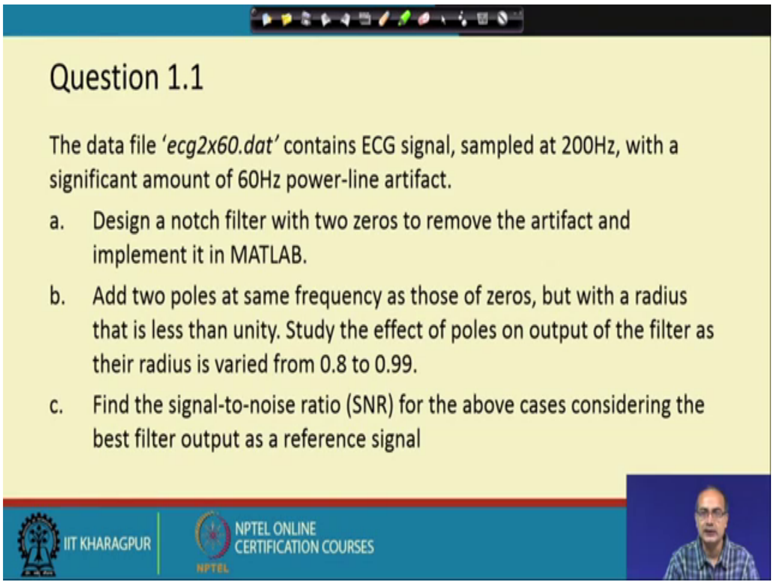
Biomedical Signal Processing
Prof. Sudipta Mukhopadhyay
Department of Electrical and Electronics Communication Engineering
Indian Institute of Technology, Kharagpur

Lecture - 47
Tutorial – I

Good morning. So today, we will start with the biomedical signal processing. Today, we will start in a different way, we will take some tutorial.

(Refer Slide Time: 00:26)



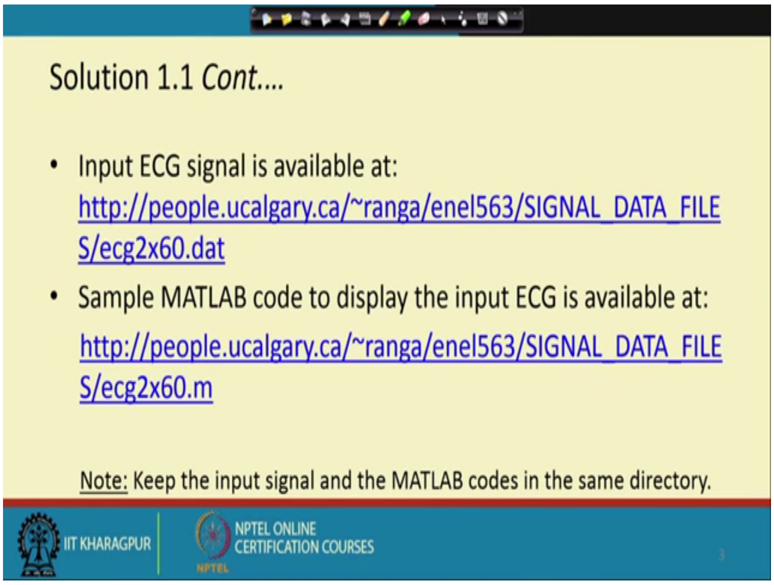
So, first we will take this topic that the questions number 1 of tutorial 1; that we have a data file of ECG sample that 200 hertz and it is corrupted with power line artifact and because the data is taken from North America, the power line frequency is 60 hertz, in India; the power line frequency is 50 hertz. So, the power line artifact would have 50 hertz frequency.

Now, the first task is to design a notch filter which two zeros to remove the artifact and implement it in MATLAB. The next task would be to add two poles at the same frequency at those two zero, but with the radius that is less than unity, the essence is that the filter should be stable for that its needed that it should be the radius should be less than unity and we have to study the effect of poles on the output of the filter as the radius is varied from 0.8 to 0.99.

Now, what would be the effect of this poles and why they are needed, we will explain that we will discuss about that later once we have the that simple two poles, sorry, two zeros for that notch filter, we will be able to appreciate it better at that moment next that we should find the signal to noise ratio of the above cases considering the best filter output as the reference signal.

In this case, we do not have any reference signal in the way that noise free signal. So, we cannot directly calculate the SNR, but what we can do? We can take the closest approximation to it is the best filter output and with respect to that we can judge that how the other cases they are actually doing. So, that is what we need to do.

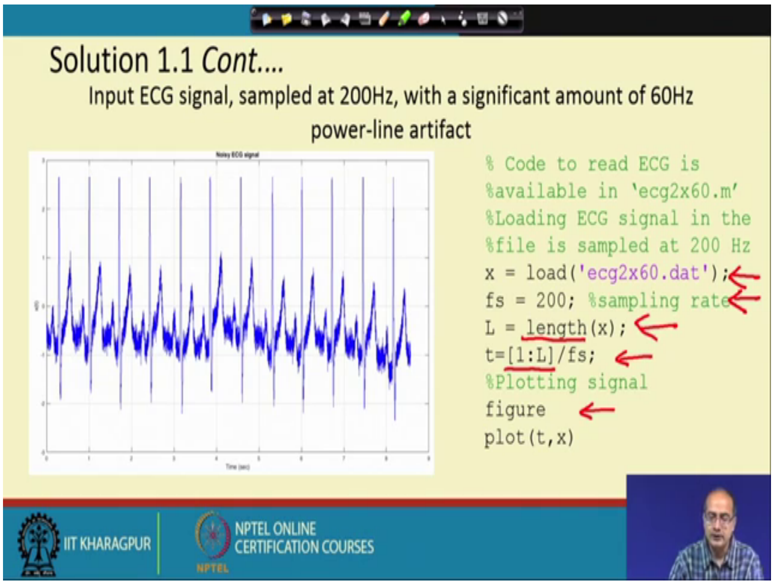
(Refer Slide Time: 02:54)



So, first task is to get that signal and the sample code which will enable us to read them. So, here we have provided the links from where we can get this data file and dot m file which will enable us to read that data faithfully. Now here that one thing, we should keep in mind that we should keep both the data file and the MATLAB file in the same directory and set the directory as the working directory of the MATLAB.

So, once you do that; then this code will work properly. So, let us proceed towards it.

(Refer Slide Time: 03:48)



So, first we would like to see that how is the signal how we can read that and to get a plot, we would like to see that how is the effect of the that power frequency noise. So, here is the code that to read the signal ECG 2 60 dot m that that ECG that 2 x 60 dot dat that is the data file and dot m file is to read it. So, it goes like that that first line is that here, we are reading that file here we are reading that file and the command is load. So, we load that one dimensional data in the variable x.

