

**Biomedical Signal Processing**  
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**Lecture – 22**  
**Event Detection (Contd.)**

So, now we get that how we first detect some of the, that the predominant rhythms. For example, let us take the task of getting 1 rhythm, we may take that as an example the alpha rhythm actually we can take any one of them alpha, beta, theta, delta. So, same way we can get that by this technique and for that, that we will take the help of that cross correlation function.

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**EEG Rhythm detection**

EEG rhythm detection (e.g.  $\alpha$  rhythm)

*Cross correlation function defined by dot product*

$$\theta(k) = \underline{x}(n) \cdot \underline{y}(n+k) = \langle \underline{x}(n), \underline{y}(n+k) \rangle = \sum_{n=0}^{N-1} x(n)y(n+k)$$

*Cross - correlation coefficients*

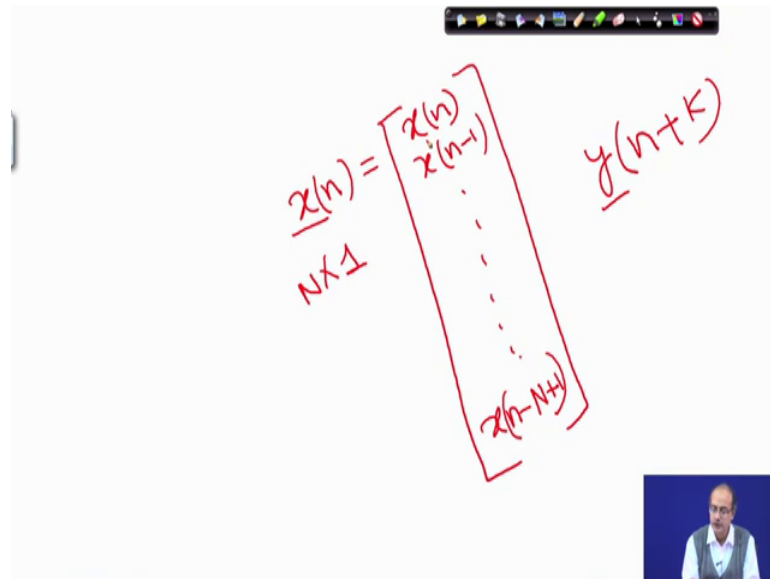
$$\gamma_{xy}(k) = \frac{\sum_{n=0}^{N-1} x(n)y(n+k)}{\left[ \sum_{n=0}^{N-1} x^2(n) \sum_{n=0}^{N-1} y^2(n+k) \right]^{1/2}}$$

$-1 \leq \gamma \leq +1$

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So, cross correlation function, this theta is nothing but the dot product of that  $x$  t or  $x$  n and  $y$  n these 2 vectors. So, there are different notations to write that, here we are just showing that thing .That what are the ways we can write that, we can use them with that the triangular brackets or we can write them as a dot product.

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The essence is, that if we look at that vector, that  $x(n)$   $N \times 1$  vector we are using that notation below we are giving a bar, so it is a vector of dimension  $N$  by  $1$ . So,  $x(n)$  is the latest sample of that and followed by couple of actually the previous signals going up to  $x(n - N + 1)$ . So, it has the  $n$  samples and for that we are computing that.

In the same way, actually we can write that other quantity  $y(n+k)$ . So, that is again a vector. So, that it will start that, the first element would be in  $y(n+k)$  and it will go up to  $y(n+k - N + 1)$ . So, that there is in simple way we can write that as a, that summation of the product of the 2 signals  $x(n)$  and  $y(n)$ . Now what is the essence of it, that  $x(n)$  is a, actually known signal. Say, we may take the help of  $\bar{x}$  to take out some portion of the signal which is known as alpha rhythm for sure.

So, that is, if that is the  $x(n)$ , then we take any other signal on which we can do the experiment and find out that whether that alpha rhythm is present or not. We can take the, this cross correlation. Why we tell it is cross because we are taking 2 actually signals one is the  $x$ , another is the  $y$ . So, otherwise if we actually apply it on the same signal or signal from the same patient and for the same set of leads, then we can tell that is auto correlation function and if actually that same kind of rhythm is they are then, what will happen we will get a match; that means, the 2 signals if they have good match this sum of product given by the cross correlation function it will have a high magnitude.

