

**Biomedical Signal Processing**  
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**Lecture – 29**  
**Waveform Analysis (Contd.)**

So, in the last session we have gone through the minimum phase correspondent.

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Minimum phase correspondent and signal length


Minimum-phase correspondent (MPC)



It follows in cepstral domain that  $\hat{\phi}_{xx}(n) = 2\hat{x}_c(n)$ , and therefore,

$$\hat{x}_{MPC}(n) = \begin{cases} 0 & n < 0 \\ 0.5\hat{\phi}_{xx}(n) & n = 0 \\ \hat{\phi}_{xx}(n) & n > 0 \end{cases}$$

where  $\hat{\phi}_{xx}(n)$  is the complex cepstrum of the ACF  $\phi_{xx}(n)$  of  $x(n)$ .

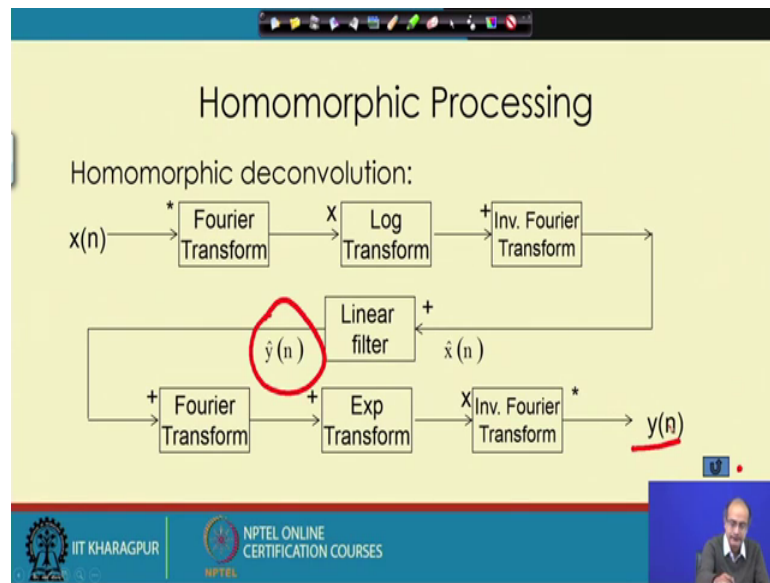


And signal length; so, we will take up from there and the first thing we will do; we will just see that where we left it last day. So, for a signal we found the expression for minimum phase correspondent that we can get it with the help of that even actually the component and that we can get from the complex cepstrum of the ACF that is  $\phi_{xx}(n)$  of the signal  $x(n)$ . So, what we actually received here in this formula that is the minimum phase correspondent in the cepstral domain.

So here why we are looking at that? Now we need to actually go back to the time domain and for that that we need to look at that.

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What is the way to get that? So, we are here; so, after the that in the cepstral domain. So, from there we need to take the Fourier transform, then we need to take the log and inverse Fourier transform to get the corresponding minimum phase and the correspondent part of the signal in the time domain. So, first we need to come to that then we can go for actually that how we can take the minimum phase counterpart from here; that how we can make use of it for calculation of the signal length.

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### Minimum phase correspondent and signal length

Signal Length

- Notion of signal length (SL) introduced by Berkhour (1978).
- Different from signal duration, which is the extent of time over which the signal exists.
- It depends on how energy distributed over its duration.
- It depends on magnitude as well as phase of spectrum.
- For one-sided signals, minimum SL implies minimum phase and converse is true.

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So, now let us see that about the signal length that it was proposed by that the Berkhour in 1978. So, first difference of these from the common notion is; it is not about the duration of the signal or the extent of the signal, it is about that how energy is distributed over its duration? So, it is about the way the spread of the signal. And whenever we talk about the energy, the first thing that comes in our mind that magnitude actually part of the spectrum but it is actually associated with both the magnitude as well as the phase of the spectrum.

So, if we have a spectrum for which the phase is modified; then the signal distribution is also changed and that we get that the signal length also will change. So, for a one sided signal for example, the causal signal which we are dealing with; the minimum phase that signal length that we will get that, the minimum signal length we will get; that it is minimum phase and the converse is also true.