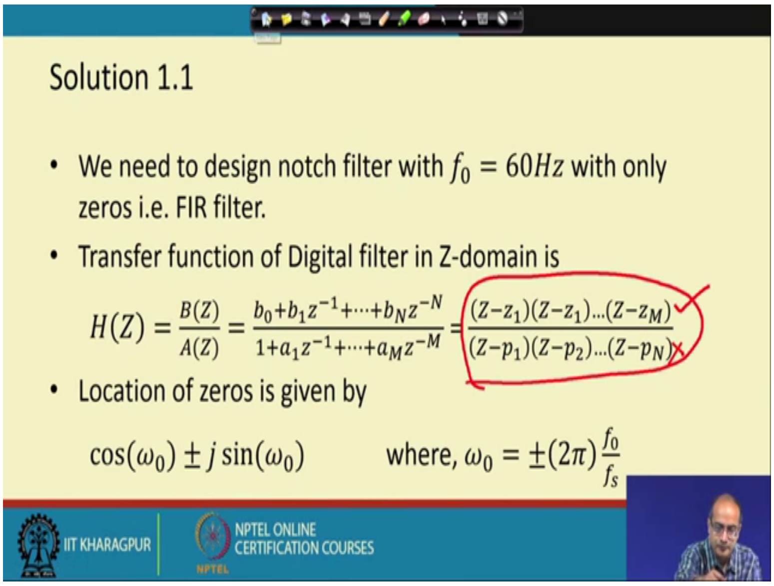
Next we make use of that information that the sampling frequency is known. So, we first assign a variable fs equal to 200. So, we keep the sampling frequency, next seeing, we calculate that what is the length of the data; that means, for how much period the data is collected and for doing plot or doing any processing, we need to know the number of samples. So, a simple command is there length which can give us the length of the vector x and we assign that to the variable L.

Next to take the plot, we actually would like to see the plot in time rather than in terms of sample because that will give us a better idea about the span of the signal and that what is the that length of one cycle. So, that we can understand that better and here is the command that here within this third bracket 1 to L gives us a RAM starting from 1 to L with the increment of 1 at each step and that is divided by fs means fs is the sampling frequency. So, 1 by fs gives us the actually that sampling interval.

So, once you multiply that with this counter means we are starting from 1 to L, we get the time instance of the sample. So, t is the variable for that which will help us to do the plot. So, to get a new plot that first we need to issue a command called figure which will create a pen and then we plot that t comma x that we get the variable for plotting. So, here is the plot, we get the plot is in between 8 to 9 seconds. So, we have good amount of data and we get that most of the part of the signal, the amplitude is from minus 2.5 to 2.5 that is the magnitude within that and there is a lot of actually power frequency noise and that is more visible in the p and the t wave because the QRS wave, we sharp we cannot get that ff that clearly here, but for the p and t wave we can get that thing in a much better way.

So, now let us proceed, let us move forward.

(Refer Slide Time: 08:05)



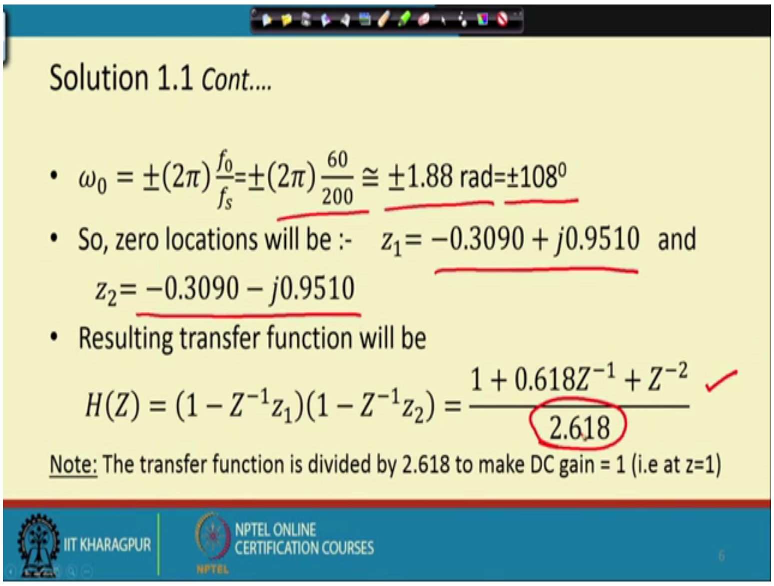
Let us see that how to design the notch filter we know the power frequency noise is at 60 hertz and now to stop them that all the signal at that frequency, what we need to do? We need to assign zeros here. Why zeros because zeros will force that the any component that 60 hertz to 0 and instead of one single 0, we are taking two zeros which are actually conjugate pair the reason is our signal is real and we want the output of the filter also should be real.

Now, for that; what is the requirement that we need to have a filter which should be real and in that case for a real filter, we should have at least two zeros that is a FIR filter with two zero and they should be conjugate pair and why we are chosen the FIR filter because the FIR filter is the simplest form of filter and in that case that we need not have to worry about the stability of the filter. So, that is why FIR filter is chosen and that is the form of the notch filter that is also could be a way to present that that we are asked to implement a notch filter. So, we are just doing that.

Now, if we look at the transfer function in the Z-domain of a filter, we can write Hz in this way, we can put that as a rational polynomial in the numerator, we have the terms that v 0 to v n at different lag and in the denominator, we have the terms that in terms of a i and the same thing can be written in the pole 0 form. So, that is what is given here that in the pole 0 forms. In fact, here it is more intuitive, we can see them as poles and zeros and in this particular case as we are talking about that FIR design, we will only have the zeros, we would not have the poles at the beginning ok. So, that is the design we have chosen.

Next is the location of the 0 for the conjugate pole pair, it should be on the unit circle. So, r would be 1. So, our location of the 0 is r cosine omega 0 plus minus j of sin omega 0 and that frequency omega 0, it would be a fraction of a phase it can be given as 2 pi into f 0 by f s where f 0 is the frequency of the that power frequency noise or the low that is determining location of the notch filter so, with that that we need to find out first omega 0.

(Refer Slide Time: 11:44)



So, we go ahead with that omega 0 is because we have conjugate actually zeros where the pair. So, we have omega 0 equal to plus minus 2 pi f 0 by fs.

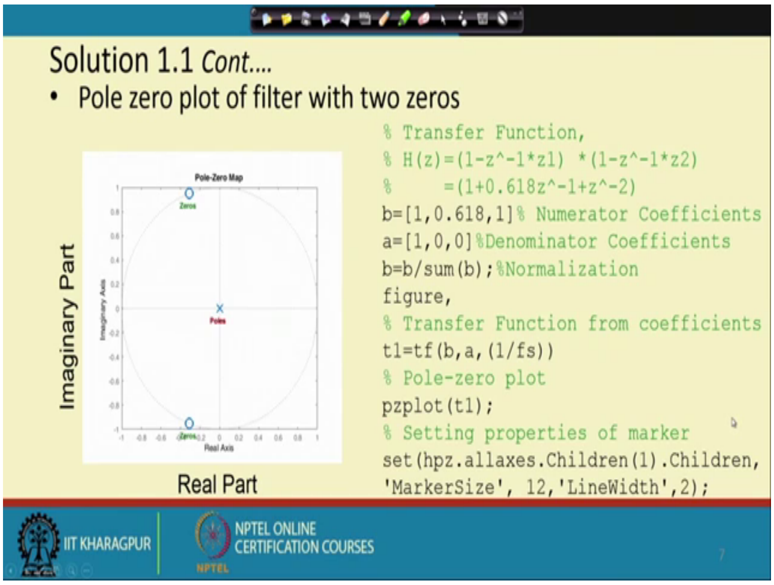
Now, when we replace the value of f 0 and fs that is 60 and 200 respectively we get that it is plus minus 1.88 radian, please keep in mind, we would get the value in radian and for our ease of understanding we may convert it into other terms like in degree so that we can visualise the location easily ok. So, that is what is done here that we have converted it, but please keep in mind that when we are computing the that sin and cosine term, the default functions, they uses the theta or the angle omega in radian unless it is specified, it should be in radian. So, it is better to keep it in radian for the sake of computation.

