Biomedical Signal Processing
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Lecture – 38
Frequency Domain Characterization (Contd.)

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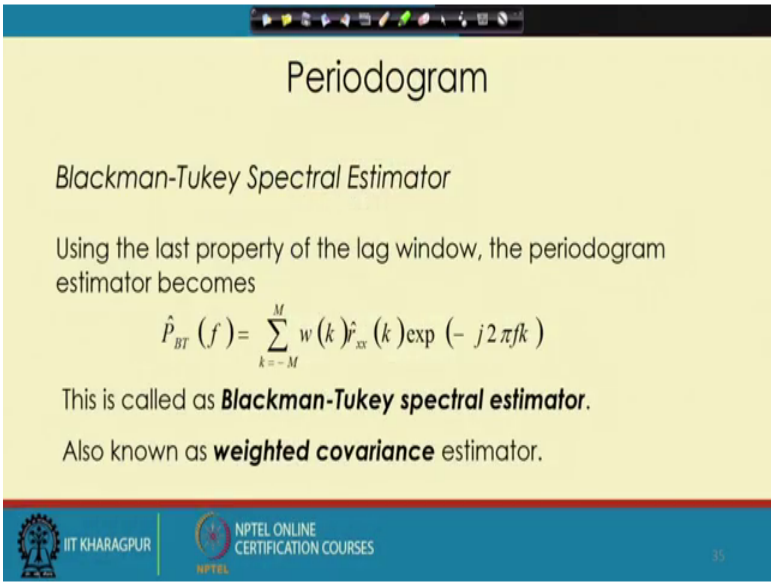

So, as we have the Blackman-Tukey spectral estimator, the other researchers; they became inquisitive about its development that Blackman-Tukey made a such a huge contribution and in scientific world actually the progress goes through review, they started reviewing the results proposed by back Blackman-Tukey which solves all the problem of periodogram at least at that moment. So, was thought by Blackman-Tukey and other researchers. So, they started the review and they came up with number of criticisms of Blackman-Tukey.

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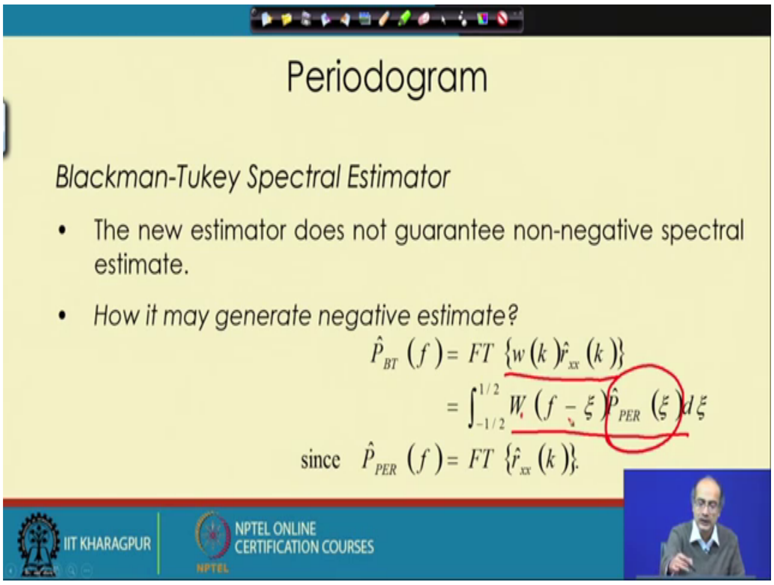

So, let us look at them. So, the first thing they found that new estimator does not guarantee non negative spectral estimate, at times that PSD value can be negative and that is an absolute no, no, we cannot explain at certain frequency the energy is negative physically we cannot explain that situation.

So, it becomes very difficult to accept that kind of estimator and then they go forward the researchers they are not criticizing just to defame Blackman-Tukey, they want to get a better actually estimator to improve the situation. So, they try to explain that why it is happening ok. So, this search for that that how it may give negative values and in that search, they find that here the estimator; what we have taken as Blackman-Tukey estimator that is that is using a window function along with the autocorrelation estimate before taking the multiplying with a window before taking the Fourier transform.

So, in the frequency domain, it can be taken as the convolution with the real periodogram, mind it, this is periodogram; this is not that true PSD with that this new window or spectrum of the window is coming into picture. So, now, with that what is the effect?.

