**Course on Introduction to Medical Imaging and Analysis Softwares**
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**Module 02**

# Lecture 08: Random Walks for Segmentation

So welcome to today's topic. It is on Random Walks for segmentation and we are going to specifically going to look into the method for segmenting medical images. Now this particular method over here is this is a sort of semi supervised method in which basically the user has to give certain seeds and based on these seeds you can segment it out in a very good fashion. And now this family of methods which we are going to discuss today comes down from a generic family which is called as graph trace methods for image processing and so the name suggests, over there you would obviously get to know that we are going to deal with images in a representation which is not necessarily a matrix but is a graph.

Now how to represent images in graph, that is one of the main interesting problem which we will solve initially and then how the whole segmentation problem is solved. Now since I said that it is a semi supervised and you need to give down seeds that does not in any way restrict the whole power of this method to be limited by a user giving down seeds, in fact there are many more advances which have happened on this particular method in more than a decade more than 10 years in the last wave (())(1:28).

And people have come up with multiple ways in which you can initialize in a automated way how to give down seeds and they also play a very crucial role in to its walks

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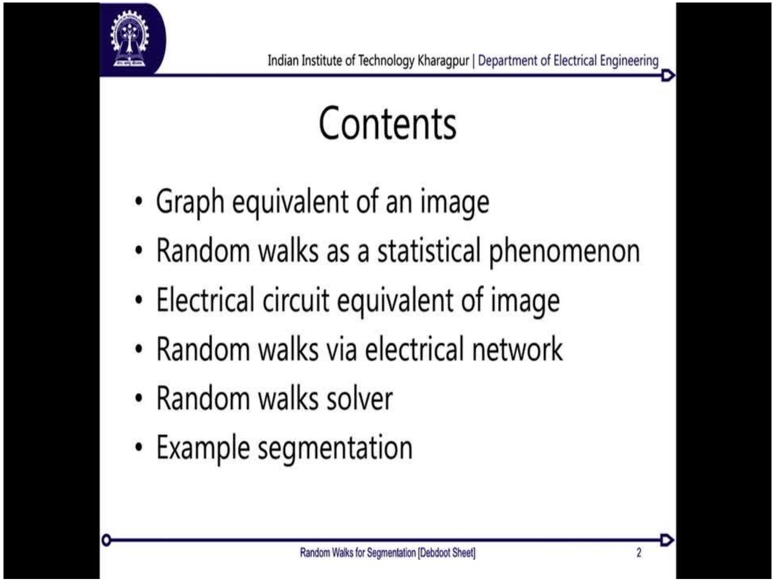

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- Graph equivalent of an image
- Random walks as a statistical phenomenon
- Electrical circuit equivalent of image
- Random walks via electrical network
- Random walks solver
- Example segmentation

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So without much of a delay how we are going to do is the initial part will be on representing an image into an equivalent graph and from there I would be speaking about what random walks is as a statistical phenomenon and what it actually means over there why this very curious names called as Random and walk coming down together. From there we enter into the electrical circuit equivalent of representing the image and from there to solving a random walks onto a electrical network equivalent.

And from there we enter into the actual solver or this is called as a solver because you have a set of linear algebra equations over there which you need to solve in order to get down the solution and then there would be an example on an MR image which we will be showing.

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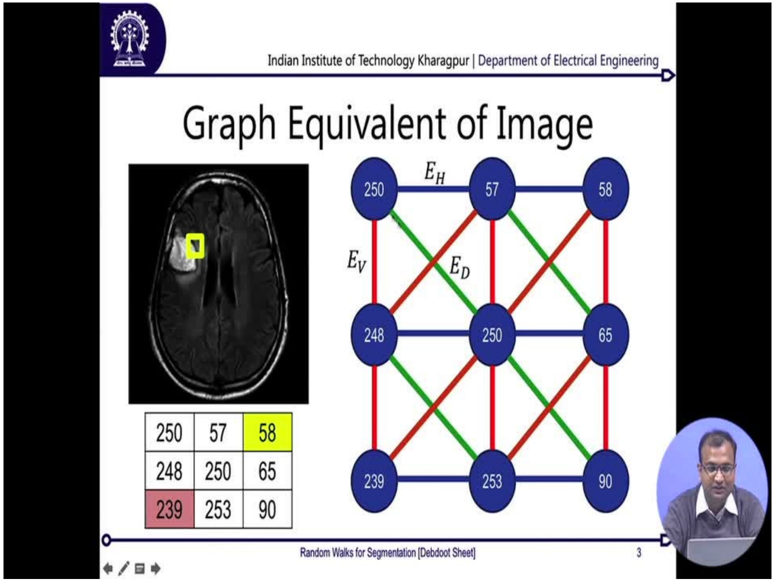