So, that is why that we are looking for the minimum phase correspondent; otherwise what can happen? We can have the same signal, but because the phase is changed; we can have actually a change in the phase spectrum and that can modify the signal length.

So, we do not want to have that happen because we want to make use of the signal length as a measure of distribution of the energy and we want to use it for certain purpose. So, let us see how that happens?

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Minimum phase correspondent and signal length

Signal Length contd.

$$SL = \frac{\sum_{n=0}^{N-1} w(n)x^2(n)}{\sum_{n=0}^{N-1} x^2(n)}$$

where  $w(n)$  is the nondecreasing, positive weighting function with  $w(0)=0$ .

The choice of  $w(n)$  depends on application and desired characteristics of SL.

For  $w(n)=n$ , we get centroidal time instant of  $x^2(n)$ .

The slide includes a hand-drawn graph of the weighting function  $w(n)$  versus  $n$ . The graph shows a straight line starting from the origin (0,0) and increasing linearly, representing a nondecreasing positive weighting function. The x-axis is labeled  $n$  and the y-axis is labeled  $w(n)$ .

At the bottom of the slide, there are logos for IIT KHARAGPUR and NPTEL ONLINE CERTIFICATION COURSES, along with a small video inset of the lecturer.

The next what we look at is a formula of that signal length, which we have actually represented here in short as a cell; that signal length. Now it is as you get it is expressed in terms of that; the energy of the signal accumulated energy of the signal. And it is weighted by some function  $w_n$  and again it is normalized by the total energy.

So, this  $w_n$  actually we are trying to get actually that how the energy is distributed out of that. In this case  $w_n$  has a special property, it should be non decreasing and positive function and at the starting point that  $w_0$ ; the value should be 0.

Now, non decreasing means it could keep on increasing or it can saturate and stay there without any drooping. So, that is what we mean by non decreasing; so what we get that as the time passes on from; the  $n$  equal to 0; as we proceed  $w_n$  will not come down. It will either increase or it will remain constant and with such a function that when we wait, we are giving actually more importance most of the time if  $w_n$  is actually non decreasing to the values, which are coming in future.

Or we can say it is in the recent past; so, in that way we can get actually a measure of the spread and that when in the simplest form that  $w_n$  equal to  $n$ ; that is  $w_n$  is as  $n$  is going over time,  $w_n$  is moving in a linear way.

So, in that case we get the centroidal time instants of  $x^2 n$ . So, we get actually centroidal actually a time instant of the instantaneous energy. So, that is the way; the signal length is defined. So, that gives us the idea that how we are actually getting that that  $S_L$  is giving the some notion of actually the time that how far it is away from the that 0 in terms of that how the energy is getting away from the that starting point. So, that is the way  $S_L$  is defined and that is actually used to serve some purpose here in this case.

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Minimum phase correspondent and signal length

Applications of SL

- QRS-T wave is action potentials of ventricular myocytes.
- The normal duration of QRS-T is 350-400ms (QRS alone is 80ms).
- For PVC, the QRS-T becomes wider.
- For PVC, QRS and T waves are not separate i.e. they lack ST segment.
- PVC and normal wave have similar magnitude spectra.
- SL depends on both magnitude and phase spectra.

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So, here we are looking at the case that the QRS-T wave of the action potential which is actually representing the action of the ventricular myocytes, it is generated by the ventricular myocytes.

So, if we concentrate on that then we get the QRS-T waveform as a result of it and QRS-T that; it has a good span 350 to 400 milliseconds span; out of which the QRS which actually having the most of the energy, it is about 80 millisecond time it takes. So, what we get from this that the initial part of the QRS-T wave; most of the energy is concentrated and in case of PVC that P Ventricular Contraction that we get that QRS-T becomes wider.

