

Biomedical Signal Processing
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Lecture - 41
Modelling of Biomedical Systems (Contd.)

So, now here we will look at a process or a model which is called as point process.

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Modelling of Biomedical Systems

Point Process

$x(t)$
(pulse train) → $h(t)$ → $y(t)$

- The series of impulse as input (to be convolve with filter) is known as *point process*.
- Let the time interval between i^{th} impulse and the previous one is τ_i .
- Let the origin is set to the first impulse that is $i=0$ and...

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In this case we get the input that is given by a train of impulses and, we have a transfer function which gives rise to our signal of interest that is represented as $y(t)$. So, when there is a train of impulse, which is convolved with a filter that is called a point process. And in this case that what is very important is the that inter pulse interval the time interval of the i -th pulse with respect to the previous pulse and we represent that here as τ_i .

So, for first pulse we can say that it is starting at the time 0. So, τ_0 is we can take 0 and after that with respect to the first impulse that for, the next impulse onward we can take the delay with respect to the previous impulse as the τ_i ok.

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The slide is titled "Modelling of Biomedical Systems" and is labeled "Point Process". It features a block diagram where an input $x(t)$, labeled "(pulse train)", enters a block labeled $h(t)$, and an output $y(t)$ exits. Below the diagram, there are three bullet points:

- Let the τ_i are i.i.d. with mean μ and variance σ^2 ; and $\tau_i > 0$ for $i > 0$.
- The time of arrival of i^{th} pulse is given by $t_i = \tau_1 + \tau_2 + \dots + \tau_i$.
- The time of arrival t_i is the sum of i independent variables; mean and variance are $i\mu$ and $i\sigma^2$ respectively.

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And that is the way we can actually define this process. Now, this τ_i what we have taken without any prejudice, we can take they are i.i.d; that means, they are independent and identically distributed and here we have assumed that they are coming from a say normal distribution with mean μ and variance σ^2 . For τ_i equal greater than 0 for i greater than 0 because, we cannot go actually back in time or the system cannot be nonconventional.

So, that is how that these two constant just have come and for that, if we are interested in the time of arrival of the i -th pulse, then actually we have to add up all the delays up to the τ_i that can give us the time instance of the i -th pulse ok. So, we can measure that and that could be a very useful information.

Now, the time of arrival being sum of the independent variables so, they are actually sum of i independent variables. So, if you are interested in the mean and the variance the mean would be i times the mean of the iid process and the variance also would be also i times the variance of the that that the constituent process, that is σ^2 .

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Modelling of Biomedical Systems

Point Process contd.

- The time of arrival t_i is also a random variable with Gaussian PDF;
$$p_i(t_i) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left[-\frac{(t_i - i\mu)^2}{2\sigma^2}\right].$$
- The input point process $x(t)$ represented by
$$x(t) = \sum_{i=0}^N \delta(t - t_i).$$
- The Fourier transform of point process $x(t)$ is given by
$$X(\omega) = \int_{-\infty}^{\infty} \sum_{i=0}^N \delta(t - t_i) \exp(-j\omega t) dt$$
$$= \sum_{i=0}^N \exp(-j\omega t_i).$$

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So, we get some actually information about that that t_i and if we try to modulate that first thing what we note, the t_i the because they are random. So, t_i it is also a random variable because t_i is represented by a sum of t_i ok. So, it is a random variable again it also follows a Gaussian PDF because each one of them, they are gaussian and the sum of them they would give rise to again a Gaussian distribution; however, in this case the mean and the standard deviation is different, mean is $i\mu$ and variance is $r\sigma^2$ ok. So, that is the difference with respect to the iid of t_i .

Now, the input point process $x(t)$ it can be represented here, as the sum of typical pulses which are occurring at time t_i , they are occurring at time t_i . So, sum of actually impulses we can take that in that way and that can help us to understand that signal better, what we can do for that that first we can take the Fourier transform of that that input point process $x(t)$ and the Fourier transform, we can actually take by taking the fourier transform of the that some of actually impulses that is this process.

So, as we take these are the impulses that integration boils down to the summation of certain exponential terms ok. So, we get some of exponentials and with that the $X(\omega)$ is defined.

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Modelling of Biomedical Systems

Point Process contd.



