**Biomedical Signal Processing**
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**Lecture - 45**
**Modelling of Biomedical Systems (Contd.)**

So, now we will look at ARMA model which is more general one. We will see that how we can compute the coefficients of the ARMA model. If really it is the case that AR model cannot do justice to that signal.

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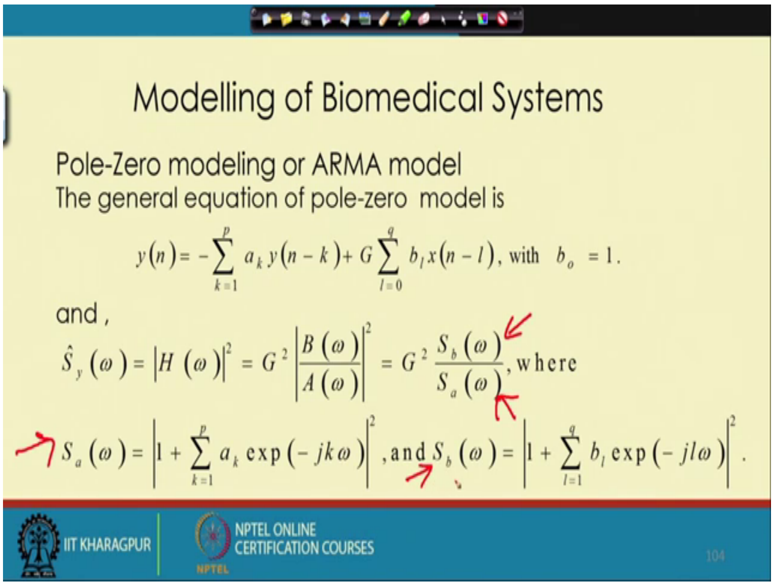


So, for that let us look back at the pole-zero model. The general model equation is given here consist of a k's and b l's and the first value here that the for the b 0, the value is 1 and corresponding to that if we look at the spectral domain, we get the spectrum or rather the estimate of the spectrum is at y. It can be given as the square of the that transfer function h omega because our input x n, we have told that x n is either white or it is an impulse response or impulse ok.

So, as it is constant, it can be represented, the spectrum can be presented only it z h z and that filter h z or h omega, we can represent in terms of the polynomials a and b; in the z domain and the corresponding actually the spectrum is S b and S a, the ratio can determine them is you can determine them ok.

So, we have given here, below that the expression of S a and S b. So, that is the the starting point in short.

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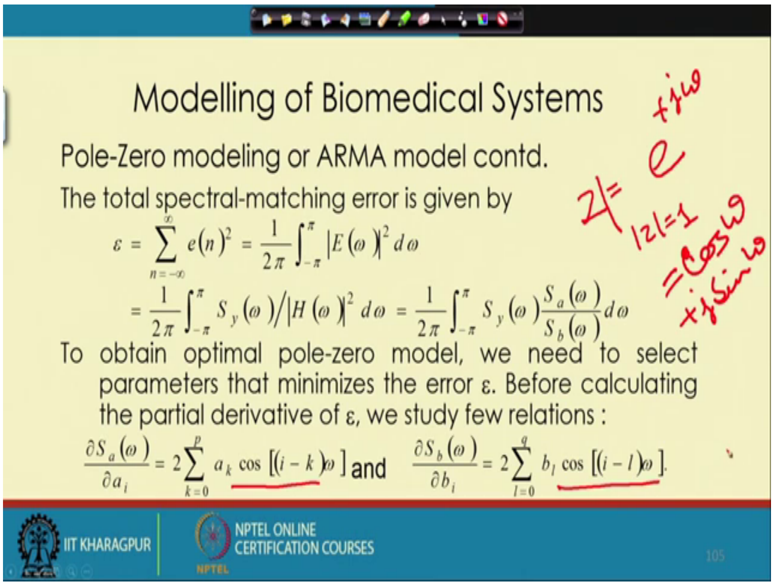


So, from here, the total spectral matching error, if we are looking at the error that what we are trying to minimise, we can again give it the error is sum of the that error in the that time domain as well as that is same as the error in the spectral domain and the spectral domain error e omega or rather e omega mode square, it can be represented in terms of the spectrum of the signal S y and h omega and h omega can be represented in terms of S a and S b.

