Biomedical Signal Processing
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Lecture – 28
Waveform Analysis (Contd.)

So, we have already seen the changes in the signal for, specially for a ECG signal and now, we would look at the different techniques to capture that morphological changes in the ECG signal.

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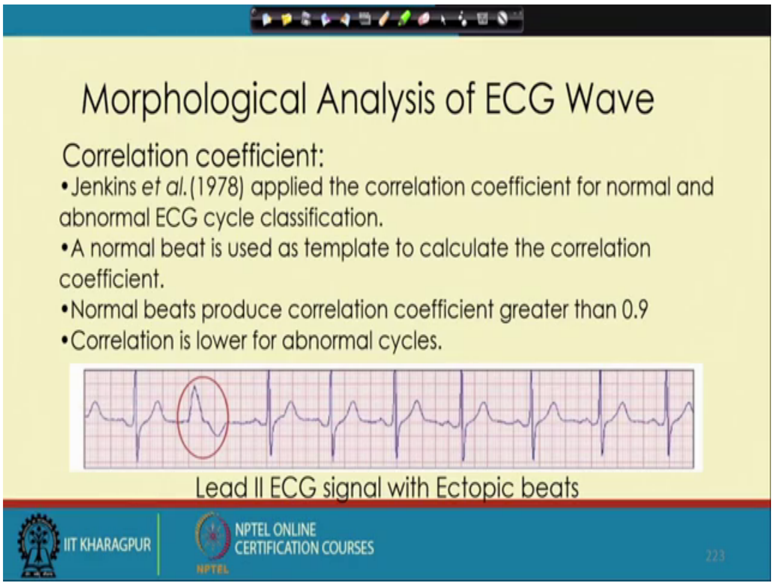

So, first we look at the correlation coefficient for that purpose that Jenkins et al in 1978 proposed that if we take actually a normal beat as a template and calculate that the correlation coefficient with the help of that template for both the normal and the abnormal ECG cycle what we will get for the normal ECG signal the correlation coefficient would be very high. So, according to his experiment the value would be more than 0.9. On the other hand the correlation would be much lower for the abnormal cycles.

So, here some example is given example of lead II ECG we take ectopic beats. So, in this case that in the red color we have shown that one such actually ectopic beat. So, as the shape is entirely different in this case for the QRS complex then the correlation coefficients will be much lower. However, the correlation coefficient we found that though it is immune to noise still it gets affected by that different kind of corruptions or the noises and that to get a very good value of the threshold becomes a difficult exercise. If we take a high value of correlation at times we miss some of the real actually beats. If we take it as a low value some of the arm abnormal cycles or abnormal beats they are detected as a normal one. So, that is the challenge we get. So, from that the search has begun that to find out better techniques. And that the people have started looking at that the minimum phase correspondent and the signal.

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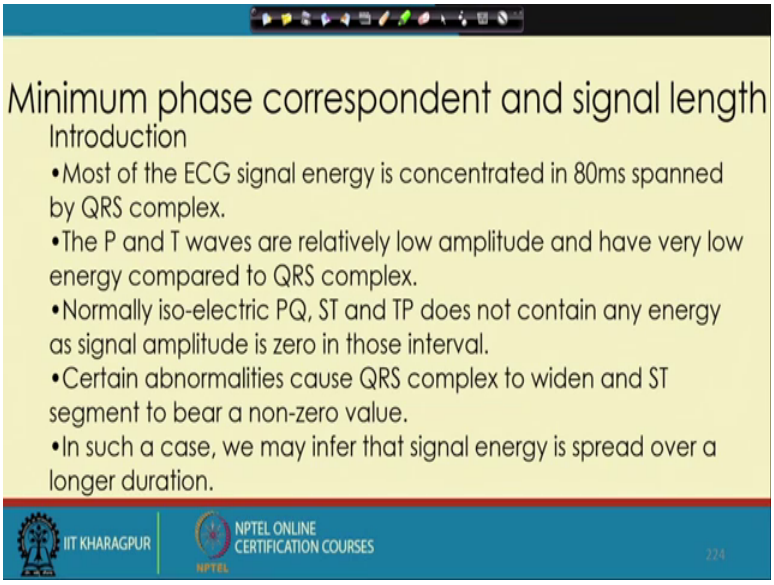

