Biomedical Signal Processing
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Lecture – 34
Frequency Domain Characterization

Good morning. So, today we will start a new chapter. So, for that we need to introduce something.

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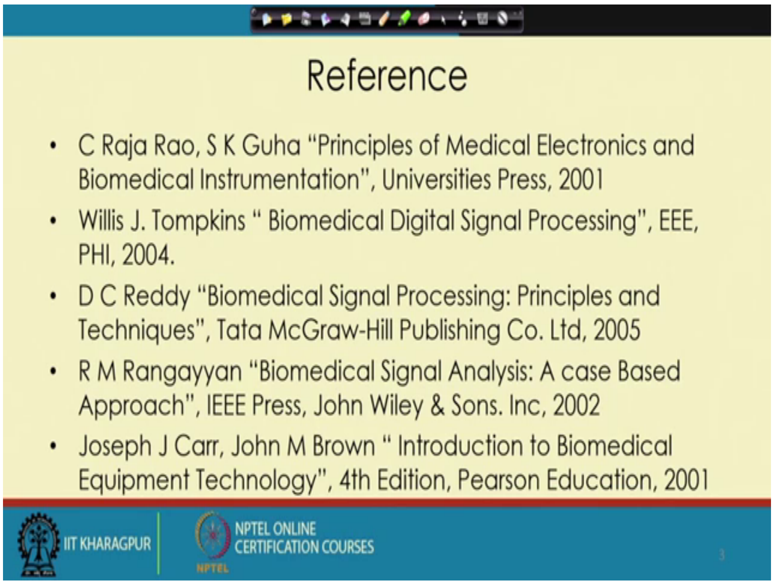

So, here we will take you through the reference list once again that all the previous references that remains.

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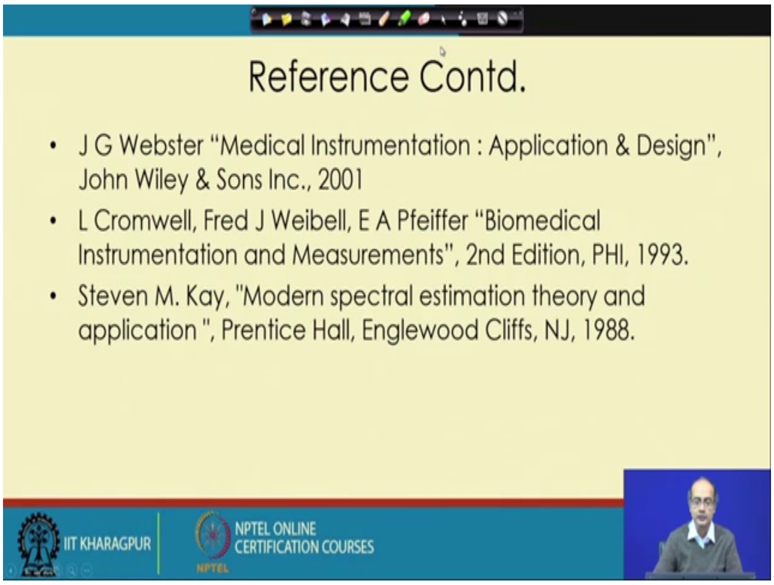

And, we will just add one more at the end that is that this Steven Kay's book on Modern spectral estimation theory, the last one ok, at times that we will refer to it and I think that would be useful in the following chapters, ok.

So, we will proceed to frequency domain characterization. Now, to start this topic the first thing we should note that we the engineers, we are problem solvers and for that we need to look for the opportunity of different kind of solution that whatever suits best for a solution. So, when we look about that that our heart rate we find it easy to express that 72 minutes, sorry 72 actually beats per minute that is the easier to express compared to the fact that if we tell that it is so many seconds or milliseconds is a cycle, ok. So, based on the convenience we pick up the thing.

