

Biomedical Signal Processing
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Lecture – 14
Artifact Removal (Contd.)

So, now we will start with adaptive filtering. First we should look at that why the adaptive filtering is required.

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Adaptive Filtering

Adaptive Filters for interference removal:

Scenarios:

- Power line interference with varying power frequency
- ECG signal of fetus & mother
- VAG (Vibroarthrogram) and VMG (Vibromyogram) signal

Problem statement : Design optimal filter to remove a non-stationary interference from a non-stationary signal.

Goals:

- Filter should be *adaptive*
- Filter should be *optimal*

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The first scenario is a power line interference with varying power frequency; let us look at that how our power system is managed. At every instance we are switching on some light fan pump air conditioner or so many other electrical gadgets.

Now, for each one of them that we are changing the demand on the actually power system, and if you look at every instant that the demand is actually the change is a sum total of all these changes, and the generating station actually feeding that power to this system. So, whenever there is a change in demand, that what we find that generator which is producing the power that it is actually having some reaction to that change.

If we try to draw more power; that means, we switch on more equipments, then it will put a demand on the generator, but on the other hand the generator is run by a prime mover; say most of the cases that genetic stations are thermal stations. So, let us take the

example of a that the turbine steam turbine, which is acting as a that that prime mover for that and the thermal power station.

Now, they are as the demand is more, then the power supplied by the prime mover there would be a deceleration of the; that rotor of the generator. And that deceleration or the change in rpm that is actually detected and that is fed to the control system of the prime mover, so that what we do we increase the feed there feed of well. So, that that again the prime mover compensate for that loss of inertia, and it feeds actually more energy so that that demand could be met. And if that we keep on switching on the load, that mean in that case what will happen that due to the certain change in the load that, but the prime mover is actually supplying the power at the same rate.

The rotor of the generator will start actually moving at a higher speed that means, it will accelerate. So, to compensate for that again the control system will detect the change and give a appropriate negative feedback in that case, that the supply to the that will the prime mover that should be reduced. So, that there is a balance is maintained between the input and output.

Now, if you look at this generators, that the part of the prime mover they are mechanical equipment. So, even if the control circuitry in the side of the generator, that is the electrical equipments or electronic equipments which are having a fast response, the end part of it when we are actuating the that the prime mover that part is slow and due to that there is a lag in the response.

So, because of that that we find the power frequency is changing all the time, and as per the law it allows actually 3 percent in India that its allowed 3 percent actually change plus minus 3 percent; that means, over 50 hertz. Plus minus 3 percent means 1.5 hertz it can go below that power frequency as well as it can go above the power frequency.

Now, under that scenario what can happen? We have found so far that notch filter is the base to actually take care of the power frequency interference. That notch filter which is designed at 50 hertz say, now as a power frequency is changing that the interference from the power frequency also will change its position and the notch filter will not be effective anymore. So, what to do in this scenario? That the optimal filtering or the notch filtering it cannot take care of this scenario, which is one of the example that why adaptive filtering is required.

Now, let us look at the next scenario that already we have told about the ECG signal of the fetus and mother. Now when the child or the fetus in the; is in the womb, then when we are trying to get the mother ECG, the fetus ECG added up with the mother ECG and acts as an interference. Some we will be interested to check the health of the fetus and we would like to take the that ECG from the fetus, but in that case also what will happen even if we put the probe in the abdomen of the mother, because the mothers ECG is stronger it will again ride on that the fetus ECG, and act as a interference there.

Now, in such cases because they are actually the spectrum is very much overlapping and again both of them they are actually cyclo stationary signal, they are changing over the time slowly; that there is no other way other than adaptive filtering to actually move remove from each other or separate each other. The third example let us look at that is the vibro ortho gram. We hear about the people old people who have to go for the knee surgery or knee replacement surgery.