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Graph Equivalent of Image

|  |  |  |
|---|---|---|
| 250 | 57 | 58 |
| 248 | 250 | 65 |
| 239 | 253 | 90 |

Random Walks for Segmentation [Debdoot Sheet]

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So say that you have an image and this is about small lesion in the brain as we have been seeing in the previous exercises as well. Now let us consider one of the regions over here which is at the boundary of the lesions and the healthy part of the brain and if you look at just a small 3 cross 3 neighborhood over there this is sort of a grey values which we will be getting.

Now I have two of these two of these pixels over here which are color one in yellow and the other one is red and this belongs basically these are the two seeds which I am placing down in these 3 cross 3 and this is the basic example template which I would be using for explaining you the rest of the concept. So this 58 which belongs to somewhere around in this region is the healthy region and since so we have colored it with this seed called as with the color yellow so that is one pixel which is marked that is saying that this is the part of the healthy brain tissue.

And then the other seed is marked in red which is to denote that this is the part of the lesion tissue present over there in this MR image. Now, this 3 cross 3 sub matrix of that image can be represented as an equivalent graph by basically plotting down each of these pixel values into one node of the graph, okay. So initially you will have one small blob or what is also called as a node and each node is what has the intensity of each of them.

Now the coordinate location of each node is also the same as the coordinate location of this particular pixel over here. So if this has a coordinate position called as x, this also has a same coordinate location called as x. Now, once we are able to put down all the nodes over there, the next thing which we need to do is draw certain connections between these edges and these are called between these nodes and these are called as edges over there. So as we keep on doing you can draw horizontal connections, so these are called as horizontal edges or what we also define as E with a subscript of H, okay.

Similarly you can have vertical connection between the nodes and these are also known as v subscript of E subscript of v which are for vertical edges over there. Now along with that you can also draw diagonal edges between nodes as well as cross diagonal ones which are together called as diagonal edges or Ed. Now if you look at any particular node over here you would see that it has some sort of an 8 connectivity to its neighbors. So there are 8 ways in which it can connect down to its neighbors and obviously it can connect down to non neighbor as well.

So if 250 wants to connect down to 239 this particular one, so there are multiple ways, one, it can pass through this connect, it can pass through this and connect, it can pass through this and connect, it can again pass through this, this, this and connect and if you look at this one and remember something from your graph based solutions or graph theory classes, you might remember that this looks something like those pipeline networks which lot of people try to solve around or when you are trying do bellmen’s optimality principle or dijkstra’s graph cuts over there in order to find out which is the max flow mean cut to be obtained.

Now we do a similar kind of an approach although they are also used for image processing applications as well once you are able to define this whole image on an equivalent graph you can apply your graph cuts over there it is perfectly okay.

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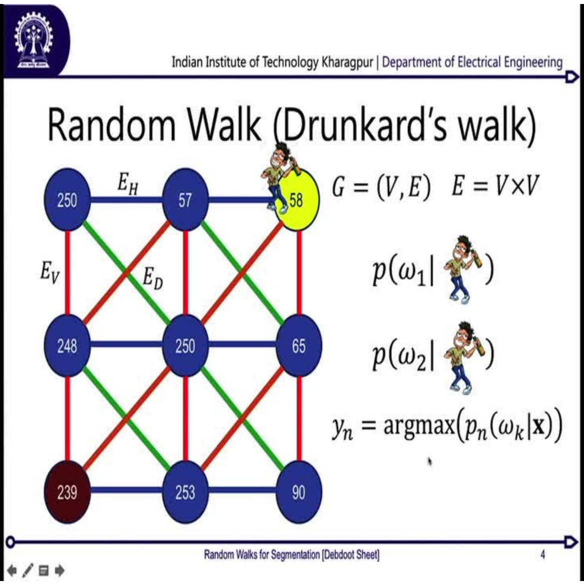

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## Random Walk (Drunkard's walk)

$G = (V, E) \quad E = V \times V$

$p(\omega_1|\ )$

$p(\omega_2|\ )$

$y_n = \text{argmax}(p_n(\omega_k|\mathbf{x}))$

Random Walks for Segmentation [Debdoot Sheet] 4

But we are going to do another very interesting phenomenon which is called as a random walk or also often called as Drunkard walk as you can see over here. So basically imagine the question to be something like this that there are two nodes over there, one of them belongs to the red class and the other belongs to the yellow class. And now say that there are multiple people who are located at each of these nodes, okay. so this let us consider only one person over here.