Now, let us start that let us see that how that the concept has evolved. For the ECG signal that the researchers have noticed that most of the energy is concentrated in the that small interval of the QRS complex and the span of it is just about 80 millisecond and if we look at the ECG signal that that from the P QRS and T that even if we take that all the waves that takes a very small part in the overall time cycle. In fact, that most of the part it would be no electrical activity; that means, after the T where the next P wave it comes after a long interval and again in quick succession we get QRS and T and again a gap, so in that way it comes.

So, if we look at that signal what we can tell that most of the energy of the ECG signal is just located in the QRS complex small interval of that. And P and T they are actually low amplitude and thereby much lower energy. So, they do not give actually much input there and then that other segments like PQ, ST and TP it does not contain any energy at all. In fact, if someone is interested in to capture the high frequency noise actually those are the places that you can look at that where there is no ECG signal is present. So, whatever we are acquiring that is actually interfering noise. So, those places does not contribute any energy.

Now, if there are certain abnormalities like ectopic beat or ischemia all such things we have seen that causes the QRS complex to widen the QRS complex becomes more wide and ST segment it becomes nonzero either it gets elevated or depressed; that means, it is moving away from 0 and having some amplitude. Now, such kind of changes we can look at in this way that the signal energy has spread earlier it was concentrated in small region of 80 millisecond of QRS complex. Now, QRS complex has become more wide and ST segment also is becoming nonzero that means the energy spread has become more. And that can be taken as a trademark signature of abnormality of the ECG waveform.

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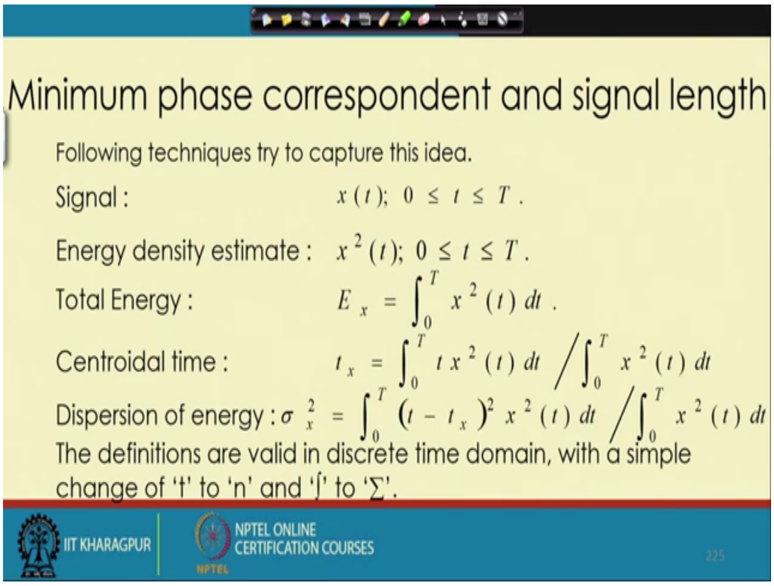

Now, first here we get that idea that how that can be exploited that. To capture these actually phenomena what we do we first mark that x t as the signal that it is taken for an interval 0 to t and instantaneous energy is taken by the square of it. So, the total energy we get by integrating that over that interval 0 to capital T and we can calculate the centroidal time or what we can call as a first moment. So, we can calculate that first moment and same way we can have the variance on the dispersion of energy that first moment is getting the centroid and with respect to the centroid (Refer Time: 07:24). What is the spread of the energy if it becomes more wide that this QRS complex is becoming more wide and ST segment also is changing its place space then that the dispersion of energy will actually show that the dispersion is more. So, that is the way we can capture actually that the change in the signal.

