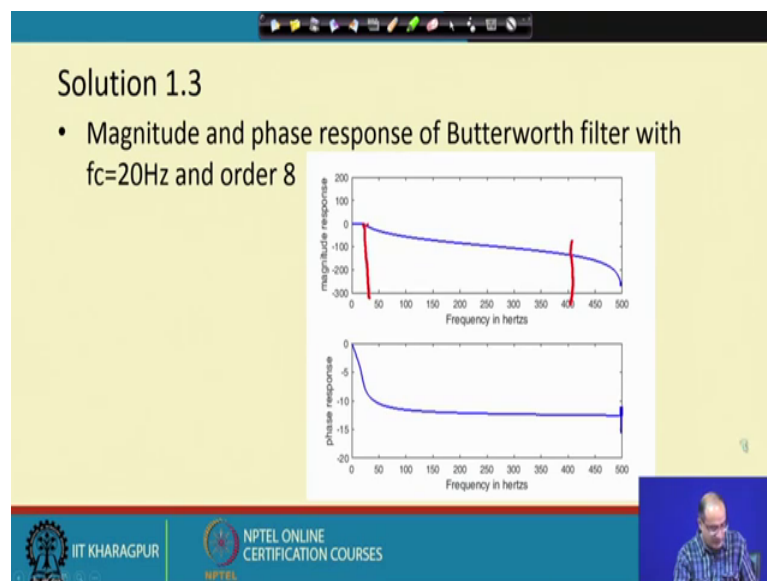
And then we issue a command `butter` which will provide us the numerator coefficients and the denominator coefficients for these i i r filter ok. So, it is not a fir filter. So, we need both. So, for that the variable, we have given the first variable is the order of the filter. Next we are giving the cut off frequency as a fraction of half of the sampling frequency ok; that means, if we look at the graph that both side, if we if we take the normalise actually frequency that it is minus 0.5 to 0 and 0 to 0.5 in the right hand side. So, to take care of that fact we are taking half of the sampling frequency and what part of that 0.5 that our, that cut of frequency is we are taking that fraction and then the last command `low` it is suggesting that we are looking for a low pass filter ok.

So, you can get these things much better and in more detail if you issue the command `help` can look at the manual of MATLAB and that lot of magnitude and phase for that first we create that pen with the command `figure`, then like the previous case, we have

use the command that freqz and we pass this variables first is b is numerator that polynomial coefficient, a is the denominator polynomial coefficient, L is the number of samples we need; that we are giving as the same as the number of signals in the time domain, but it could be some other value it need not be same as the number of samples in the time domain signal and fs is the that sampling frequency ok. So, that is the command actually we are using and with that here we get the two things; one is frequency as well as the phase ok, both we can get in the form of plot and we have given it in the figure here.

What we get; that cut off frequency is ten; that means, somewhere here up to that the amplitude is constant 0 to 10 after that that there is a very slow decrease that mean the transition is not very sharp and the model order is actually filter order is responsible for that we have only order is two and the phase it has some discontinuity at that part and then it is constant nothing much to note in the phase part.

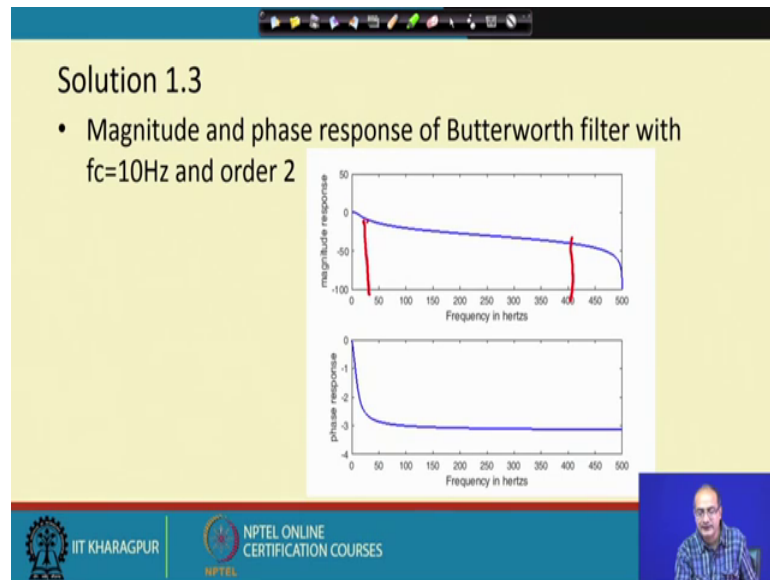
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Next, we look at that it more clearly in a bigger way, we see that that magnitude and the phase plot and then we would change the cut off frequency and the order go for the second case where we have the model order is 8 and the cut off frequency is also increase. So, with a increase with a cut off frequency, we see that we can see these constant portion more clearly and it has come up to say 20 hertz is constant up to that after that there is a decrease and the decrease is more pronounced ok.