Now this person has a tendency to either go to this or to go to this, okay but then if this guy is drunk, so basically his cognitive decision making is impaired at that point of time. So if this guy is drunk, he does not know or he is not going to necessarily take a very informed decision as to which route he is going to go. So he can basically j walk, start j walking over there. Now one of his yeah so now that we have this particular graph over here we represent this j walk over this particular graph which is termed as a collection of set of vertices or nodes and edges where your total number of edges is basically a V cross V set because we will have one edge linking to two vertices.

So that will basically be the space of the total number of edges. Now, over this space what will happen is each of his walking phenomenon is something which will be traversing over a set of V's and E's on this graph. So one way may be he walks straight to this red node, this is one possibility. The other possibility is he can traverse through this path as well, there is another possibility that he goes in a sort of rounded up manner over here, okay.

Similarly, there are multiple possibilities in which he would end up over here, okay. Now what will happen is that given that he is at a particular node there is a probability that he will reach this first class which is omega one, okay and that you can obtain it very easily . By just counting the number of times he has reached this omega one via all the possible paths over there divided by the total number of times he is trying to attempt to reach some particular point over there, okay.

So the other point where he would like to reach to this yellow class, that is the only other point which he can do, so one way he can go along this particular way there can be multiple ways he can go straight, he can go this, this, this and come like this. So together with all of those ones you will have another probability of this drunken guy going down to another class called as omega 2. Now such that this total sum of this p of omega 1 given this person and p of omega 2 given this person for a particular node.

So it is always probability of a person's starting at a particular node to reach any of the 2 class that is what is defined as. This total summation will again be 1, okay. Now what we define as a segmentation problem is quite simple that if you are able to somehow find out these probabilities, then the label associated to this particular node where the random walker was starting is basically the is the argument over the maximum value of these particular probabilities which you are obtaining.

So if there are total number of total k number of classes, so you will be obtaining a probability over all the k number of classes for starting at this particular point and n is the nth number of the node where you are going to do. Now if you find out this maximum value and whatever the label associated with that that is going to give you the segmentation result over here, okay looks easy right? Should not be looking too complicated? Except for that fact that how do you know how this drunken guy is going to walk.

And one option is basically you lay down some sort of a field created from a equivalent of your image then you get a few people drunk over there and let them walk around, that can be one way of doing it. In fact scientists do a similar kind of experiment in simulation which is via something called Monte Carlo estimation method in which you find out different ways in which a physical phenomenon might happen so there basically you simulate the different paths a drunken walker can take and then try to find out what is the probability of reaching down a particular point. So this is one way of doing it.

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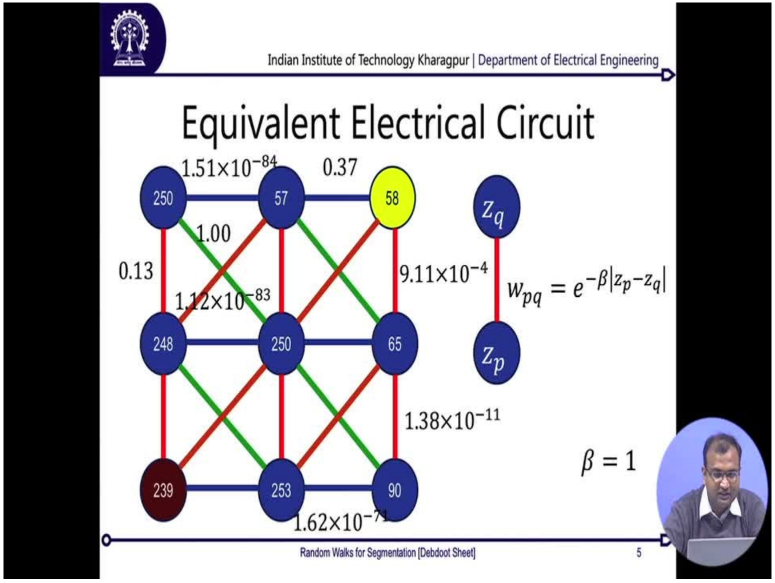