And this definition is also valid in the discrete domain the only thing. That will the changes what we have to make that the T variable would be replaced by the index variable n and the integration would be replaced by the summation operator. Otherwise the concept actually remains intact.

So, now, let us see that how we can proceed.

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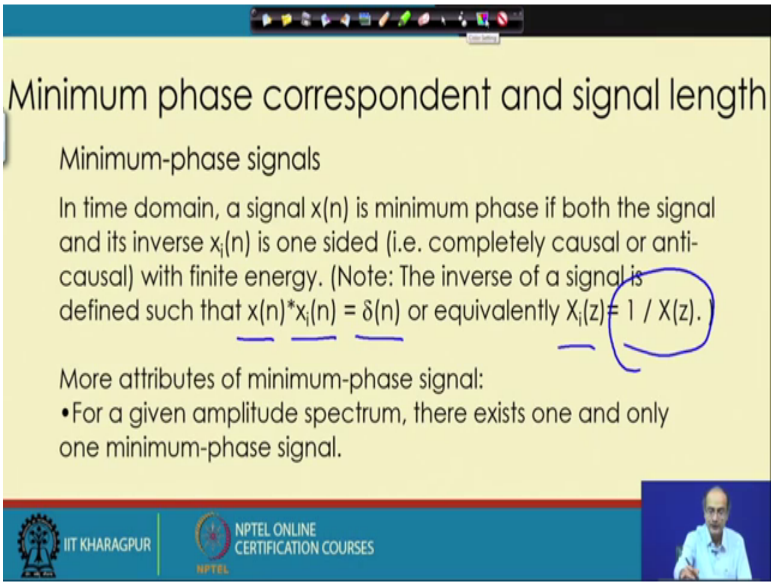

So, for that first we have to know the concept of the minimum phase signal. This is an important concept. So, before getting into the mathematics let us try to understand that what it means.

All of us we are aware about that the z domain a transfer function we can take into the z domain. And if we draw the unit circle we know for a stable seed system. That all the poles and the 0s they should be within the unit circle.

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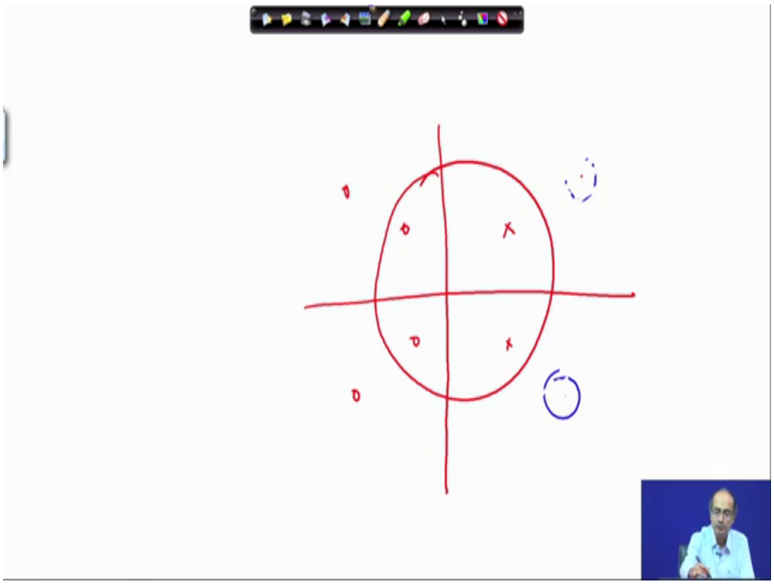

If the poles are going outside then it becomes unstable. Now, even if the poles are not outside if the 0s are going outside then it does not actually remain minimum phase. So, long all the poles and 0s they are inside it is not only stable it is the minimum phase signal and if it is outside that the specially the poles are outside, then we can say that it is becoming unstable and even if there is no such pole is there that we raise these poles even if we have some 0s outside they would create the problem that we would get actually that non minimum phase signal. Non minimum phase signal means what will happen the signal energy would be having more spread. So, that is the simple way to we can look at the thing. Now, let us look at the description to get more details about it.

