Biomedical Signal Processing
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Lecture – 25
Homomorphic Processing

So, today we will go for the homomorphic processing. Before that we would like to actually draw your attention to a phenomena, I hope all of you have tried to take some photo of yourself of or for your friend. Now if you try to take the photo within the room, and say that it is a good day; that enough light is there outside. So, instead of depending on the artificial lights; that if you depend more on the light coming out actually coming inside through the windows. If you take the images, you will find that when you take the image that the friend who is close closest to the window; that after taking the image received that that person has the brightest complexion.

Now, if you doubt the fact, then you can actually change the position that and you can that put another friend closer to the window and take that picture group photo you will see; that again the same phenomena, that if a person moves near to the window and that it is actually that the complexion improves or becomes lighter. Now how that happens is that that as the light is coming, the diffused light specially, not the direct sunlight that from the window. We get actually more intensity close to the window and as we move further that the intensity of the light actually decays, and thereby due to that non-uniform illumination, any object that is nearer to the window looks brighter.

Now, if we take that as an example what we find that it is little different from the other kind of noises, that was present in the discussion. The noises what we have dealt with so far, they were additive noise any interfering signal we told that is noise and they are actually that corrupting the signal of interest, and they are actually corrupting in an additive way. But in this case that if we look at the intensity of the light coming from the window. That is actually getting multiplied with the that the object intensity. In fact, that is getting reflected on the object and that is what we capture through our eye or through camera.

So, here the noise we can say it is multiplicative in nature. So, such kind of case; that is present in the nature and unfortunately the previous techniques what we have learned, none of them they are effective there. But we have seen that the linear filtering it has a lot of actually scope and lot of development. So, we are always tempted to make use of the tools they are developed for the linear filtering. Or we can say that in other words that what knowledge we have learnt we want to make use of them one more time. And from there actually the concept of homo morphing the processing has come into play.

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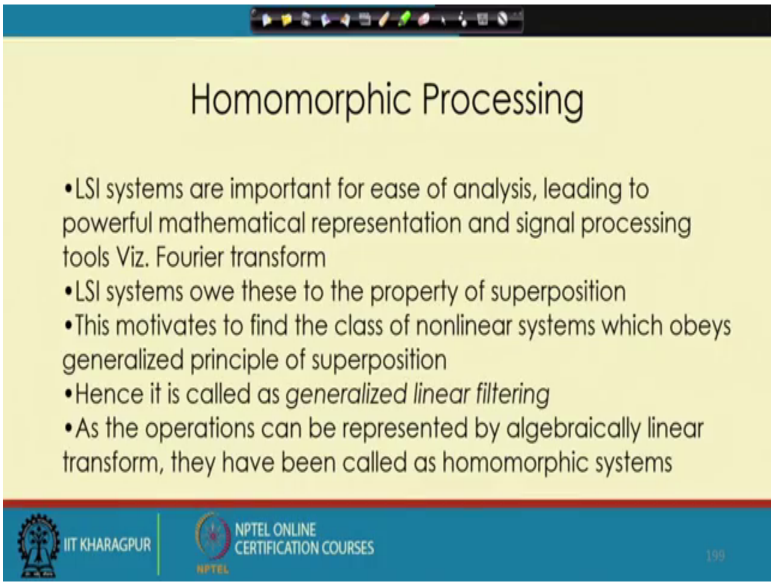

So, here that we get that first that that linear that system, linear space invariant system LSI system. They are important for ease of analysis, and we have a very strong mathematical background, and a lot of tools are developed out of them. That which falls into this category linear shift invariant system the for example, Fourier transform we have lots of filters. So, they are all actually getting to these categories. And if we dip actually deeply look into it that; why that linear shape invariant system is so effective or attractive. The reason is what we find that it is making use of the theorem of superposition. So, that is actually is a cause of the popularity that makes a life much more simple.

Now, these actually motivates us to find out a way that if we can make use of similar actually system, which can be used for the non-linear actually systems or non-linear signals, and for that we are ready to extend this concept of superposition, and a name is given generalized principle of superposition for that. And under these actually that generalized principle of superposition, whatever the tools we use we call them as generalized linear filtering. They are little different from that the previous filters what we have designed. And for that actually that we need to have some algebraic mapping or algebraic transform which are called as homomorphic transform, and overall, we call it is a homomorphic system.

