Biomedical Signal Processing
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Lecture – 39
Frequency Domain Characterization (Contd.)

So, today, we will look into the new periodogram estimators. In the last session, we have seen that Blackman-Tukey, they have proposed the theory that we should depend more on those ACFs which are more accurate and we should give less weight to those which are less accurate or we have less averaging on them.

So, we are not sure about their that estimated value and in fact, some of them may be discarded which are not good at all, but all the things together, it comes down that that same as that previous the different kind of windows that we can apply for that and it gets at that meaning at the end that it is using the window means that is giving us some that true PSD convert with some window Fourier transform.

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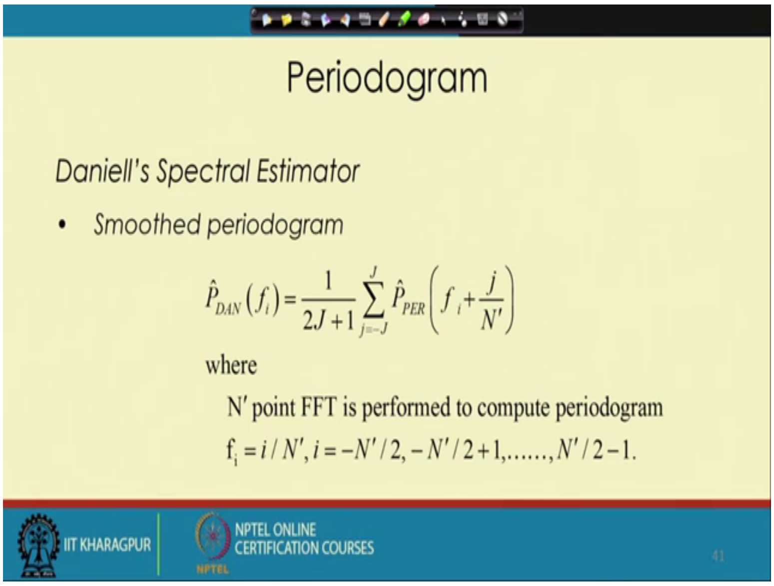

So, from that we get that another suggestion from Daniell; Daniell suggested a smooth periodogram and for that; he has suggested a very easy means what he told that let us take some Fourier transform of index points and after that we simply apply a MA filter in the frequency domain. So, if we apply that MA filter in the frequency domain. In fact, what he have suggested that using a that that unweighted averaging or simple averaging.

So, J plus 1 points, it is actually averaged and to normalize that 1 by 2 J plus 1 that factor is multiplied.

So, the idea is that for every point in the frequency domain you take both the sides adjacent J points in both the sides including that the point of interest. So, we have 2 J point, we just take the average and replace the true value of the PSD that what we got through a (Refer Time: 03:13) with that average value ok. So, that is the suggestion of Daniell. In fact, the beauty of it, it makes the work very simple and if we are sacrificing we are ready to sacrifice the frequency domain accuracy then why not directly go for averaging ok. So, that was that is the suggestion given by Daniell ok.

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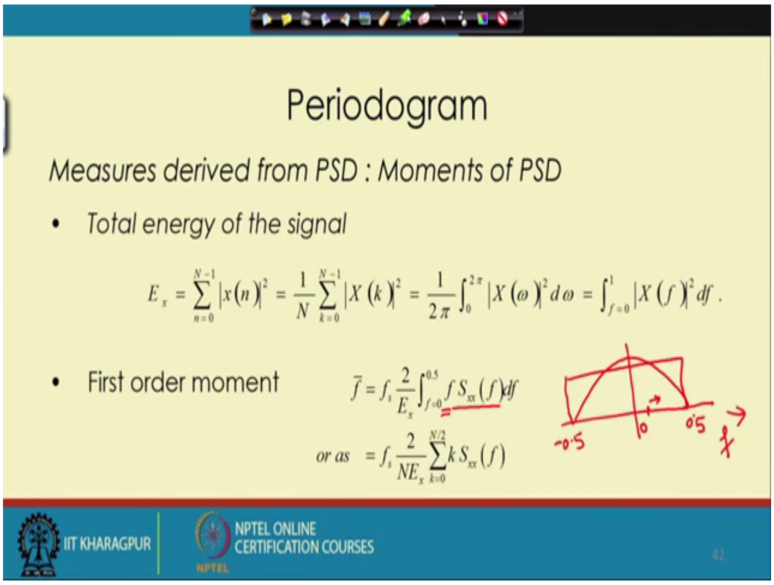