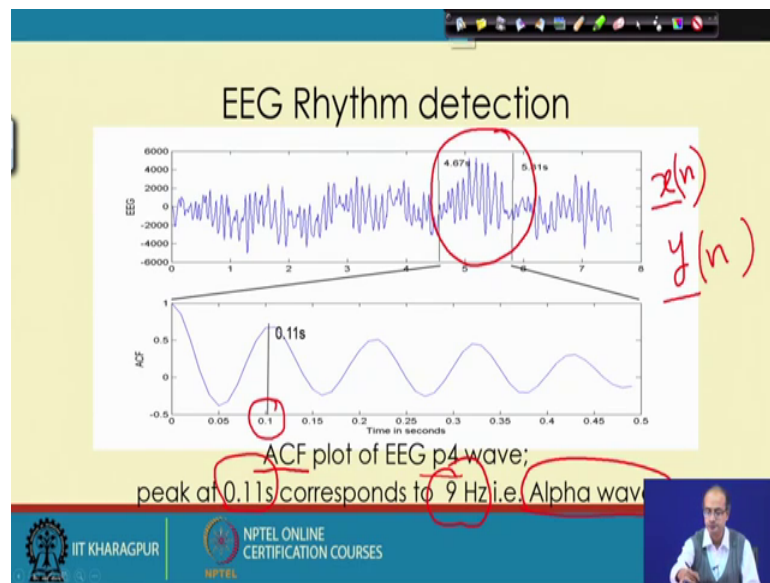
So, that will signify that these 2 signals  $x$  and  $y$  at this particular lag they have a good actually correlation and why we are taking actually different lags varying this  $k$ , the reason is we do not know actually where they will align with each other. So, that is why we need to try out with different lags and find it out, that where we are getting the maximum value of the cross correlation. However, this technique has a, that the challenge that the when we are talking about that  $\theta$  has high magnitude, everything is becoming subjective.

In the sense, if the signal strength is more that is of  $x$  or  $y$ , then that overall that peaks as well as those small values everything will change. So, to get rid of that kind of bias what we do? We divide it by the square root of the energy of the signal  $x$ , there is a square root of the energy of the signal that this is a sum of energy of the signal  $x$  and this is the estimate of the energy of the signal  $y$ .

So, we take the square root because both are energies and that takes care of the dimensionality, we need to take the square root for that reason and we get some quantity called cross correlation coefficient. Now, what we gain by this is that, this cross correlation coefficient this is actually varies in between minus 1 to plus 1. So, it will be within this range only. So, it get normalized.

So, if we have to take a quantitative approach, we want to write a program and decide that there is a match or not. This makes it simple to do that quantitative evaluation. Otherwise if we want to actually look at the signal and come to a decision then cross correlation itself is good enough, that cross correlation coefficients or normalization by the energy of the signals, that helps us to quantify it and take the decision based on this output of the cross correlation function.