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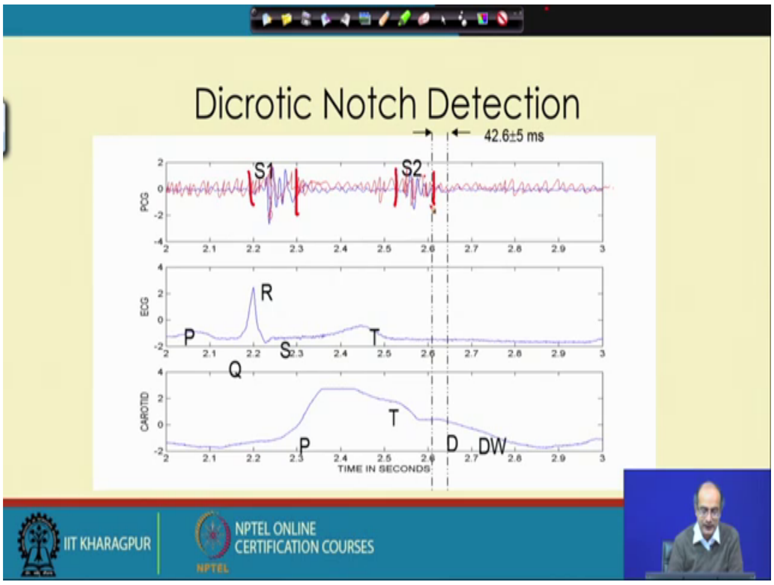

And, here let us look back that to one of the examples and say we have that the phonocardiogram signal which has two components S 1 and S 2 and the way it is shown here actually it is a filtered signal, clean signal that in real life it will have a lot of actually that noise as superimposed on it, ok. Now, if you think of that a lot of noise is superimposed on it then and not just here, but here also you get that that S 1 and S 2 does not come so clean.

Now, once it is not so clean the problem is that what we get there that we find that it becomes very difficult to actually get the characteristics of S 1 and S 2 and two different phenomenas and so first thing what we do, we try to eliminate the most of the noisy portion, the portion in between say S 1 and S 2 and we want to isolate this part that what is the portion of S 1 and S 2 and eliminate rest of the part because we know there is no activity, but only noise is there. So, for that we took the help of the QRS complex of the ECG signal and the notch in the carotid pulse, ok. So, in that case that we found that we can confine them in the time domain and read off the problems there and, we can get rid of the problems there.

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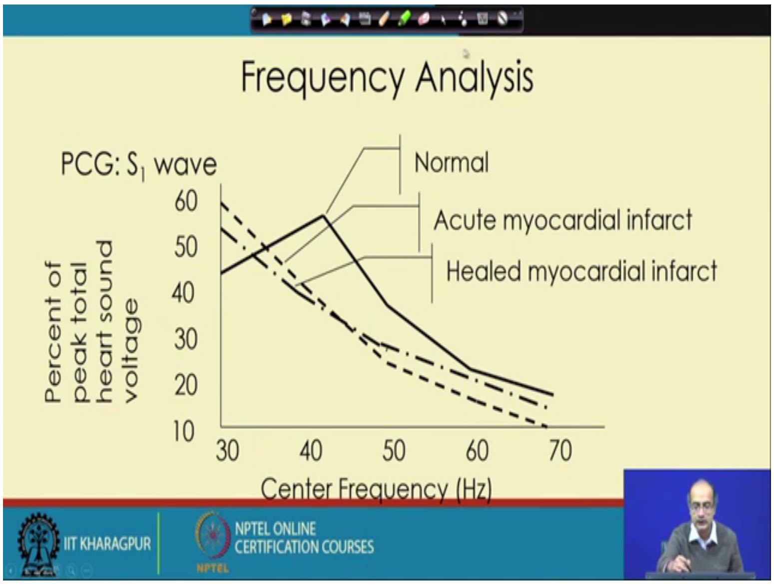

So, we can get a first confine them in the time domain. Now, after that when we try to characterize them what we find that if there is any abnormality say in S 1 wave which consist of number of components first of all that that the blood is coming out of the ventricle at high speed and opening up the valves towards the arteries. And so, it is causing some turbulence next the opening of the valve that makes some sound, then the valves actually which were so far giving the blood from outer to the ventricle they are getting closed. So, if there is any leakage rare that will also contribute some sound, if there is actually constriction in the arteries which are supposed to carry the blood from the ventricles to the next stage. So, all these things also have some component in them.