So, now, let us go for some measures of our PSD that how this that PSD or power density that how we are going to use it ok. So, this power spectral density first of all it gives us the total energy of the signal. So, if we think that energy of the signal has something to talk about the signal, then we can get that in time domain; that means, we can take it by summing the energy of all the sub samples in the time domain. At the same time, if we are taking the PSD, whether, it is in that discrete domain or the continuous time that frequency domain we can compute that ok.

So, that is a way we can get that. So, from PSD also we can get the energy and at times the energy is a good measure; for example, that if we have to decide whether this part of the speech is a voice speech or unvoiced speech, we know the energy of the voice speech is much higher than that of unvoiced speech ok. So, in that case the energy of the signal could be a good measure and if we are taking PSD, we can simply aggregate the energy at actually the different frequencies and we can get that value ok. So, that is the first thing we get.

Next, we can look at that first order moment first order moment, we can simply get by multiplying the PSD that the PSD is multiplied with the frequency and if we integrate, we are we are able to get the first order moment here 1 point, just to note that usually the frequency is computed from either 0 to 2 pi that is if we are looking at the that that the radial domain or if you are looking at the frequency then it can be taken as minus 0.5 to plus 0.5.

But for computing the first order moment that they have taken it from 0 to 0.5. So, that say that is a difference than the normal that the way we compute the that moment the reason could be that we are talking about real signal all the signals in the biomedical signal processing they are real. So, by default what we will get if we look at the spectrum the spectrum would be symmetric now in that case if we try to take the moment it will be coming to the 0. So, it will not give us the energy distribution. So, they have taken it from say 0 80 0.5 that within that here it is the frequency scale.

So, that will give us the distribution of the frequency and if the distribution changes say if it becomes uniform for the same total energy you will find that the first order moment it will move actually outward ok. So, that is the difference from the normal formula. So, we should keep it in mind that we are not actually taking it over the full range, but we are taking it from 0 to 0.5 ok. So, that point we should keep in mind while applying it.

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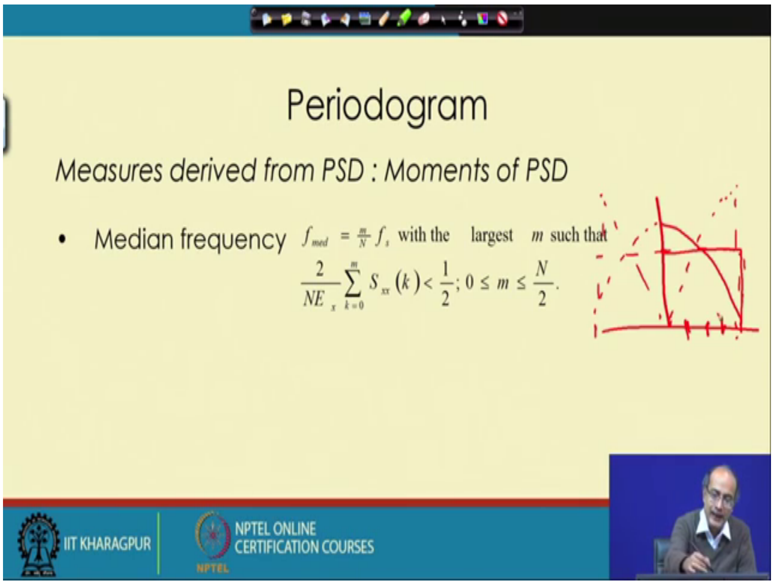

Next, we look at median frequency; what is the meaning of the median frequency that when we compute the distribution then we can find a frequency such that that in the left hand side and the right hand side, the energy distribution of the accumulation of energy is same; that means, it is in the middle in that way not in terms of frequency, but we can tell that above the median frequency the total energy is same as below that actually median frequency ok.