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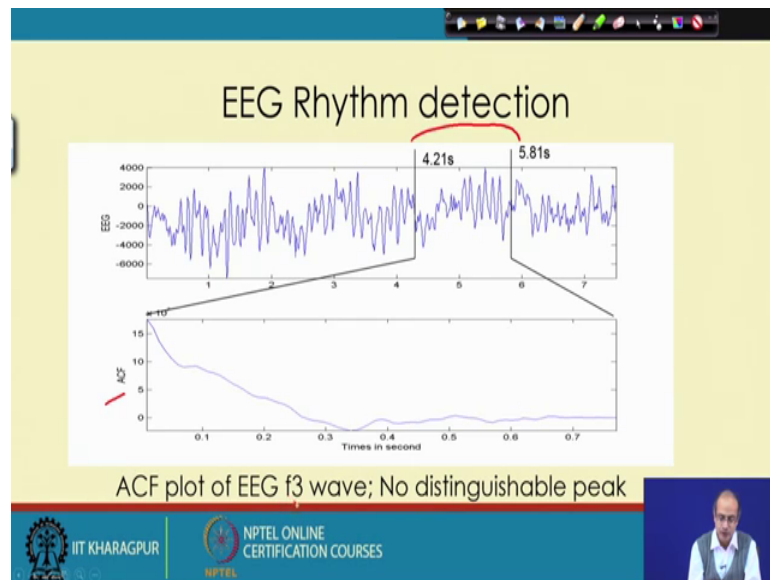
So, now, let us see some examples that how we make use of this fact. So, for this purpose, here we have taken a signal that is p 4 is the lead and some part of it, that 4.6725.81 once again, within this range that doctor has actually suggested that alpha wave is visible. Now we would like to know whether the alpha waves is actually present throughout this recording of 8 second or only in this portion alpha wave is there, because we are asking this question because we are not experts or we want to device an algorithm to have an objective estimate of it.

So, what we do, we take these part of the signal that we take as  $x(n)$  and the overall that EEG signal or p 4 that we take as  $y(n)$ . Now, because it is taken from the same signal we are calling it as a special case that is ACF. So, we compute the auto correlation function or ACF and we get that. ACF is having number of peaks, the first peak that is occurring at about 1.11 second. So, point 0.11second that if we take that the inverse of it, that gives us 9. That means, that 9 hertz signal is predominant here and as that is within the range of the alpha wave we call actually that alpha wave is present in the p 4 signal.

So, that is what, we could actually get out of this exercise and auto correlation function is a just a special case of the cross correlation function which is more generalized. In this case that because the same signal we have applied, we have taken the template and we are applying the correlation we call it auto correlation coefficient. So, we get some idea that

how this auto correlation coefficient can help us.

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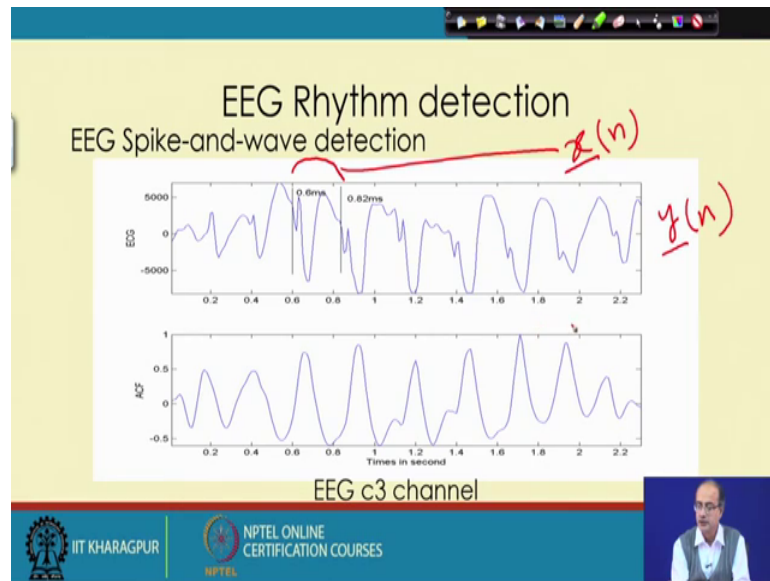


Now, let us take one more example. In this case, we are interested to know the presence of that and that alpha rhythm that within this region. This is the region we are looking at 4.21 second to 5.81 second and that we want to see that these kind of wave, whether it is actually present in other places also.

So, visually it looks similar kind of thing is there, but when we take the ACF plot, auto correlation plot just in the same way for this easy recording that is f3 wave. We do not find any peak unlike the previous one. In the previous case, that we could get a peak here, that at 0.11 second, but in this case we do not find any peak for the auto correlation function of the f3 signal, you see only the valley.

That means, we can say that, this signal whatever was present between 4.21 second to 5.81 second, it is not actually getting repeated. So, that is a way, that because no distinguishable peak is there, we can come to that conclusion that the signal is changing and we are unable to get the repetition of that particular rhythm present within this interval. So, that is the conclusion we draw.