So, using this fact, we can obtain actually that pole-zero model or the coefficient required for that by minimising the error, the total square error eta ok. So, before actually going into that let us look at some actually small formulas that which would be useful for to do that work that if we take the partial derivative of the spectrum of the filter a z, that is S a omega at mode z equal to 1, we get it in terms of the error coefficients, sorry that autocorrelation coefficient a k and some cosine term.

Here, we actually make use of the fact that when we are talking about z, z is at when mode z equal to 1 ok, then it is represented by e to the power j omega or rather we take z z to the power minus 1. So, minus j omega will come. So, e to the j omega this exponential can be represented in terms of cosine omega plus j sin omega ok. From there the cosine term is coming into play here ok. And for S b also, if we take the partial derivative, we get the summation of the terms with b l and some cosine terms along with it ok. We can derive that. We need some time just to do that.

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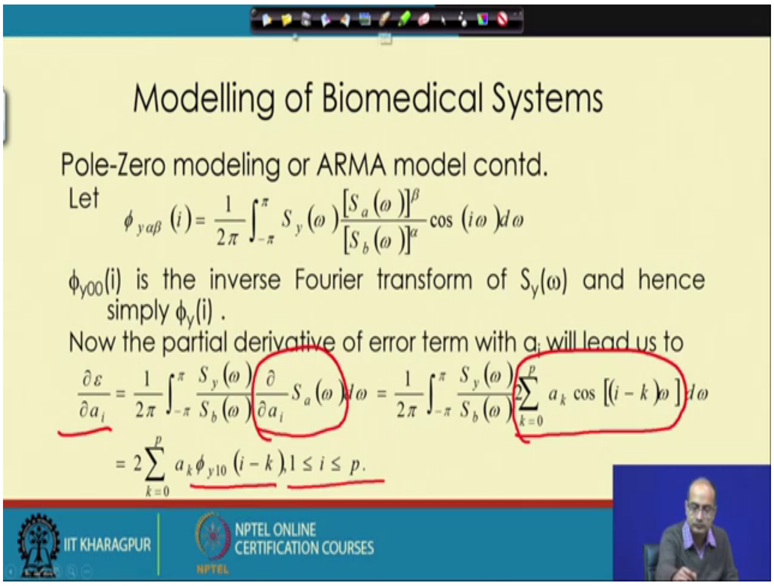


And then, we take another expression which will help us to write the things in a more compact way. We defined something called phi y alpha beta. So, it is very similar to phi y, the only thing what we have done, we have used alpha and beta to take the power here that we have taken in the denominator for S b the power alpha and for the numerator the term S a as the power beta ok.

So, that is that is the special thing what we have done and phi y 0 zero i is nothing but the inverse Fourier transform of S y or simply we can write that as phi y; that means, it is a more generalized form when we take alpha and beta both are 0, we get actually the same old autocorrelation function of y.

Now, why we have taken these, it will be more clear as we proceed. Now the partial derivative of the error term with a i that is delta eta with respect to a i, it will give us that the that terms S y and S b, they are not actually having any term with a i. So, the partial derivative will be applied only with S a and we have taken it in that way and here we make use of that the previous result we replace it with the summation of the that a k coefficient, the previous result and using that that we can write it in a more compact form.

We make use of that we get the results are coming in terms of some variable phi y 1 comma 0 ok. So, phi y 1 comma 0 means we have here S b is there that we got one term of S b but S a is not there in the numerator ok. So, we are getting the equations in that way.