So, it provides a midpoint in that way and using that median frequency we can again characterize the that spectrum for example, that if we look at say a spectrum the one we have looked at the beforehand, here the spectrum probably median frequency could be somewhere here. Now, if it becomes now we are drawing actually only the right hand side otherwise the median frequency will always be the 0. So, we need to keep in that thing in mind. Now if I take it is an uniform distribution. So, if it is an uniform distribution, then it will be the midpoint here if the distribution is say like a triangle which is a triangular one like this one, then median frequency will come here.

So, median frequency would be an easy way to talk about the shape or the distribution without getting into the detail, it could be a single value which can talk about the distribution of it and the beauty of median frequency is that it is pretty robust just like the median of any distribution that whenever some out layers are present that we find that median is still the perturbation or the change in the median is the least compared to other measures.

For example, if you have taken the mean or first order moment compared to that that change in median is actually much less. So, that way median is a very attractive measure. So, now, we look for more such features we look at the variance or the second order central moment ok.

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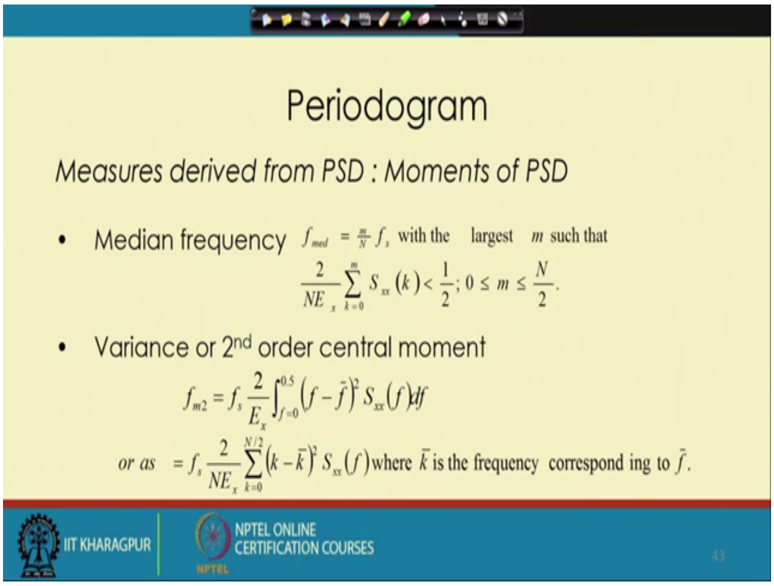

Now second order central moment means that the simple the variance; what we compute, we know that that the meaning of variance, in this case, the only difference is we are looking at the variance in the spectral domain; that means, if we know that what is a that mean of the energy with respect to that how much is the spread variance will give that and again, when we are computing it, we are taking the that range as 0 to 0.5, not minus 0.5 to plus 0.5 because we all the signals are real in that case again the mean would have been 0 ok.

So, second order moment even for that we are restricting to that the right hand side of the that spectra or we can say the corresponding analytical signal spectra, we are looking at and you can compute that in the continuous frequency domain or in the discrete domain either way that can be computed and we all know the meaning of variance. So, we will not get into more details of it we will move forward to find out more such measures.

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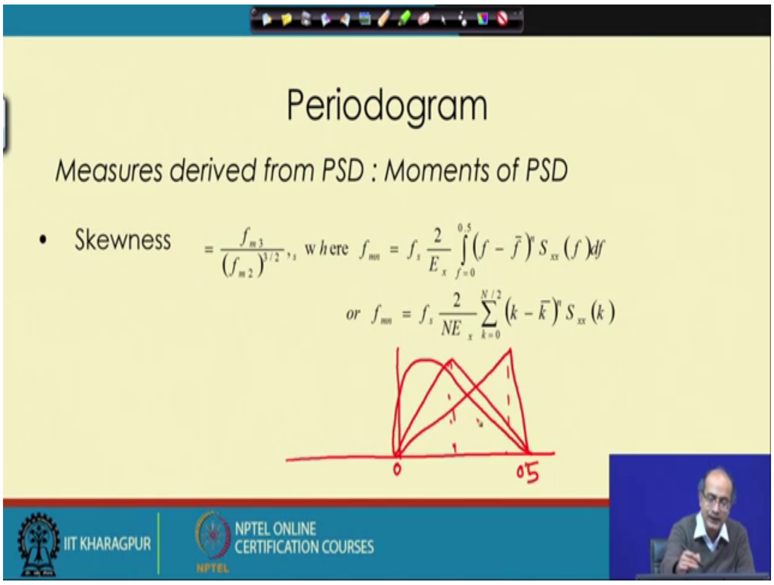