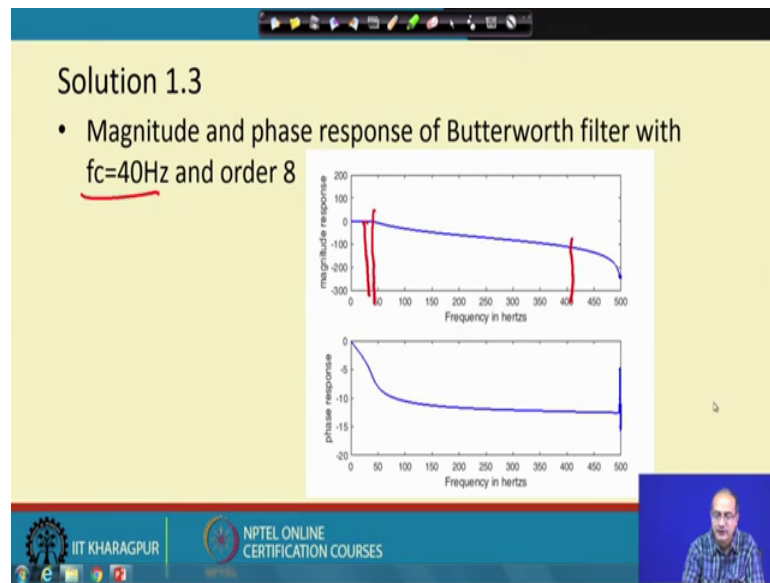
If we look at that at 400 hertz, what was the decrease? If we go to the previous case; would be able to appreciate that that it was actually less than it was more actually Less than 50.

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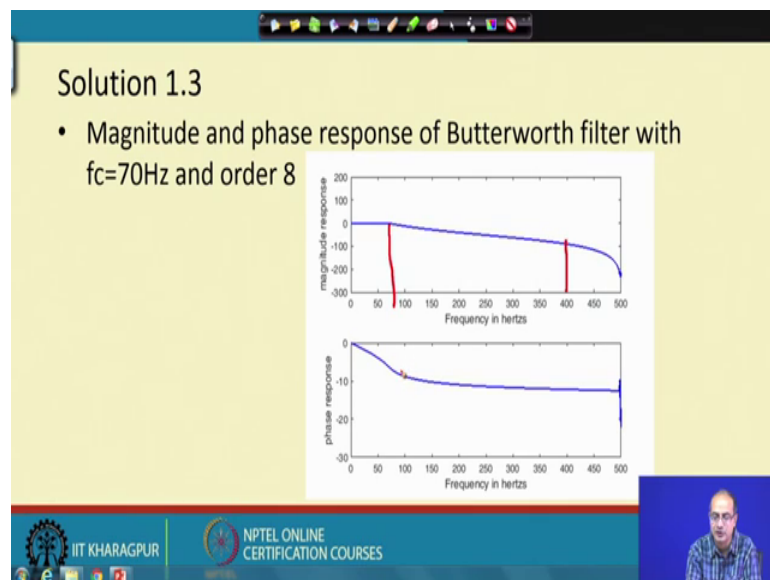
the decrease in the previous case when the model order is to now though the cut off frequency has moved towards the right the decrease is much more pronounced and I think, it is about minus 100 here have a phase part that is not much thing to note, but only thing; what we can get the knee position is changing with the that cut off frequency ok. So, that is the change.