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Equivalent Electrical Circuit

$$w_{pq} = e^{-\beta|z_p - z_q|}$$

$\beta = 1$

Random Walks for Segmentation [Debdoot Sheet]

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Well there is actually another way of solving it out in a much elegant way and that is why basically treating this whole image graph in terms in an equivalent electrical circuit and from where this whole concept is that since electrons are always in a thermal (i) thermally agitated state so these electrons always follow a very random path of walking down from one point to a another point. Now assume that these two are the nodes which have some different potentials and this is a resistive connected network and you are asked to find out what is the potential at any of these unknown nodes.

Now if you are trying to solve a similar kind of problem, then you would be solving out your Kirchhoff voltage laws and current laws over here or and n by input network solution in order to find out an objective. And that is where the inspiration comes down for our analytical solver as well. So what we do is say that there are two nodes, one of them is Zp and there is another node which is called as Zq, okay.

Now in between them you have an edge which is connecting these two vertices and this edge has some sort of a way, now these are not isotropic weights each will have a different weight of its own. the other point is that these graphs are all undirected graphs, there is no directivity. So you can go from p to q you can go form q to p and the weight associated with either of the traversals is called as Wpq.

So Wpq is equal to Wqp and if you look at this particular equation that is what it gives down. If I interchange p and q there is no change in the whole equation coming down over there. Now, say that this is the condition and there is a factor called as beta which is called as the conductance phenomenon over this network and this beta is what guides how easy it is to move from one node to another or how hard it is to move from one node to other and Wpq so if this value is very close to 1, then it is easiest to move, if it is very close to 0 then it is the hardest to move.

So imagine this to be the conductance value of a some sort of a resistive network over there which is specially conductance is opposite of is inverse of your resistance over there. So higher the conductance the lower the resistance and the easier it is to move this is the analogy which you need to keep in your mind. Now from there let us assume a value of beta is equal to 1, now with that if you want to look between this node 250 and 248 you will be computing your value of this weight to be 0.13, okay.

Similarly if you are looking between this particular node and this node which has a value of 250 and 57 your conductance value comes down to 1.51 into 10 to the power of minus 84. It is actually a value which is very close to 0, you could have in fact approximated this to be 0. Now between 57 and 58 you again have a much higher value which is 0.37. Now between 250 and 250 there should not be any problem, it should be easiest to move and that has a conductance of 1, between 248 and 57 it has a value of 1.12 into 10 to the power of minus 83 which is again a value which is very close to 0 but again this value is higher than this particular value.

So the amount of effort taken to traverse from this is this is basically much more than to traverse from this to this, this is what it definitely indicates over there. And that is why we are not putting those values as 0’s. Now between 58 and 65 it is a 9.11 into 10 power of minus 4 which is obviously a value which is much higher than any of these ones. okay now between 65 and 90 it is 1.38 into 10 to power of minus 11 so this impedance over here or the total resistance which is forcing is much higher than the amount of effort to traverse in this one.

Now this is how you can keep on populating this whole graph each of this edges it should not be hard as such to do.

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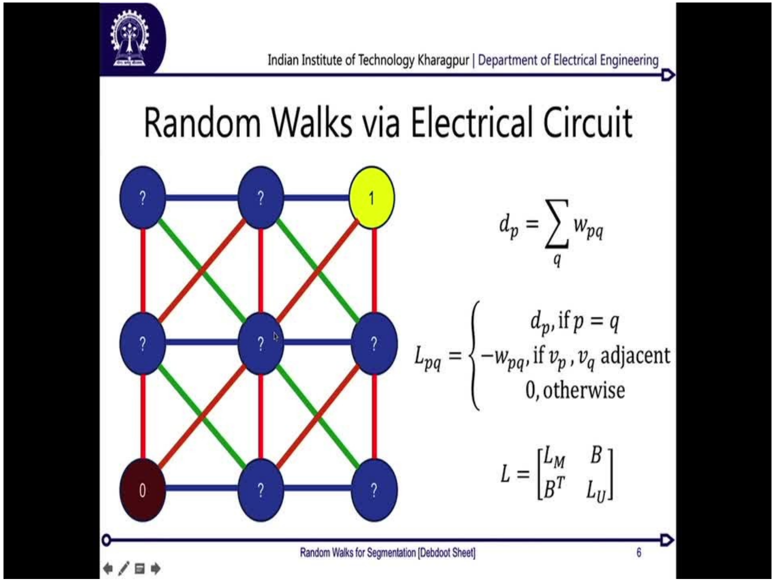