Next, we look at skewness; skewness actually provides us that whether these distribution that is symmetric or asymmetric. For example, that if we take a distribution here that we are taking the corresponding analytic signal that is. So, we are looking at 0 to say 0.5 and if you are taking the real signal, then there would be a replica, we are not just drawing that in this case, yes, I assume that this triangle is a actually symmetric triangle both the sides are same.

So, in that case the skewness would be 0, but if I draw the triangle in this way, then we get both the sides, they are unequal this is the third point. So, in this case the skewness actually will be nonzero ok. So, we can have different kind of distribution need not be just the triangle we can draw a distribution like this where we get it is not yet at all symmetric. So, how much actually the symmetry is maintained or the lack of it is given by the that the measure called skewness.

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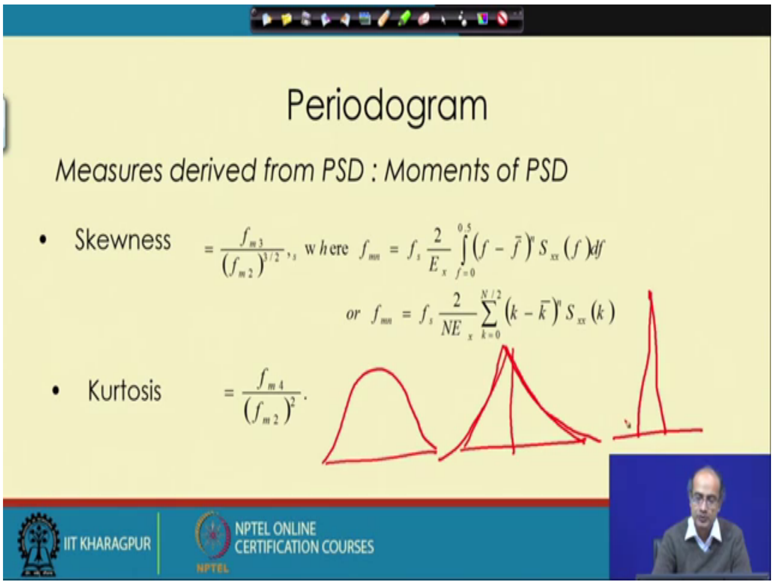

Next we get the fourth order moment or kurtosis which is normalized by see that fourth order moment is normalized by the variance and this one; it gives us another nice property that when we are talking about the distribution let us take couple of distribution.

First we take normal distribution then we can take say double exponential or we can take actually a spike which is again, this one is a double exponential is a triangular wave with very high peak. Now each of this case we can have the normalization of the energy; that means, energy total energy could be same, but there is a difference in the shape the first one, it is a having a blunt head, it is a bell shaped curve compared to that that the double exponential, it is more peaky and if we actually try to have it; actually a triangle which mimics our say impulse response kind of thing in that case that the peak would be much higher though the totally energy within that triangle would be the same.

So, in that case out of this 3, we can say the third one this one is having the highest peakedness followed by the double exponential and the least would be in case of the bell shaped curve. So, kurtosis gives us the measure that how peaked it is or how sharp is the peak. So, that gives a good description about the wave shape or the energy distribution in that way. So, we got another actually scalar to talk about the shape of the distribution.

Next we find that that spectral power ratio before that spectral power ratio first the thing we define that what is the power within a frequency band, in this case, we have taken a frequency band that f 1 and f 2 that is the frequency band taken.