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Now, let us look at more examples. Now we will go for the cross correlation. So here, now instead of going for that the predominant rhythms like alpha, here we are taking one such that long duration discrete event that is spike and wave detection. That is a particular kind of shape, we know this actually that waves are known by their shape. So, we have taken that and we have demarcated manually the boundary of it.

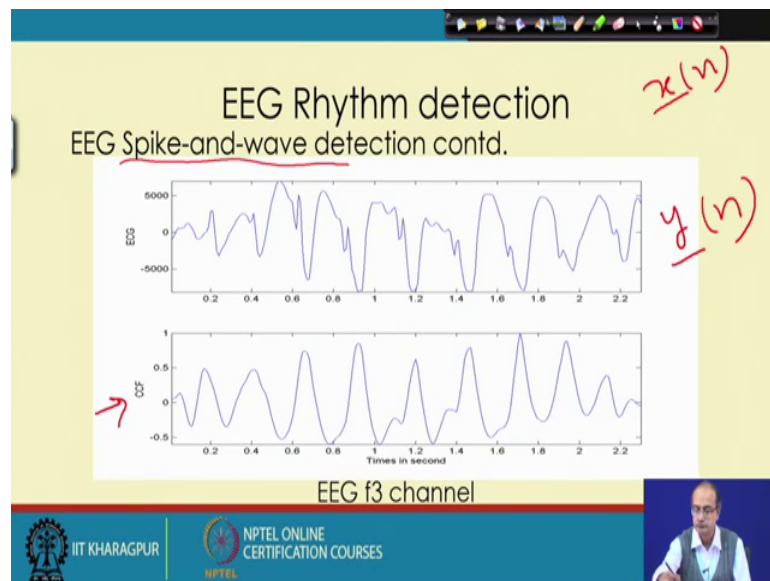
Now, while demarcating the boundary that we need to just keep in mind that spike and wave that as per the description it should contain 1 spike, following followed by that smooth wave low frequency wave. So, we should capture the both, if we take actually more than that it is fine, but it should at least capture 1 actual wave completely. So, that we can get good output in the auto correlation, so using that as a template. So, these part it is taken as our  $x_n$ , underscore that. Though that is giving us the, we can say 1 signal and the overall signal c 3 that is giving us that  $y_n$  that vector.

So, when we take the, that correlation at different lag we get distinct peaks and that suggests very clearly, that our spike and wave this wave form is repeated. In fact, visually also if we look at, we get that spike and wave is repeated in the that range say 0.42 say that 1.8 or 2 seconds within that range many a times it is getting repeated. So, now, we could get that easily and because we have taken auto correlation coefficient it is getting normalized between minus 1 to 1. So, we can quantify that also and can have a

quantitative decision based on these findings.

Now we take another example, where that we have changed that the signal, earlier we are looking at c 3 channel. Now we are going for and we have taken a template out of that. That from the x part we have taken a template from 0.6 millisecond to 0.82 millisecond.

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Now we go for another channel, that is f 3 channel and there also we see similar kind of pattern that spike and wave detection, but as we are not expert, we are not sure that whether we can call it that spike and wave is present or not. So, in this case we take the that cross correlation, why we call it cross correlation, that we have taken the x bar from the previous one that the x vector and y vector is the one, that we have here we are getting from the f 3 channel. So, the cross correlation coefficients is given below and we see that cross correlation function has the same nature.

It has very prominent peaks and many of them are more than 0.05. Now, one may ask the question that, why the value is not going to 1? The reason is that if we look at the spike and wave each of these they are not exactly same or matching with each other, they have a similarity. They are close enough to be described as spike and wave, but they are not 100 percent matching with each other.

So, that is why, the value is less than 1 because we are not getting 100 percent match, but they have very strong match and that is why we are getting the cross correlation coefficient is more than 0.5 at many places. So, that gives us the faith, that in f 3 channel also that we have the presence of spike and wave that signal. So, we get some that use of it.