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Random Walks via Electrical Circuit

$$d_p = \sum_q w_{pq}$$

$$L_{pq} = \begin{cases} d_p, \text{if } p = q \\ -w_{pq}, \text{if } v_p, v_q \text{ adjacent} \\ 0, \text{otherwise} \end{cases}$$

$$L = \begin{bmatrix} L_M & B \\ B^T & L_U \end{bmatrix}$$

Random Walks for Segmentation [Debdoot Sheet] 6

So now once we have that, let us look into the electrical equivalent over here. so now here what we assume is that for each of the nodes which are marked down for a particular class label over there, those particular nodes will be given down a value of 1. so say we are looking at the yellow class over here so I am going to give it the voltage 1. So assume that this the resistive network over there and for each one particular class you have one node denoted ,now you connect it to a vcc of 1 volt over there, good.

Now you will be having the other classes over there as well the nodes which are connected down to all the other classes. Now each of them you connect it down to a ground voltage or a 0 volt over there and if this network is in a steady state condition all the other nodes which were not connected down explicitly or to a voltage level will be getting down some value and now whatever is the value over here that value is going down to lie in a range between 0 and 1 volt.

So obviously one good thing about this phenomenon is that these values when it is restricted in a 0 to 1 volt, that is in the probability space itself. Now, looking into another concept over here if I interchange this 1 volt and 0 volt between the nodes, I am just going to interchange the way how I am exciting this network and now from reversibility of my whole network I will be getting down some values over here which will basically be 1 minus the voltage which I had obtained in the earlier case over there.

And that does preserve this whole network in the probability space as well such that each node is going to give you a probability for each class coming down. Now in order to solve this kind of a network one way is that you can use your Kirchhoff's current laws and voltage laws and solve it out but that is obviously not a very efficient way of solving this kind of a large network. So in order to do that what we do is we start defining it via a graph theoretic measure.

Now we define a particular value which is called as dp or the degree of a particular node or a vertex with a label p and the degree of that node is basically the sum over the weights which are all incident upon that node. Now since we do not have a directed graph over here so obviously these weights are there is nothing like a signed weight assigned over here, it is basically an unsigned weight because of being an undirected graph. So this is what the degree of a particular vertex at with a label with location p will be looking like, okay.

Now from there we can define another matrix on which you will have elements called as p's and q's. So basically p is the coordinate or the or the location for one of the nodes in this graph and q is for any other node over there. So if you have some n number of nodes in graph so if it is say some 10 by 10 graphs or in over here it is 3 cross 3, so basically there are 9 such nodes over here and if there are 9 such nodes over there then Lpq so p will be varying between 1 to 9, q will also be varying between 1 to 9. So you will basically be having 9 cross 9 or 81 elements in that particular matrix.

Now each element of this matrix which is defined by this ordinal value Lpq is given something like this that if p is equal to q, then the entry is basically dp. If p and q are adjacent, then the value is minus of Wpq and 0 otherwise. So basically for non-adjacent node that is for this node to this node you have an entry of 0. For everything else it is populated like this. Now once you are able to populate this one what you will see is that with a perfect ordering or you can obviously since it populates as a matrix so you can do your translations along the rows and column appropriately without changing the properties of the matrixes over there.

Now if these changes are appropriately done, then you can divide this whole matrix into 4 sub matrixes which his called as Lm or what stands is basically the laplacian values corresponding to the marked nodes. So basically whenever this say this particular node and this particular node they are all marked nodes where you have said that they belong to one particular class over there. So all the entries for L which corresponded to marked nodes will be defined within Lm, okay. Now B will be the other side of the entry so over here what will happen is that this is the relationship between one marked node to another unmarked node, okay.

