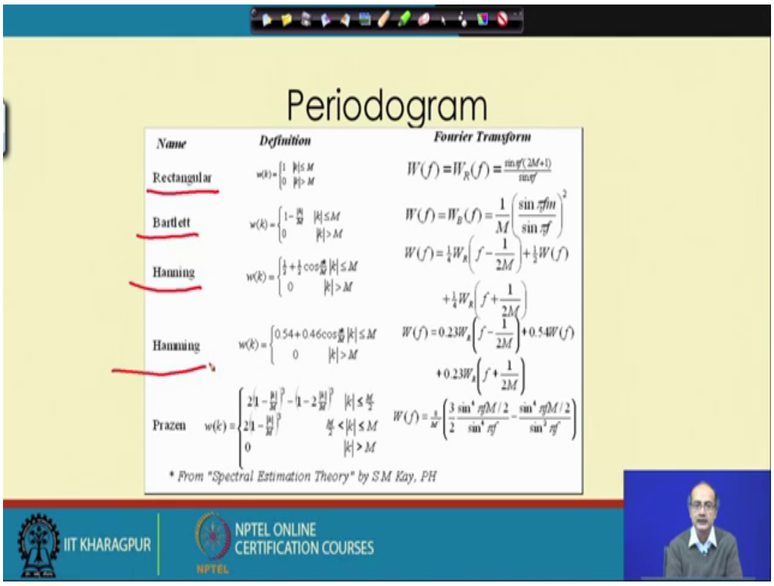
Biomedical Signal Processing
Prof. Sudipta Mukhopadhyay
Department of Electrical and Electronics Communication Engineering
Indian Institute of Technology, Kharagpur

Lecture – 37
Frequency Domain Characterization (Contd.)

So, now we start the periodogram session again,

(Refer Slide Time: 00:23)



First we would like to show different kind of windows and for a different kind of windows, would like to notice you the forms. The simplest one is a the rectangular window that we have finite amount of data. So, window varies up to that and then it become 0 in either side.

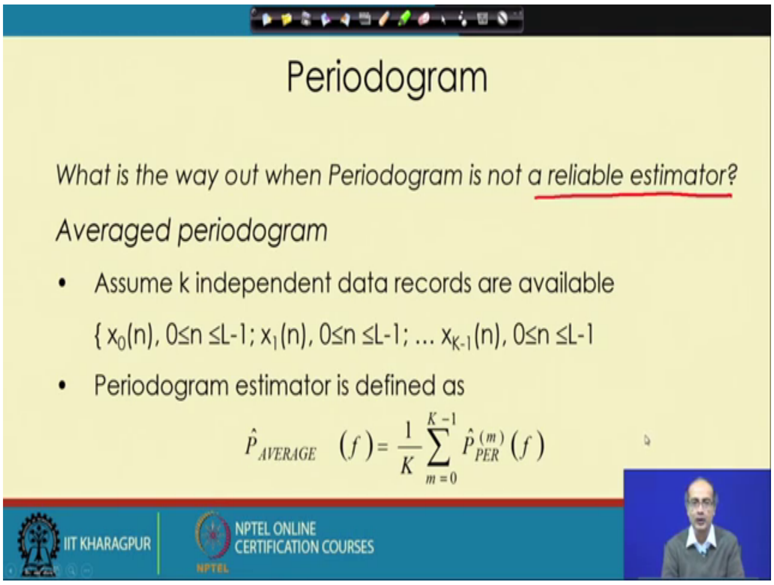
So, corresponding to that; the filter we get that is sin function and next comes that that Bartlett window instead of a rectangle, it is a triangle, we get that at the centre, it is having the maximum value one and both the sides, it is coming to slowly reducing linearly and beyond a point that value more than a M, it will come to 0 next we get Hanning window instead of a linear decrease in the window value again it is maximum at the middle and I think few properties are common for all these windows that they have maximum value at K equal to 0 and they are symmetric and also they are even function ok.

So, this properties are common. Now the first one is having actually 0 slope, the second one having linearly, it is decreasing from 0 to as we move either side, increase the lag that having window, it is using a cosine function. So, they decrease this following the cosine function and that cosine, we know when cosine 0, it is 1 and as we increase the angle, the value will decrease the hamming window is also very close to Hanning window, only thing the proportion is changed the DC value and this value that it is changed then the Prazen window it has the polynomial form of k ok.