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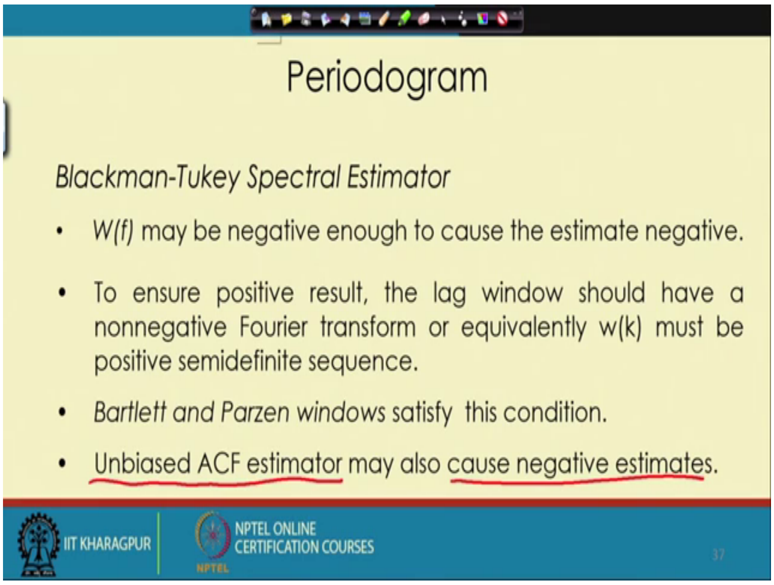

This W f that is the spectrum of the window, it can be negative enough to cause the estimate negative actually this is the main culprit that it can if it is negative, it can pull down the that convolution result to negative ok. So, we need to first take care of the window function, we need to be careful about the choice of window.

So, let us see; what they have suggested to ensure positive result the lag, window should have a non negative Fourier transform or equivalently w k must be; that means, in the time domain that window must be a positive semi definite sequence semi definite means the value could be 0, if we tell positive sequence; that means, the that the result should be always positive, it cannot be even 0 ok.

So, that is a new thing we get and fortunately, there are some windows which can give positive semi definite sequence that Bartlett window and Parzen window, they satisfy this condition ok, they satisfy this condition. So, we are relieved that some of these windows, they are good enough that we can get the better estimate; however, the problem can come from other sides also see, we are using unbiased autocorrelation estimator because we are using the windowed version of the estimated autocorrelation which is unbiased these can also cause negative estimates ok.

So, we cannot guarantee actually that these estimates would be always positive ok. So, all the development that is done hopefully it will give us estimate, but we cannot give any guarantee on that.

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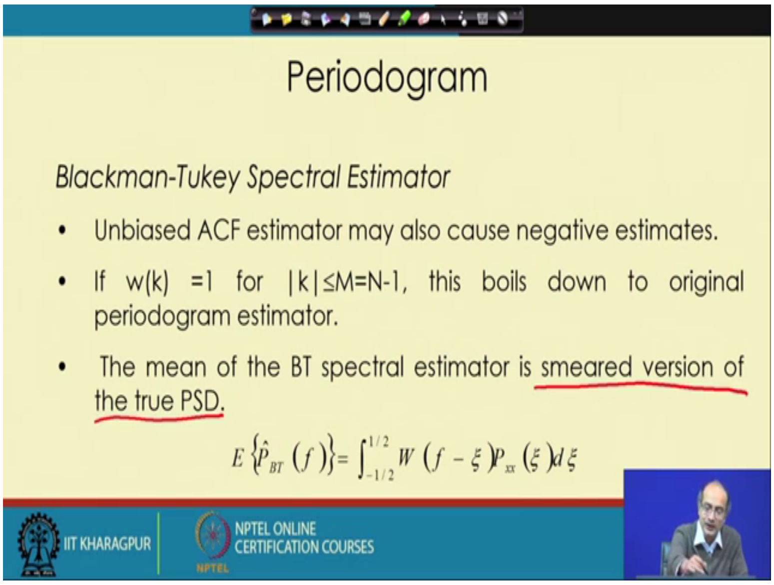

So, still unbiased estimate can give nonnegative estimates. So, now let us look back that w k equal to 1, if we look at the initial one that window that a rectangular window as we are going back, we can come back to the original periodogram estimate. So, Blackman-Tukey; it is giving us some flexibility by some choice of window, we can go back to the original periodogram from where he started, he wanted to improve and from there, we can do something better by a better choice of window at best; what we can tell that Blackman-Tukey spectral estimator is a smeared version of the true PSD smeared version means it is convolved with a that spectrum of a window.

So, it is called smeared version ok, it has the word smear that has a negative connotation because the other researchers they have criticized. So, they have used this term. So, if you do not like it, you may tell that it is smooth version of true PSD ok, but the fact remains the same that we are using a smoothing function and we are conforming with the true PSD that we get we are supposed to get, first of all, it is taking the original periodogram estimated, then using some other window. So, twice it is getting smooth or smeared whatever way you would like to take it. So, this is the expression we get that combining the two that different estimates that we get a smeared version or smooth version of the original PSD ok. So, that is the evaluation we get about the Blackman-Tukey estimator.