So, if we look at even the S 1 signal it is a multi component signal and lot of different causes are there. One of the thing we know that when there is murmur then the frequency of the S 1 wave increases, but then again already noise is present and that our 0 crossing or that kind of technique that that the turn turns count they are not very accurate actually measure of frequency. They give a rough measure of the change in frequency. So, it becomes difficult actually to capture in that way that what is the change that is occurring from that a person who is normal and the person who has having the disease.

In fact, at the early age also that for the people that who are pretty young they are also some murmurs are there ok, but when they grow old actually that subsides only for the pathogenic cases we get the murmur and that murmur has some different actually frequency components. So, what the people found that instead of looking the signal in the time domain if we can look at them in the frequency domain it becomes much easier to diagnose them. So, here the first thing what we note that if we look at the S 1 signal that the PCG signal S 1 wave. In that, that for the normal one we will have actually that that we will get here that at around 40 hertz we will get a peak.

So, most of the energy that would be concentrated on the near the 40 hertz band, but if there is any abnormality in that the heart and that causes the characteristics of that the tissues especially the and the muscles there. So, we have actually change in the spectral characteristics we get the power they are at the 40 hertz band it actually subsides and we get increase in the lower band and first when we go for the case of echo accurate acute myocardial infarct infarction, that myocardial infarction means that we have lack of blood actually lack of oxygen in those muscles.

So, then we get that the changes are first occurring and that the DC component or near the thirty hertz that the frequencies they are increasing there and it has a steep decrease as the frequencies are going down. When this myocardial infarction is getting healed then again there is some more changes what is happening towards the 30 hertz the that lower frequency one that components are actually again getting decreased, but there is no increase near the 40 hertz brand band. But for the higher frequency that you see the that frequency components they are increasing going towards the normal patient.

So, if we look at that frequency domain description of the S 1 wave it becomes much more easy to get actually the changes for the normal the characteristics is clear that it has maximum frequency in the 40 hertz band and for the acute myocardial infarction we get the that signal is more at near 30 hertz and that other higher frequency above 50 or 60 that it is much lower at this region it is much lower. When it gets healed again, at the higher frequency 50 to 60 it is actually moving upward and the component which was there near 30 that the increase is again getting actually subdued here in this portion.

So, frequency domain description in this case is becoming more effective and that is why that we find as an engineer that instead of looking at time domain at times we should look at the frequency domain description of the signal and from that point of view we are looking at the frequency domain analysis of the signal and we know that classical definition of the frequency actually that the way for receive the signal the frequency is described it is with the help of the auto correlation coefficients. We know the auto correlation coefficient and the PSD it has one to one relationship through the Fourier transform, but calculating that autocorrelation function for all the lags itself a big task. So, we find we try to find actually an easier way so that directly from the signal can we get the PSD and from that point of view that we look for new technique that is called periodogram.

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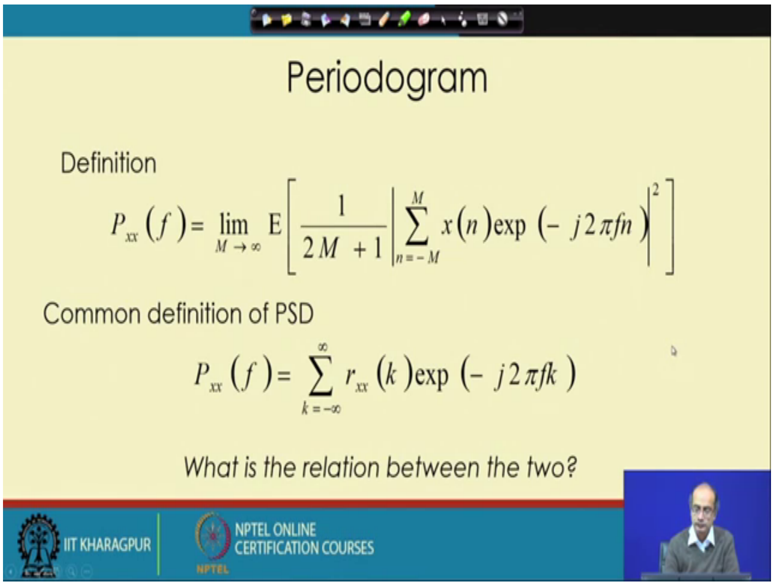

So, here the definition of periodogram is given that we take here that the Fourier transform you can say of a sequence starting from minus M going up to plus m; that means, 2 M plus 1 points and we take actually for that we take the mod square of sum of actually that this components and it is averaged divided by the number of samples and after that we need to take the expectation because that x n is a random variable. So, we need to take expectation and then that we take the limiting case, where M tends to infinity. So, that is the definition of periodogram.