So, these are the different choices of window to actually take care of the side lobes, but the one which gives more attenuation in the side lobe we need to keep in mind the main lobe width increases for that window more among these if we take the simplest one that rectangular window it has actually smallest 3 dB bandwidth ok, but the attenuation of the side lobe is very small at the same time.

So, it has very strong side lobes that is a demerit of it. Now as we move forward, you go for the other windows, we get improvement in the separation of the side lobe, but at the cost of increase in the width of the main lobe. In fact, both has their negative effects if the two harmonics are there close by that side lobe frequencies that can accentuate and actually merge them together, again if the separation of them is less than the main lobe width, this two can become actually merged this two harmonics. So, you will not be able to separate two harmonics as a two different one, they would look like one single frequency in the periodogram. Now let us move forward from here.

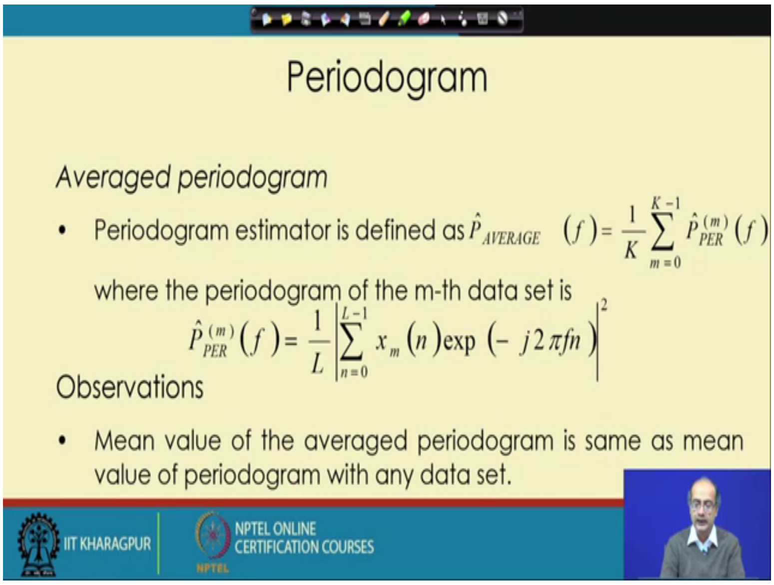
(Refer Slide Time: 05:15)



That we get a dilemma; so far, we have found that periodogram makes a life easy and we all use it in real life, but we get to know after taking this previous session that it is not a reliable estimator and we gave you the example of white noise to test it out. Now what is the way out? Can we throw away periodogram and go back to the old classical definition of autocorrelation may try that, but again the problem does not end because when we have finite amount of data how do we get infinite lags of autocorrelation that is not possible to calculate ok.

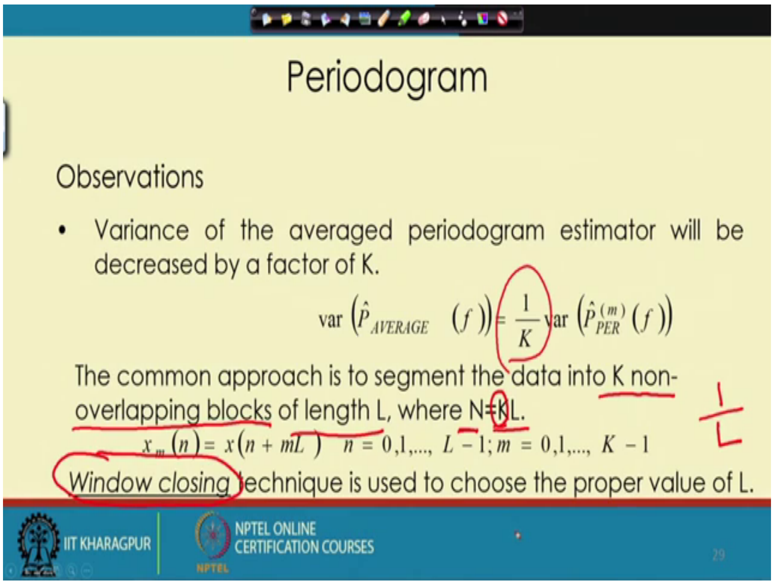
So, we do not have much fall back option. So, look for the way explore that how we can move forward and the solution came as in the form of averaged periodogram; what it suggests that let us take K independent data records and let us compute first record them that x 0, x 1 2 x K minus 1 independent data records and compute the periodogram of each one of them and the new periodogram estimate is the average of this individual periodogram. So, that is the new estimate in front of us to take care of the problems generated by the original definition of periodogram ok.

