Biomedical Signal Processing
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Lecture – 12
Artifact Removal (Contd.)

So, for the wiener filter, now what we do? That, we have the constituent that comes. So, we with the help of that we can actually write this the mean square error J w in a mode compact way.

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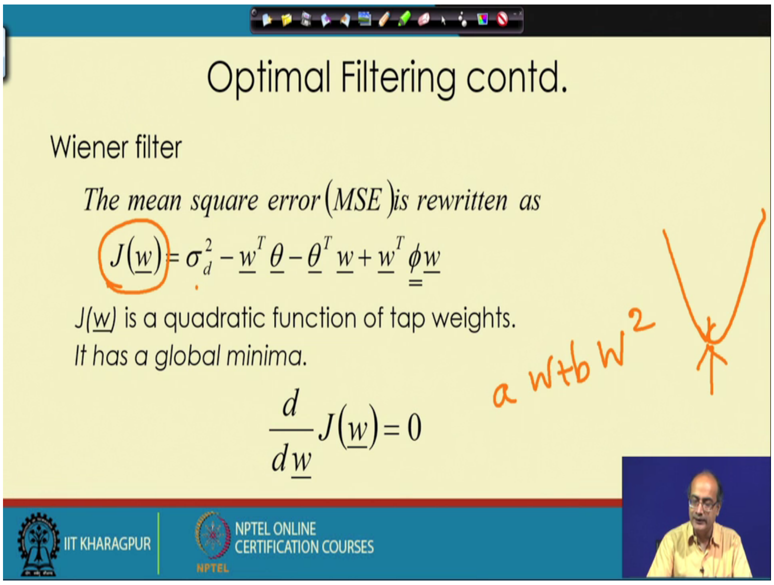

The first term is the variance of the desired signal, the second and the third term they nothing but the cross of that, that weight and the vector theta is the cross-correlation vector between the observed signal and the desired signal. And the at the end we get that the term involving the phi, which is the auto correlation matrix of the that observe signal excel. So, what we get here? That we get a expression which is quadratic function of w, the w at the tap weights here. And what is the specific? Actually, the quality of that quadratic function that it gives us a global minima.

So, to understand that let us take just a simple case that were the dimension is 1. So, if I take a function in dimension the tap weight dimension is 1; that means, it involves with only say w square and w also could be there. So, it is a function of the form a w plus say b square. So, it would be a parabolic function, a parabola and they straight line a combination of that. Now being a parabola it has a minima here. And these global minima that global we can reach there by actually taking the derivative. To be more specific, in this case we need to take the partial derivative for each of this weight. When w is having actually the multi-dimensional 1.

So, what we get actually that J w, it is a surface it is say or we can tell it is say a hyper plane, and this function has a global minimum of it. And at that minima the gradient at this point would be 0, at this point. So, we exploit that situation. So, we want to find out the value of the weight, where the gradient would be 0. And thereby we would like to find out that the point where that value is minima. So, we are using that fact so, we have to take actually the derivative of that objective function J w. So, let us now proceed with that. And let us see that if you take that derivative.

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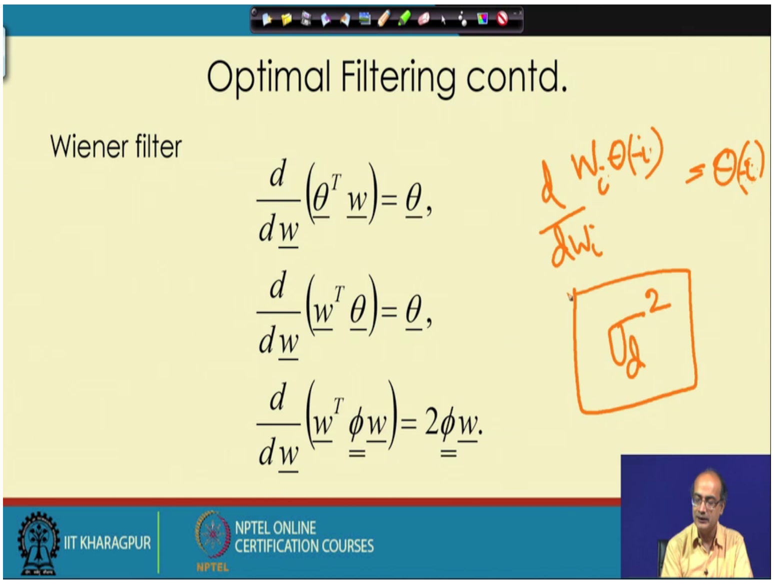

What are the terms we get here? So, we get first that, if you take the derivative of that the term involving the theta transposed and the weight. That for each of these component say w 0 w 1 and all, that taking the derivative with respect to the vector means that we are taking actually the derivative with respect to each of these terms, and then putting in back in the vector form. So, if we do it manually we can easily check that for each of them that we get the corresponding value of the theta; that means, say if I look at this term it would look like that say with; say w 0 into say, theta 0. And if we take derivative of that with respect to w 0. So, we would get theta 0. So, if we perform it in this way or let us take it in a more general way, if we take it that with respect to say for each of these term, say we have theta i here. So, here it would be minus i. So, we get the terms like this, each of these term.

