Biomedical Signal Processing
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Lecture – 35
Frequency Domain Characterization (Contd.)

So, what we have found that Periodogram is something

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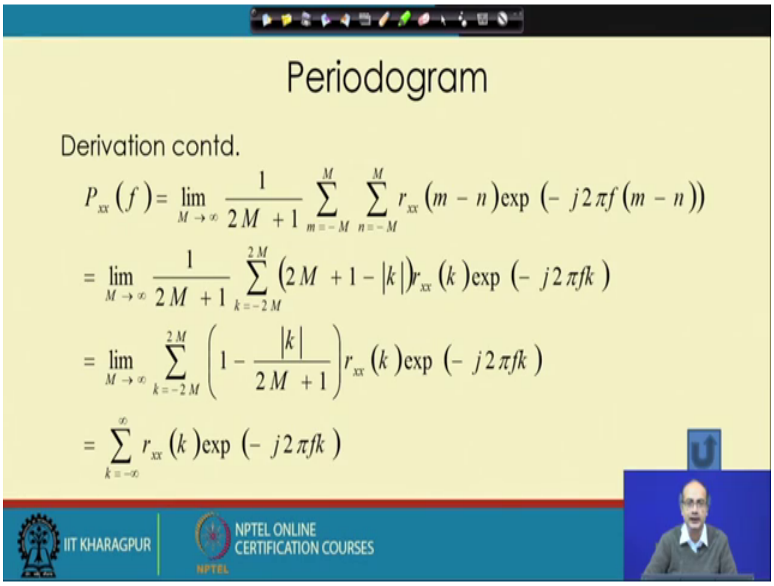

Which can replace now our old P h d and give us a benefit of directly using the data to compute the P h d.

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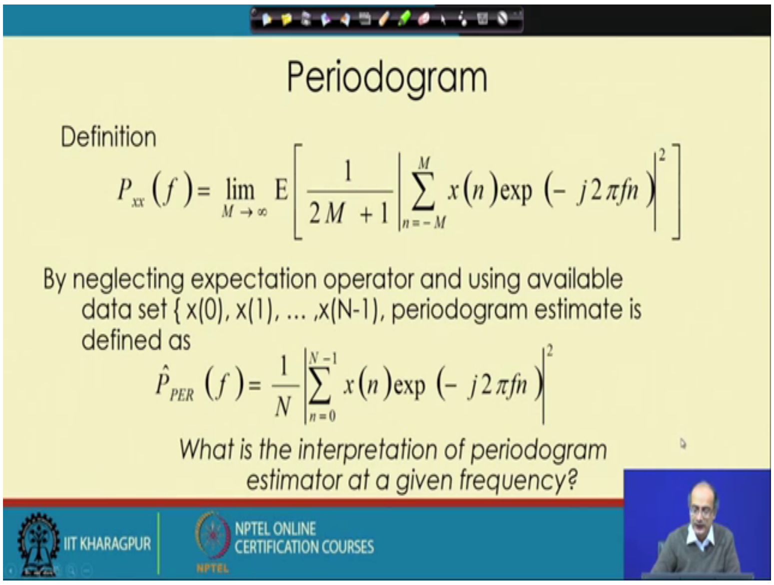

So now, we try to make it more convenient instead of taking the sum from minus M to plus M; that means, a centered data which is somewhat artificial. We would like to actually change the indices for the data set usually given in the form starting from 0 to N minus 1, that Periodogram form would be a little different. We would compute the sum n equal to 0 to n minus 1 that Fourier kernel would be there along with the term x n and we will take mod square of the summation and again we will take the that this scaling factor 1 by n like the previous one.

Earlier we also had that 2 m plus 1 that is the number of terms of we are adding here, it is 1 by n. So, many terms we are taking and for the practical use, two more things are done that is we have always limited data. So, and we have used all of them; 0 to n minus 1 data was there we have used all of them. So, there is no point in keeping this term. So, this limit is dropped to make it usable.

And next thing is that expectation operator, this is also is a difficult thing that we told that what is a meaning of expectation operator; we need to do multiple experiments if there is a luxury of that. For example, if you can have multiple coins we can try them at a time to find out what is the probability of getting the head and we can collate all the results that that will give us the effect of expectation that averaging in the random space.