(Refer Slide Time: 07:52)



So, let us see how far that this average periodogram that take care of the problems we had earlier. So, periodogram estimate or the averaged periodogram, we have defined and for that that if we look at that that each of the individual component we get it in this form that it was like the previous estimate, we are taking just the average of them. Now, let us have some observation out of that if we take the average the average the mean of the averaged periodogram, it would be same as the mean of the periodogram of the each data set; that means, if you take the expectation of the averaged periodogram and expectation of the each of the individual that periodogram they will give us the same value ok.

(Refer Slide Time: 09:20)



So, that is the first observation, we get next is about the bias the bias of the averaged periodogram will be decreased by a factor of K, here actually we have taken K data sets. So, we get one good thing after taking the averaged periodogram that variance is decreased and the property of bias remains to be same that if more data is there that we have that lays bias and with increasing the data we were suffering from high variance in the periodogram and high fluctuations in the that frequencies we can get rid of them or we can suppress them at least and the decrease would be proportional to the or inversely proportional to the number of such data sets we get.

But in real life actually the solution is not. So, easy what we really have that we have one data set we do not really get actually K independent data sets, but to make them independent as far as possible out of the same data set what we do we take K non overlapping blocks because if we take overlapping blocks be a straight away contradicting the assumption that they need to be independent. So, we are taking K non overlapping blocks of length L. So, the total data if it is n that is equal to K into L; that means, were implies in all the data what is given to divide into K subsets or non overlapping subsets.

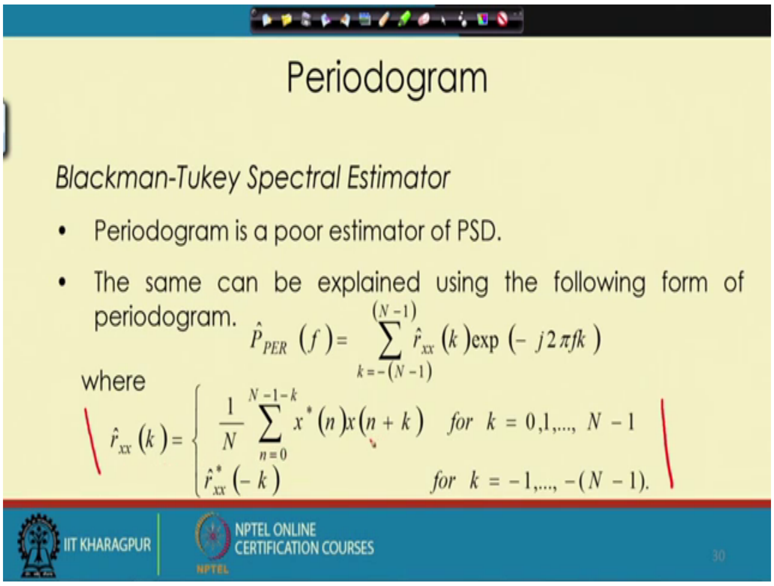
So, in this way that we have generated the individual data sets which are assumed to be independent at least that is true. So, long the observations are independent from each other now to get the proper value of L, we take a technique called window closing. Now what do you mean by window closing for that we need to go back to the bit of understanding that what is the effect of choice of K and n, we have seen that if we increase the value of K, what will happen for a fixed n?

L will decrease; that means, length of individual data sequence will decrease what will be the effect of it we get that if we do that then we will have actually that band pass filter which comes out of the finite amount of data set, it will it has a bandwidth that is proportional to the number of data that is 1 by L in this case. So, frequency domain resolution will decrease, if we reduce the L. So, that is not a good option.