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Now, the third one is that we have the cut off frequency at 40. So, it has moved here up to this part that is it is constant and then it is decreasing.

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So, and then we go for; that cut off frequency 70. So, it is coming up to this point constant and that we will see that there is some impact that probably it has gone little less than it is higher than minus 100. So, as cut off frequency is moving right that the suppression is not that prominent, but the model order has more impact that we can get and then knee point also that from where it is becoming constant that is also moved that

is come near the 70. So, these are the different cases we get.

Now, we need to look at that how the output signal is affected by the change in the magnitude spectrum and the phase spectrum of the Butterworth filter. So, for that first we need to filter the ECG signal that noisy ECG with the, that each of the butter Butterworth filters one at a time.

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Solution 1.3

- Noisy ECG and filtered output of Butterworth filter

```
%% Input ECG and filtered Output
x = load('ecg_hfn.dat');
fs = 1000; % Sampling freq
L = length(x); % Signal length
t = [1:L]/fs;
figure; subplot(2,1,1);
plot(t, x); % plot input
output1 = filter(b,a,x);
subplot(2,1,2);
plot(t, output1); % plot filtered
output
```

Note: In this example, for SNR calculations, output of Butterworth filter of Order 8 and $f_c=40$ Hz is considered as a reference signal

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So, for that; here we show the code for once case that already we have loaded the signal. So, that part is there ok. So, first part of it that loading the signal, then noting the sampling frequency the length of it and the time axis that is there and then we plot that in the top row of the pen using the figure and the subplot command ok. So, we use that plot command for that.

Now, next part is the is to calculate the output and the already we have computed the that the numerator polynomial coefficient and the denominator that polynomial coefficients b and a for the Butterworth filter. So, we make use of that and use the command filter and the third variable is the input signal that is noise corrupted ECG and with that we get the output what we stored as the output one.

So, we plot that signal here and we try to appreciate that what is the change ok. So, as we see here the signal that the signal is looks pretty clean in terms of suppression of noise this is the case that where our cut off frequency is 10 hertz and model order is 2, but the

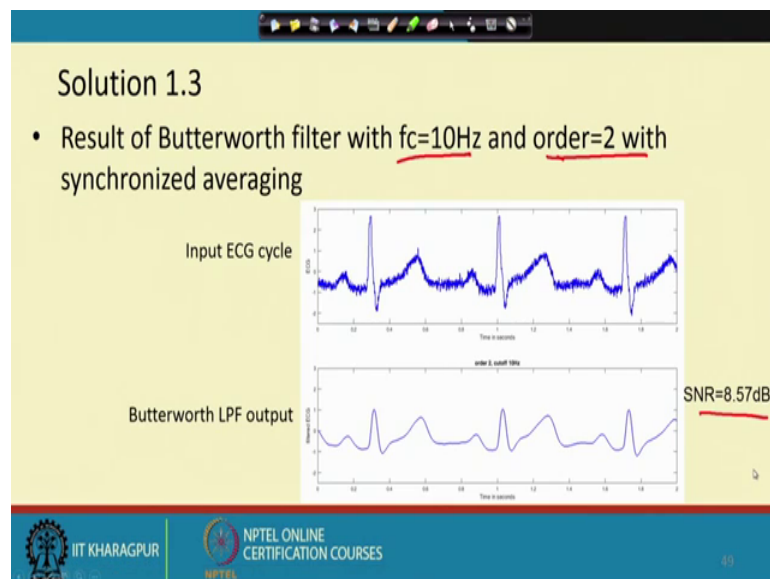
signal looks like a little misery or distorted that you can get, if you look at that the amplitude of the signal the input amplitude, say it is varying from minus 2 to plus 2 here, it is reduced it is minus 1 to plus 1 kind of thing that is one thing.