So, now what we would like to know that if we compare it with the standard definition of the PSD which is again a sum from minus infinity to plus infinity that containing the all the lags of the autocorrelation coefficient r r r xx and we take the Fourier transform of it to give the PSD. Now, in this case what is the commonality of these two or do they have anything in common in between? So, that is the first question we asked that we have a standard definition and we are taking a new definition where, we need not have to compute explicitly the autocorrelation coefficient, but we are directly going to get the PSD with a new definition called periodogram, will they give me the same thing or it will be something different. So, we start with this question now.

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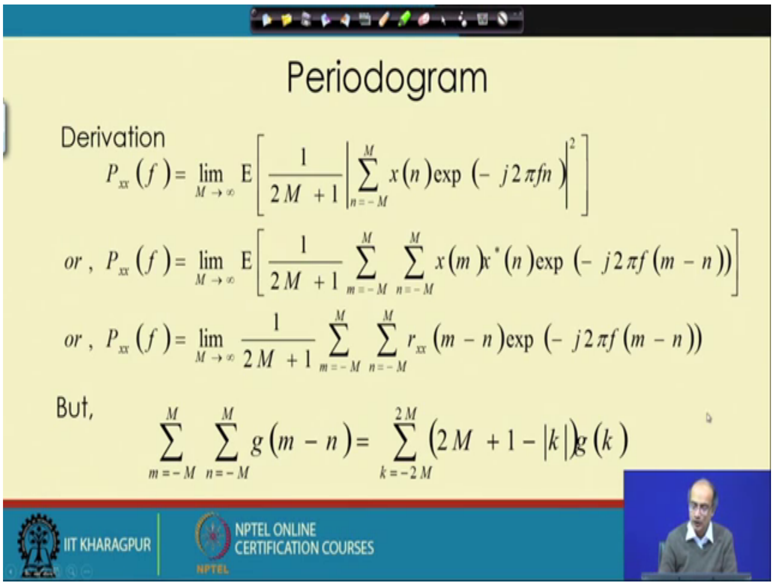

So, for that we actually do the derivation of that first we take the periodogram formula. That periodogram formula that we have the mod square here, we have taken the mod square. So, mod square can be represented as the simple quantity multiplied the conjugate of that quantity, ok. So, we take the conjugate of the same expression and just to make sure that two indices does not get mixed up we take the two summations with two indices one with small m another with small n and one of them that gets conjugated here x conjugate that n here we have conjugated and because of the conjugation we get that here in the exponential term we get that one term called m minus n, ok.

So, we get a expression like this after simplification or getting rid of the that the square term, ok. We get a quadratic term like this involving x m and x star n. Now, let us look at the exponential operator what is there. Now, exponential operator will work on the random variable rest of the thing they do not change with the probability space. So, if we take it inside then we will get actually a term exponential of x m into x star n, we will get a term like this. So, from that that we get that is nothing, but our autocorrelation function for m minus n f lag, ok.

So, that we look at that and here that m minus n the way we could write only because; that means that this is equal to r xx m minus n we could write this with some assumption. First of all that we have taken the assumption that this is a stationary process and because it is stationary it is shift invariant. Over a time if we change it will not change with the time even if we change the shift the that indices of time that is m and n and it depends on the difference of these two time instances and that is why we could write it in that way otherwise we need to go for a more complicated kind of thing we have to write m comma n, ok. So, that property we abused. So, when we are taking this kind of derivation one thing is implied that is we are dealing with the stationary signals only then these derivations are strictly actually that they will follow otherwise the things could fail.