Again to increase the L, if we reduce the K what will happen frequency domain resolution will increase, but the variance also will increase in the frequency domain because the variance is proportional to 1 by K. So, we are in a dilemma. So, what we do really in the window closing method initially we start with say one sequence of the whole data and we see how the frequency estimate looks like or the PSD looks like it will give highest amount of frequency resolution at the same time maximum fluctuations also at the PSD. So, we will get very fluctuating estimate of PSD. So, after that what we do we keep on increasing the K and thereby as from the value 1, it goes to 2, 3, 4, the L decreases and we see that how the change takes place in the PSD and we continue the operation till it becomes smooth, but we do not want the prominent peaks in the PSD that will actually merge together and the PSD will look somewhat different.

So, we want to preserve the frequency resolution at the same time we want it to get smooth or the variance should come to lower come down. So, we keep on changing the K till that point and stop there when we see that it has become smooth enough, but the separate peaks they are not got merged ok. So, that is the only way by manually tuning them to go forward to actually change these values and that thereby select the ideal value of K.

(Refer Slide Time: 16:40)



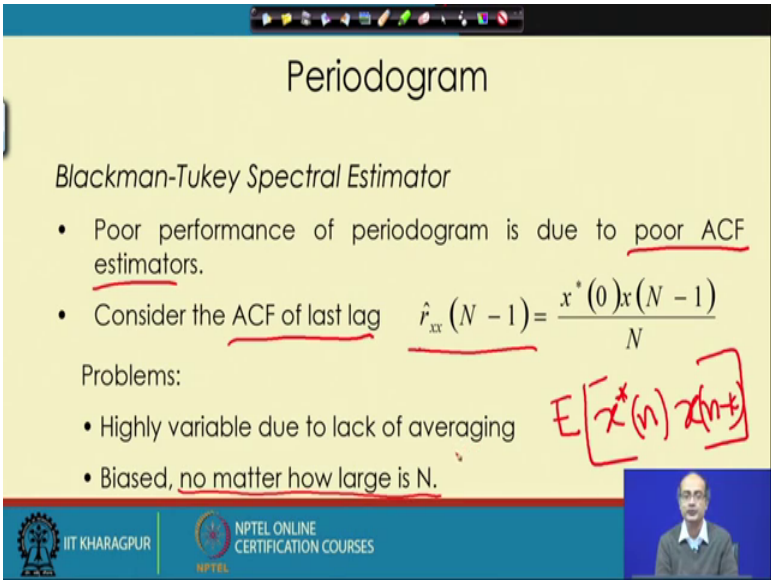
Now, we get another suggestion that averaged periodogram could not solve our problem, we can actually accept that it is better than the original periodogram, it helps to reduce the that the variability in the PSD, but it does not completely eliminate the problem. So, this two gentlemen Blackman and Tukey; they came forward with their analysis and suggestion to give a better solution. So, let us see; what is their offering, first, they point out that what is the cause of the problem and from there in a very logical way, they build up their theory.

So, let us look at their proposition. So, the first thing what they have suggested that periodogram is a poor estimate of PSD and thereby, it gives a scope for improvement for Blackman and Tukey now we can have some expression for them and for that we need to look back the formulas of periodogram, if you look at the periodogram formula, what we get in the periodogram; what we are actually taking that is the estimated autocorrelation sequence instead of the autocorrelation sequence, we are taking the Fourier transform of the estimated autocorrelation sequence to get the estimate of the PSD.

So, that is the source of the problem; where here is the expression of the autocorrelation function here is the function of the autocorrelation, the estimated autocorrelation function from a finite data set for the positive lags, we have computed them as the window is changing, what we notice that number of terms is decreasing as the lag is increasing because that data beyond x N minus 1 is not available. So, that is assumed to be 0. So, even if you write here it does not make any sense and for the negative lags you simply have taken that they are related by a conjugate transposed.

So, from that positive terms we can get the negative terms negative lags and for a real data as conjugation does not make any change. So, it would be there is a reflection of the same set of coefficients ok, for a complex input data, we will get actually the conjugate otherwise, we will get just the a even function by such estimate ok. So, now, here let us proceed for the analysis.