So, at the end if you combine each one of them, then we get the vector theta after taking that derivative. And because that this thing is a scalar, actually this the product is a the scalar quantity that w T theta and theta T w both gives a same result. So, here also we get after the derivative the vector theta. And for the term that the last term, that we involving that the autocorrelation matrix, we get it to that 5 w. And here we have actually omitted one part; that is, what happens if you take the derivative of this one. Actually, we have not mentioned about it. Because see these one is what it is the standard deviation of the desired signal.

Now, it is already given and whatever may be the tap weight it will not change. So, if we take the derivative the derivative would be 0. Or in other word, there would not be any change in the variants of the desired signal even if it change the tap weights. So, this way we get actually the derivative of all the 4 terms involved in J w. So now let us then proceed and see that what we get out of it we substitute those values.

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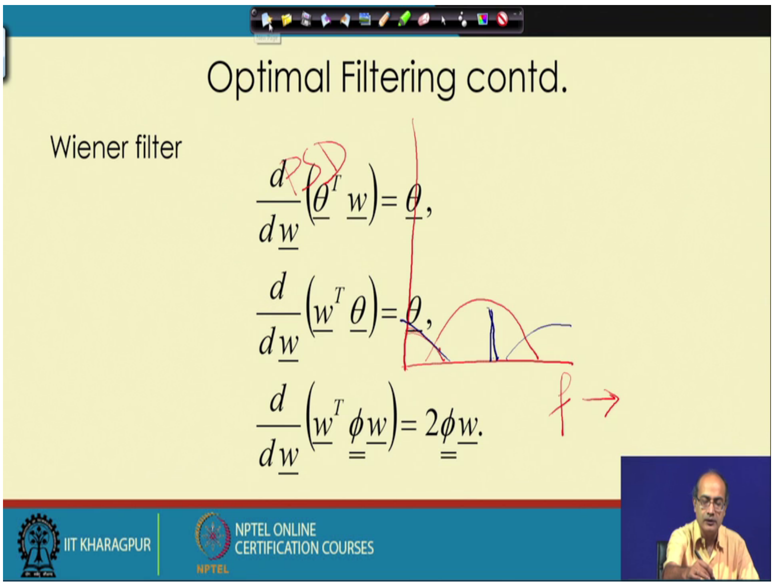

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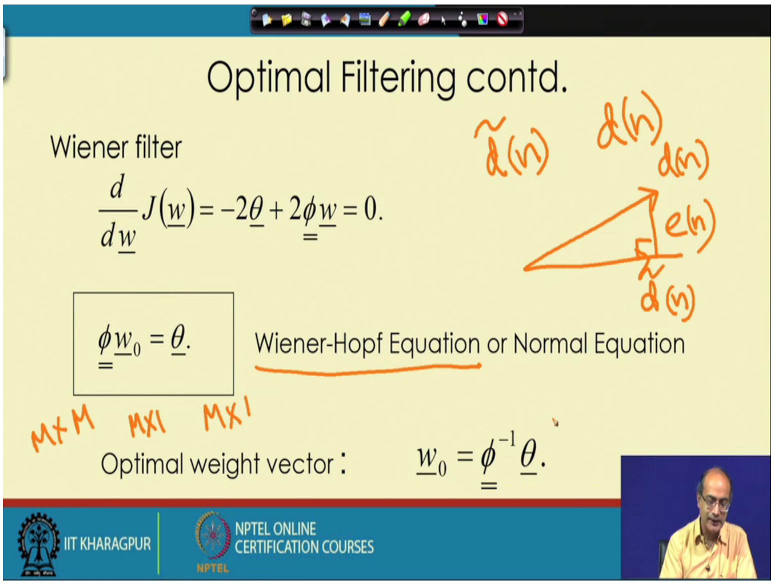

That by substituting those values of that derivative of those 4 terms, we get for the wiener filter that. We get the first time that derivative of the standard deviation of the desired signal contribute to 0. And for the second and third term that is contributing minus 2 theta. And the 4th term is contributing this part that 2 5 w.