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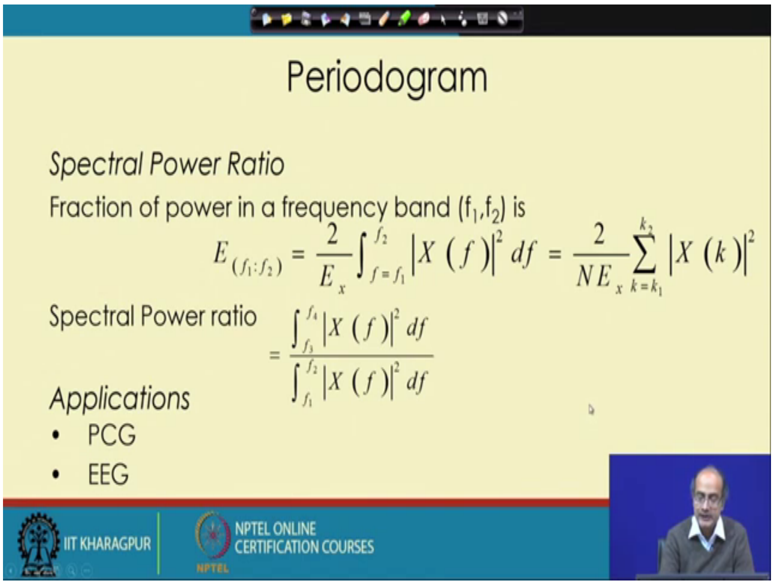

So, what is the energy within that what could be the typical use? We have seen for e g that depending on that what is the predominant actually the band where the energy is concentrated depending on that we name that signal as alpha beta theta delta ok. So, that energy can give us that important information, if we select the f 1 and f 2 in a meaningful way, next, we find out the ratio of it ratio usually is taken across two different bands. Now if we take a choice that whether it is a alpha wave or a beta wave; what we can do we can take the ratio of the energy across the two bands.

In fact, if it is alpha, then we know in one particular band the energy would be more and if the signal energy is preserved; that means, it remains to be the same for beta, it will move to much higher frequency band. So, in that case, as we know the energy shifting from one band to the other, if we take the ratio, it can capture it in a much better way, another thing, it can do that if there is any scaling of the signal that is that may be because of that say instrumentation say amplifier we are using so many in between. So, that also when we take the ratio that scaling actually we can get rid of ok. So, that makes the decision simple. So, that is the use of the spectral power. So, we get such number of actually measures which can actually help us to use power spectral density which we have learnt through or estimated to so many different spectral estimators.

Now, let us look at some of the applications first is the PCG signal, in case of the PCG signal, we know that we are looking at the heart sound and they are the main concern was that at 40 hertz band, how much energy is there, this is the energy is maximum, they are at that if it is that case, then we can get the heart is actually normal, otherwise, if it is moving towards the low frequency band may be at the 30 hertz band, then there is some problem either that it is going through actually that myocardial infarction or that time also has passed.

It is now getting healed and trying to regain its previous ship next is EEG signal the EEG signal the applications are even more straightforward because the description of the EEG goes through that how much energy is there or what is the predominant frequency band or the predominant frequency band in terms of the energy ok. So, there the application is even more easy or intuitive for the power spectral density ok. So, we have completed these topic.

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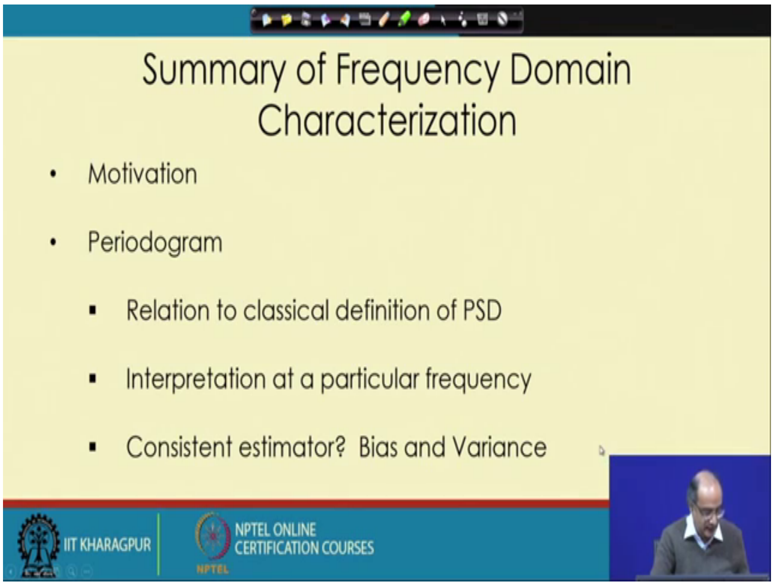

So, before completing that we should now go through the summary of it. So, first we look at the motivation of it for that; we have seen number of cases that we have seen that the first part is that we have looked at the EEG signal.