So, now let us proceed from there, next step is to find out the 0 location using the value of omega 0, we find the location of the that the 0s z 1 and z 2. So, they are actually conjugate pair the two complex numbers which are conjugate to each other and with that that we need to proceed for the design ok.

So, the resulting transfer function in this case would be in this form that Hz is 1 minus z inverse z 1 into 1 minus z inverse z 2. So, now, replacing the value of z 1 and z 2 with the values what we have calculated that is these two values, we get this polynomial here and here we have used a term for normalisation, it is just to take care of the fact that if we look at that the degree centigrade gain that is that z equal to 1.

What is the gain? We are getting if we do not do any normalisation, we get the value would be 2.618. So, just to make the DC gain equal to 1, we normalise it or divide it by 2.618. So, that is the normalisation term we have used. So, we have the transfer function ready.

(Refer Slide Time: 15:09)



Now, we should like to see that how the pole 0 plot of the filter looks like before applying that we would like to ensure that how it looks like and one of the intention is by mistake, it should not be going out of the that unit circle, if it happens, then we know that we make some mistake. So, we want to do that check. So, here is the code, first we have assigned the numerator coefficients that is given by b that we have seen the terms that one comma that 0.618.

And that next lag z to the power minus 2 is again 1. So, we have taken that numerator term the denominator, we do not have any polynomial. So, it is a scalar. So, we have kept a equal to 1 and we could have given actually the 2.618 also instead of 1 to do that normalization, but for pole 0 diagram that will not actually change the scenario.

Next, actually we have done that term that b we have divided by. So, as we have taken it want to keep it simple, we have done it in the next step b is normalised by that sum of b ok. So, that is taken cared of now, we create a new pen as figure if we do not issue this term what will happen that the previous figure was there on that pen, it will actually repent and that previous figure would be lost. So, while doing the work many a times you would like to go back and see that what was the input how it look like. So, in that case every time we take a plot or draw a new figure we should issue this command to create a new pen for the figure.

Now, the next thing is to create the transfer function out of the polynomial coefficients as and bs. So, here is the command for that that we assign that filter as t 1 and that command is tf the first variable is v that is the numerator polynomial coefficients that a is the denominator coefficient and 1 by a fs is the actually sampling interval we know fs is the sampling frequency. So, 1 by fs is the sampling interval we have given that.

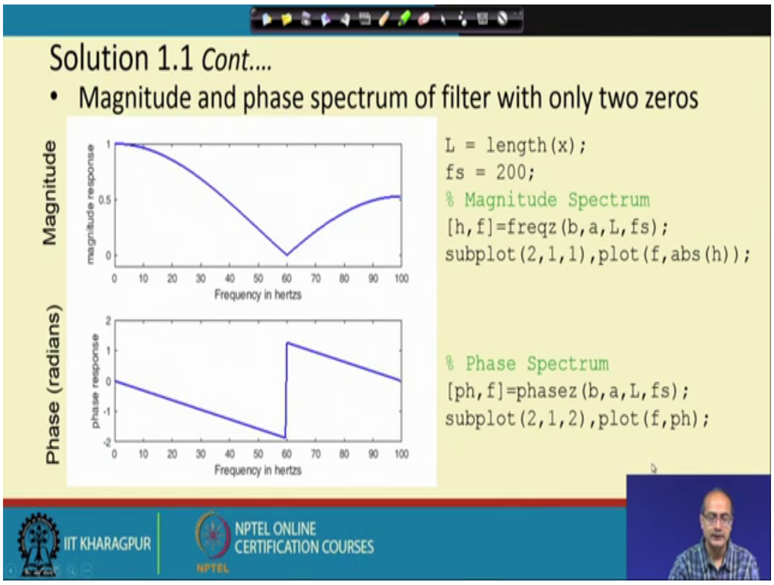
Now, the pole 0 plot, we have a direct command that it will calculate and plot do everything in a single command with the unit circle that is pz plot t 1. So, issue this command and after that we have a small tinkering that we are setting the size of that marker and line with it to make it more visible. So, please keep in mind that these last line; it is not actually mandatory here, we want to make it visible in the presentation.

So, that is why it is required, but just for learning if you want to see that you will get those actually markers will smaller. So, that does not affect actually your learning. So, you would be able to see it yourself on the console. So, in the last part in the last line, if you do not actually need to make any PPT or so, you may skip that part also ok.

So, in the left hand side, we get. Now the pole 0 plot; what we get that we have the zeros as expected, they are on the unit circle and both of them that they are in the left half of the plane because it was 108 degree more than 90 degree that angle was there with respect to the 0 axis and in this case that because with the polynomials in terms of z inverse.

So, we have poles the poles are at the centre ok. So, we have that imaginary part and the real part of the z plane and the unit circle is shown. So, we get the pole 0 diagram, in this way ok. So, that confirmation what, we wanted to do to look at the stability of the filter that we have done.

(Refer Slide Time: 20:35)



So, next we would proceed to get actually the magnitude plot. So, for that what we do that again we check that that what is the length of the signal. In fact, it could be there already here we have just written it just to keep in mind that capital L is giving the length of the that signal; that means, how many points at there in the signal fs is our sampling frequency. So, these two; we have rewritten here if you are continuing the work then you need not have to rewrite because already they are assigned in your program.

Next is we would like to have the magnitude spectrum and for that that the first thing is that to get those points that here we use the command freqz and we give the transfer function in the form with the help of b the numerator polynomial coefficients denominator coefficients L is the length of the that signal or rather here actually this length is how many points, actually, we want to find out because once we have the transfer function that how that what scale we would like to sample that to draw the plot. So, these L is giving that and here we have chosen the same number of points as the signal length.

Now, if you find that it is too much that the frequency plot is continuous you can reduce that if you think that know it a sharp changes, I want to view it at higher resolution in frequency. So, we can increase that. So, it need not be actually L it could be any other value you can put 100, 200, 5000, whatever you would like and the last is the sampling frequency because that will tell us that what is the that the part that what is the frequency scale usually we normalise it minus 0.52 plus 0.5 out of that because it is a real filter one half is enough.

So, what would be that 0.5 here fs is 200. So, our plot would be 0 to 100, if we just look at the right hand side and next, what we should do we issue, if we want to do not want to read actually eliminate our the previous plot, we should again issue a command figure otherwise, if you think that the pole 0 plot is not required, we can rewrite that. In fact, that is what it is done here that we have we should now a command called subplot the subplot command is interesting we are dividing the pen into two parts here, two rows are there, the first variable is giving the number of rows. So, we have divided into actually two part one row and here is the second row.