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**EEG Rhythm detection**

Cross-spectral techniques

The PSD  $S_{xx}(f)$  of a signal is related to ACF via Fourier Transform:


$$S_{xx}(f) = FT[\phi_{xx}(\tau)] = X(f)X^*(f) = |X(f)|^2$$


- PSD shows peaks at the frequency corresponding to periodic activity.
- PSD shows activity over a band.


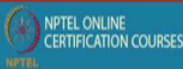
The CSD  $S_{xy}(f)$  of a signal is related to CCF via Fourier Transform:

$$S_{xy}(f) = FT[\phi_{xy}(\tau)] = X(f)Y^*(f)$$

CSD shows peaks at frequencies that are present in both the signals being compared.





Now let us see some other technique. So, here we go for another technique that is called cross spectral technique. So, so far we were working in the time domain, now we are switching to the frequency domain and in the frequency domain that we expect there should be some change, some advantages would be there. So, now, to correlate that thing first what we notice that, the what we call spectrum or power spectral density, we write that as,  $s_{xx}$  and  $f$  is a frequency. It is nothing but that the Fourier transform of the auto correlation function of the signal.

Now for that, this is the formula that is given. that we can take the Fourier transform of the auto correlation function or simply what we can do, we can compute the Fourier transform of the signal and we can take actually the dot product of the that Fourier transform and the conjugate of Fourier transform or in other word we are looking at the that amplitude spectrum or the square of the amplitude at every frequency, because we want to know that how much energy is there for a particular frequency and for a



continuous spectra that PSD is the best way to get that estimate.

So, here that the PSD if there is any periodic activity, that the PSD would give a peak at that frequency and if there is a band over which the activities is there that will also be visible. That means, a part of the description if you look at that the predominant waves we talk about, that alpha, beta, theta, delta all of them they are having that energy, maximum energy present in a particular band. So, that there we are more interested not a particular frequency or, but the distribution of the energy within a band and that part we can also get directly from the PSD.

So, we get PSD is it natural match actually for that what we are doing and very similar to that power spectral density something we have, that is called cross spectral density or CSD and it is represented by a very similar, that name is  $x y f$ , where  $x y f$  is nothing but the Fourier transform of the cross correlation and you can also get it by taking the product of the Fourier transform of  $f$  and that the Fourier conjugate of the Fourier transform of  $y$ .

So, that is the way that CSD can be computed and here the interpretation remains to be very similar to that what we are doing, that interpretation is, that if we are having 2 signals they are matching at a particular frequency or a band of frequency then we will find peaks in that cross spectral density. If there is no match will get very low value.

So, that is the general concept of it and now, we get actually 2 techniques 1 is time domain technique another is a frequency domain technique. For the time domain technique, we had the cross correlation coefficient and in the frequency domain we have the cross spectral density and both of them they are actually Fourier transform pairs. Now let us take some example and let us see, that how they are actually match with each other or what advantage we get, if we replace 1 technique by the other.

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EEG Rhythm detection

Cross-spectral techniques contd.  
The normalized coherence spectrum is given by:

$$\Gamma_{xy}(f) = \left[ \frac{|S_{xy}(f)|^2}{S_{xx}(f)S_{yy}(f)} \right]^{1/2}$$

The phase of coherence spectrum is given by:  $\varphi_{xy}(f) = \angle S_{xy}(f)$