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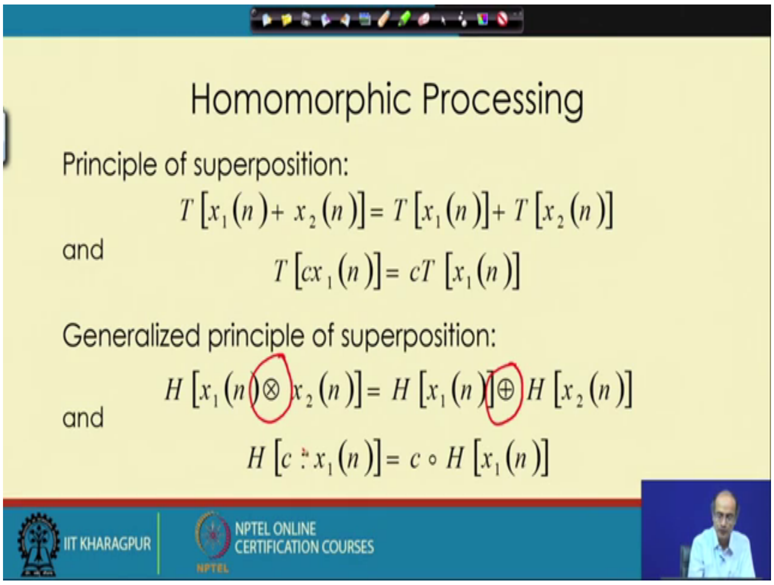

So now we look into it more precisely. First, we look at principle of superposition, that what it suggests that if we have 2 constituent signals x 1 and x 2 and we have a operator that is addition. If we add them then we get that a resultant signal x 1 plus x 2 and after the transform, that what value we will get out of it that the mapping of x 1 plus x 2 after the transform T, it would be same as if we transform the that x 1 and x 2 first and then add them up. So, that is one principle.

Next is that we have that principle of that where that the scaling is defined; that if we have a scalar c which is multiplied with the that the input x 1, and then transform the resultant value. It would be same as if we take the transform first and scale it up by the scalar c. So, these 2 together we call as the principle, principle of superposition, these 2 rules. And they are the cornerstone of the linear system.

So now let us look at generalized principle of superposition. In this case that the same kind of rule is defined, when we take the 2 inputs x 1 and x 2, the only change in this case the generalization, that in the input domain we have a algebraic operator cross it was defined. Now after the transformation with the map H homomorphic transform what we are calling that we get into another domain and there the operator changes we get a plus operator.

But the structurally they will remains the same. If we apply the operator in the input space or the domain of definition of H, that in the same way that like the principle of superposition we get another operator in the range space, with the help of that we can actually add the 2 that the maps of x 1 and x 2. And we can get the same result. Same way for the scalar also, that we have something defined that first we take that input space that we have something colon that is an operator, which is defined for the that one scalar and once a vector x 1. And after the transform that a new operator comes in it is place; that is, in between the scalar and the vector in the range space, that is that the mapping of x 1 after the transform, that we get the same relationship exists there.

The only thing the operators are different in the 2 spaces, but structurally it is same as the principle of superposition. So, that is why we can call it as a that generalized principle of superposition. More so because now that the previous principle of superposition just becomes a special case of it; where there is the same actually operators are used in both the space. That is the domain of definition and the range space. So, that is the difference we get between the principle of superposition and the generalized principle of superposition.

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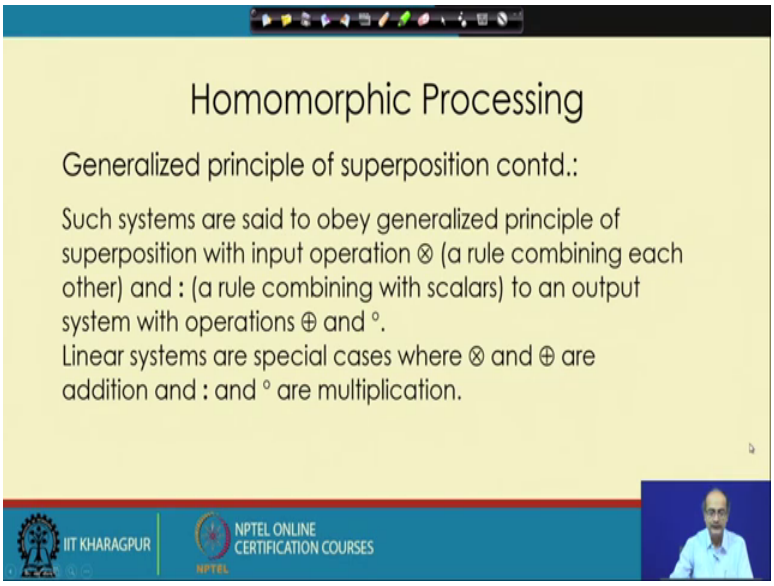