Next that we are talking about number of column; so, it could be actually divided in the form of a matrix; so, here we need only two slot; so, we have divided into two rows and we have given the two; one is the grid we have chosen and out of that the first actually the place, we would like to make this plot what is this plot that with respect to the frequency axis f; what we get out of this function freqz.

We would like to plot the absolute value of h that is the magnitudes spectrum and for that we get it here right hand side we get there is a deep at 60 hertz and we are just looking at the right hand side part please keep in mind that if we want to draw that spectra it is actually a real signal. So, the symmetric part would be there in the negative side also.

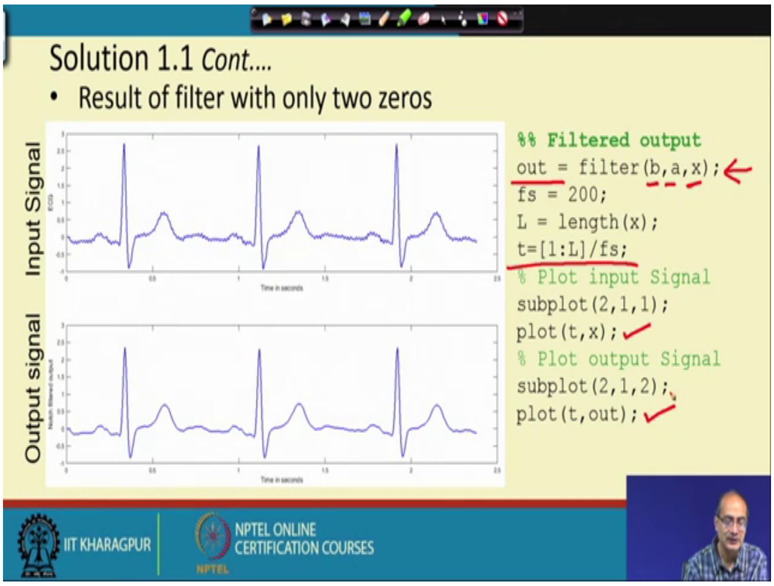
And next we look at the phase spectrum phase spectrum is given by the common phase z again the that the input variable or the signature remains the same we have the numerator polynomial b that denominator polynomial coefficients a, then al is the number of samples we need in the phase that is the resolution in frequency we can say and fs is our sampling frequency.

And now this time we would like to the put it below here. So, we would use the second slot of the grid two one. So, first we have to direct that using the command subplot that where our plot would be given and we issue the command again the same plot command plot f comma ph ok. So, we get this command this output here and what we get that the that phase output is linear with a discontinuity at the location of the 0. So, this is what we get and now what we realise that though that in this portion that the power frequency would be eliminated at if it is located at 60 hertz.

If we have the signal energy close to it that will also be actually reduced because of the that the particular shape of the notch filter, it is not very sharp we would like actually just to eliminate the 60 hertz component. So, that the match of the signal is actually can be maintained we do not want that signal at 59 hertz, 50 hertz, 61 hertz or 70 hertz to be affected ok.

But in this case, we cannot ensure that and that is the prime reason that we told that we should have some pole; now what would be the effect of the pole as a 0 is trying to force that particular frequency amplitude to be 0 in case of a pole, it will try to take it to infinity now if we put the poles and 0 on the same frequency; what would be the result we have to see that rather than the speculating let us go and find it out.

(Refer Slide Time: 28:41)



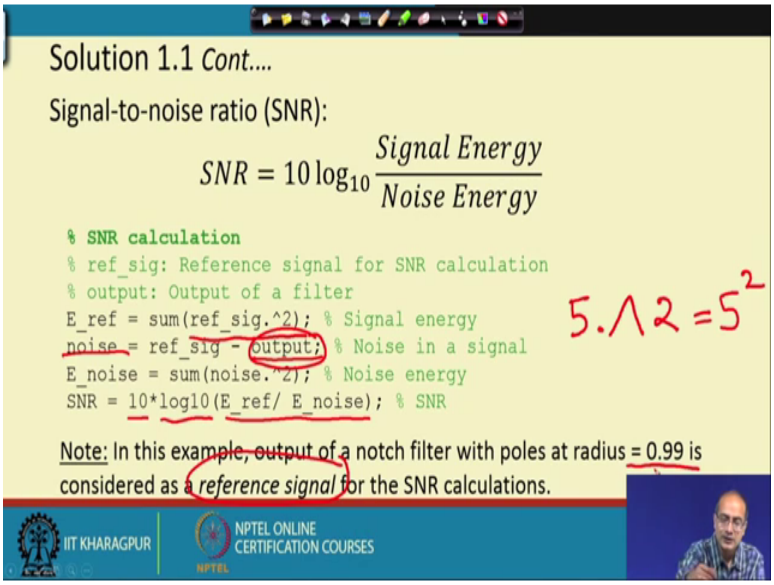
So, here let us processed towards it. So, first let us look at that what is the output that how successful, it was we see that that output signal in the output signal what we find that that as we have taken small number of cycles, we can see the undulations more clearly that undulation has gone. So, this notch filter with two zeros it is pretty much successful in eliminating the power frequency noise ok.

Now, how you got the how you can get this output. So, let us look at the code. So, first we have use the command filter that output is we get by filtering the input x which is a one of the input to the command filter and we have given the that denominator numerator and the denominator coefficients b and a. So, we got the output of the filter as out the variable here name and for that fs and L already, we have discussed again to get the frequency axis, we have assigned t which will give us that the time axis and we use the subplot command, we have used the first actually the row for the input and the second row for the output. So, that is how we get the this plot.

And if you look more carefully that what is the impact of that notch filter apart from eliminating the ripples due to the power frequency what has happened please check that the magnitude of the QRS complex which is easier to find it out that because it is pretty prominent in the input, it was little more than 2.5. Now though we have normalize the filter it has come down it has come little below than 2.5 other part of the changes, there are other changes also that the changes are not that sharp and it would be difficult to actually quantify or get that easily, but just from this time domain observation also you can make out that there is some change or some loss in the signal.

So, our next job would be to look at that how we can make up for that ok. So, let us proceed towards it.

(Refer Slide Time: 32:07)



Now, before we proceed we show you the signal to noise ratio that formula here one point, I would like to just mention that there are different formulas that many times, you get that it is given as 20 log to the base ten that amplitude of the signal and then noise amplitude.

But here we need to keep in mind and noise is random. So, amplitude fluctuating and the signal also is not a sinusoid, it has a varying actually amount of actually amplitude or strength. So, in that case we cannot use that formula, we should use this formula the ten log to the base t and we should take the ratio of the signal energy with respect to the noise energy ok, if we do not do that that we will make some mistake. So, you should keep this in mind.

Now, once we have the reference signal first job is to calculate the signal energy and that we can do it by this term that first what we are doing the reference signal this command dot, then hat two, it means each of these coefficients say if we tell that here I have a value say five. So, it will give us actually it is equivalent to 5 square ok. So, each of this respective sample points we are taking square of them and then we take the sum which gives us the energy of the reference and for the noise.