(Refer Slide Time: 20:43)

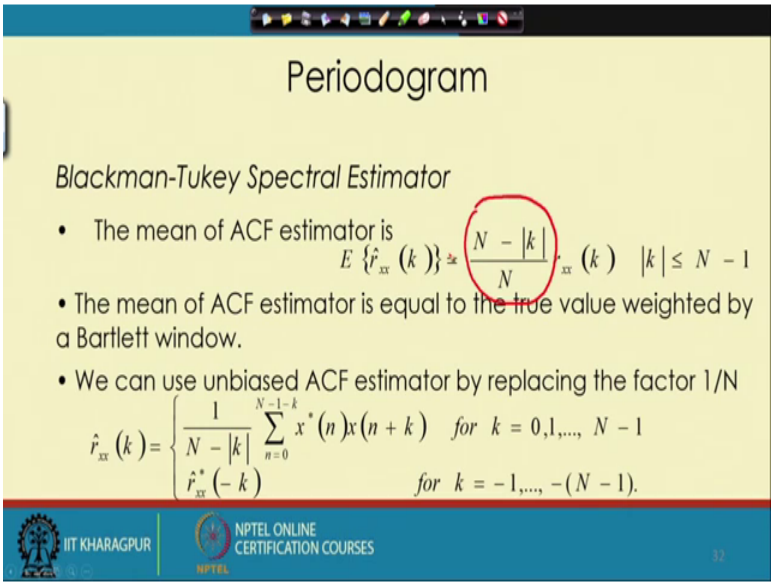


That the Blackman Tukey, they suggested that the poor performance of periodogram is due to poor ACF estimator, it is true that we have not taken the ideal estimator we have a taken estimated ACF for doing the job. So, they have suggested that the ACF is the real cause of the trouble the poor estimate what we have taken for periodogram and for that they have taken the weakest point of periodogram, they have taken the last lag of ACF, for that what we get, we get the estimated value is x star 0 into x n minus 1 by n. Now what is so special about it? Here we do not have any averaging. In fact, we have the all the terms to average; that means, N terms, we had to take average for the lag 0 as we move increase the lag, we see the number of terms is reducing linearly.

And for the last lag there is no averaging at all now these cost us in terms of fluctuation if there is no averaging that estimate what we are taking the ideally, we are suppose to take actually expectation operator expectation operator of the two terms, that x star n into x n minus K or you can take the star in the second one leaving the first one the way, it is defined, now that expectation was actually replaced by the summation here using the concept of the sequence that; this is a stationary sequence and that the averaging in the random space can be replaced by the averaging in the time domain.

So, that averaging being missing, we have to pay for that penalty and we get high variability in the estimate and unfortunately this is going to be there no matter, how large is the N, only thing what will happen that for that lag this lag will also change; that means, if the value of N was 10 initially, then you would get the maximum variance at tenth lag. Now if N becomes 100, we will get that at hundredth lag, but qualitatively if we look at that what is the betterment for the last lag, we cannot say anything positive ok, qualitatively, there is no change this situation would be there and in these set up we cannot do any change the mean of the ACF.

(Refer Slide Time: 24:46)