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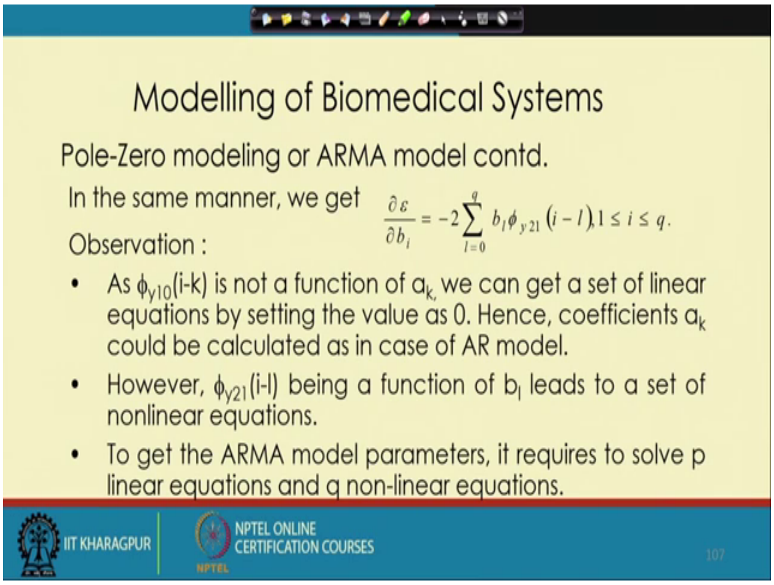


Next in the same manner, we can take the derivative of eta with respect to b i and we get the equations in terms of phi y 2 comma 1 ok. So, we can give a compact representation we get two sets of equations. Now if we look more carefully, we observe something the terms that phi y 1 comma 0, they are not function of a k. So, we get a set of linear equations by equating that partial derivative to 0. So, by that way, we can compute the coefficient a k by those p equations because p equations and p unknowns are there.

Now, if we look at the next set of equations, we get unfortunately phi y 2 comma 1. It is a function of again b i. So, it is not a set of linear equation, by equating that delta eta partial derivative of that with respect to b i, equating that to 0 the equations what we get they are not linear equations in terms of b i but it becomes set of non-linear equation.

So, what we need to do to solve the AR model or to estimate the AR model parameters, we require to solve p linear equation and q non-linear equation. So, let us see first how that can be performed.

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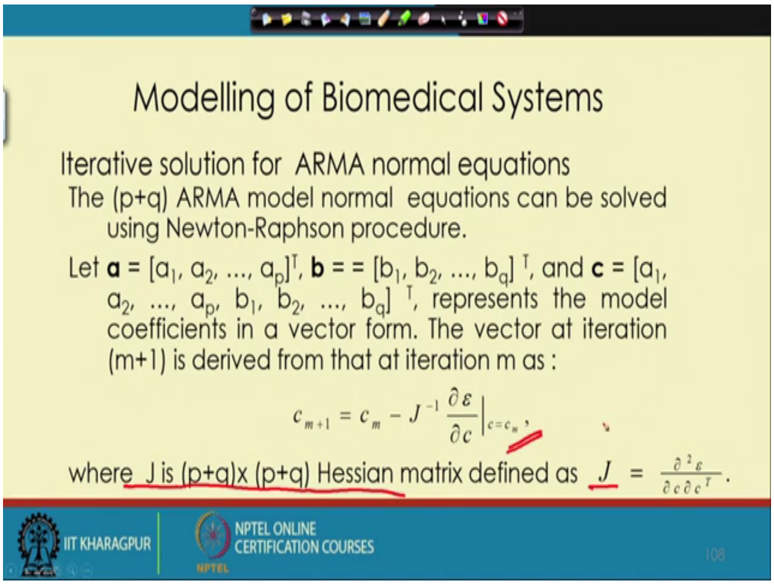


First we take an iterative technique because we are to deal with a set of non-linear equation. So, we have taken the Newton-Raphson technique and using that we can compute the values of a vector a and vector b and combining these two together rather concatenating them, we get a new vector c that is the vector of actually the model coefficients and we need to find that using this iterative process.

So, we have some update rule at every iteration; We update the value of c, the vector c with respect to that the previous update value c m, we get c m 1 and we make use of the Jacobean of eta rather we are using of the hessian matrix here j is the hessian matrix; that means, we are taking that double derivative of the error eta with respect to c and we compute that at a point c equal to c a ok.