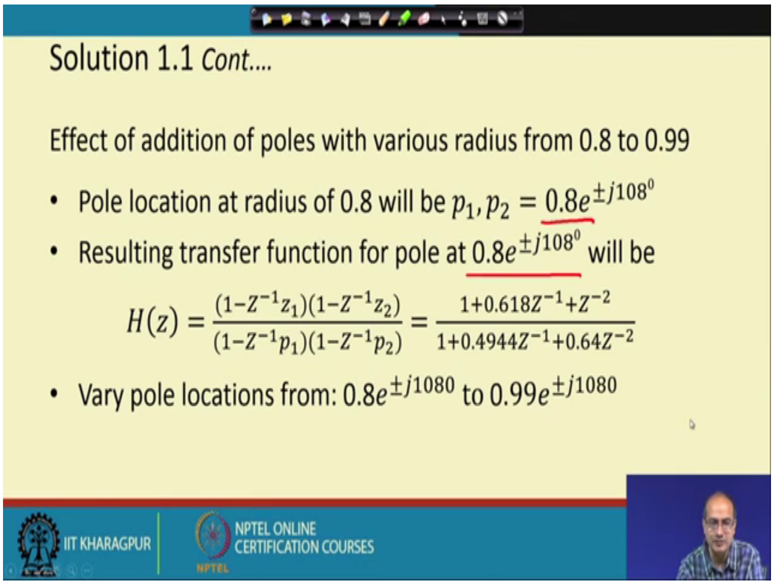
We are taking the difference of the that signal and the output. So, the difference is giving us the noise here, what has happened? The reference has changed usually we take that the corrupted signal. So, and the noise is energy of the noise we can calculate in the same way and here that output means is noise corrupted actually signal ok, this output is not the filtered output or you can take that in other case that filter output this one is the signal plus noise.

So, we are getting in the difference the noise. So, we calculate then the noise e noise or the energy of the noise in the same way as the signal and then we have implement the SNR in MATLAB 10 multiplied with log 10 which gives the logarithm with respect to the base 10 and then we take the ratio of the that the energy of the signal by energy of the noise which will give us the SNR.

And in this case that output of the notch filter with pole at radius 0.99 is considered as the reference actually after the experiment we need to find out that where we get the best result and that is taken as the reference in this case that was the direction. So, we found that that is at 0.99. So, that is the signal output of the filter is taken as the reference; however, this decision comes not just now it would come actually at the end of the experiment ok.

So, we need to actually wait to get this value when it is the best.

(Refer Slide Time: 36:39)



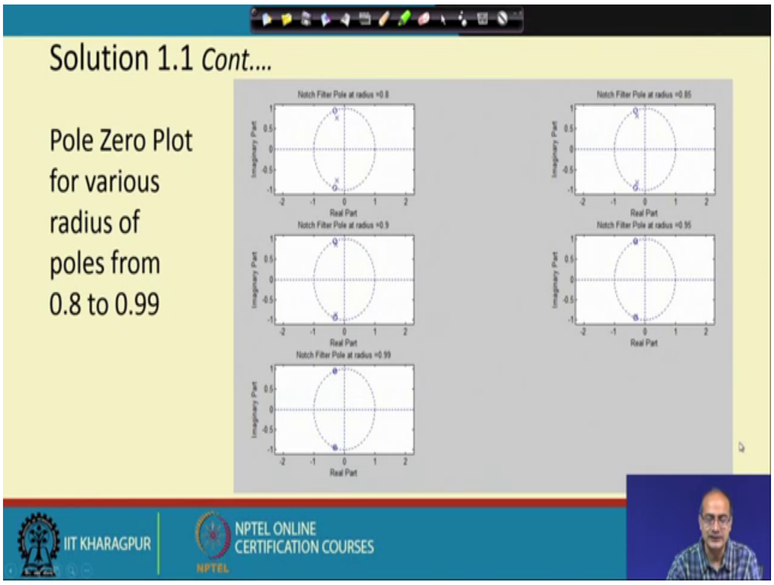
So, here the effect of adding poles with varying radius 0.8 to 0.99 we need to look at. So, for that first if we look at that 0.8; how the poles will look like that they would be 0.8 into e to the power plus minus j 108 degree ok. So, that would be the same location the angle would be the same only thing that radius has been actually reduced as we have suggested to go for that 0.8.

Now, if we want to go for some other value, we can do it in the same way now first look at that 0.8; what is the transfer function. So, transfer function would be in this case; that now we have some denominator polynomial. So, we have added that and again, let me remind you that while calculating that thing that we should here for the sake of understanding we have given 108 degree, we should take the value in radian to compute the sine and cosine term unless we take special care ok.

There would be some special function to compute also with degree, but they are not the default choice as you have used any programming language you know that sine and cosine value if we have to take the angle should be given in radian.

So, with that we get the denominator polynomial and the that the numerator polynomial we already knew. So, we proceed with this transfer function and in this way, if we can vary the pole locations in steps from 0.8 to 0.99 the angle remains to be the same, whereas the location varies and what is the effect of it.

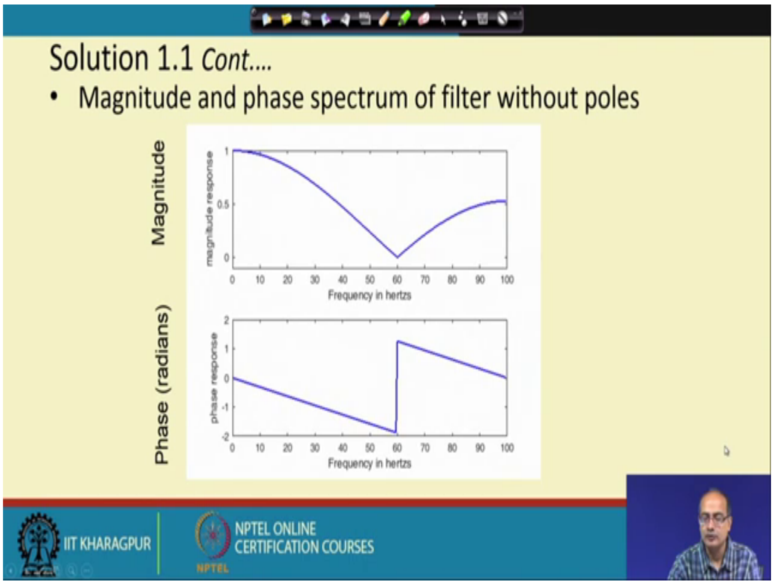
(Refer Slide Time: 38:40)



Let us look at the plot here the pole 0 plot for various radius of poles from 0.8 to 0.99 that first, we get here for 0.8, this is the case that the pole and the 0, they are pretty far though at the same angle, then it is reducing the gap, it is the radius is becoming 0.85. So, it is nearing the that zeros, then it is coming further near to 0.9 then 0.95, it seems it has actually caught the that the 0 and here it is when 0.99 is the radius, it Is very difficult to find out the difference between them, it seems they are cancelling each other.