That first let us look at that the description in the time domain that x n the signal is minimum phase if both the signal and it inverse x i is one sided that is completely causal or anti causal. We know for a causal signal the signal is 0 before the starting point; that means, for a causal signal x n it should be that for n is negative the x n should be 0. So, for a completely causal signal it will have only positive values for n greater than 0. And if it is anti causal then just the opposite thing will happen for n greater than 0 the value would be 0 it is only having that nonzero amplitude when in less than 0. And in that kind of finite energy signal that inverse of a signal is defined in this way that if we convolve x n with its inverse x i n we get actually an impulse response or in the z domain what we can tell that the inverse that signal its transfer function is the inverse of the that z domain representation of x n that is X z.

So, in that case what will happen? If we have some the poles inside they will go outside and if the 0s were also inside they will also go outside because we are taking the inverse and there are however, more actually attributes of the that our this minimum phase signal, a lot of qualities it has. For given amplitude spectra they are exist one and only one minimum phase signal. In fact, that that from here what we can get that whatever may be the phase of the signal the amplitude pair spectra is the same and only the minimum phase one that, that is a single minimum phase signal can be there corresponding to that spectra and you can actually that the changing the phase what that effect it brings you can do with a small experiment. What you can do?

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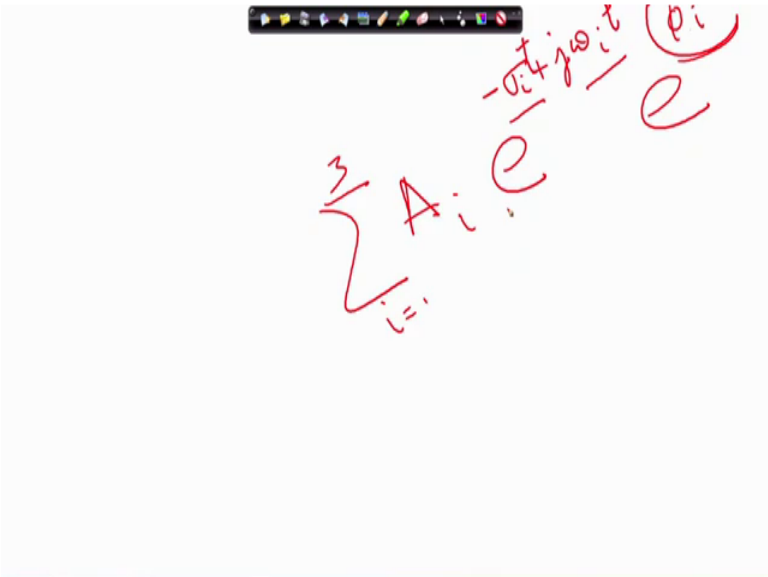

That you can take some complex exponential signal sum of say several complex exponential signal say A i e to the power sigma i plus j omega i. We can write minus to tell that it is damped exponential. So, that they become stable over i. I could be say starting from 1 to 2 or 3 in such kind of signal we can get some spectra and you can take the plot of the time domain signal and here we should have a j omega i think t sigma i yeah t would be there t or n whatever.

And there would be another part that would be phi j or rather it should have the same index. So, it should be phi i. So, this phi i is giving the phase. Now, if we take the signal keep the value of this sigma i omega i they are constant, but change the phi i. So, by changing this phi i you will see that it does not change the spectra you can take the

Fourier transform and look at the amplitude spectra, amplitude spectra does not change. But the phase spectra will change because of phi i and another interesting thing you will notice when we look at a time domain signal, they look so different from each other that it becomes very difficult to actually comprehend that they are having the same constituent frequencies. So, that could be a very interesting exercise to look at to get convinced that what is the effect of phase in the shape and the look of the signal.

Now, let us look at more properties of the, this minimum phase signal.