Now, let us proceed and see that what we get next. That here what we can tell that such systems are said to obey generalized principle of superposition with input operator a rule combining each other each other; that means, it is a binary operator, that cross and the colon is a rule combining with scalars. That and to an output system with operator plus and dot for the case of that the scalar and that the vector that output space. And linear systems is nothing but a very special case of this general generalized that system, generalized linear system we can say.

So, what we get first, that what all we have done it is nothing but a very special case of a generalized linear system. And now we are going to expand it for the generalized linear system.

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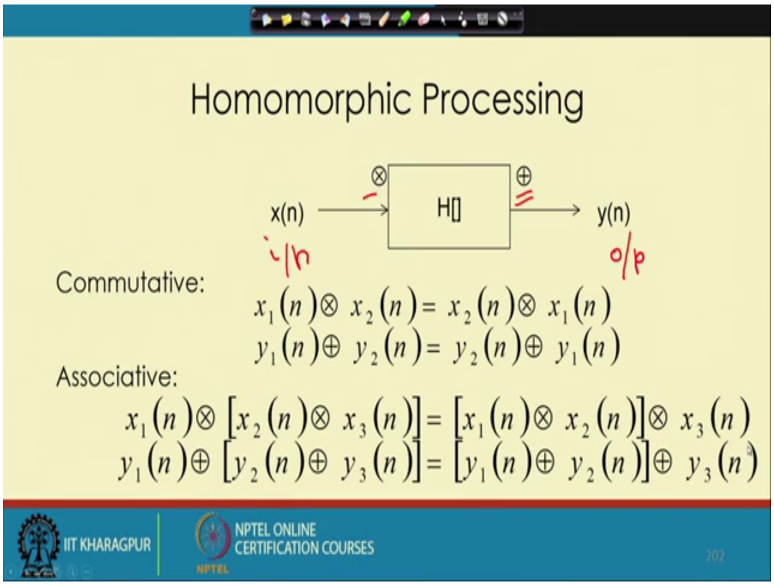

So, here we are showing a diagram about the mapping edge. Now in this mapping the point to note that x is the input this is the input and y is the output. And these 2 spaces are different which are actually having a we have a map defined from x to y in the input space. That is the domain of H. We have a cross operator. The equivalent to it in the output space is the plus operator.

So, the relations what we had that for the previous over that in the input space, that structure would be preserved, but the operator will change. So now, this case it also actually these operators they have some more properties. First is that it is cumulative; that means, sorry commutative. Commutative means the order of operation can be changed. And that in real life if you look at, many things are not commutative. If we look at operations say every day to day tasks that operations, there are lot of them they are not commutative in nature.

For example, we wear the socks before wearing the shoes. Now if we take them as 2 operators that wearing the socks and wearing the shoes there are 2 operators or 2 actually items, they have they are following each other. Can we reverse the order can we wear the shoe first and then go for the socks? Probably we cannot go for that. So, what we see that in real life or all that things, what we do, everything is not commutative. They there the order is very important order of operation, but here what we get that some of the operations are there where this order is not important.

For example, when we multiply the 2 numbers. Whatever order we take, 3 into 5 or 5 into 3 gives the same result. If we add 2 numbers, a plus b we can not we change that order b plus a we get the same result. So, here these operators that cross and the plus, both are commutative. That is one of the property they need to actually follow. Next is they have to be associative. Associative means, the order in which the operators are actually that used to combine the items; that order if it is changed. We do not have any change in the ultimate result. In the first example what we are showing that x 1 x 2 x 3, 3 members are there in the input space, and we have the operator cross so that combining them.