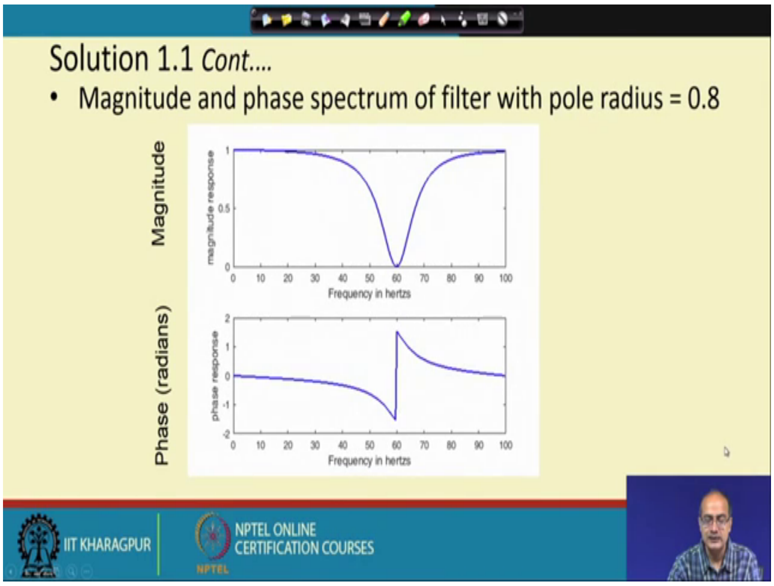
But we need to keep in mind that they we need to place them in such a way they should not cancel each other because if they cancel each other, then it will become an all pass filter. So, we have not get the effect of the notch filter and our part pass would be defeated ok. So, come whatever may be it can be as close as the 0, but they it should not cancel each other. So, if you like you can take it to 0.995 or 0.999, you can try with that, but even by mistake do not take it one. So, that they cancel each other.

(Refer Slide Time: 40:21)



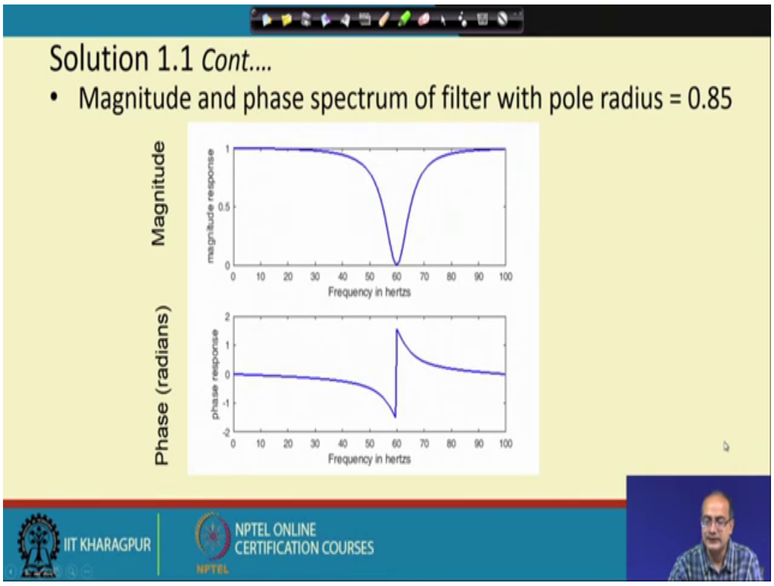
Now, let us go back that; how was the notch filter earlier without the poles; here is the magnitude and the phase spectrum for that right hand side and now let us see that how it changes with the addition of poles.

(Refer Slide Time: 40:45)



Next that when we have added the pole at 0.08 radius, then we see that notch has become more narrow, it means that the nearby and neighbouring frequencies would be less affected than the case where no pole was there at the same time we see, there is a some effect in the frequency the frequency at far from the notch filter that they are it has become more actually constant there is no changing phase, but there is the discontinuity remains at the location of the notch filter or the location of the 0 and the pole and there is some non-linear behaviour near that.

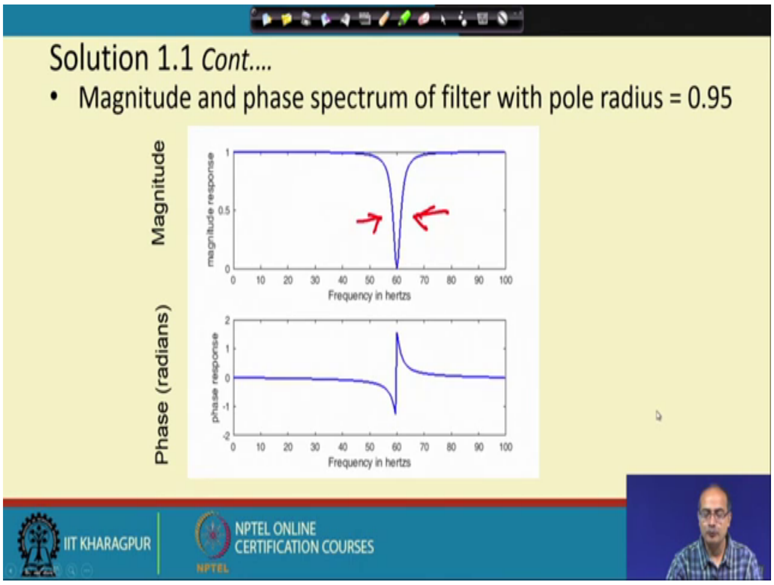
(Refer Slide Time: 41:44)



Now, let us proceed and see that if we choose the pole near the 0 what happens now it becomes more narrow you we can check that by going one step back and if we just draw the boundary that where was the location of this one ok, this was the gap. Now in the next case, if we proceed we get that it has become narrow and same way, the phase read the discontinuity remains and it becomes more sharp the transition becomes more sharp.

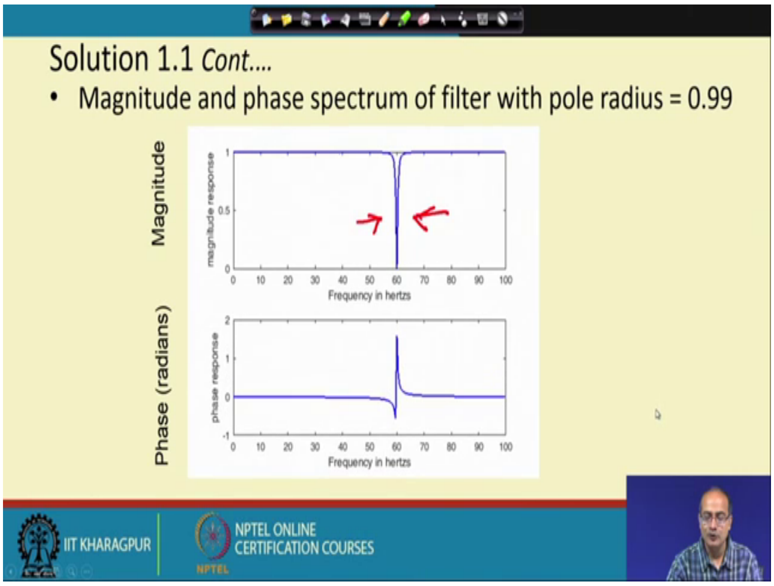
Now, let us go forward push it forward to 0.9, we see the notch becomes even more sharp and the discontinuity in the phase also becoming sharper.