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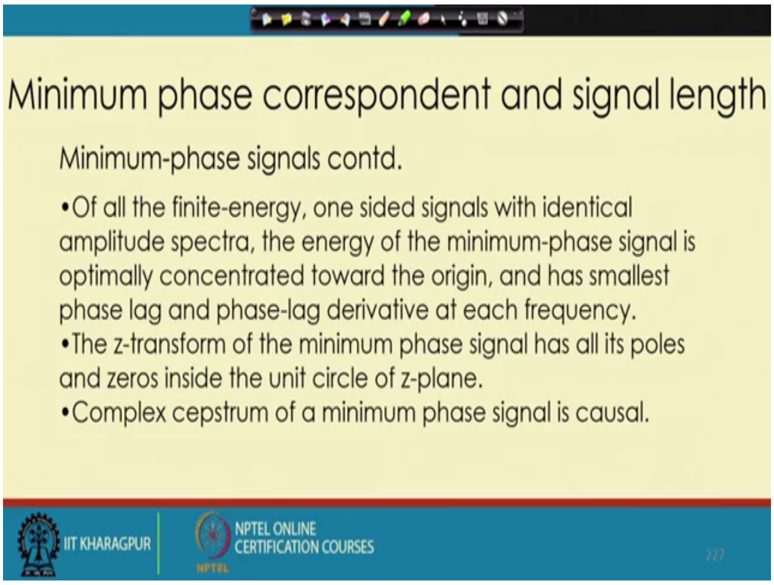

Of all the finite energy one sided signals with identical amplitude spectra the energy of the minimum phase signal is optimally concentrated towards the origin and has smallest phase lag and phase lag derivative at each frequency. So, what all it is suggesting that we know for finite energy signals that we can have that different phases of the signal; that means, it may be minimum phase it may be maximum phase, it may be mixed phase kind of thing; that means, if all the poles and 0s they are inside they are minimum phase if all the poles and 0s they are outside that is called maximum phase and if it is in between its called actually mixed phase kind of signal.

So, in those cases all of them they have the same amplitude spectra and for the minimum phase signal what we get the energy of the signal is concentrated towards the origin or the starting point that is 0. Now, when we look at the z domain for such signals the minimum phase signal has all its poles and 0s inside the unit circle and the complex cepstrum of the minimum phase signal is also causal. That is another property that for minimum phase signal the cepstrum complex cepstrum is causal. Later we will make use of this property.

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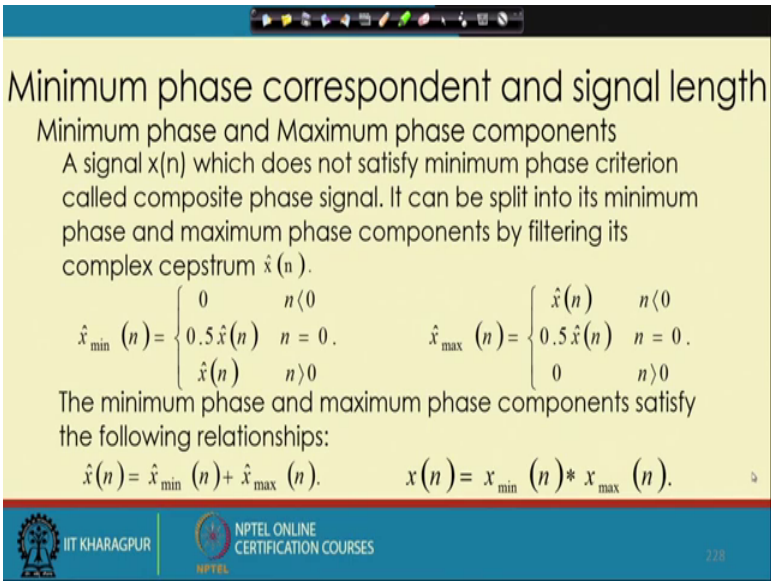

So, now, we try to get the thing clear we started with a signal x n which does not satisfy the minimum phase criteria. So, if it is not a minimum phase signal the most general form would could be. That it could be maximum phase or the mixed phase kind of thing. So, we call it as a composite phase signal, that is a safe enough definition that it is composite maybe some part it is minimum some part it is not.