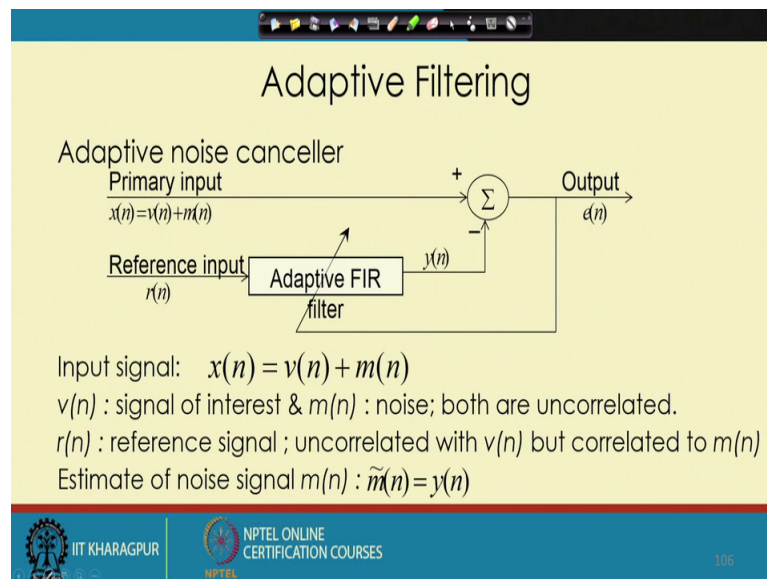
Now, the first stage of it what happens that the knee joint which is a hinge, that the both the sides of the bones they are having some cartilages and that actually is lubricated with some fluid there. So, if it happens that due to some reason maybe that overweight or some other disease, that degeneration of the cartilage happens and it becomes rough then the moment you try to actually fold the leg; that means, a hinge is operated there will be a much higher friction and that gives rise to some noise, which could be audible by a microphone. So, that is actually the vibration that is picked from that knee that is called the Vibroarthogram.

But as we are moving the knee there must be some signal given to the muscle, and that the vibration generated by those muscles that is the Vibromyogram that actually also picked by that our; that the mic there. So, what we find that while we are interested to know that what is the amount of noise generated by the knee or what is the condition of the Vibroarthogram, we are getting another unwanted signal and both of them they are non stationary.

Because they are started actually the generation started when we started moving the knee and as soon as the moment is completed they subsides. So, here again our; none of the field test what we have read, they are actually could give a good result to separate them.

So, now let us look at the new problem statement that we have already learnt about the optimal filter. So, we do not want to lose on that; that here the problem statement is that we would like to have an optimal filter, but it should work in a non stationary environment; that means, the signal is non stationary or the interfering noise or interfering signal is non stationary. In other words we want an optimal filter at the same time it should adapt with the time or add up with the signals it is dealing with. So, that is the motivation for having a adaptive filter in this case.

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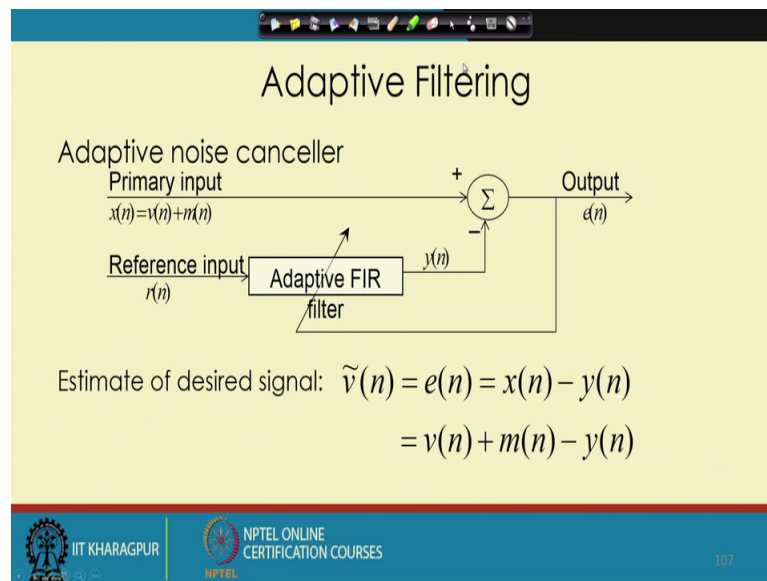
Now, let us look at the structure of the adaptive filter. That first the application we have taken it is adaptive noise canceller and it is interesting enough that here like the previous case, that we have the input $x(n)$ that is the observe signal, it is a sum of $v(n)$ that is our signal of interest; this is the signal of our interest and interfering signal or noise is $m(n)$ here in this case. And the these 2 that they should be uncorrelated with each other and here the other input what we have taken the reference, it is the some signal $r(n)$ which is correlated with the noise.

Now, what is the advantage in taking the signal which is correlated with the noise? I think it is much easier to actually get for example, if the primary that noise here is a power frequency harm, in that case that what we can do? We can take that of the that the power line and take the reference because power frequency harm would be related to the power supply voltage, which we can directly take from the that power supply and then

what we can do this reference input which is correlated with the noise, it can be passed with an adaptive IIF filter and we can generate something we or rather we can synthesize something called $y(n)$ which is actually would be a an estimate of the this noise $m(n)$.

So, if you subtract this one from the observe signal, we expect that we get somewhat closer to get the signal of interest or $v(n)$.

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So, here we look at the scenario that we have this error $e(n)$, this error is the difference between $x(n)$ that is the observed signal minus the estimate of the that noise, features come synthesized by the adaptive fir filter and with the help of the reference input.