(Refer Slide Time: 42:43)



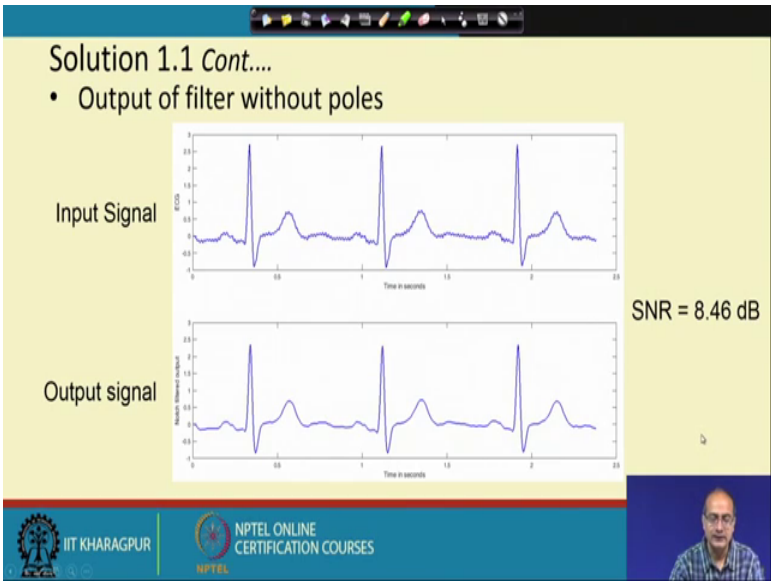
Going for 0.95, we see clear difference. now it looks much better that we see that it is much more uniform and probably at 50 hertz and 70 hertz, there is no attenuation almost no attenuation.

(Refer Slide Time: 43:01)



Now, going for 0.99, we get the best among these results. Now you see that the nature of the all pass filter has come, we have come very close to it except for 60 hertz where we need a sharp change and eliminating that particular frequency. Now it has become very close to the ideal notch filter and phase also has sharper change that the increase that going to the very high value that has become even more sharp ok. So, this is the change in the spectrum and the phase we could get.

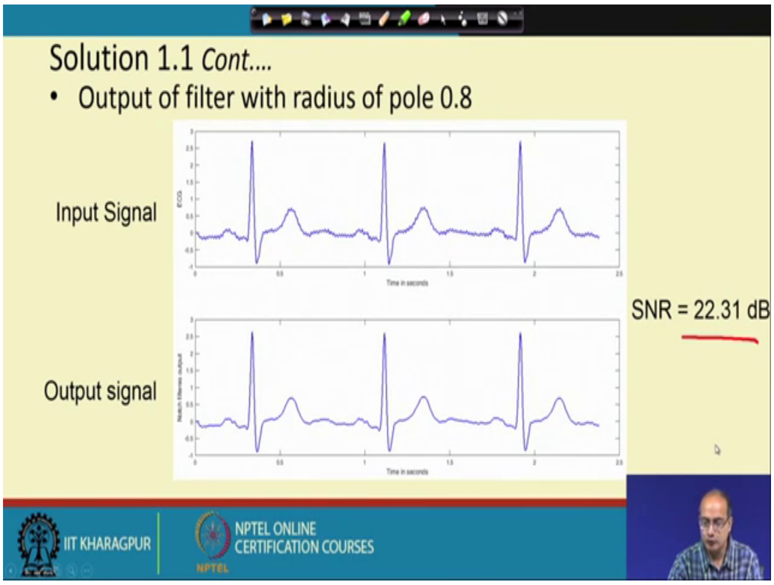
(Refer Slide Time: 43:49)



And let us see that what is the impact in the signal, first we go without pole, we have already described that and the base result we expect for that the filter with poles at 0.99 radius. So, with respect to that when we compute the SNR of this output signal. So, we get the SNR is 8.46 ok. So, SNR is not very good.

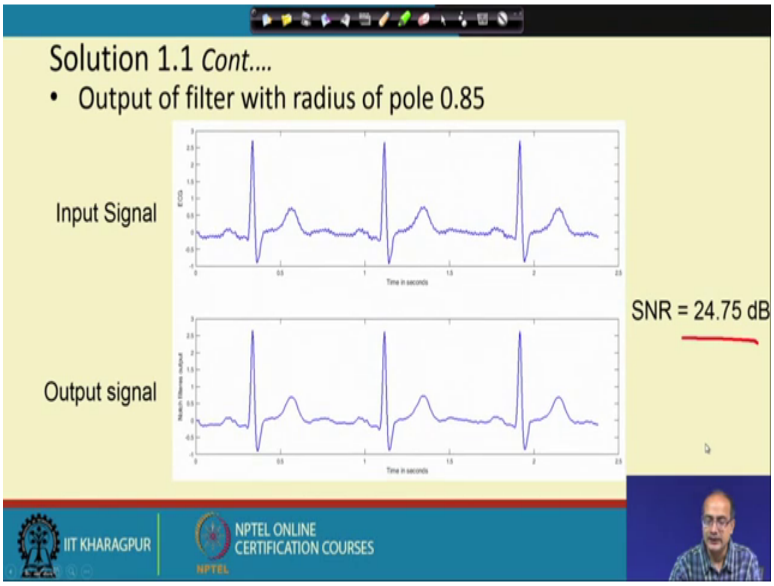
Now, why it is so because we lost actually some part of the signal along with the noise the noise elimination, I would not say that it is bad noise elimination is good, but the signal got distorted and that has pulled down the SNR.

(Refer Slide Time: 44:43)



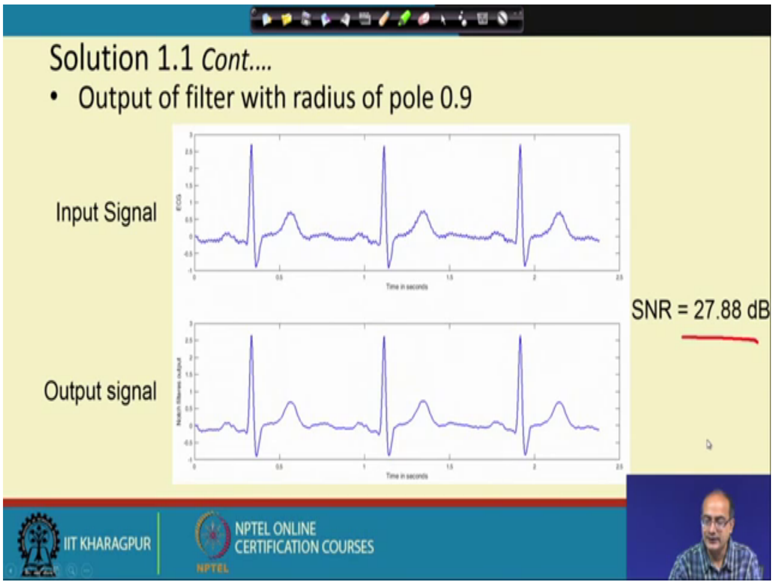
Now, let us go for that first the pole is added at 0.8 immediately we see a drastic change we get the signal amplitude you see, now it has increased and we it is a allowing more changes actually that more detail in the signal and the SNR has also moved from 8 to 22, it has become more than double near to we can say 2.5 times.