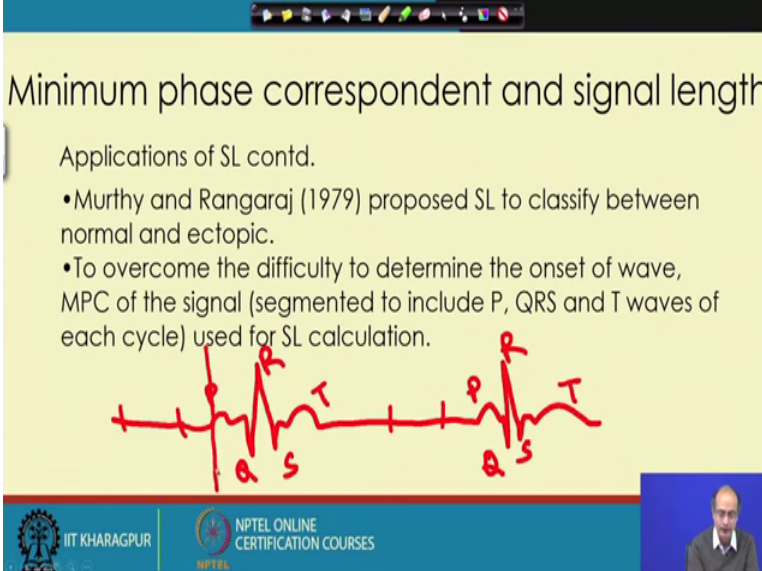
That the QRS complex it becomes wide and the energy gets spread and that will actually change our; that S L and in case of PVC, QRS and t they are no more actually separable because now the ST segment, it actually does not remain isoelectric; so, or isopotential it does not remain at 0.

So, because of that; that QRS and the P, they get actually connected. So, what happens in this case? That compared to that when we look at the normal wave; the ST segment does not have any change. However, if we look at the spectra of the signal that spectra of an PVC and a normal wave does not have much change. The reason you can think of the amount of energy outside the QRS is small.

So, it cannot actually bring a measure change in that magnitude spectra; however, if we notice it carefully we can actually see the change actually in the that location of that energy. Now the energy is outside the QRS complex and thereby we can make it noticeable by using a appropriate measure.

So, that the S L that the signal length; it depends both the phase and the magnitude of the spectra that is the beauty of it. So, we make use of it to find out the difference between a normal wave and a PVC.

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Minimum phase correspondent and signal length

Applications of SL contd.

- Murthy and Rangaraj (1979) proposed SL to classify between normal and ectopic.
- To overcome the difficulty to determine the onset of wave, MPC of the signal (segmented to include P, QRS and T waves of each cycle) used for SL calculation.

The slide features a red ECG waveform with two cycles. The first cycle is a normal sinus rhythm, and the second is a premature ventricular contraction (PVC). Vertical red lines mark the onset of the P wave and the R wave for both cycles. The signal length (SL) is indicated as the duration from the P wave onset to the end of the T wave. The PVC cycle is shorter than the normal cycle. The slide also includes logos for IIT KHARAGPUR and NPTEL ONLINE CERTIFICATION COURSES, and a small video inset of a man in a suit.

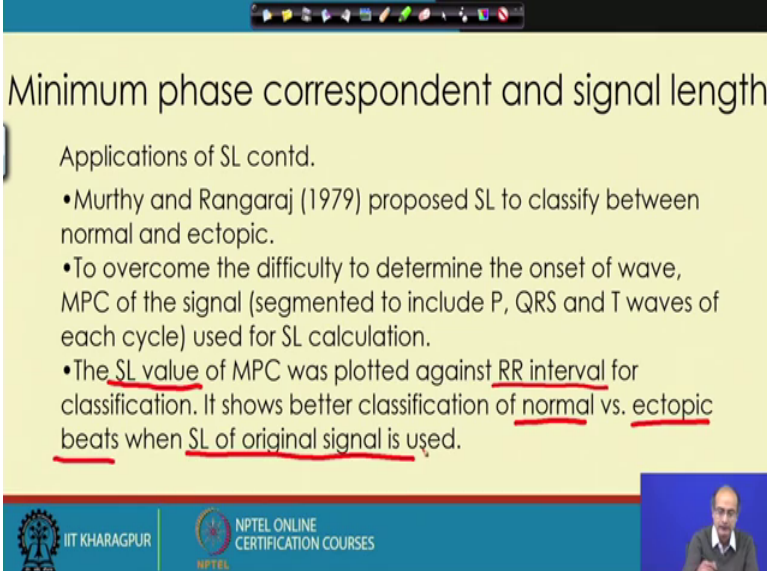
So now let us see that how that can be used; Murthy and Rangaraj; they in 1979 proposed that the signal length to classify between the normal and the ectopic bit; later they made this observation and they reported that in the journals. Now there is a difficulty; the onset of the wave what we mean by that? If we look at that our ECG signal; the ECG signal the energy actually is limited within very small part and most of the period, there is no activity.