Now, in such a signal we can split into two parts one part is the minimum phase one and other part is a maximum phase component. And for that we need to do filtering in the complex cepstrum of excited. How we can do that? We have already told if it is minimum phase it should be positive what we have not told that if it is maximum phase then it would be only negative. So, now, taking the origin as that separating point we can separate these two parts very easily. So, we are going to do that now.

That we take now, that signal at the cepstral domain that or the complex cepstrum that accept n that how that is a complex cepstrum what we have computed, out of that we are taking the part which is in the positive part. So, we call that as a minimum phase signal or minimum phase component this part and for that the signal energy at n equal to 0 which is at that juncture we are taking half of it with the minimum phase component and same way the part which is in the negative side and 50 percent of the energy at n equal to 0 we take separately and we call that that gives us the component which is corresponds to the minimum phase component. This is the cepstral part corresponding to the sorry this is the part for maximum phase component this is for the minimum phase component. So, we separate the two parts of the cepstrum of the signal x n and the cepstrum was represented by x hat n. We already know how to compute the cepstrum in the previous session. So, we make use of that knowledge and we proceed.

Here that what we get that they satisfy the following relation that we get the minimum phase component and the maximum phase component, these two if we add them together again we can get back the complex cepstrum of the signal x. So, that is another relationship we find here in this case. And if we look at the time domain signal if you go back we see that the corresponding parts that from the x hat mean we can get x mean, in the time domain and from x hat max we can get x max that is the maximum phase component and the previous one is the minimum phase component if we convolve them we get actually the original signal observed signal x n. So, this is the relationship among these components and we can make use of them for our processing.

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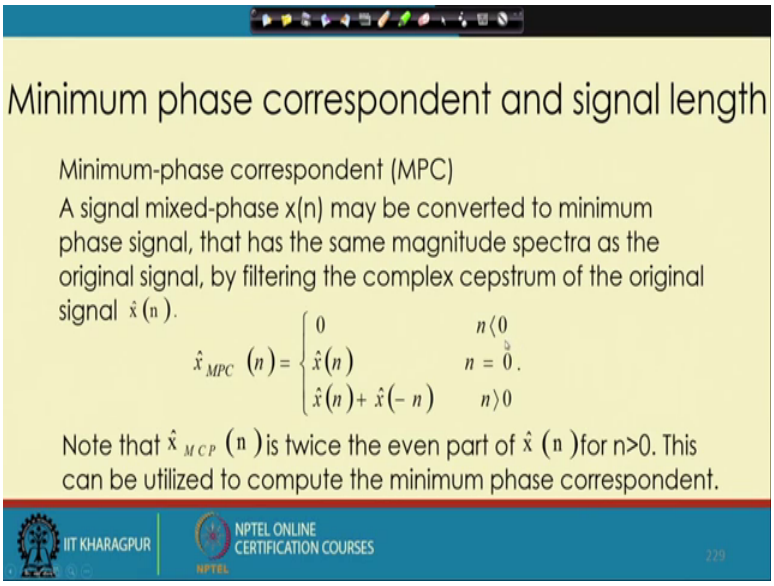

Now, let us look at the minimum phase component. A signal is mixed first may be converted to the max minimum phase signal that has same magnitude spectra as the original signal by filtering the complex cepstrum of the original signal x hat n. So, starting with that signal which is composite phase we not only can separate the two parts that minimum phase component and maximum phase component we can convert it to the minimum phase component altogether which preserves the amplitude spectra of the signal. However, that we are actually making the part that in the way that it gives us only the minimum phase part is preserved.

And to do that what we have done, we have taken the negative part of the cepstrum and flipped it back and added to the positive axis. So, this is the new formula with the help of that we are getting the exact minimum phase counterpart M C P or minimum phase correspondent signal. So, here that we force the negative index to 0 and we preserve the value of x at n at the 0 th index and for the positive value of n that is in that positive part of n, what we can do, we take x at n plus x at n minus 1. So, the values what we had in the negative part of the original cepstrum we are flipping it back and adding it to the positive part and x hat at M C P is actually providing that twice the even part of the signal for n greater than 0.