(Refer Slide Time: 45:15)



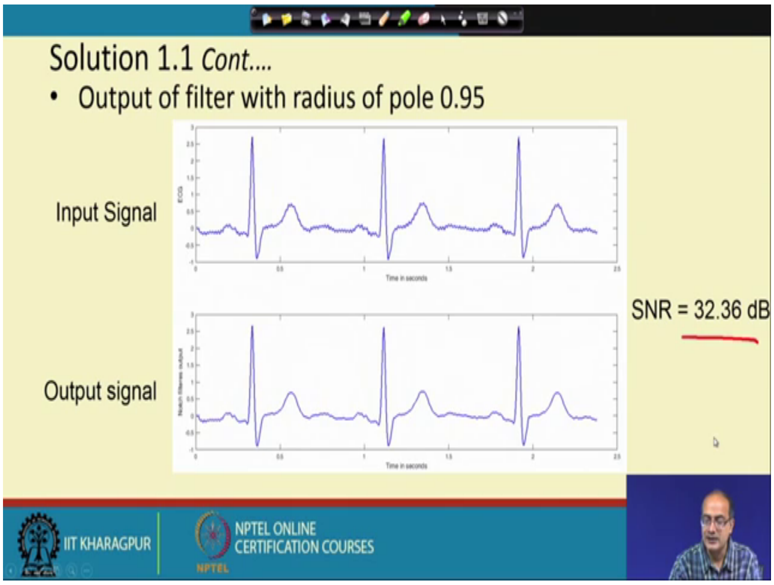
Now, 0.85; it has again pushed further going for 0.9, it has become now 20; near 28 dB.

(Refer Slide Time: 45:24)



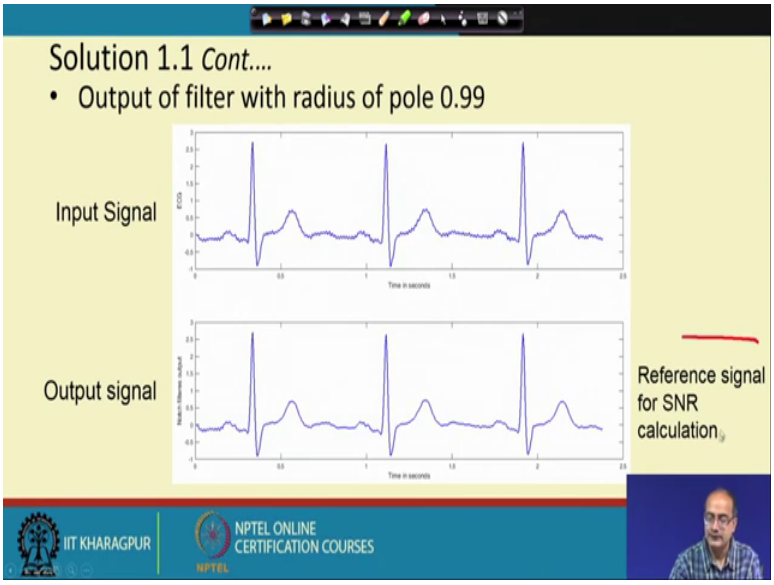
So, now it looks like it is a good one.

(Refer Slide Time: 45:33)



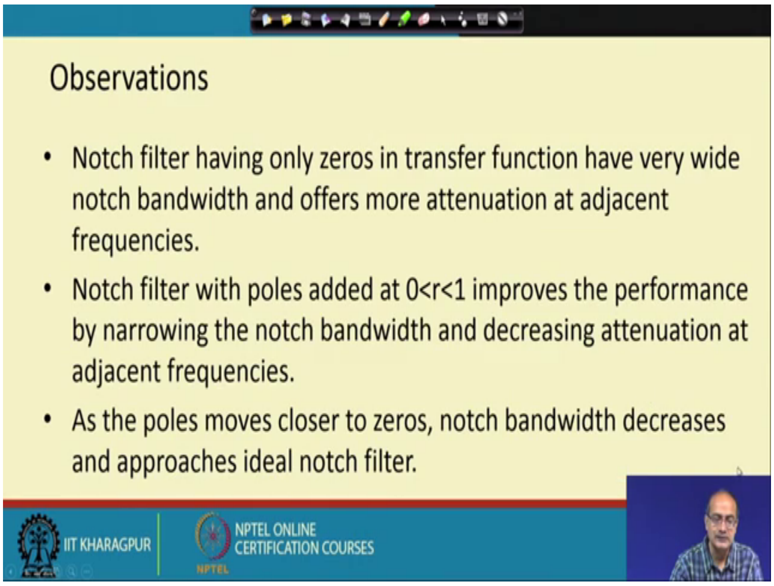
And the last one that that what we can compare that is 0.95 we get, it is more than 30 dB ok.

(Refer Slide Time: 45:44)



And at the end; that we have the reference one that what we have taken as the reference the best output that is for point that 9 9 would be the pole location ok.

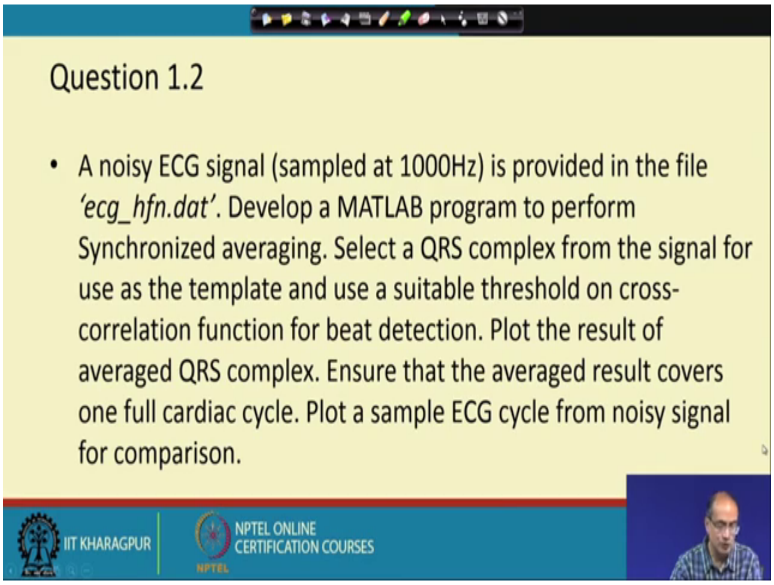
(Refer Slide Time: 46:00)



So, we get the results and now, let us make some observation the notch having only the zeros in transfer function, it is a very good concept, it is a FIR filter, it offers good attenuation at the that notch point, but it has a actually broader that that band and because of that that it is attenuating the adjacent frequencies also.

The next part is that we have added the poles between 0 to 1; to increase the performance and how that performance is increased the notch actually bandwidth should be narrowed. So, that the attenuation at the adjacent frequency is decreased and with that we are able to do that and as the pole is moving near the 0, the notch bandwidth is decreasing and approaching the ideal notch filter. So, that is the take away of this experiment ok.

(Refer Slide Time: 47:06)



So, with that; we complete this presentation and after a small break, we will take up the next one.

Thank you.