What we have done first, we have combined that x 1 and x 3, and then it is combined the resultant is combined with x 1, then that the same result will get if we first combine x 1 and x 2 and the resultant is combined with x 3 with that operator cross. And the same way goes for the range of the half that that map or transform H, and that is what we get in the next line that for y also; when we use that the add operator we get the same associative rule that is followed.

So, these operators have to be associative and commutative in nature. That is another thing we need to keep in mind.

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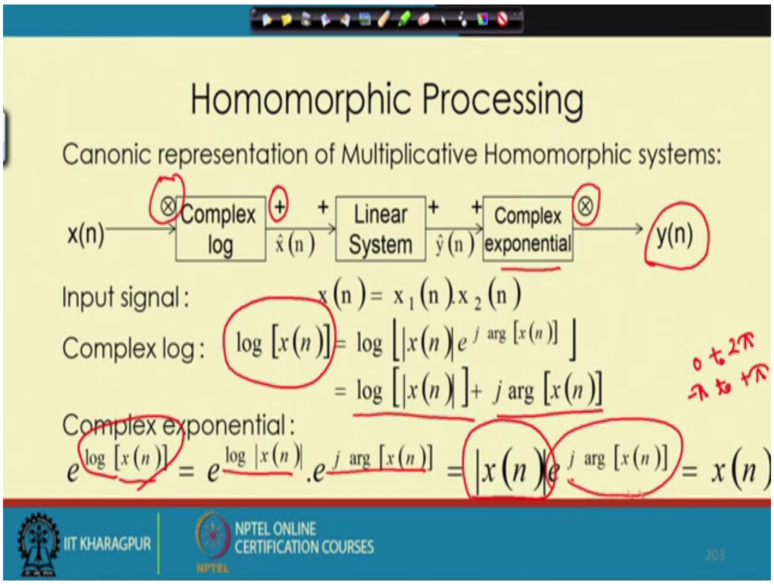

Next, we go for a system to build a generalized filter. Here the challenge is that the one we told earlier that. Sometimes we have multiplicative noise. So, here are also the same kind of case that, we have a multiplicative actually signal, the components are getting multiplied with each other, and we want to separate them out. So, what we do? The signal or we can say that the observed signal x which has multiple components in it, added in a or not added rather they are combined in a multiplicative way.

First what we do we pass them through the complex logarithm function. So, after taking the log, what happens that initially if there was an operator that is the multiplication here. That becomes an addition in the output space of the complex log. So, we get the signal x hat where the that the operator becomes addition, and now we can have the linear filtering and after the filtering we can get y hat, again we have to go back to the original domain. So, what we do we apply a complex exponential, and go back to the that original domain we get the answer is yn. And that again the operator cross comes into play. So, that is the way it works.

And let us now go step by step through it. First is let us look at the input signal, input signal xn or the observed signal, we can say it has multiple constituents x 1 and x 2 for simplicity we have just taken the 2 components, they are combined in a multiplicative way. Next is that we have that complex logarithm. First, we need to know that how to compute the complex logarithm, if we take a complex number and take the log that what we have to do first, we can express that complex number as that absolute value, and we can take that as a angle as associated with it, or what we can call that it is that radial actually the form we can express them.

And when we take the log what we get actually the log of the absolute value; first, that is the this term, and we get the angle as the actually that complex term. Or imaginary part of it is the angle and the real part of it is a log of the absolute value. So, that is the output of the complex log. Now here complex log makes one more thing that it brings one more challenge, and first let us point it out and then we get that how to fix that later. That when we take the log operation and find out the angle we need to define that what is the range of that angle. The value could be defined between 0 to 2 pi for example, or it could be from minus pi to plus pi. So, within an interval it is there; however, the real angle it could be actually outside this range.

But once we take it through the complex log, it will always mapped into that 0 to 2 pi that kind of actually domain. So, it goes through a modulo operation, and because of that we get some discontinuity in the signal now. For that we need to understand that what is actually these phase means. Phase means there is a lag when we are talking about some signal multiple components are there.