Now, this error we can name that as $\tilde{v}(n)$, because we expect that would be close to something of our signal of interest $v(n)$. And if we actually replace $x(n)$ with $v(n) + m(n)$ we can write it in that way also. So, that is the basic form we get for that desired signal. So, interesting enough here that, by changing the reference input to the that reference of the that noise; that means, a row correlated signal that with a noise, the output what we call as a error that is actually is coming close to the desired signal of or signal of our interest.

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Adaptive Filtering

Adaptive noise canceller

Mean square error (statistical):

$$E[e^2(n)] = E[v^2(n)] + E[\{m(n) - y(n)\}^2] - 2E[v(n)\{m(n) - y(n)\}]$$
$$= E[v^2(n)] + E[\{m(n) - y(n)\}^2]$$

as $E[v(n)\{m(n) - y(n)\}] = E[v(n)] \cdot E[\{m(n) - y(n)\}] = 0$,
using the fact that $v(n)$ and $y(n)$ are uncorrelated and zero mean.

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So, now, let us look at that form more closely; as you know in case of optimal figured when we talk about the optimality, we need to tell that what is a sense of optimality and once again that sense is that we are trying to minimize the mean square error. So, let us look at the expression of the mean square error. So, it is expectation of E square n and by replacing that E square n by the expression what we got in the previous page that is v n plus m n minus y n the whole square of it.

Now, we can make it in actually 2 terms one part is v n another part is a m n minus y n . So, we can actually expand that the square operation, and the square term and we can get 3 terms here; out of them that one term that is crossed between the v n and m n minus y n . On the other hand first 2 terms s square of v n and that m n minus y n . Now this cross term this is that this one that v n is uncorrelated with m n and because y n is correlated with m n , v n is also uncorrelated with y n .

So, in this scenario what will happen that we can get actually that expression as that expectation of v n into expectation of m n minus y n ? Now as the noise is zero mean and signal is also zero mean both these terms; that means, the expected value of them would go to 0. So, these term the cross term goes to 0 and what we all need is actually uncorrelatedness of the signal and the noise. And please note that there is no more demand on the signal we do not even ask for the stationarity of the signal.

So, with that we can show that the mean square error it boils down to 2 terms, one is that we have the energy of the signal and the energy of the difference of the estimate of noise and that estimated noise.

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Adaptive Filtering

Adaptive noise canceller

The minimum power output:



$$\min E[e^2(n)] = E[v^2(n)] - \min E[(m(n) - y(n))^2]$$

and

$$e(n) - v(n) = m(n) - y(n).$$

When $E[(m(n) - y(n))^2] = 0$ we have,

$$m(n) = y(n) \text{ and } e(n) = v(n); \text{ i.e. we get noise free output.}$$

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So, that is the expression we get at this point and let us move forward. Now when we try to minimize this error or m s c, that expectation of e square n what we get here actually that e square n is that we get a term with the; that the first time we do not actually if we try to take the minimization, we do not find any effect on that. That the reason is that the minimization is taken by what we change the tap weights for the fir filter.

Now, that does not have any effect on the standard deviation of the input signal. However, it can change that the estimate y n and thereby it can change the second term that m n minus y n. So, that that part when we minimize the error actually the second time can be minimized, the first term remains unchanged. And another thing we get that from this that we are just rearranging, the sides that error minus v n that is that error is the output of the that noise canceller and v n is the signal of interest, this is equal to m n minus y n m n is the interfering noise and y n is the estimate of the noise.

So, if we can achieve these that by minimizing the second term, the minimum value can happen 2 0 because we are taking the square that it cannot be negative. So, the minimum is 0. So, in that case what we can get that m n equal to y n; that means, there is no more error there, and as a result of it because there is no error in the estimation of the that

noise, then that whatever the output we are getting from the adaptive filter $e(n)$ that becomes that $v(n)$ or the signal of interest.

In other words, actually in this scenario; we are able to get the noise free signal. So, that is actually the goal of our adaptive filter.

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Adaptive Filtering

Adaptive noise canceller

We have $y(n) = \sum_{k=0}^{M-1} w_k r(n-k)$; where $w_k, k = 0, 1, 2, \dots, M-1$.

and error $e(n) = x(n) - y(n)$.

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Now, here we again come back to the that the adaptive noise canceller structure, and we get that the emphasis will go now to that filter, we have taken that the reference signal $r(n)$ that is fed to it a tabular filter and for different actually lags the value of $r(n)$ we take, and multiply with the tap weights and we sum them up, which gives rise to our estimate of the noise that is $y(n)$. And as we subtract that from our observe signal $x(n)$, we get the output $e(n)$. So, here that $y(n)$ is actually sum of the present value of the reference as well as past $m(n) - 1$ values of the references. And this w_k is a weight which is actually if it is properly tuned it can actually bring $y(n)$ close to the interference additive signal or additive noise we should say that is $v(n)$.