But in practice, we are actually most of the time left with one single actually set of data ok. So, we do not have the luxury to do the averaging over that random space. So, we simply drop that expectation operator and earlier in case of the LMS algorithm, we have done that and we have found that it did not give very bad result ok. May be that has encouraged us to do that. So, we get a somewhat working definition of the Periodogram here ok. We get a working definition of Periodogram what we can use for the day to day work and with that we proceed.

Now the question that comes to us that; what is the interpretation of Periodogram estimator at a given frequency? We know that p h d is defined in such way and p h d has an interpretation. It is p h d gives us the energy at a particular frequency but Periodogram is somewhat different though they have a relationship, when we can take those limiting condition M tends to infinity and taking the expectation operator, we can show it is same as the that classical definition of P h d but in practice, we do not have them.

So, still can we get any interpretation of this Periodogram in the frequency domain; that means, for a particular value of a f, what the Periodogram value or the estimate we will give. And here 2 actually signify that here it is an estimate, we have used actually these hat operator, you see over the p we have used a hat. It is just to signify that this is an estimate ok. So, what is the meaning of it we would like to know that?

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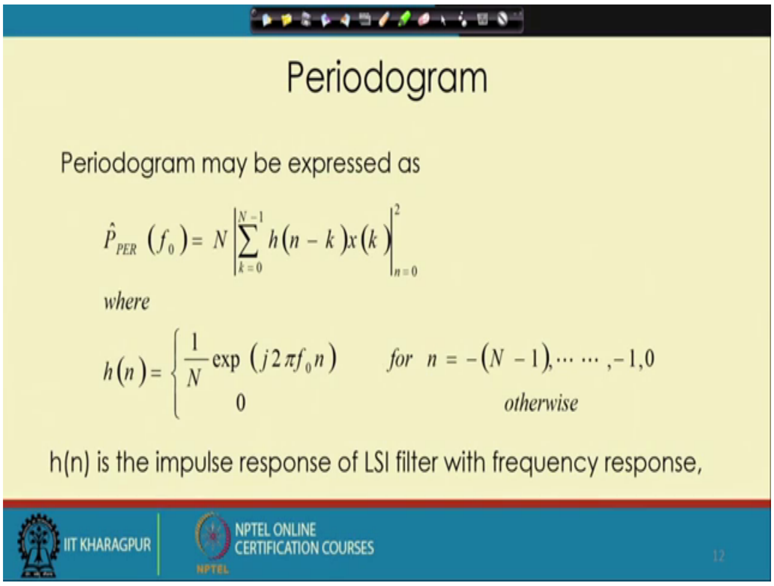

So, we will do some more small derivation for that. The Periodogram, we will rewrite in a little different way.

What we will do; we will write it instead of the that the exponential term directly, we will write in the form of a convolution; convolution width that N impulse response h n where h n has the terms 1 by capital N exponential j 2 pi f 0 n for the minus lags starting from minus N plus 1 to 0 and it would be 0 for the other lags. Now here, we have chosen a particular frequency f 0 for our actually derivation.

However, it could be any frequency, but at this moment, we are just trying to concentrate that at a particular frequency, if we can find the meaning of the Periodogram estimate then we can generalise it, we can compute it for any other frequency there is nothing more in these use of the term f 0 instead of f ok. So, it is just to tell that at this moment, we have chosen a particular frequency and it is not a variable at this moment ok. We are dealing with other variables to compute the value of the Periodogram estimate at this fixed frequency f 0 which is arbitrarily fixed. So, if we need we can change it. So, now we get that h n is the impulse response of the linear shift invariant filter with some frequency response.