- $X(\omega)$ being a function of random variables is a random process itself. The mean value is obtained by

$$\bar{X}(\omega) = E[X(\omega)] = \sum_{i=0}^N E[\exp(-j\omega t_i)].$$

$$E[\exp(-j\omega t_i)] = \int_{-\infty}^{\infty} \exp(-j\omega t_i) p_i(t_i) dt_i,$$

$$= \frac{1}{\sqrt{2\pi i \sigma}} \int_{-\infty}^{\infty} \exp(-j\omega t_i) \exp\left(-\frac{(t_i - i\mu)^2}{2i\sigma^2}\right) dt_i$$
- Substituting $t_i - i\mu = r$, (where r is the temporary variable), we get

$$E[\exp(-j\omega t_i)] = \frac{1}{\sqrt{2\pi i \sigma}} \exp(-j\omega i\mu) \int_{-\infty}^{\infty} \exp\left(-\frac{r^2}{2i\sigma^2}\right) \exp(-j\omega r) dr$$

So, $X(\omega)$ is also we cannot say random variable because, it is a function of t_i which is random. So, $X(\omega)$ also is a random process and we can compute the mean of it to get the understanding about the average behaviour of that process $X(\omega)$. So, to get that what we do, we take the expectation of $X(\omega)$ and represent that as $\bar{X}(\omega)$ and as we take the expectation, expectation means we need to take actually that integration over the real line, real line is starting from minus infinity to plus infinity.

And we need to multiply it with actually the probability and, here what we have done, we have taken just one term out of this summation and we are performing that integration. And after replacing that $p(t_i)$ which is again a gaussian we know with that mean is $i\mu$ and variance is $i\sigma^2$, we get that some expression to make it simple what we do we do some substitution of variable, we take $t_i - i\mu$ as r that helps us to get a more actually compact form after the change of variable, we get this is the expression and we get one term outside that that integration sign.

And here what we notice that here it is actually the previous expression if we look at that this is the that Fourier carnal and, this is a carnal of a the first one that this is the that normal distribution the carnal and this is the Fourier carnal. So, what we are doing here, that we are taking the Fourier transform of the normal distribution ok

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Modelling of Biomedical Systems

Point Process contd.

- Using the property that the Fourier Transform of the expression $\exp(-\frac{t^2}{2\sigma^2})$ is $\sqrt{2\pi}\sigma \exp(-\frac{\omega^2\sigma^2}{2})$ we get:
$$E[\exp(-j\omega t)] = \exp(-j\omega i\mu) \exp\left(-\frac{i\sigma^2\omega^2}{2}\right).$$
- Hence,
$$\bar{X}(\omega) = \sum_{i=0}^N \exp(-j\omega i\mu) \exp\left(-\frac{i\sigma^2\omega^2}{2}\right).$$
- The ensemble averaged Fourier transform of the output signal is given by: $\bar{Y}(\omega) = \bar{X}(\omega)H(\omega)$,
where, $H(\omega)$ is the Fourier Transform of $h(t)$.

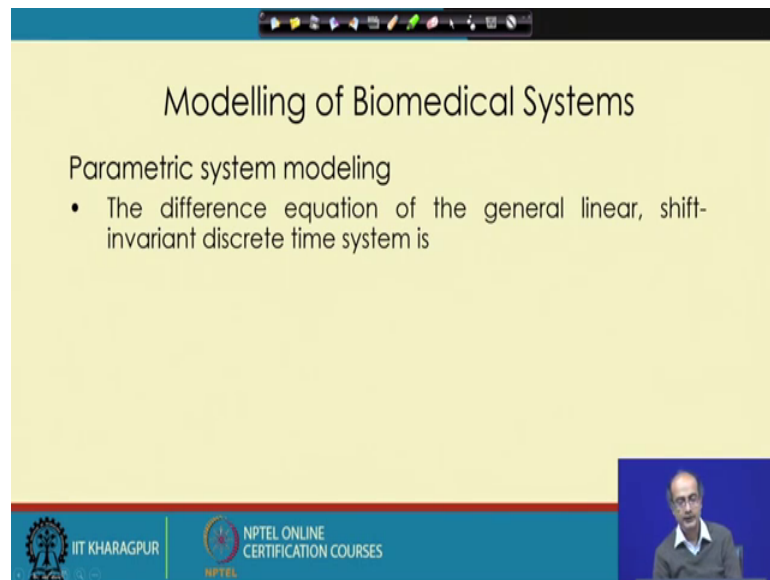
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So, for that that if we take the Fourier transform of the expression that. So, we are taking this is the expression. So, if you take the Fourier transform, we make use of that fact that the standard form of Fourier transform and by that we get that expectation of the exponential term in this way ok.

So, now using that what we can do, we know it was summation of such exponentials. So, that mean of the that variable that X omega, we can write it in this way ok, it is the sum frequency multiplied by the that sum damped exponential, we can say that that the damping in the frequency is provided by sigma.

So, that kind of expression we get and the ensemble average Fourier transform of the output signal, we can get by convolving the mean of the input with the h t , or in the Fourier domain. If we simply multiply by H omega we can get that, where H omega is the Fourier transform of the impulse response h t . So, that is the way we can get to know about the output.

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Modelling of Biomedical Systems

Parametric system modeling

- The difference equation of the general linear, shift-invariant discrete time system is

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So, here we get that enough about the point process and with that we will leave our topic here today. And we will start the next session, that next part in the next session.

Thank you.