In fact, the signal we can have the even part and odd part even component and the odd component this one is giving actually twice the even component of the signal for n greater than 0. So, that is another observation we have made here and let us see that how we make use of it.

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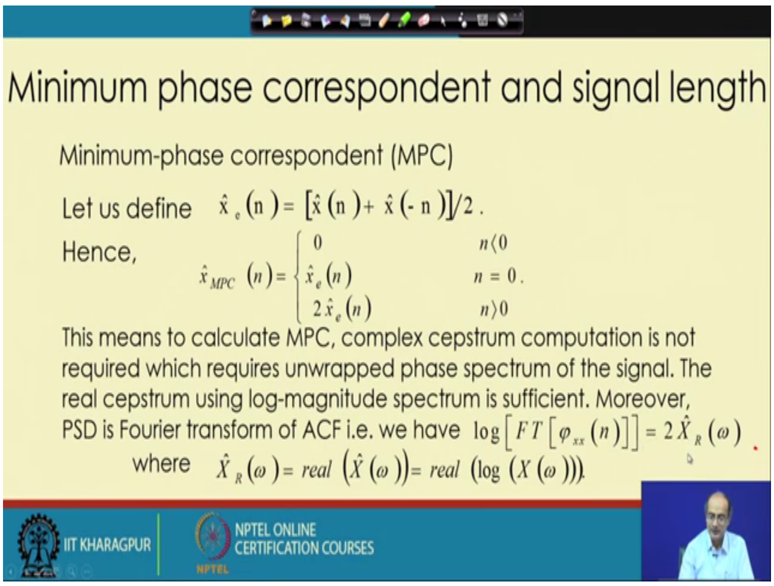

Let us define the even part of the signal or to be more specific of the complex cepstrum x at n is x at e n. So, that is nothing, but the sum of the negative part and the positive part and we take the divided by two; that means, we take the average of that and even signal that it will we see the reflection across the point 0 will get the same value in both the sides. So, that is the even signal we know.

So, for that what we can get that as our new definition of that minimum phase correspondent matters with that what we get that for that M C P we can write in terms of the that even component of the complex cepstrum. So, which can be written as M C P can be written as 0 for index n is negative. For n equal to 0 it is same as the even part of the complex cepstrum and for the positive part it is twice the value of that even part of the complex cepstrum. So, we get another way of expressing that cepstrum of the complex cepstrum of the minimum phase correspondent.

Now, what this means the complex cepstrum computation does not require unwrapped phase cepstrum of the signal. The real cepstrum using the log magnitude spectrum is sufficient because for that case that if it is that for real signal we know it is real and we need not have to bother for the phase. So, what we can do that we need not have to do the unwrapping of the phase and moreover for PSD of the Fourier transform of ACF we get actually that if we take the log of the Fourier transform of the ACF we get the that twice the real component of it and there that way we can make use of the fact to find out actually that that even spectra and let us see that we can how we can compute that even spectra.

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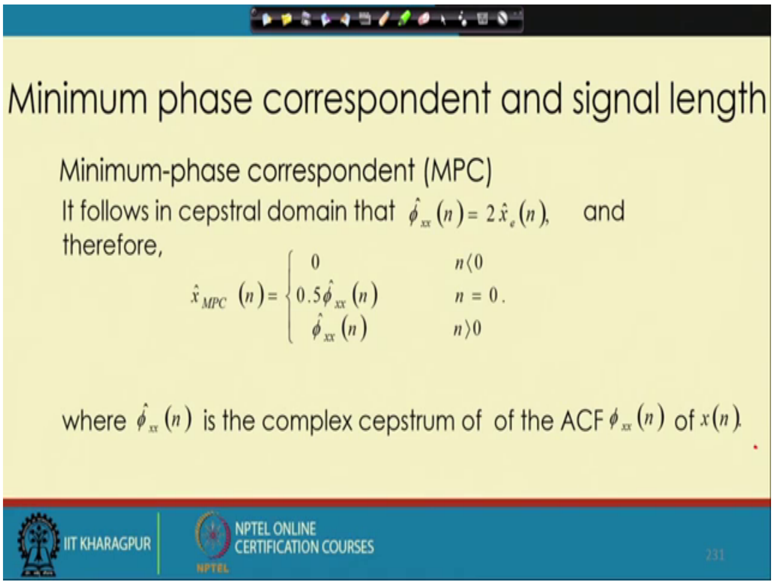