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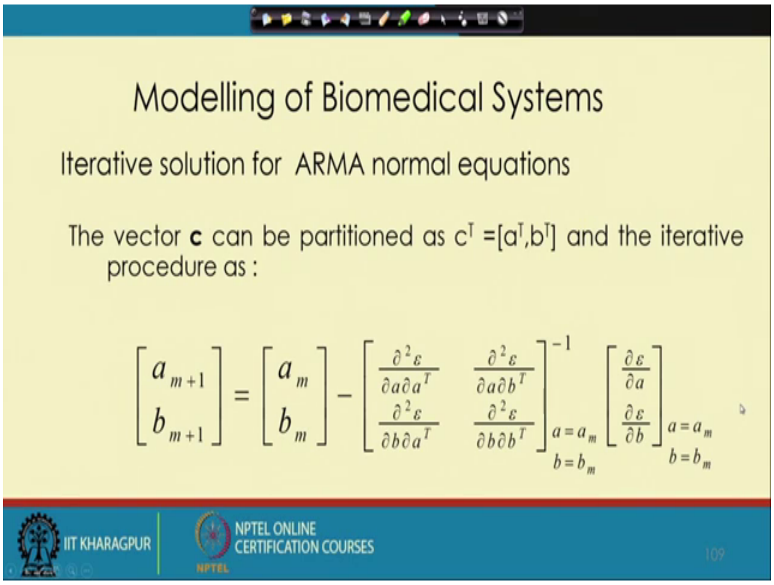


So, once we compute that, we can update that value and this way we can actually compute and for that part. We can actually partition also the 2 parts to make it simple that we can write it in this way that c is concatenation of a and b or rather a transposed and b transposed, we can write the update equation in this form where we have to compute the inverse also here.

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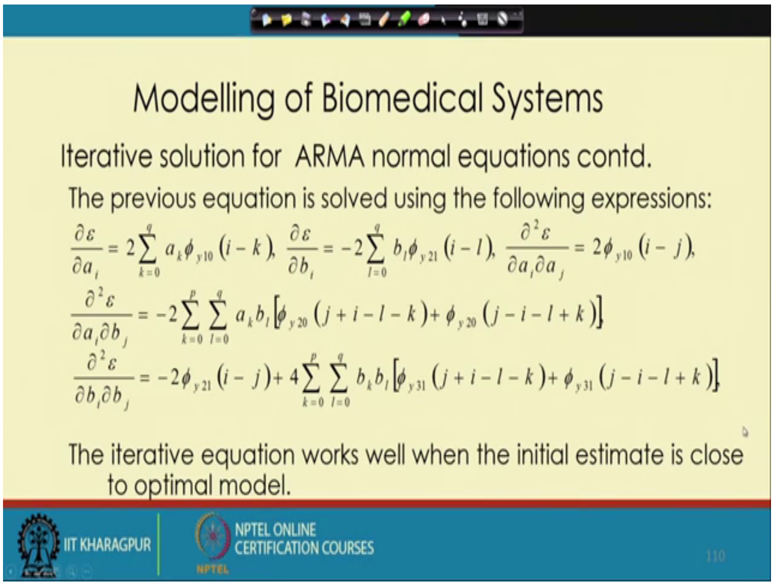


So, here we can using that we can get a number of equations using the first and the second order derivatives and that we can solve though these equations using these equations we can compute the value of a k's and b l's; however, the as we are using some set of non-linear equation, the results are very much dependent on the initial condition. If we have good initial conditions, we will get the optimal result otherwise we may get actually something much inferior than that.