Now, what we find that when we try to simplify them we end we have ended up with double summation which does not make it that simple and it in fact, is longer than the initial expression. So, we try to do something about it and here we notice something that if we take a double summation of a function g m minus n of this form, then it can be expressed in terms of a single summation of almost double the that lags. If we take m minus n the term that is the lag ok, here the summation was the double summation was from minus M to plus M, here we are going minus 2M to plus 2M, ok.

We have that many actually lags we have here in this case. So, we have the replace that part now that m minus n we have a index for that that is given k and however, these term g k it is weighted. It is weighted here, that we have a weight that is 2M plus 1 minus k; that means, for 0 at lag we will have most of the that terms they are getting accumulated when k equal to 0 will have 2M plus 1 such terms when we are going away from the 0 in either side then the number of terms will reduce. Any way that whatever may be de-scaling one thing is clear that we could replace this kind of function which can be expressed in terms of g m minus n with the help of having double summation with M and n with a single summation when we take m minus n is the new index. So, we apply that in this case because we get in this term that r xx is having m minus n term and exponential also has m minus n time, ok.

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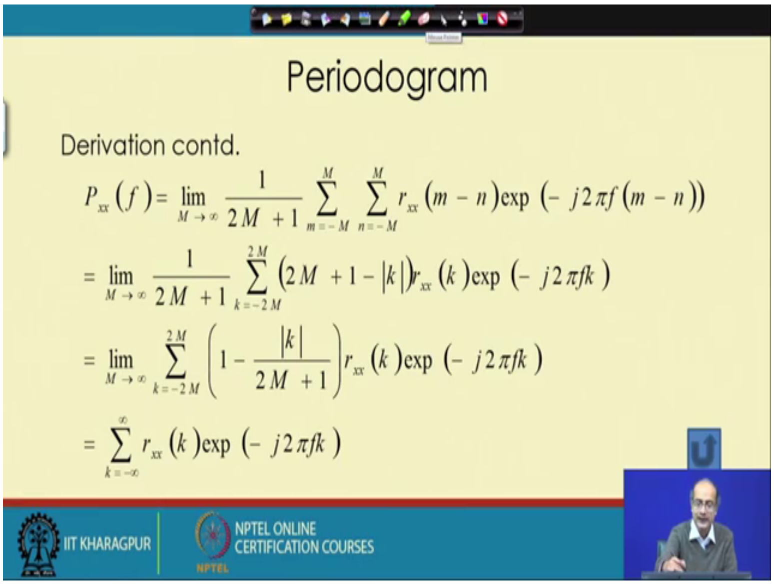

So, using that we simplify and we write this expression in a simpler way we see it in the second line that we take the new index k and with the help of that that we can write it in the form of a single summation, ok, of r xx k and exponential minus j 2 pi f k and here one thing that we need to keep in mind that f is a actually continuous variable we are getting a continuous spectra. If we get spectra in that sense what we are supposed to get for the classical definition using auto correlation function.

Now, in the next step what we do we take the term 1 by 2M plus 1 inside and we combine with this scaling 2M plus 1 minus mod k, ok. So, we get a new scaling here 1 minus mod k divided by 2M plus 1 and we get here that r xx k minus j 2 pi f k. Now, if we look from here itself that this term that looks very similar to our autocorrelation function. In fact, this is the autocorrelation function of the signal and this is the that taking the kernel of the Fourier transform so, we already got somewhere near. Now, taking the limit, so far we have not considered the limit. So, we apply this limit now.

Now, limit is on M and we see these two terms they are not directly affected, because they do not have terms involving M what will change first of all the index k that limit that minus 2M to plus M it will become minus infinity to plus infinity. So, we have a change here and for these term what we get at as M tends to infinity these term second part of it this one it goes to 0, for all finite value of k. So, we can simply replace this term that the limiting value would be 1, so, we get the classical definition of the PSD here. So, what we get that the expression what we have derived for our case that periodgram that is nothing, but a new way to find out our that PSD without directly going to get the autocorrelation function, and then going to compute the frequency domain transform to get the PSD. It is a direct way to get the PSD and that way periodgram gets a legitimate actually place in the literature.

So, with that we stop here.

Thank you.