We can compute that with respect to that ACF or the complex cepstra the twice the even spectra is same as that our, that complex spectra cepstrum of that autocorrelation coefficient and using that we can compute that the complex cepstrum of the minimum phase correspondent with the help of the complex cepstrum of the ACF. So, we get a new way to find out the minimum phase correspondent of the signal.

Later will get that why we are interested to get the minimum phase signal, but before that we would like to take one more point that we told you here that every time we are mentioning that we need not have to compute the phase unwrapping with lot of joy we are expressing that. Now, what actually makes so much difference to find out the phase, it is a very simple operation.

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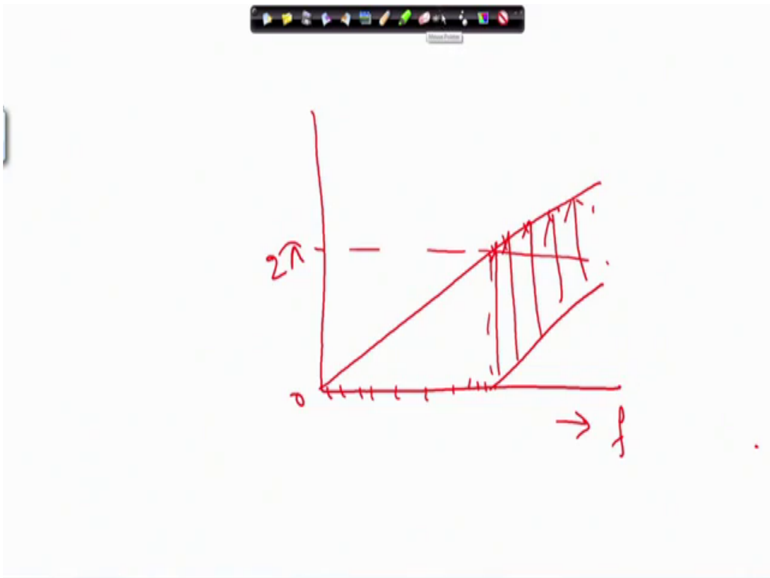

Say the phase is changing linearly and that if this is the frequency with respect to that that different component the phase is changing and if it say goes above 2 pi. So, it is getting truncated and it is starting from here a discontinuity is coming into play. Now, it is not a very difficult operation to do, but the fact is that why we detest this part that this is a sequential operation. When we are doing the other part of the operation or any other operation we look for the opportunity of parallel processing and we know that Fourier transform we can have parallel implementation of it. So, there are many efficient implementation and however, the phase unwrapping is actually spoiling the game.

Here for each frequency component we have to look separately and consecutively. If a phase change is happening here that we need to put it back here we need to have that same effect for all the following components. So, I cannot do the test here and this side paralally and to start with I do not know where that change is happening. So, I need to start from the beginning go to the end in a sequential manner and there is no scope of parallel processing. So, that is actually the sole the problem the people are concerned about it and do not like these kind of actually the work that which makes it a the problematic one and that is why we keep on mentioning again and again and we have with a actually relief that if we need not have to compute the that phase unwrapping part, because there we can go for the parallel processing there is nothing to stop actually from using it. However, the moment of phase unwrapping will come it will force us to stop all the operation and goes in a sequential manner.

So, with that we stop today and in the next session we will look at the applications of the minimum phase signal and understand that how it helps us to change the, change in the shape of the waveform.

Thank you.