So, in this case for these diagram, we get a error is $x(n) - y(n)$ or the difference between the observe signal minus the estimate of the noise.

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Adaptive Filtering

Adaptive noise canceller

$M \times 1$

$$\underline{w}(n) = [w_0(n), w_1(n), w_2(n), \dots, w_{M-1}(n)]^T$$
$$\underline{x}(n) = [x(n), x(n-1), x(n-2), \dots, x(n-M+1)]^T$$
$$\underline{r}(n) = [r(n), r(n-1), r(n-2), \dots, r(n-M+1)]^T$$

Note:

$$e(n) = x(n) - \underline{w}^T(n) \underline{r}(n).$$

- No assumption on $v(n)$, $m(n)$ and $r(n)$, other than independence of $v(n)$ and $m(n)$ & correlation between $m(n)$ and $r(n)$.
- Can be extended where $m(n)$ and $r(n)$ are deterministic.

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So, these part the adaptive noise canceller, now we can write into a very compact form and first what we can do that we can write the tap weights w . That this one is a m by 1 dimensional vector and please note one thing that unlike the optimal filter here, the tap weight is having an index of n ; that means, it is changing with time so that it can adopt and give us the optimal input that optimal estimate of the noise even if the that interfering signal or the signal of interest they are changing with the time and maintain that optimality.

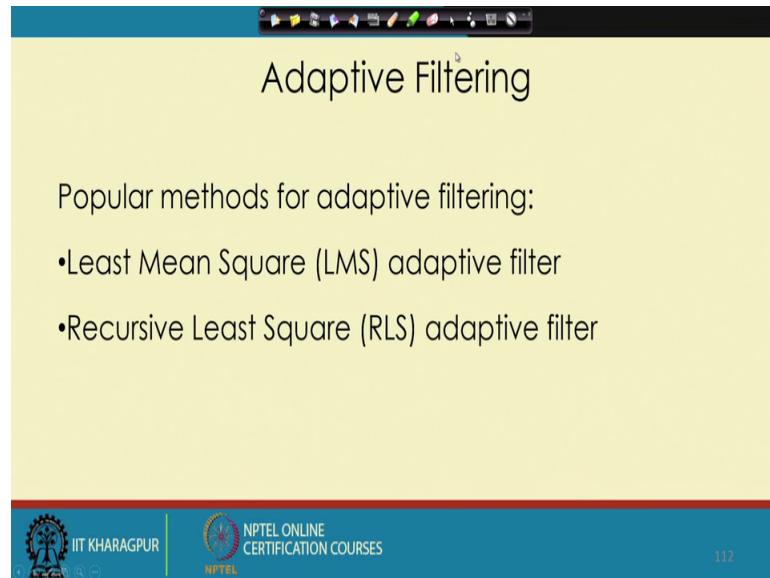
So, w n we get as a m by 1 vector and it will update with time and that is the difference with the previous wiener filter. The x n remains with the same it is again m by 1 vector that and it is consisting of the present value of x n as well as the past n minus 1 values.

The reference signal for the noise canceller r n that its again m by 1 vector and it is the present and last m minus 1 values, and here the r n is chosen in such a way that it is correlated with the that the additive interfering noise. And in this scenario that the output of the noise canceller that is e n it is nothing, but the difference of the observed signal and the dot product of the weight and that reference input r n . And here couple of things that we should note that there is no assumptions on v n , m n and r n .

Other than independence of v n and m n and correlation between m n and r n . Actually independence is a stronger condition we can say just that they are uncorrelated that is good enough to show this actually theory or derive these theory. Again that same set of

equation they can be extended where m , n and r are deterministic, they are not actually random processes even in that case, we can actually use these framework or same set of equations and apply the adaptive filter for that better noise cancellation.

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The slide is titled "Adaptive Filtering" and lists two popular methods for adaptive filtering: Least Mean Square (LMS) and Recursive Least Square (RLS). The slide is part of an NPTEL online certification course from IIT Kharagpur.

Adaptive Filtering

Popular methods for adaptive filtering:

- Least Mean Square (LMS) adaptive filter
- Recursive Least Square (RLS) adaptive filter

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Now here we look at that the different kind of adaptive filters, there are 2 common actually are very popular adaptive filter filters. One of them is a least mean square that algorithm or LMS adaptive filter. The other one is called that recursive least square adaptive filter or in short we call it RLS adaptive filter.

Thank you.