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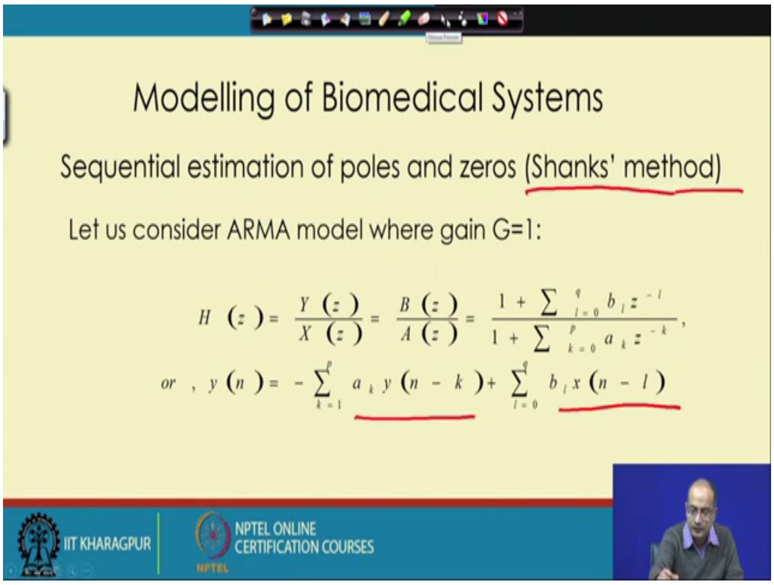


So, we look for some alternative and for that, we take one technique proposed by one scientist name Shank. So, it is known as Shank's method ok. So, let us look at that that what is the Shank's proposal.

Let us go back to that ARMA model. In the ARMA model that we have taken h z in terms of the ratio of the two polynomial of b z and a z or in the time domain we can taken that as a linear prediction equation using that few previous values of the output and present and the few previous inputs.

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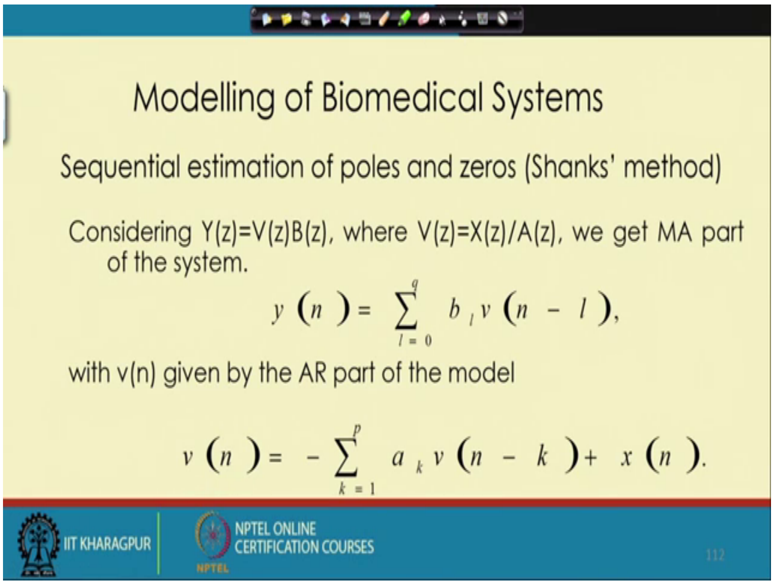


So, here what we do, that we compare y z or rather what we are doing we are decoupling that two parts that in the ARMA process one part is the m a part and one part is the AR part. So, we are decoupling the thing in a way we are for that purpose we are choosing a variable v which is helping to an intermediate variable which is helping to separate the object that this general equation into two parts; one part is the only m a part, another part is only the AR part.

So, if we separate these two parts, we can tell that v n is the signal what we get by the AR model and using that v n in that ma equation, we can get the output y n ok. So, we are actually dividing the two parts using this new variable v n ok, that is the trick actually is done we could decouple the two things.