So, if we draw the ECG signal; it will look like this. Now for most of the work we are happy to tell that we have taken a cycle of the ECG and we are happy with that. But here because we are interested in the phase part of it; the starting point becomes very important and so where we start, we stake the cycle from here to here or here to here.

So, that can actually make a huge difference; so, what it is done that they told that we should take actually where the real activity is starting; we should start from here and that is the way we would take that this should be the starting point. So, that is the way they suggested that we should calculate the S L starting from P then QRS and T.

Because rest of the part, there is no activity; so, if we take that it will just include some amount of noise. So, that is of no use rather it will actually bias the result; so, you should concentrate or limit as from.

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Minimum phase correspondent and signal length

Applications of SL contd.

- Murthy and Rangaraj (1979) proposed SL to classify between normal and ectopic.
- To overcome the difficulty to determine the onset of wave, MPC of the signal (segmented to include P, QRS and T waves of each cycle) used for SL calculation.
- The SL value of MPC was plotted against RR interval for classification. It shows better classification of normal vs. ectopic beats when SL of original signal is used.

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P QRS and T waves which is much less than the overall the one cycle, but this is the main part where the energies are concentrated. And within that please keep in mind that P being the first one and that is taken as the starting point. So, at the beginning of P we would actually take that that the starting point of the signal. And what they suggested the signal length of that minimum phase correspondent; it should be plotted against the RR interval of the classification.

The reason is that in case of ectopic beat, if we look at the RR interval also would be modified; it will become irregular whereas, for the normal wave RR interval would be within a very small range, it would be almost constant. And what we expect? The S L also will change; so, taking together in a 2 D plot; it will help us to get a better classification between the normal and the ectopic bits; where the S L of the original signal is used.

It is expected to give better classification accuracy. So, that is a way that we actually started and that was proposed by Murthy and Rangaraj, so let us see how the results we get.

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The slide is titled "Minimum phase correspondent and signal length". It contains the following text:

Applications of SL contd.

- The classification is done using linear classifier.
- In the experiment, for first patient, 132 out of 155 normals and 48 out of 53 PVCs are correctly classified. For second patient, all the 89 normals and 18 PVCs are correctly classified.

Below the text are two scatter plots. The left plot is labeled "SL of signal" and shows a scatter of points (x and o) with a red circle around a cluster. The right plot is labeled "SL of MPC" and shows a similar scatter of points with a red circle around a cluster. The y-axis for both plots is labeled "RR interval" and the x-axis is labeled "SL of signal".

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Here we get that classification is done using a linear classifier; the reason for using a linear classifier is that we want to show that the classification is easy and the features what we have taken they are effective.

So, we have chosen a simple classifier; a linear classifier and then an experiment is conducted where in the experiment there is large number of actual data where 132 out of 155 normals and 48 out of 53 PVCs are correctly classified; that is for the patient 1. So, for the first patient most of the, that PVC's and the normal they are actually classified, but they are properly classified, but there are some misses.

In case of the second patient, the results are little better for 89 normals; all of them and eighteen PVCs all of them they are properly classified. So, that is a result we get from them and here we are showing a, the plot just to show that how actually the schematic came, it is not the actual the data. But what we see in case of that signal, if we have taken the original signal; we get there is some overlap in this part; between the normal and the one with our, that PVS's.



But when we use the minimum phase correspondent; then we can get well separation between the two clusters. However, it does not mean that they would be completely separated some small overlaps could be there; like here it is shown one. So, such kind of small actually gaps could be there, but they becomes much more separated and that gives us actually that and hope that using that signal length, we can separate out the PVC's and our, the normal bits. So, that is the contribution made by the researchers.

Thank you.