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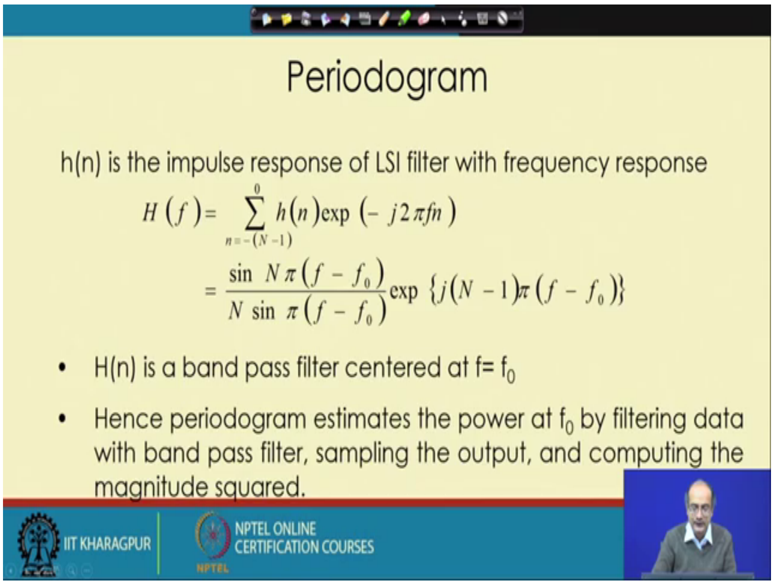

So, we can write it in this way that we can take the Fourier transform of h n to find out the frequency response and what we get that we have some term here, that is sin function and then we have an expression here exponential j N minus 1 pi f minus f 0 ok. So, that gives us the term.

Now, what we get interpret from here that h n is a band pass filter centered at f equal to f 0. From these actually we are getting it that, it is centered at f equal to f 0 and we that we get that h n is located at frequency f equal to f 0 and the periodogram estimate from here what we get gives a power at f 0 by filtering data with a band pass filter that we have the data the samples and from there, we have computed the squared magnitude of that. The filtered output if we sample and take square, that output is giving us the value; that means, we are getting the energy if we apply a particular band pass filter which is centered at f equal to f 0; the energy at f equal to f 0 of this particular band pass filter ok. So, we get some idea that what we are expected to get out of our data.

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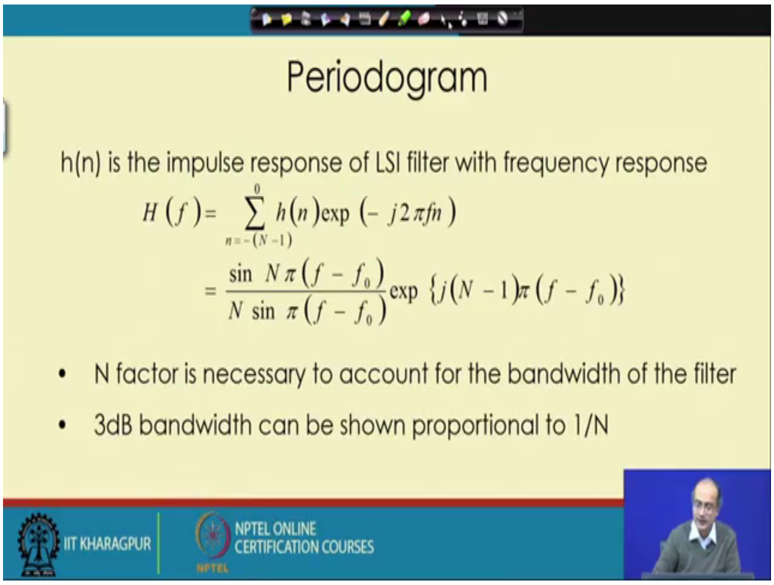

And we get some more thing from here. The term n that capital n which gives the number of samples, this factor is necessary to account for the bandwidth that how wide or narrow would be that band pass filter that is determined by n and we get the 3 dB bandwidth of that band pass filter it is proportional to 1 by N. So, what we get, if we have more and more data then that bandwidth would be smaller and smaller. So, it will slowly converge towards the impulse function in the frequency domain. So, if we could have infinite amount of data, it could give us the exact energy at frequency f equal to f 0 and that is true for any f 0.

So, what we get with finite amount of data, periodogram gives us the energy of the band pass filter with certain impulse response that is we have given here and around f equal to f 0 and as we have more and more data, it will go towards the ideal condition that band pass filter will collapse towards a delta function and it will gives us the exact energy at that particular frequency ok. So, it is moving towards the ideality. So, that is the second actually result we get and that gives us much hope about the Periodogram.

Now here, we will take a small break to observe this idea, and again we will come back soon.

Thank you.