Next we have looked at that the PCG signal both the case that power spectral density can makes it much more easy to find out the what kind of actually that wave is there or what is the status of the that that heart ok. So, with that we got the motivation and we started with different kind of things that primary thing is that how we estimate the frequency content when it is not a monotone kind of signal monotone means that if we talk about say the frequency count for the normal heartbeat we know it should be say 72 per minute, but if it is a multi component signal we cannot actually do it with a single measure we need to look at the frequency content over a band.

So, first thing was suggested that; what is the easy way to compute that? So, compared to the classical definition where we need to first compute the autocorrelation function and that too for infinite number of lags to compute the PSD, we wanted a easy way out and what we found that we can use the Fourier transform and we can use different kind of windows on that.

So, as a primary step to it first of all we need to find out that if we take Fourier transform kind of approach of the data what is the relationship of it with the classical definition of PSD. And what we have shown that in a limiting case that the way we compute that true the that periodogram or in simple terms, we can say that Fourier transform of the data, it actually converges to our true PSD in a limiting sense; that means, if we can take infinite amount of data and if we can take the expectation of it ok. So, in that case the two estimates would be the same.

Next we look at that what is the interpretation of the that periodogram at a particular frequency. So, what we found there that that a particular frequency the periodogram value is providing us the output of a band pass filter centered at that particular frequency and how would be that band pass filter that is determined by that what would be the size of the or the shape of the window, we have used when we are taking a limited data and we may think that there is no window, but in that case actually we are using a rectangular window on the infinite amount of data ok.

So, by default; when we have a finite number of data is actually we are using a rectangular window and on that for different purpose, we can use the other windows also. The primary reason is that if we take a rectangular window, we get a good actually the main peak; that means, the main lobe the width would be small, but when we look at the rectangular window it has very high side lobes.

So, if we want to suppress those side lobe; then we should look for actually other windows ok. So, we got some interpretation of the frequency and next question comes that whether it is a consistent estimator; that means, when we are looking for the ideal case that if I have infinite amount of data will my periodogram, estimate will give me that true value; that means, the bias should actually go to 0, the variance also should go to 0. Now what we have found that as n tends to infinity as per our expectation that bias is going to reduce and it is going to 0, but not the variance.

In fact, the variance of the periodogram estimate we found it is as big as the mean and if we look at the smoothness of the periodogram, then it actually is affected by increasing increase in the data size more the number of sample points the periodogram becomes more fluctuating ok. So, that tells us that program is not at all a consistent estimator and we need to do something more to make use of it.

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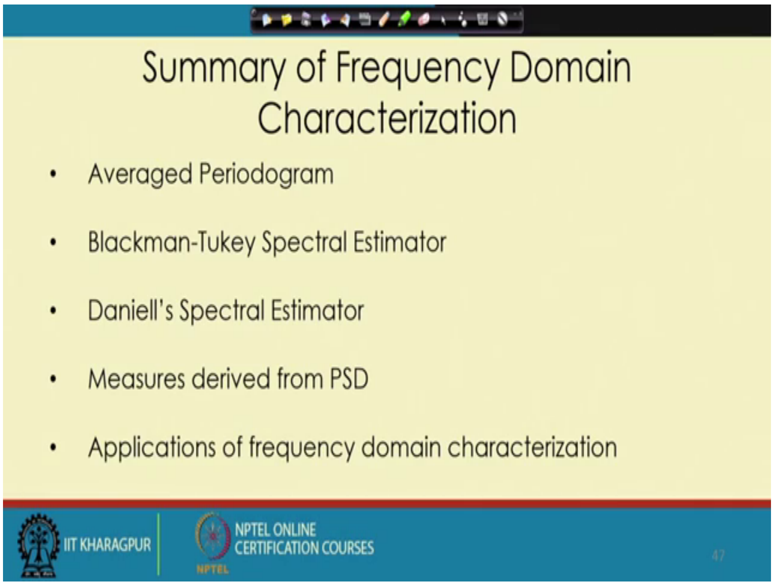

So, there are number of suggestions came the first suggestion was to use a average periodogram what; that means? That we have a finite set of data which is the case in the real life, we do not never have the infinite amount of data. So, that data can be divided into multiple non overlapping blocks and for each one of them we can compute the periodogram and we can take the average over this actually periodograms of different blocks now what is the benefit of it?