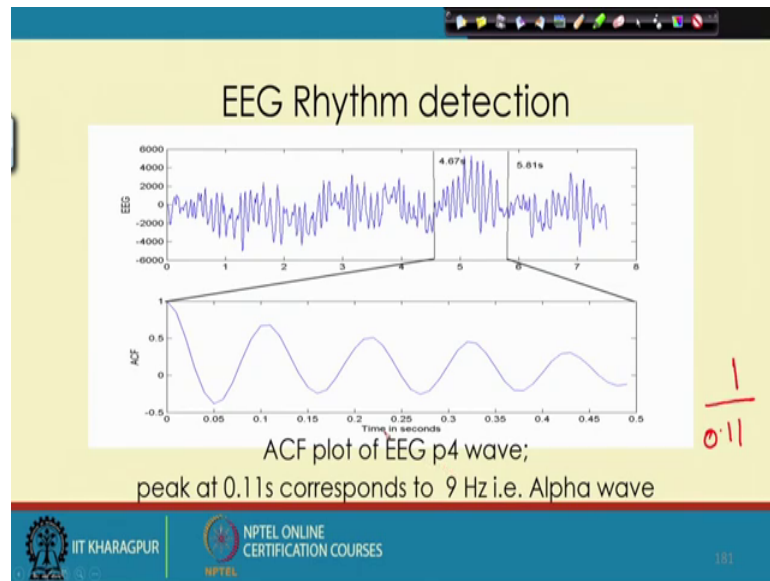
This represents the average phase difference (related to time delay) between common frequency components in two signals.

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So, now before getting into that examples, we have a term that is called that normalized coherent spectra. So, earlier we have that auto correlation, that ACF, that auto correlation function and cross correlation function. Same way that here we are normalizing that signal with the help of that the energy of the signal of x and y and the motivation is the same. That it should be within a band rather than getting affected by the scaling or the amplitude of the signal.

So, that same way we can look at the phase of coherent spectrum, that is given by the phase of  $S_{xy}$  and it talks about that average phase difference between the 2 signals x and y. However, this phase part of it, have actually found less application in use. So, we will primarily concentrate in the normalized coherent spectra or the cross spectra of the signal.

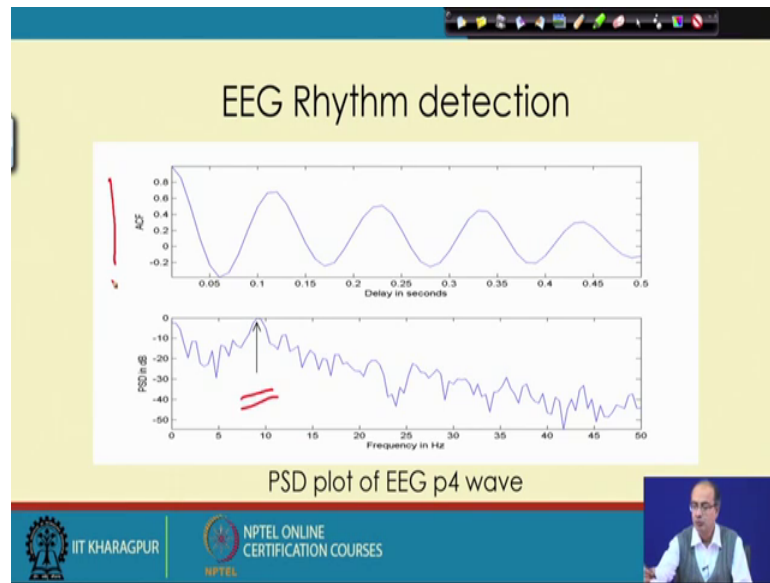
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So, now we go back to the signal p 4, that what we have taken earlier. That our, the way we have defined that problem that we have that this part of the signal 4.67 second to 5.81 second that is marked as alpha signal by an expert and we have applied that taking that as a that reference. We have computed the auto correlation function of the overall signal of that we have 7.5 second.

The total duration of the signal we compute that ACF and we could get peaks, the first peak was at 9, when that 1.11 second corresponding to, that 0.11second if we take that, the reciprocal of it, it gives us that 9 hertz. So, and from there we could get that alpha wave is there. So, that was the last f of what we did with the time domain technique.