The transpose of this will be filled up over here which is relationship between an unmarked node to a marked node over there and that is why this basically comes up as a transpose of this B over there and Lu is basically the relationship between all unmarked nodes over there. So Lm will have entries something like this one to this one if there was a connection involve over there but that is not there. Then Lu will have a entry of this one to this one, then this one to this one which are all unmarked, whereas B will have an entry of this to this, this to this, this to this and this to this, this to this and this one to this one. So these are the particular kind of entries which will be present in this matrix called as L or the laplacian matrix.

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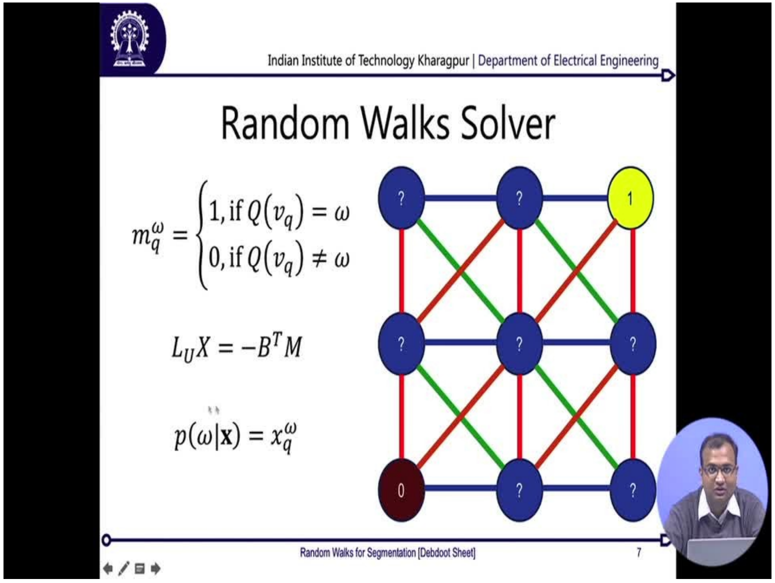

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Random Walks Solver

$$m_q^{\omega} = \begin{cases} 1, \text{if } Q(v_q) = \omega \\ 0, \text{if } Q(v_q) \neq \omega \end{cases}$$

$$L_U X = -B^T M$$

$$p(\omega|\mathbf{x}) = x_q^{\omega}$$

Random Walks for Segmentation [Debdoot Sheet] 7

Now from there how we end up doing is what we do is we define another particular matrix which an element relation as m, now how this is defined is that q is the number or the index for each of the nodes present in this particular graph over there, okay. And I am going to define it as a 0 1 relation based on for which class I am going to define it. So if I am defining mq for the yellow class, then all the nodes where this one is truly marked as belonging to the yellow class will be given a value of 1, everything else will be given a value of 0. So from this one if I am trying to do something so like I am just going to have a matrix the size of all the marked nodes over there of which there nodes which correspond to a particular class will be marked as 1, everything else will be marked as 0.

So unmarked nodes will not be part of this matrix called as m in anyway. Now from there we have a inversion equation which we can solve to get it out so say x is what we want to solve, so this Lu times x is basically equal to minus B transpose times of M this is the relation which you get by solving the earlier total systems equation. And from there what we end up getting is each value of x which is called as x at a particular node q for a particular class label over there omega is the probability of obtaining a class omega given a particular labeled value over there.

So now once we have this probability obtained for each class, next point is that you will get your class label as arg max over all of these probabilities over there as per the earlier slides which we had seen.

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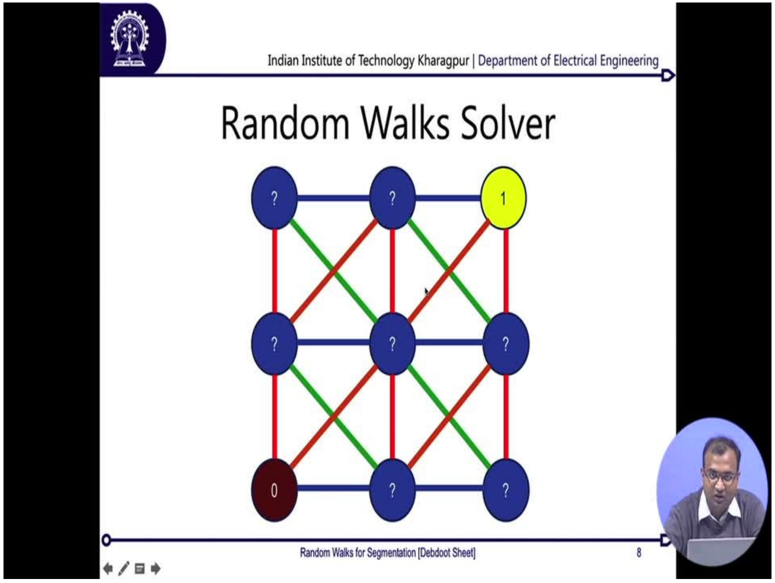