For example, when we talk about the Fourier transform of a signal we have multiple frequencies present, each one of them has some magnitude as well as the phase. What that phase means is that at the starting point, what was the phase difference between them each one of them say if we take that. Each one of them is a complex actually sinusoid they are not starting from the 0 phase. They have some initial phase, and usually what is found that the phases are changing continuously, and because of that there is no guarantee that it would be actually contained within minus pi to pi or 0 to 2 pi whatever range you may take.

So, either would be a continuous change in that and it can go beyond that; however, after the that complex logarithm, it would be mapped into one of these range depending on that how we are calculating that angle. So, that brings to a discontinuity. The previous the smoothness in the phase that is lost, so later we need to take care of that. So, next that we go for the complex exponential in case of complex exponential what we do? If we take the exponential of a that complex number, that is x n again, that what we need to do, here we have taken actually that part that what we have taken complex log we have taken the complex exponential of that.

So, that gives rise to that we can expect actually write that that complex that log into 2 parts that one is the absolute value another is a that is a real part, and the imaginary part. And with the help of that the first part what we get it is giving the absolute value mod x, and the next we are getting the angle part of it and together, it gives us the xn. So, we just show here that if we take the that the complex exponential after complex log, we will get back the same signal except for that the problem created by this addition of discontinuity.

Now, if we feel that that can create a problem that discontinuity in the phase, then what we need to do we need to do phase unwrapping. We will show that later when it actually would be required..

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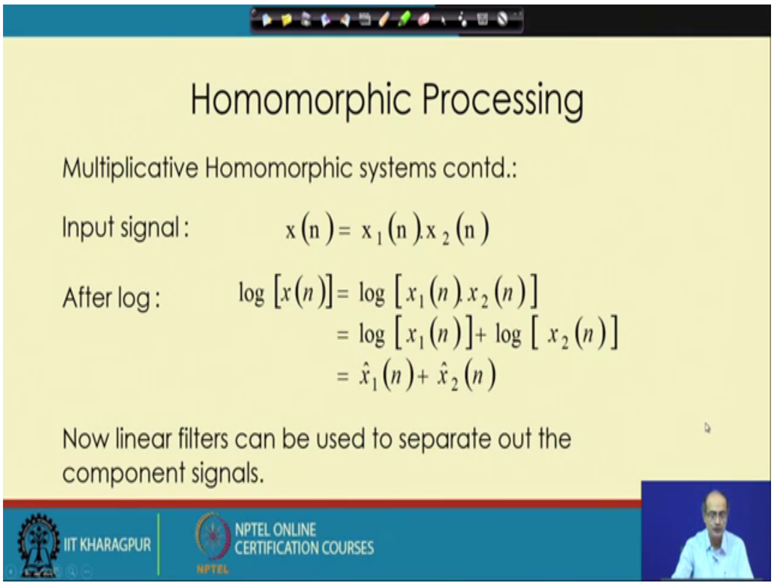

Now, let us look at that that part that we have the 2 components of the observed signal x. Which are x 1 and x 2, now if we take the logarithm, what happens for them. That as we take the logarithm of the signal xn, and we replace that with these 2 multiplicative constituents; x 1 and x 2, we can simply add they actually write them as additive terms after taking the log. So, which was actually that multiplicative term, previously now they become additive, and we get x 1 hat and x 1 x 2 hat in a complex notation we write them in that way.

Now, comes a question of filtering, and here it need to be mentioned clearly just like any other filter; that means, the linear filter that x 1 hat and x 2 hat need to have some say for a separation in some domain. Only then we would be able to separate them. For example: that we look for that filtering to separate out the high frequency noise or power frequency noise from the biomedical signal of interest. And there we need to first assume that the signal of interest, and that high frequency noise they are separated in the frequency domain.

So, here also x 1 hat and x 2 hat they need to be separated in that frequency domain or some other way they need to have some separation, for successful separation of them if that does not happen our attempt will not be successful. Fortunately, in many cases they are actually separated and we can separate one from the other. Once that is done that filtering is done, then we can actually look at that the next part of it that linear filtering first we use to separate this constituents. And then we go for that next part of the operation.