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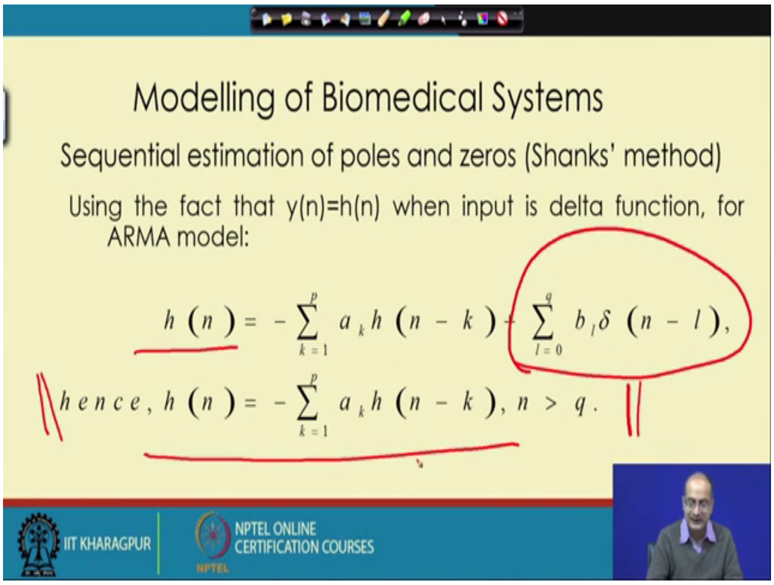


Next, saying that Shank has assumed that let us take that these signal is nothing but the impulse response of that ARMA model that y n it is nothing but the impulse response of the ARMA model what we are trying to estimate. So, in that case if the input is delta, then output y has to be h n the impulse response of the filter.

So, the equation of the ARMA model we can write it in this way that h n in place of y n, we can write it that for first few values for n equal to say 0 to q this equation will hold and when n becomes more than q these part none of them will be present all of them the value would be 0. So, we can drop this part and it becomes a simple equation like this.

Now, what is the benefit we get out of this process or this assumption is that that in this way, we put actually separate the two sets of coefficients. Here, we see in this case that we do not have any b these set of equation that is having only the a k. So, that is the major gain we have done.

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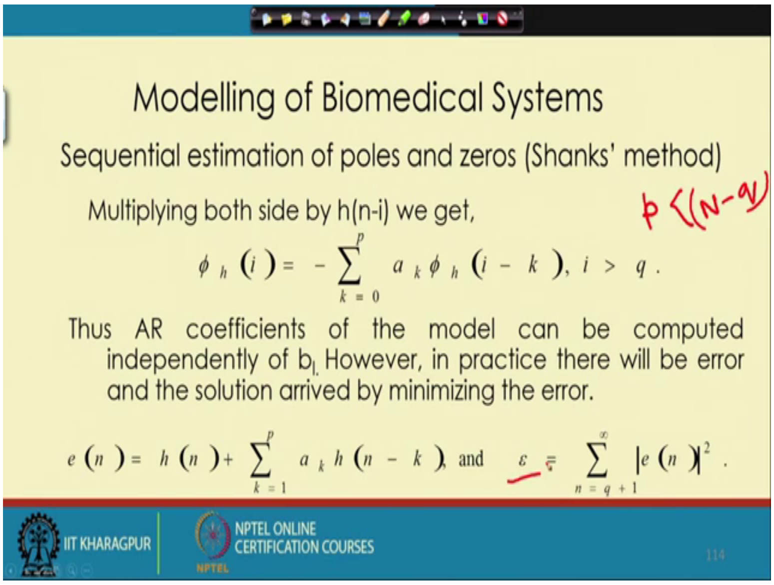


So, now if you multiply both the side by h n minus 1 in the second equation, we get actually it in terms of phi h is an equation using phi h and the a r coefficient a k’s thus, what we get that using these equations, we have actually the n is much larger than capital N is much larger than p n q. So, in that case, we have a number of actually such equations and these equations they are independent of actually the blocks.

So, we can first compute the a k’s using the value phi h and that is what is actually done first. So, because there would be some error approximation error, so, it has been written in this way that we would have some error e en that is the prediction error of the impulses here and we need to minimise these again eta that is gives the sum of error square from q plus 1 onward ok.

So, we are taking the sum of error square because so many equations and they are equation should be here in this case precisely we have actually p unknowns and we have that n minus q equations which is much actually larger than p. So, more unknowns are there it is over determined system. So, there we need to take care of the error in that error this over determined system and the error is defined in this way we try to minimise that to find out the best possible solution.