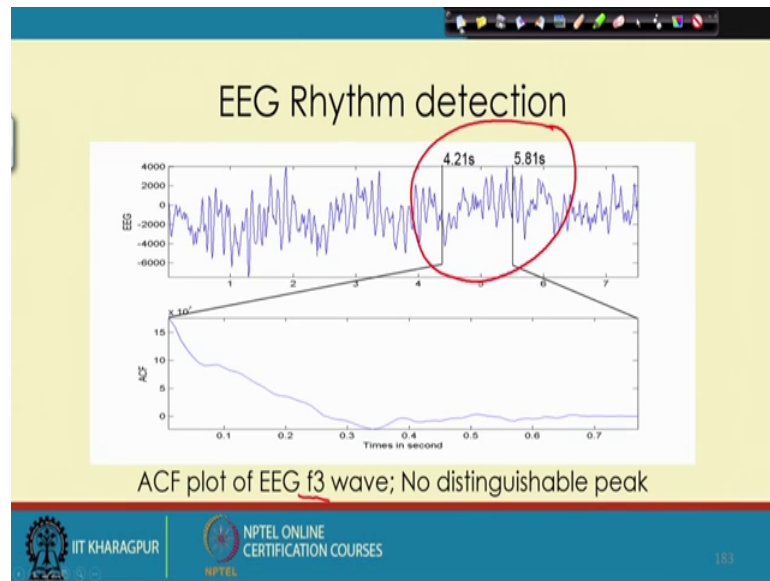
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Now, we are repeating these, theme that we would like to compute the PHD. So, we have taken the frequency plot and we have computed the frequency amplitude. Now, why we are talking about amplitude specifically? Because we know that while taking the Fourier transform, we are using  $e$  to the power  $j\omega$ . So, the real signal can give the complex output. So, we are just interested in the strength of it or the amplitude of it.

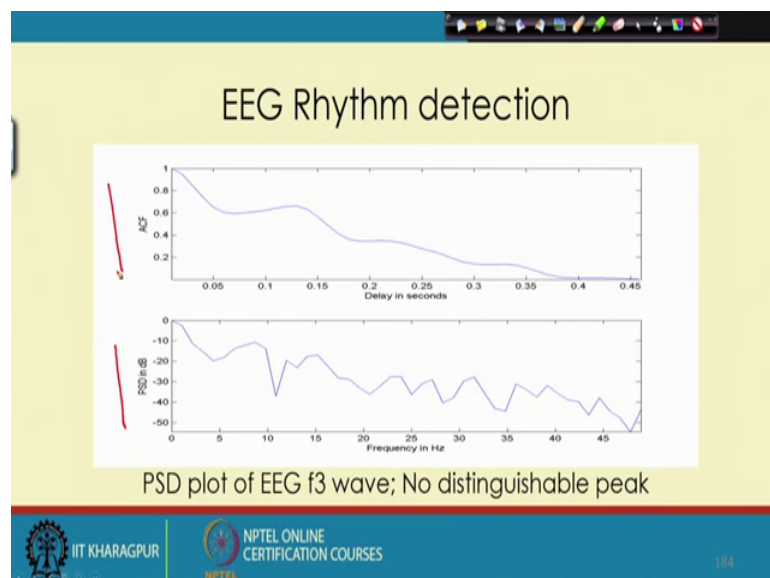
So, we have looked into that part and what we get that we are getting a distinct peak at this point, above 9 hertz and that way we can get that, alpha wave is present because the range of alpha is coming with the range of alpha and it becomes a direct way for getting that decision. In fact, just above it we have kept the ACF, the primary reason is that we should be able to compare the 2. We should have a comparison of the 2 and what we get that here also (Refer Time: 27:01) gives the same, they actually amount of information. Only thing PSD gives that in a more ready form for our use.