Now, by scaling it, and changing the side; that means, taking this theta in the right-hand side. We can write it in this form, that phi w 0 equal to that the vector theta. So, these actually equation, this equation is known as that wiener Hopf equation. And here one good part actually what we get out of this exercise, that the weights what we told the weights we are able to get, in terms of a set of linear equations. And if you check the dimensionality, what we have? Phi is the having dimension M by m, and this w 0 having M by 1. And theta also is having dimension M by 1. So, for that M weights we have actually a set of M equations. So, by solving simply this equations set of M equations we are able to get that the value of the w 0 or optimal value of the weight.

Now, here we added this suffix here that 0. That is to indicate that this is the optima weight, or the tap weight which gives the minimum for that the error function. That these J w. And here is another name for this this is called also the normal equation. Hope you recall that, with the tap weight what we are trying to do we are actually getting something called d tilde n. That is a desired value. If it should be as close as possible to d n. There by we are minimizing the noise. Now let us take that d n is a vector, say if ddn this is the dn that is a vector. And say d tilde n is going in this direction. Now the error would be minimum, when actually this is the error that if this is d tilde n, then this one would be the error. The error would be minimum when actually these vector is perpendicular. So, that that is why that we call this equation as normal equation; that means, the prediction what is happening and the error these 2 would be normal at this condition. And that is how the name came as normal equation.

So, this is how we get this wiener Hopf equation or the normal equations. And by solving them this equations set of equations, we get the optimal weight vector w 0 equal to phi inverse theta. So, at the end of this exercise, we get the value theta. Now let us try to address that how actually we can have some d, and for the set of actually solving this equation or to calculate the tap weight. Let us go back to our that the old example of e c g signal, I hope we recall that.

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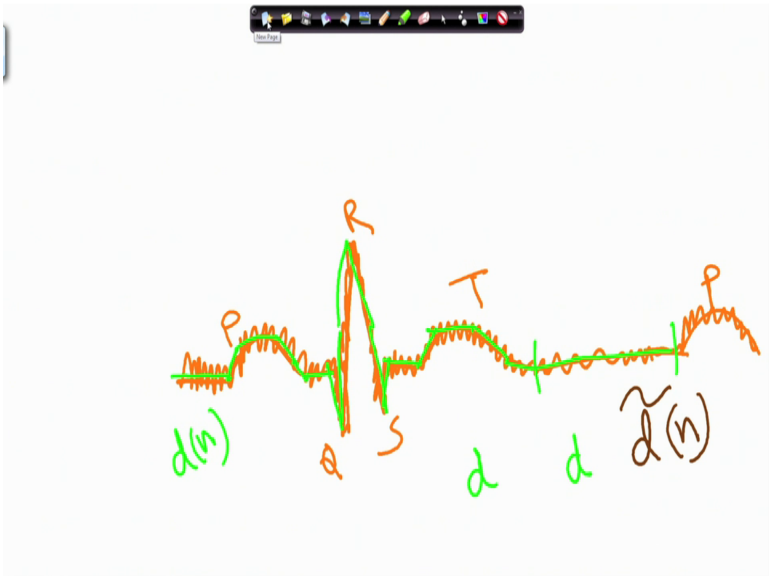

The ECG signal is having that P Q R S and T waves. This is P Q R S T and because of different kind of noises say high frequency noise, then we have that power hump. Actually, lot of noises are added with it and it got smeared in this way.

We can say, again for the next way the P is coming it got smeared this is the signal. So, in this case, we are unable to figure out that the desired signal if you could get that then we do not need to do all such job of optimal filtering. What we can do we can actually easily get some estimate of the desired signal from this corrupted signal by some approximation. For example, that from 1 T to the P wave of the next beat. In between these part is actually ISO electric line. There is no activities there. So, if you want to take that characteristics of the noise, we can just observe this part. We can take it out as a as a sample of the noise block. Where no signal is there. Now if you want to get, the desired signal that is the ECG signal, in this case, then how we can approximate we can put it with a straight line, and that then to actually eliminate the effect of a actually noise. Or to reduce the effect of noise we can have straight line approximations that will by few line segments we can approximate each of the soon.

So, we can get an approximate value of the ECG signal, which may not be very accurate, but close enough to serve as a that the desired signal dn. So, in practice we can actually get some desired signal out of the corrupted signal, but such kind of a approximation is not natural. So, they cannot serve as actually for the real processing. With the help of that that what we would like to do? You would like to calculate actually that d tilde, we would like to calculate d tilde n and that one would be smooth and after the optimal filtering, we can actually make use of that and that we can proceed with that for further processing. So, let us now move forward we see that after this part that what we have to do.