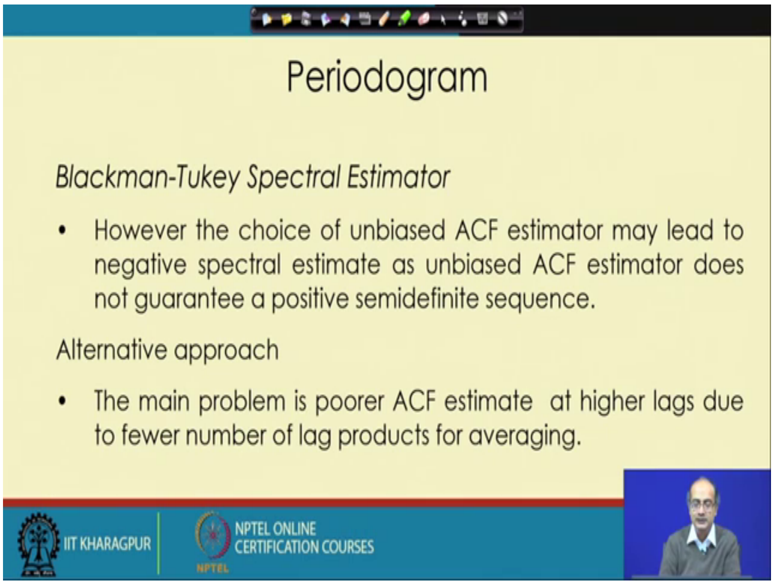


What we get? It is actually it is a scaled version of the true value, if we take the expectation on both the sides; that means, of the estimated autocorrelation function r hat x x K, we get this expression, this is the two autocorrelation and we get these term and this is nothing, but actually triangular way multiplied with the real autocorrelation ok. So, we get that there is a bias also is present by taking the expectation we are not getting the true value ok.

So, the mean of the ACF estimator at best, we can say equal to the true value weighted by a Bartlett window that is a maximum positive, we can say and the first peaks that comes to our mind that what we can do to make it unbiased ACF estimate, we can change actually the formula of the real that estimate of the autocorrelation instead of using 1 by N. We can actually modify that at least we can use the number of terms present in the summation and here is the new formula we get that new estimate estimated autocorrelation we can take as summation of the terms within the given sequence please keep in mind we cannot increase the number of terms here because we have the value from for x n for n equal to 0 to capital N minus 1; that means, n terms are there, there is no way to increase this one, what change we have done instead of taking 1 by N which is insensitive to the lags that number of terms is reducing with increase in the lag.

We are taking here the proportionate value here 1 by N minus mod k; that means, how many terms are there, we are scaling in that way and the relation between the negative lags and the positive lags remains the same there is no change there ok. So, that at least, we hope can take care of that fact that we would not have the extra term here, what we are getting in this form that Bartlett we know that that would be eliminated ok. So, that is the expectation. So, if we can do it in this way.

(Refer Slide Time: 28:20)

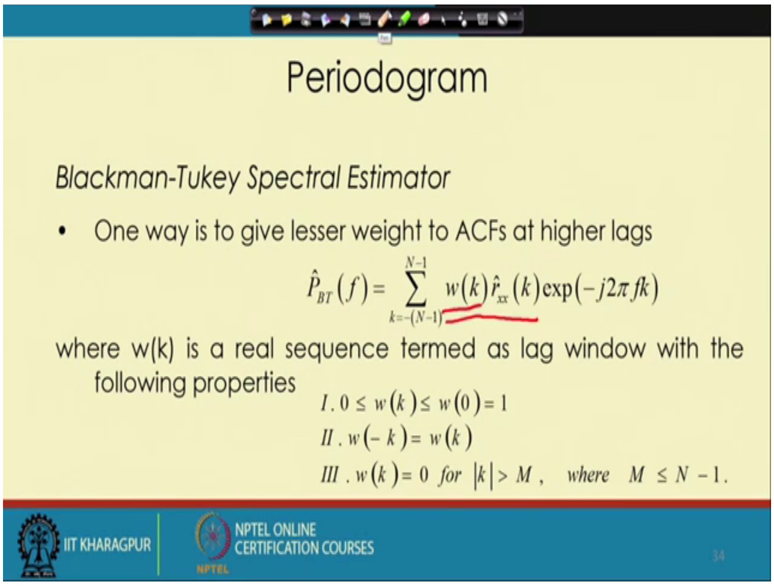


Now, let us see, what we get that in this way that the choice of unbiased ACF estimate we made the estimate of ACF unbiased, but it gives raise to another problem that it can give negative spectral estimate as these new unbiased ACF estimator does not guarantee a positive semi definite sequence a; what we mean by that that as this is not a positive semi definite sequence if we take the Fourier transform and take the absolute value.

The absolute value may become negative, now what is the problem with that the problem is we are talking about the energy of a signal and energy cannot be negative, if we get a negative spectral estimate, we are unable to explain that that how the energy becomes negative. So, we wanted to do something very positive we wanted to fix the problem of bias, but it has come out in a new way which is again detrimental to the that performance of the estimator. So, let us let us look forward, let us see that what alternatives can be made. In fact, Blackman Tukey, they suggested these alternatives.