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Random Walks Solver

Random Walks for Segmentation [Debdoot Sheet] 8

Now once you are able to solve all of this together for this particular graph which we had in question so basically you have the whole graph over there, you had your weights which are created down over there, now after that you find out your laplacian matrix you rearrange the components of your laplacian matrix then you find out your m matrix over there, then you solve your probability space equation then get all the values of those x and they are supposed to content probability for these unmarked nodes over there, that is what it will have.

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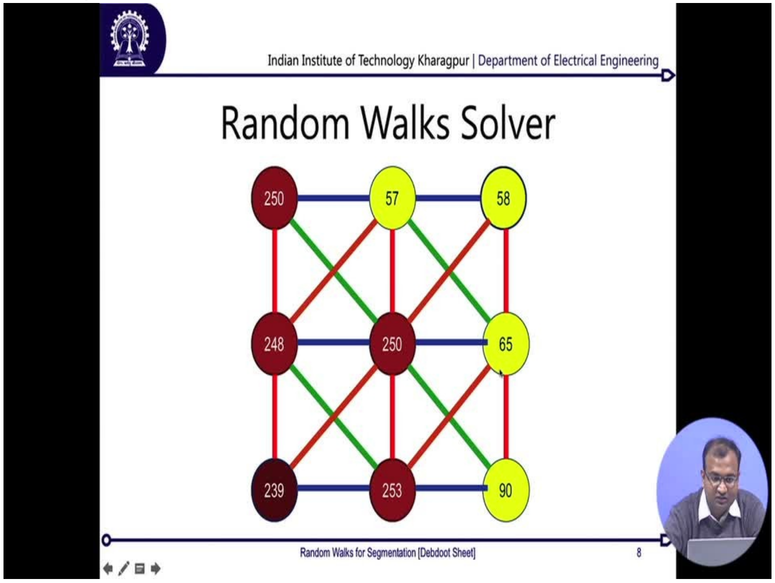

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Random Walks Solver

Random Walks for Segmentation [Debdoot Sheet]

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Now once you get the probability, it will solve out to be something like this. So these particular nodes got marked down as yellow so this particular one was already marked and these nodes where what where to be marked over there and they also turned to be in the yellow class. This node was already marked to be in the red class and then we computed them out and we found out these were also found to be in the red class and this is what your segmentation result is now going to look like. Now on the image space what you will have to do is basically re-transpose it back because each of your edge or vertex of the graph is related back to one of the pixels over there.

So if you can transpose it back to one of the pixels so you get the segmentation result for that particular pixel over there and this is how a Random Walk Solver basically works.

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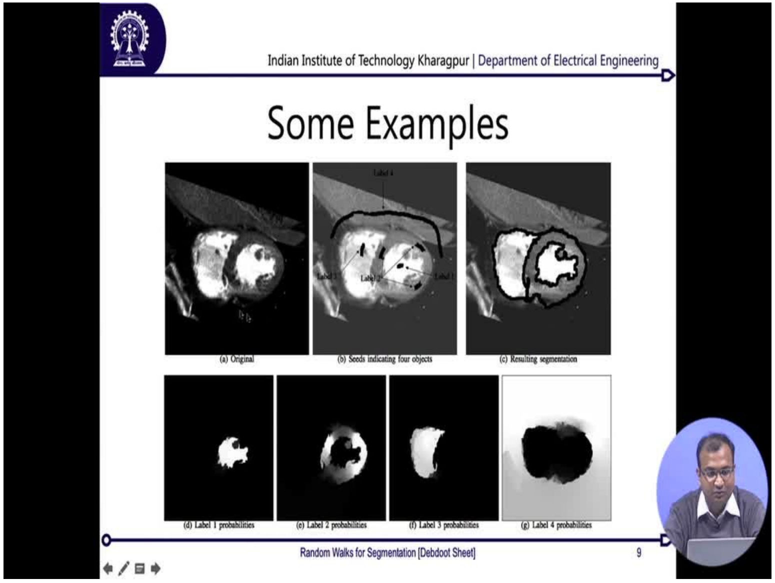