In both the case the bias remains to be the same; that means, if you have taken the periodogram of the total data and if you take the average of the that the multiple periodogram average periodogram the with n tends to infinity the bias will reduce, but along with that another thing is happening because we are averaging over different periodogram the variance actually goes down; if we take L such blocks and we average over it the variance will become 1 by Lth time than the previous or the original periodogram variance.

So, we can get a much smoother periodogram that how much smoothing we should do or what should be that value of L that remains to be a question. Because the more the number, we take or more number of blocks we take actually we are taking it with the same finite range of data. So, when we take more blocks; that means, each block will have less data and each block having less data means the frequency domain resolution will actually go down because the frequency domain resolution goes up with the number of actually the points in that block ok.

So, actually what we are doing? We are sacrificing part of the frequency domain resolution and we at the cost of that we are improving the smoothness or reducing the variance or the fluctuations in the spectrum. So, people increase the number of blocks and see that when it has become smooth enough, but no prominent peaks are mixed. So, in that way through trial and error that number need to be found.

Now, that is not very attractive proposal. So, people thought number of actually alternatives and Blackman-Tukey spectral estimator came with an explanation that all these problems are created by the autocorrelation function when we are computing the autocorrelation function with finite set of data as the lag increases the number of actually samples to compute the autocorrelation function it is reducing and at the end we will find for the maximum lag will have only one sample.

So, in that way, what we can get the ACF becomes more and more unreliable. So, what Blackman-Tukey suggested that let us first of all chop of the some of the autocorrelation functions we take only those which are more reliable and within that also we should wait they actually use a weighting function to give more importance to the ACF which are more reliable; that means, those which are having lower lag or near to 0.

So, that was a suggestion by Blackman-Tukey, but people or researchers found that it also has some problem first of all here again that we are applying a window. So, instead of a true PSD, we are getting a convolution of the window function and the that that the periodogram PSD. So, frequency domain resolution we are actually sacrificing here and along with that some more concern came with a choice of the that window function that unless we are very careful of the window this may lead to actually the negative value in the power spectrum.

Now, the negative value of the power spectrum at any point it becomes something very artificial we are unable to interpret it because our interpretation of the spectrum or spectral estimate or in this case periodogram is that; it should provide us the energy at that particular frequency or as the periodogram is given that it is the energy of the band pass filter centered at that particular frequency.

So, the negative value cannot be explained and it becomes a big actually negative for this kind of approach. Now taking the that the information or the learning from the Blackman-Tukeys experiment that; we got number of actually window functions also to do the trade off to select that what the window functions are better for this approach.

So, Daniell made a simple suggestion what they have suggested that if the goal is to get a smooth spectra and for that we are ready to sacrifice some of the that resolution in the frequency domain that simplest approach could be that let us go for the periodogram which is nothing, but the Fourier transform of the data and on that; let us apply a MA filter a simple MA filter, let us apply on that and that by controlling that what would be the number of points that by that we can decide that how much smoothness we need ok. So, that was the suggestion of Daniell spectral estimator which is also a good estimator and. In fact, you will find that that is pretty often used in practice rather than the original definition of periodogram.

Next we looked at the measures derived from the PSD; we have seen different kind of measures like energy we looked at that first order moment, second order moment of the that central moment that is the variance, we got the skewness, we got the kurtosis. So, number of measures we got; we got the median frequency which gives us the midpoint which divides the energy both asides, it should be the same in the above that median frequency and below the median frequency.

So, we got a number of measures and then we looked into the energy within a band which is very important to know that actually the kind of signal for the EEG kind of signal that is very important and in some cases where there is actually shift in energy just like in case of the PCG signal that whether the that energy is concentrated more in the 40 hertz band or the 30 hertz band there the spectral energy ratio has been taken over the 2 bands.

So, all such measures, we have learnt and at the end that that we have looked into that applications of frequency domain characterization. In fact, we started from that point from the motivation. So, that was the coverage of our that the chapter on periodogram.

Thank you.