Another thing if you just take that if you assume that know some somewhere some something is wrong with the scaling if we assume that that then I would request to look at the relative amplitude of the say r and the t, then gap between these two has reduced ok

So, what that signifies that everyone can have their own interpretation and then at the end we need to see that from the trend that which one is correct my interpretation is because the cut off frequency is low, we lost a good part of the ECG signal or the part which had some high frequency component of the ECG signal is lost and thereby we lost the amplitude of the qrs complex and that is the reason for reduction of the height of the qrs complex ok.

So, let us see that move forward and here another thing that after doing this experiment we found that for order equal to 8 and cut off frequency has 40 hertz, we could get the best results. So, that is taken as the reference signal for computing the SNR in this case ok. So, that will come later that and let us see that how the outputs that come.

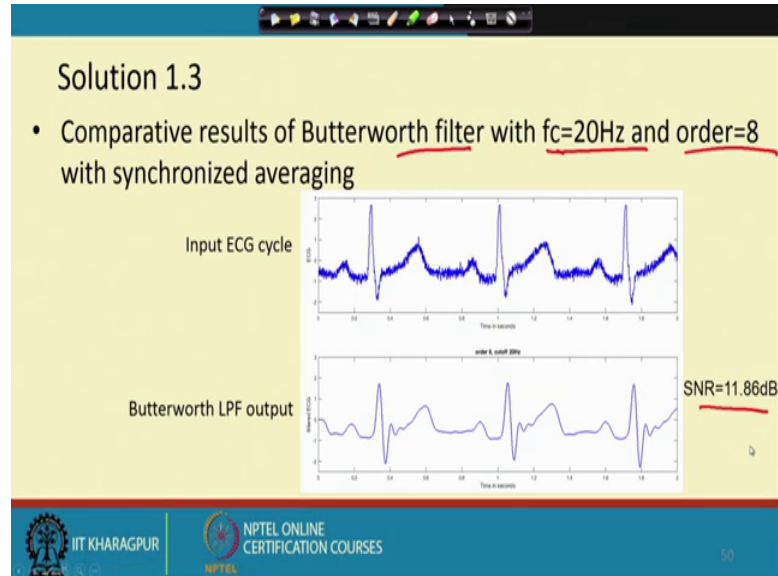
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So, first case; we have taken the Butterworth filter with cut off frequency 10 hertz and

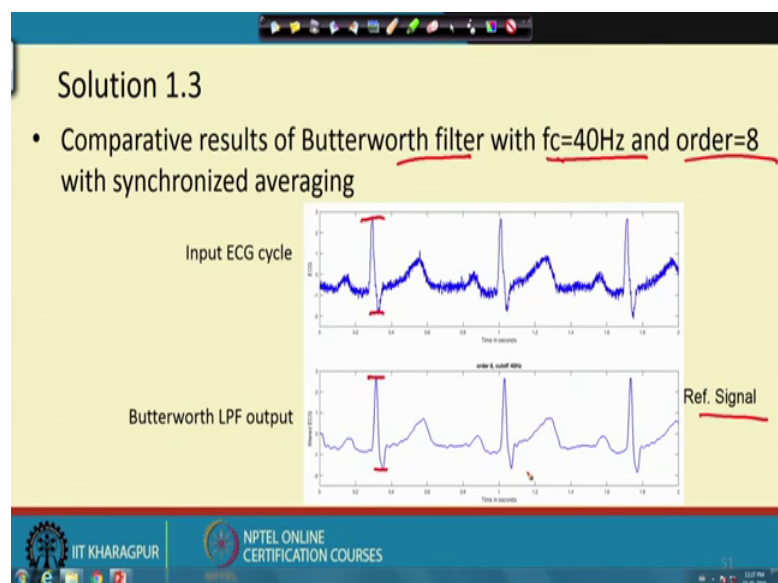
model order equal to 2 and with that that when we compare that low output the Butterworth filter we see that SNR is 8.57 ok.

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So, it is not a very good one and we look at that next case that frequency is that we have that 20 hertz and model order is increased we see there is an increase in the that qrs complex value, it has become more than one which was about one earlier and SNR also is increased ok.