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## Some Examples

(a) Original (b) Seeds indicating four objects (c) Resulting segmentation

(d) Label 1 probabilities (e) Label 2 probabilities (f) Label 3 probabilities (g) Label 4 probabilities

Random Walks for Segmentation [Debdoot Sheet] 9

So now let us look into some of the examples over here so this is basically the MR of a beating heart over here and the point is to segment out 4 different regions as marked over here. So if you look carefully there are basically small seeds user defined seeds over there, so this called as label 1 which is in this region then there are label 2 seeds which are placed over here.

Then there is a label 3 seed which is placed over here and there are seeds for label 4 which are placed over here. The final segmentation output is what something looks something like this which is the boundary marked over there and if you look into the probability of each of them so since you have the graph on which you have obtained the probability for each node so you can recast it back to the image matrix space over there and look into the probability for each of these nodes as well.

so for each of the pixels you will be getting a probability space which looks something like this. Now if you look carefully over here you would see that this segmentation is pretty intuitionistically correct with what is present on the whole image over there and makes a perfect sense as well as giving you a very good segmentation output in the presence of a lot of heterogeneity over here as in particularly these regions and these regions where you have lot of heterogeneity and in fact over here as well.

So wherever in these kind of problems generally say like this region is supposed to be different from this region as per the labels marking over there. But if you are trying to do a region growing kind of a segmentation approach over here which is based just on looking at intensity values these two intensities would be of the same range so they would basically club down into the same region.

Whereas for these kind of problems even if you have similar looking regions but you want to discriminate them out by doing this sort of a enforcing criteria you can actually segment it out in a much better way which is a one of the strongest points of random walks for its use. The down side is obviously you need lot of manually initializations clustering and other approaches do not serve this particular purpose over here.

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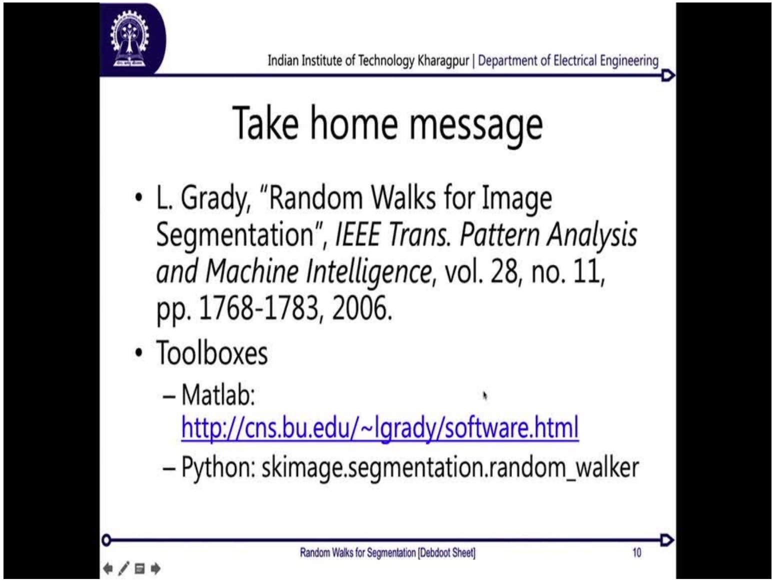

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Take home message

- L. Grady, "Random Walks for Image Segmentation", *IEEE Trans. Pattern Analysis and Machine Intelligence*, vol. 28, no. 11, pp. 1768-1783, 2006.
- Toolboxes
  - Matlab: http://cns.bu.edu/~lgrady/software.html
  - Python: skimage.segmentation.random_walker

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So with that we come to an end for this particular lecture. In order to give you more readings so you can go through this particular paper by Leo Grady which is on Random Walks for image segmentation it is a IEEE family paper which has much more detailed discussion on whatever has been presented here today with details of each of the analytical solvers over there.

Along with that you basically have two tool boxes so Leo Grady himself released out a Matlab based toolbox which you can find out on his website over there which is quite useful for segmentation of medical images and it is a very properly defined out tool kit for a lot of basic users.

And python also has its own Random Walker solver which is within the sk image segmentation routine so you can make use of this particular one as well for doing your random walks segmentation methods for medical images. So with that we come to an end and thank you.