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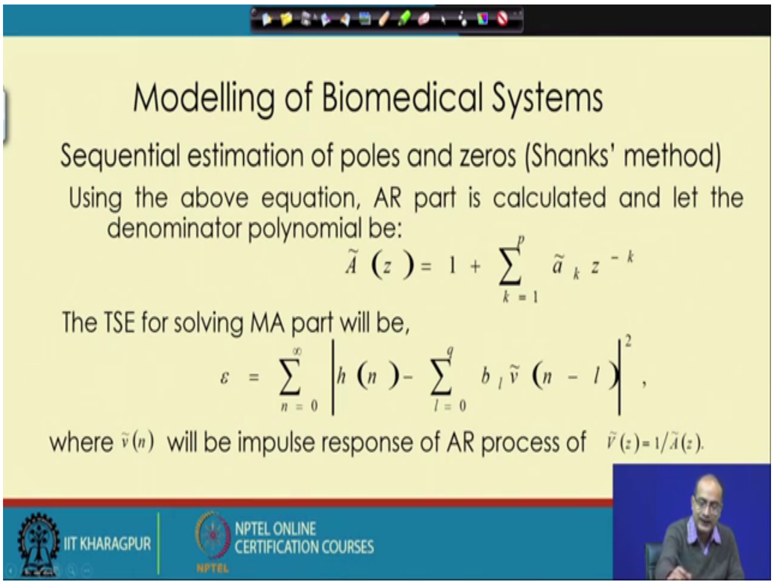


Now, using that equation first the error part that AR coefficients are computed and then, we get the estimated values of a k which is represented by a k tilde and we compute the corresponding the polynomial a z or the estimate of that is a z tilde. Now, as we have the a z tilde, we can look for the total square error for the m a part now.

We can concentrate in that and if we try to find that again we can represent that as the error in the m a part where we would need the value of v tilde n minus l and if we know that because the h n is known we can compute the b l's again in the same way.

Now, v tilde n is nothing but the impulse of the AR filter and AR filter already we have an estimate. So, using that we can compute actually the value of v tilde n using the that the polynomial a tilde z, we can compute that.

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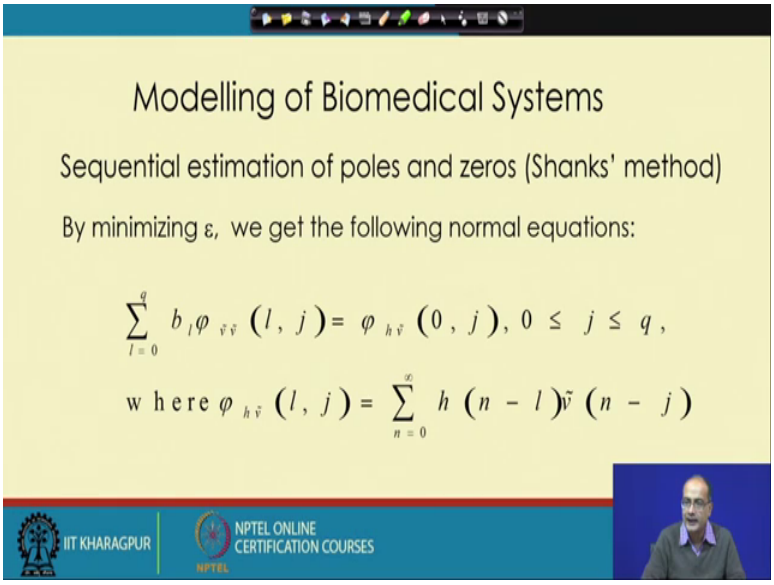


And by minimising this we can the eta, we can get this is the set of linear equations that with the help of that again, we can compute the value of b l

So, Shanks' method again, we t we are getting a set of linear equations; however, in this case it is it is just we have the q unknowns and actually q that equations ok. So, we can actually solve them and we can find out the m a parameters. So, in that way we can get both the values of the AR parameters and the m a parameters using linear equations.

Thank you.