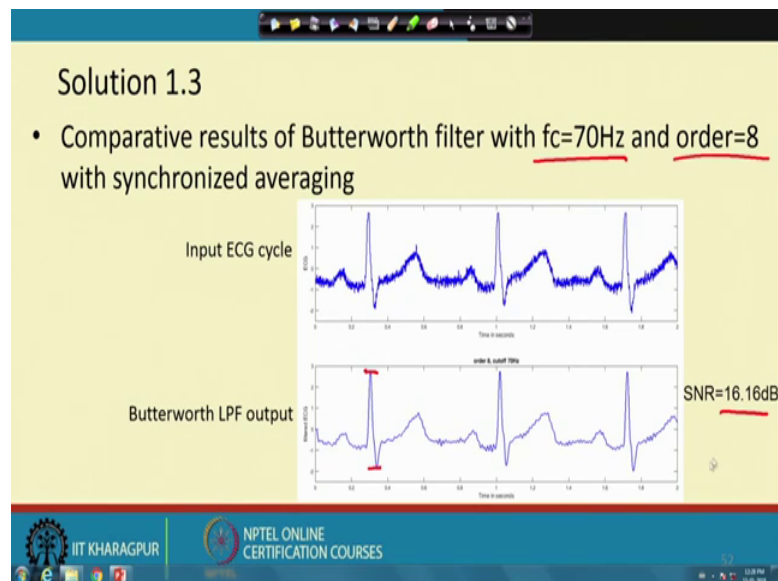
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Now, we get that the cut off frequency 40 hertz and model order 8, we see that some

undulations are there and probably there the details of the signal and signal has got actually the amplitude, if you look at the amplitude, here it is about 2.5, here the same height it has got ok. So, that the lower side amplitude also we get the same thing that. So, we can actually tell that this is closest to the original signal and that is why we have taken that as a reference ok; however, we find that the details part of that thing is there and the undulation that is there in the single that is not of that high frequency. So, we actually assume or accept that as a part of the signal.

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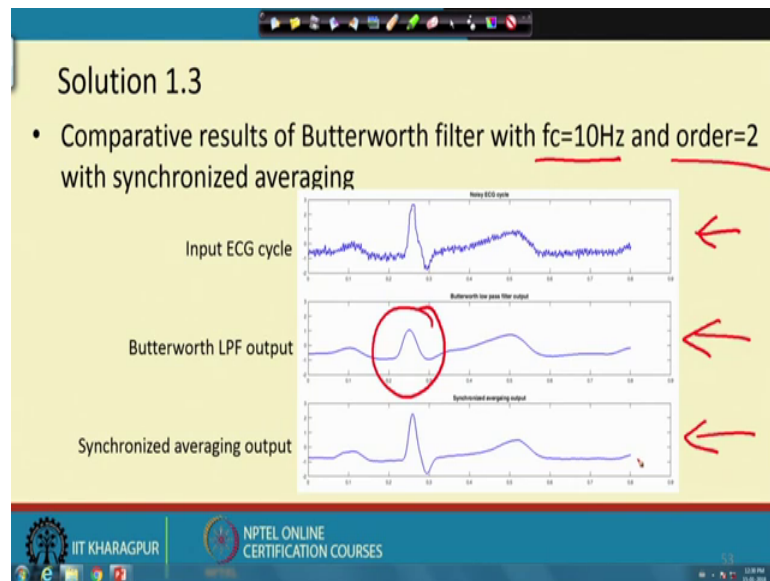


Next, we move for cut off frequency as 70 ok, a big jump model order remains the same as 8, in this case, what we find that there is not much change in the that signal height though, it may look a little better and where we more closer to the qs complex is close to the original signal, but a low not of noise is keeping in, but still it is better than probably that cut off frequency equal to 20 and the SNR also tells as the same thing ok.