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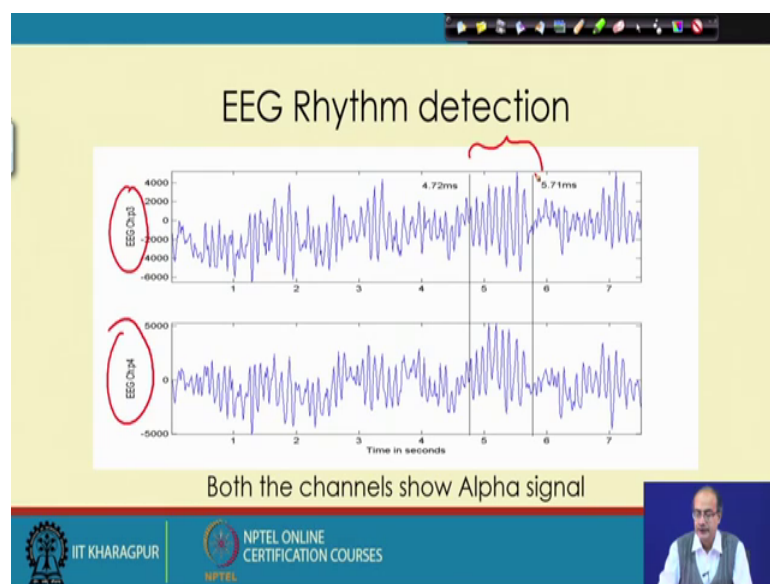
Now, let us look at that the second example, that we have taken the f3 wave and for that f3 wave, earlier we have computed the ACF to know whether the signal present within this region that 4.21 second to 5.81 second, that part, that it is getting repeated throughout the signal of f3 and we found that there is just a valley there is no peak. So, we concluded that there is no distinguishable peak and there is no such repetition of that same kind of signal.

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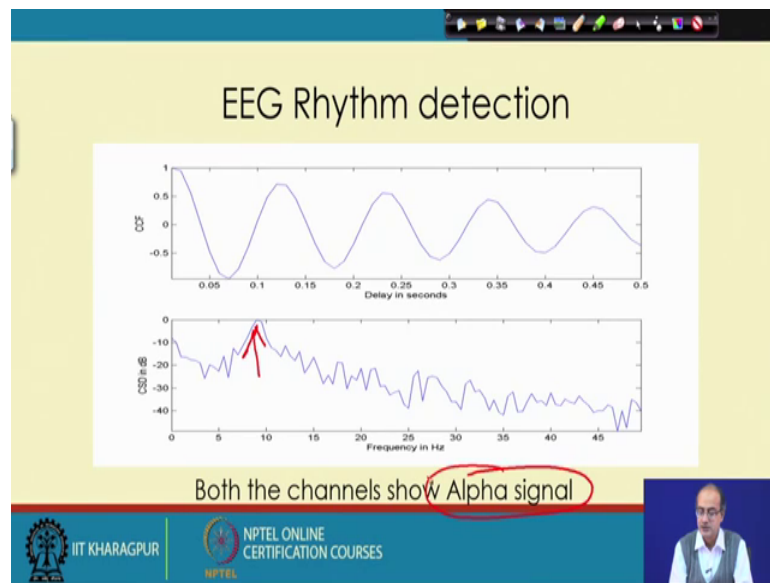
So, we compute now, that the PSD of that, here we give the PSD plot, we get the same kind of nature, a drooping PSD we do not get any peak. So, the maximum is at the 0 frequency that only signifies that there is actually some DC component in the signal, it is nothing more than that. So, we should not consider that as a peak. So, if we look at that there is no prominent peak in the signal and we have kept actually the ACF also just above it, to compare. So, we come to the same conclusion from the PSD as with ACF, that there is no distinguished peak or the reputation of the same kind of actually the rhythm.

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Now, let us take another signal that we take actually that signals from the 2 channels. 1 is channel 3 another is from channel 4 and they are simultaneous recording and in both the channel we get actually that some similar kind of waveform here, in between 4.72 millisecond to 5.71 millisecond. Now if there is a doctor present, they can give this thing but as a biomedical engineer how we could be so sure, that if 1 is alpha the other 1 is also alpha or what is the span, how long we can say that alpha wave is there.

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So, to do that what we do, that we can take the cross correlation coefficient and the cross correlation is again giving a peak, you get that in between that just above the 0.1 second and here it is in the cross PSD we are getting that peak somewhere here, that is near the 9 hertz. So, that signifies that in both the cases that we have the alpha signal is present. So, even though we are not expert, that with the help of that, the technique cross correlation coefficient and the cross PSD, we could actually that make the diagnosis that the presence of alpha wave is there in these 2 channel of signals.

So, this is the take away for this moment and would like to thank you for the session.