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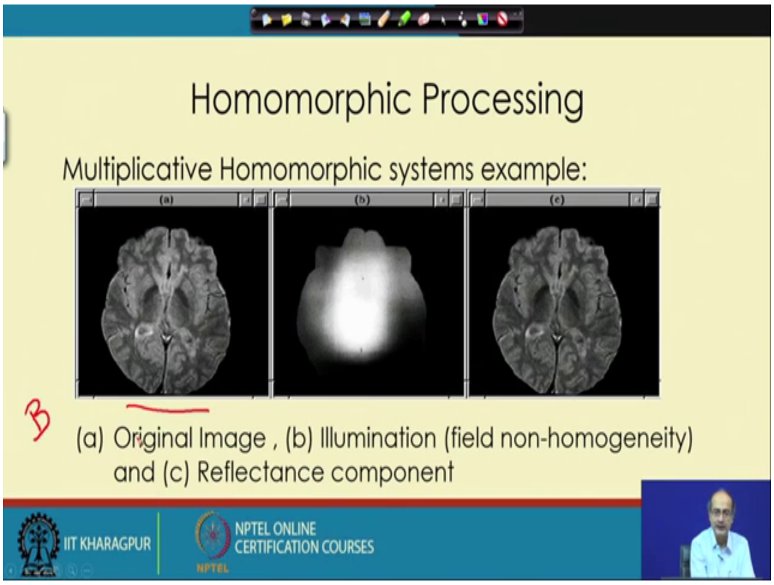

That is we take that the output back in the using the complex exponential.

Now, here we show one such example actually in 2D. Here we are showing some MRI image, in the left-hand side that we see one original MRI image, that the it is a image of a of the brain. And we know for the MRI image, one thing is very important that we have the strong magnetic intensity that actually helps to generate that MRI signal or that MR, or that image magnetic resonance that what helps to build up. And it need to b that magnetic intensity it need to be very strong, and usually that is made out of that superconducting magnets.

Now, it is very difficult task to keep that, the magnetic intensity uniform over the space. Space means that where that that gantry is there within the gantry that the we can say the cavity that should have that uniform intensity or that b magnetic that flask. Now because of the in homogeneity of that, if we look at the map of it we get that we have that here a huge in homogeneity in this case.

Now, because of these in homogeneity, there is an impact we get the inside part that is brighter and the outside, the part it would be darker. And now if we can use the homomorphic processing, we can separate them out..

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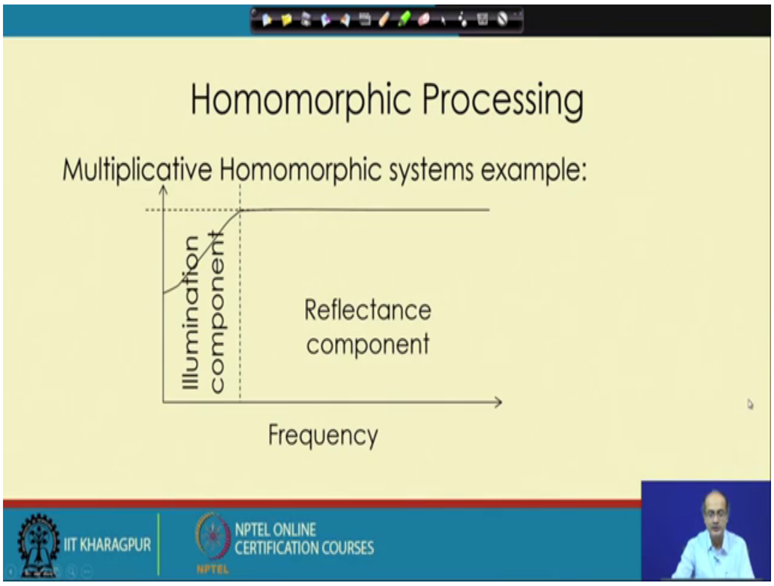

And how that actually is possible, because that in this case that what we see that the detailed component, that is due to the activity of the magnetic resonance. And the that flask or the magnetite magnetic intensity that magnetic field intensity rather that part we can say the illumination component. It is very slowly varying. Whereas, the part that different parts are having different amount of actually activity. That part is changing very quickly over the space.

So, that part the reflectance components is actually in the high frequency domain we can say, whereas the intensity is slowly varying. So, we can have simple filtering to separate them in the homomorphic domain. And thereby we can separate the low frequency part, and get only the reflectance component which talks about the anatomy of the gray matter or our brain.

So, with that actually we would take a break here, and we will come back again with homomorphic processing.

Thank you.