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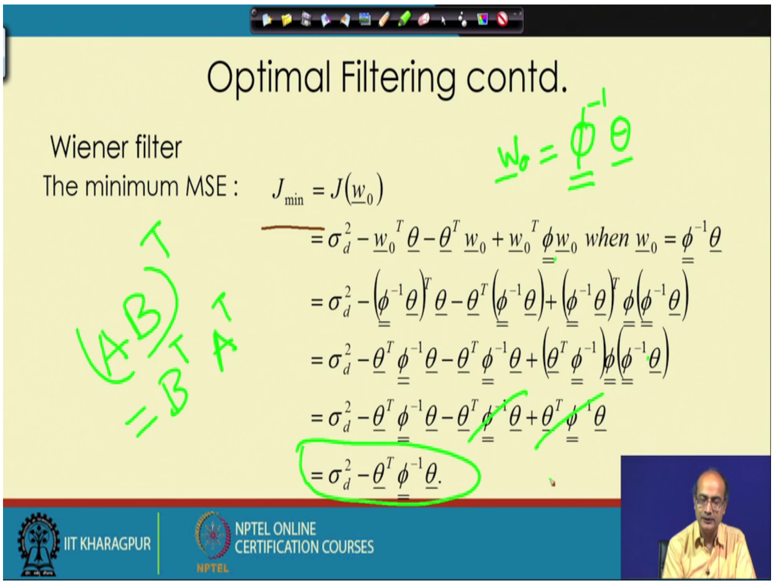

The fasting we are interested in that what is the minimum value of the error. We wanted to minimize that J w and we found that tap weight where the error will be minimum. So, first thing we would like to know that; what is this minimum error.

So, this minimum error, we have taken here that J min, and for that we are a simply replacing the tap weight with the optimal value of the tap weight, and that is w 0 we know that is equal to 5 inverse then theta. So, we need to use that proper notation for that, that and as we take it that for these first we replace actually w with w 0. So, it has been known with 4 terms, the first term is that variants of the desired signal, and then we have the 2 cross terms; that we did in the theta and that w 0 and theta is the cross correlation between the that observed signal and the desired signal. And then we have the auto correlation matrix this 4th term using the autocorrelation matrix.

So, with that as we replace it the w 0 in the third actually line, that here we actually replace w 0 with this value. And then we make use of the fact, that if we have 2 matrices A and B transposed the result would be B transposed A transposed. So, we make use of that. So, that way that we keep it here out of that there is no change in actually that phi or phi inverse. Because phi is a actually symmetric kind of matrix. So, here the transpose does not make any difference. And we rewrite that using this equation, and what we get that these 2 term is the second and the third terms they are equal. And after simplification of the 4th term we get that in magnitude the 4th term is also same, as these 2 terms only thing the sign is different.

So, 2 of them they cancel with each other, and we are left with this quantity. So, this is the minimum error we get. So, this is the interesting thing that we have to look at.

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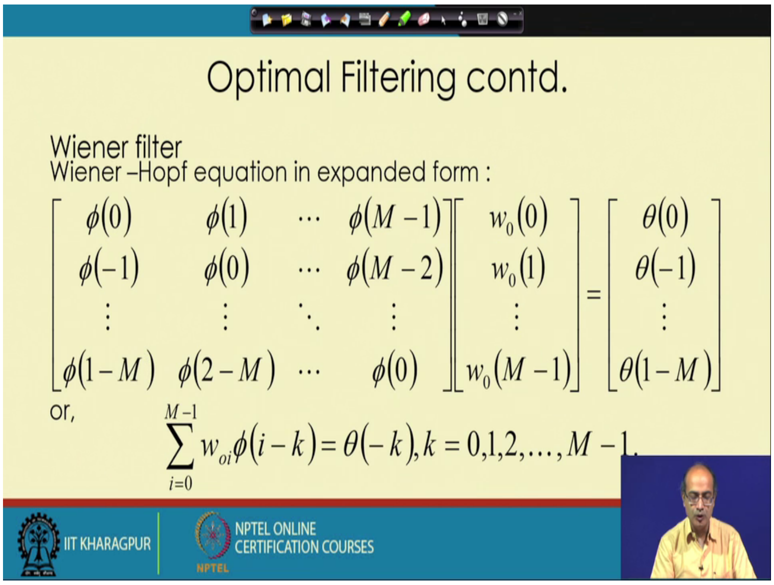

And then we proceed, and we see that the wiener filter for that the wiener Hopf equation it can be written in different ways; that one of the most popular way is the that in terms of matrix and in terms of vectors, that is what we have given. That we have actually given the autocorrelation matrix, multiplied by the tap weights, and in the right-hand side we are getting that cross correlation vector. And here for the simplicity sake you can actually write it in terms of the terms, that n minus 1 and 1 minus M they ares. That these 2 terms there are is the same. So, we can replace it in that way also. So, then we can appreciate that that it is actually that the that if you take transposed that it will not do any change. And the same thing can be written as a set of linear equations. So, both are having the same amount of information. And this is the way we can actually express that that wiener filter or the optimal filter.

Thank you.