If we go back we can see that the reference signal and that the previous two values that it was about 8.5, then near twelve then reference signal that is and now it is becoming 16. So, it is better than that, but we get the high frequency noises that keeping in because we are getting a lot of undulation there in the signal and that is the reason we have chosen the cut off frequency 40 is a better choice because in this case, though we are preserving more high frequency component of the signal we are allowing part of the noise signal to keep in which is not a good thing to do ok. So, that is why we do not take that cut off

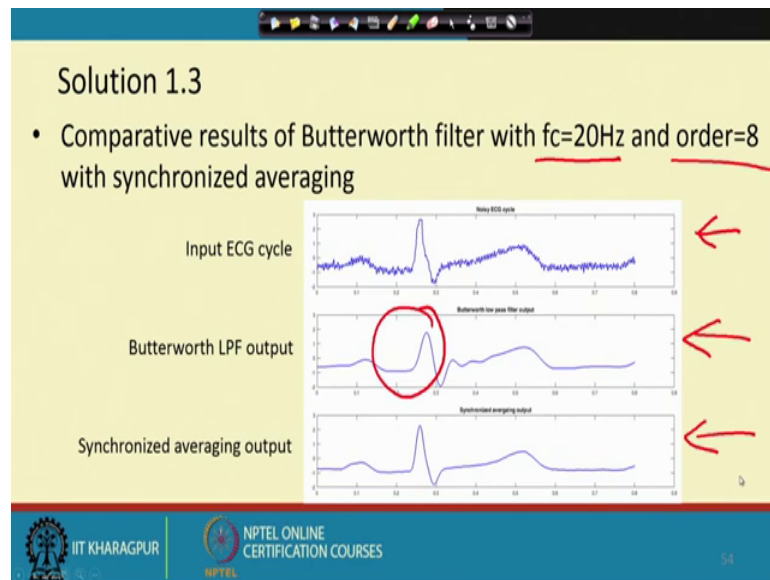
frequency 70 hertz as a good choice or the right choice.

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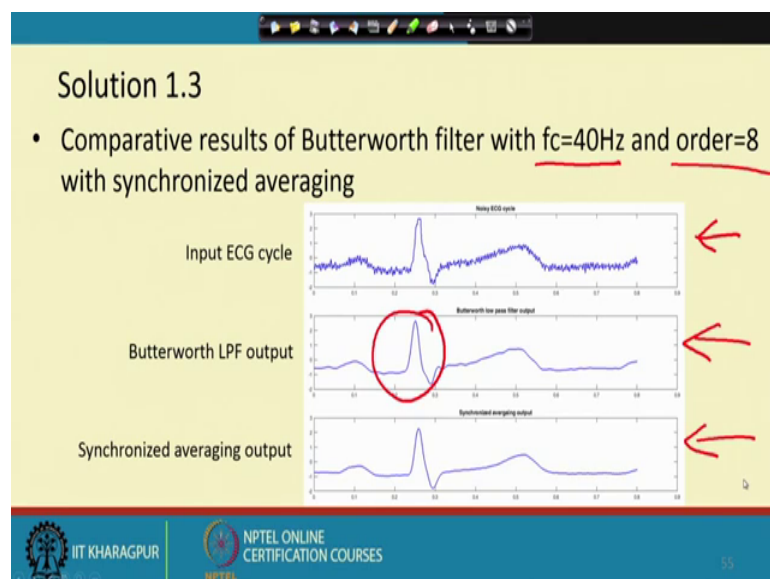
Now, let us compare with the, that synchronous average output ok. So, synchronous average output what we got first thing, we get that we have just taken a cycle at the top we have the noisy signal here in the middle, we have the Butterworth filter output and the below is the synchronous average output taking that as a reference we are comparing and we have that cut off frequency 10 model order equal to 2 for that we see the signal is very smooth; that means, noise substrate operation is very effective, but we lost some part of the that high frequency part and that is evident here looking at that your peak that is becoming evident ok.

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So, now let us go for the next case we get the next part that is the cut off frequency is moved to 20 and model order to 8 again suppression is we would say, it is very good same as the synchronous average or maybe even better smoothing smooth signal we are getting and there is some improvement in the qrs complex shape the height has increased, but still it is much further and wider compared to the original qrs complex as well as the synchronous average ok.