So, we can take that that these are the new propositions of Blackman Tukey here, what he suggested what or rather, what they suggested that the main problem is in the poor ACF estimate at high lags due to fewer number of lag products for averaging the higher the value of lag we have less terms for them and we get more and more poor estimate of the ACF when we go for higher lag because of lack of averaging. So, we need to think from that able that do something. So, one way is to give less weights to the ACFs at high lag is very reasonable kind of assumption that were we have less confidence let us use less percentage of information from there ok. So, that is the thing Blackman and Tukey suggested and for that what they have told let us take.

(Refer Slide Time: 31:49)



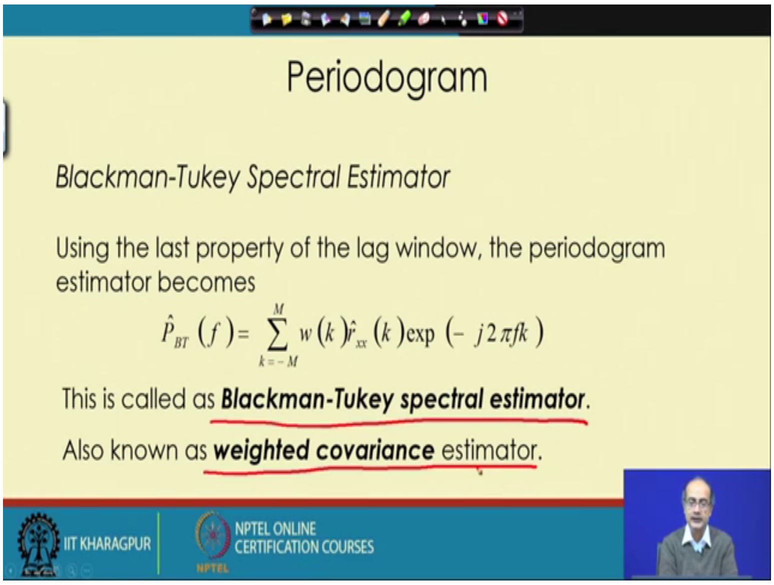
The estimate of estimated autocorrelation function as we take in the case of periodogram and apply a window, earlier, we have introduced you with number of windows. So, they gave a actually solid footing for them that why they should be there let us take that window multiplied with the estimated autocorrelation estimate with a purpose to actually give proper weighting to the different lags of autocorrelation function, we know near the centre that autocorrelation function estimates that better and they become more and more unreliable as we move away from the zeroth lag. So, w the windowing function should actually look at that phenomena and take appropriate action for them.

So, they should have some property first of all that for different lags that the first property would be that w 0 should be maximum that is equal to one and window should be positive because we do not want to change the sign of the that autocorrelation estimate. So, all the other lags are value should be in between 0 to one next that as the autocorrelation estimates are even for the real data same would be true for the window.

So, it should be a even function third thing is that it should go to 0 beyond some m what should be the value of m m should be less than N minus 1; that means, we could compute actually the lags up to N minus 1 autocorrelation up to this point, but we know when you go near to N minus 1 the number of terms to average would be very less. In fact, there would be only one term at N minus 1 at lag. So, we need to get rid of that situation, we should stop it much before that we should stop it at some value m beyond that we know the estimates are so unreliable, there is no point in using them because if we try to take them we may have more noise rather than real input.

So, we cartel that we use only the central portion of the estimates and weight them to give maximum weight to the zeroth lag and for the other lags, we are giving some appropriate weights and that window should be a even window. Now let us move forward using the last property of this window lag.

(Refer Slide Time: 35:33)



Now the periodogram estimates it becomes gets a new form where instead of taking the value of K minus N minus 1 minus N plus 1 to N minus 1 we restrict it from minus M to plus M or using less number of lags and this is what is called as Blackman Tukey spectral estimator this is what they proposed and there is also another term, it is called weighted covariance estimator because they have suggested a weight to the that autocorrelation function ok. So, this was the proposition of Blackman Tukey and we will stop here for thus this session.

Thank you.