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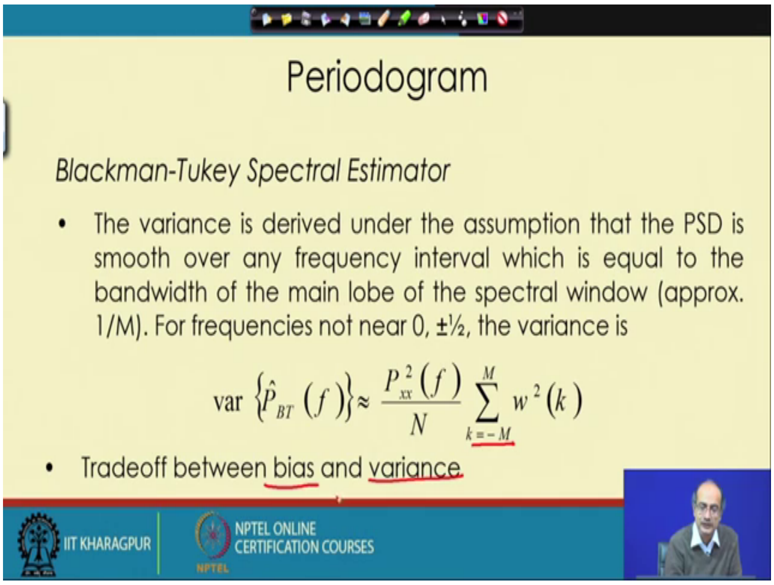

So, let us look at the variance of it the variance under this assumption that the PSD smooth of over the interval is equal to the bandwidth of the main lobe of the spectral window; that means, if the aim is high the main lobe bandwidth is low which is good for the frequency resolution, but it gives us more variation ok.

So, we get some expression for the variance also what we get that it is proportional to the that true PSD. In fact, that was the criticism for the original periodogram that variance is as big as the beam, we get some term with that that is the bonus, we get something in the numerator we get some of the windows from minus M to M and below we get N.

So, by choosing M lower than N, we can reduce this term also by appropriate choice of these window values we can reduce it further. So, the variance would be less than the original periodogram estimator that much certainly we can say and we can have a control over the variance so; however, there is a trade off between the bias and the variance, when we try to reduce the variance, what we can do? We can reduce the value of M that will reduce the variance, but increase the bias so that we need to keep in mind that we cannot get good result for both of them ok.

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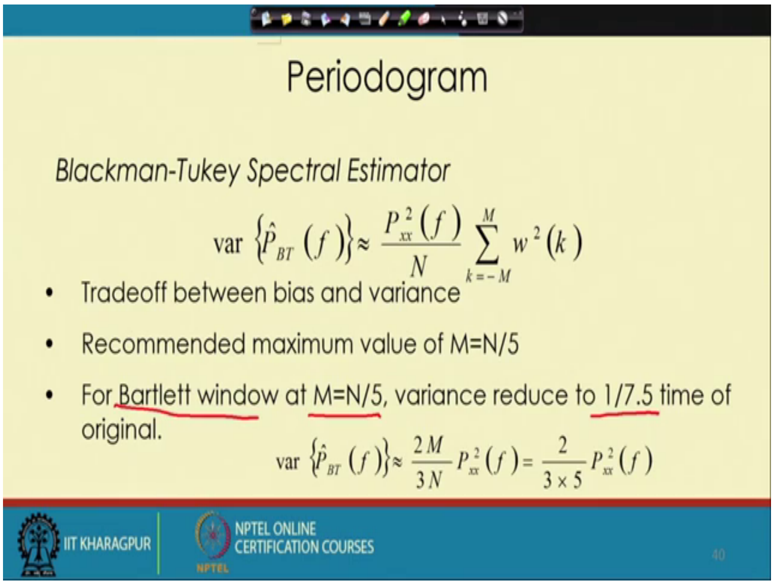

So, there should be some compromise in such case and empirically, what the people have found that if we take M is one fifth of the length of the data, it usually gives a good compromise between these two because some of the standard window, for example, Bartlett window; if we take for that at that lag that M mind M equal to N by 5 variance has a good decrease by a factor of 7.5.

It is reduced and we get it in this way that it becomes 1.75 times of the estimate and there would be of course, some increase in bias, but as a variance has reduced that gives us some better estimates and we get that we cannot solve both or improve both at the same time bias and the variance. But we can tune them to get as best as possible result with that we actually complete our discussion about the Blackman-Tukey spectral estimator.

Thank you.