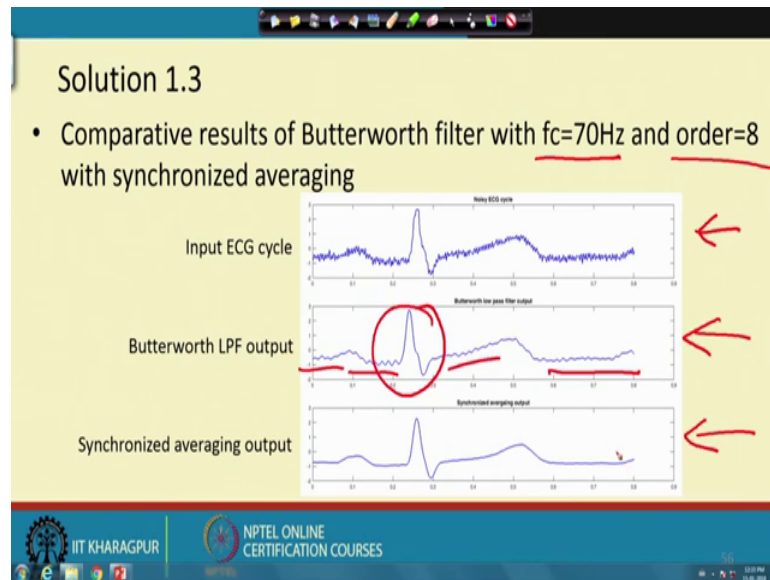
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Now, we are going for the that part where we have taken it the reference that is 40 hertz

cut off frequency there we see that now the synchronous average output that qrs complex and the that Butterworth filter output they looks very close. So, that is the reason, we have told that cut off frequency as 40 and the model order of the Butterworth filter order as 8 is giving us the best signal ok.

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So, as if we proceed further we go for that cut off frequency 70. Now we get the smoothing has become impaired in this case ok. So, this things would be more clear if we go back through the slides if we look back that we see that here that undulations are yet to come some change in shapes or details are there in the signal ok, if we go back further cut off frequency is 20, it is very smooth 10 also it is very smooth ok.

Now, at 70 though the qrs complex shape it looks again very good it looks very close to the synchronous averaging we are getting lots of undulation in other places that is if you look at this position beyond t before t or even the part of the t and p; we see a lot of undulations are there that suggest that the noise suppression has not been that effective the cut off frequency has become too high and the noise are started keeping in ok.

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Solution 1.3

Observations:

- Butterworth filter with $f_c < 40$ Hz gives distorted output as the high frequency components in ECG signal are lost
- Butterworth filter with $f_c = 70$ Hz has some high frequency noise present in output signal as noise frequency lies in pass band of filter
- The best results are obtained at $f_c = 40$ Hz and order = 8

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So, now we would look for the summary of this experiment. So, first thing what we note that the Butterworth filter with frequency cut off frequency f_c less than 40 hertz gives us distorted output ok; why this distortion that the component of the signal is lost some part of the signal energy is also lost.

Next is that we have f_c equal to 70 has some high frequency noise present and as the noise is keeping in the signal, it comes in the pass band of the signal. So, we are getting noisy output. So, that is not a right thing to have then what we get that the middle part is the best when we have that cut off frequency is in between that is we have taken 40 and model order is 8, we get the best result and for that we have taken synchronized averaging as the reference for us ok.

So, what we get in this experiment that we know that high frequency noise can be eliminated by Butterworth filter or low pass Butterworth filter successfully, but we need to be very careful in selecting the two things we need to appropriately select the cut off frequency. So, that we will not lose part of the that signal of the interest at the same time it is low enough to eliminate the noise signal and we would also look at that model order of the Butterworth filter should be should not be very low if it is very low, then the attenuation at the high frequency would be very small.

So, for an effective low pass filtering, it should have sufficient model order and we found model order 8 is good enough for this purpose with that we would like to conclude that

the problem number three of tutorial one.

Thank you.