Course on Introduction to Medical Imaging and Analysis Softwares
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Module 01
Lecture 02: X Ray and CT Imaging

Welcome, today we will be learning about x ray imaging and computed tomography and this is one of those early modules which we are going to cover or the first module on actually major of medical imaging. Now as you have seen that the way medical imaging has gone over the ages, initial was visual inspection and from there the first time was to look within the body without even having to cut the body and the first particular one which allowed to do is was x ray imaging and thanks to Wilhelm Rontgen that he really took the initiative of doing the first x-ray scan on his own hand and then then trying to do it eventually.

And from there on how it has progressed is that as images started coming down and we have lot more of images and that did more of analytics, the whole field of medical image analysis started growing. And some of the early contributions on medical image analysis which are way back from around 1980's or early 80's I actually relate it to the field of x-rays itself and that brings us to this immersive importance that we need to treat this modality with a good amount of justification and initially stand with it.

Another couple modality which goes along with planer x-ray imaging is computed tomography and this is the first one which allows you actually look into the 3d projection of every single part of your body or whereby you can actually create a virtual anatomy of your own body or any other person's body in the 3d space together.

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So we will be staring initially with how x-rays work, how so the initial topics are basically on x-ray generation, then we have this particular topic on photon matter interaction and when we say photons over here they are all x-ray photons. So the energy band in which they operate are necessarily for x rays to operate down. And from there I am going down to how attenuation works out in different inhomogeneous materials, from there we would be entering into the first topic of CT image formation which is from using multiple x rays or projection images how can you get down on computed tomography together.

So after that we enter down into one of the fundamental aspects called as Radon transform and Fourier slice theorem which actually establishes that given that you have sufficient number or technically speaking infinite number of projections available, you can actually construct each and every single point within the body doing an inversion of the whole process. From there practical CT instrumentation how does it device look like, how does it work like and from there I will be entering into certain principles of reconstruction because although I said that in theory it is possible that you have infinite number of samples and then you can do it.

But in practical situations you will never get an infinite number of samples, you will always have a finite number of samples. So reconstruction is a specific mathematical framework which allows you to have the same sort of a signal retrieved or same sort of a resolution of image retrieved given that you have much lesser number of samples than what you would be requiring over there actually.

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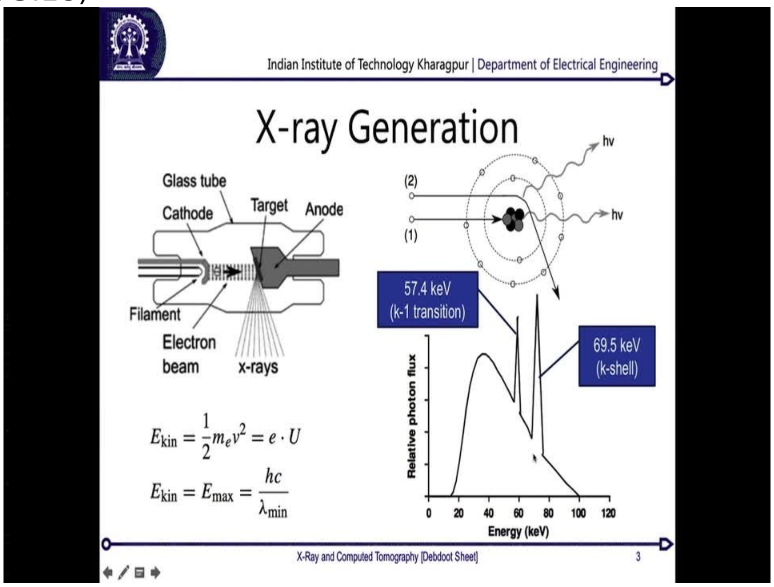

So let us start with the basics over here, the first part is on x-ray generations so how it works out is this is a bit of those first year engineering level physics where you had studied for x-rays at any point of time, so I will be just be giving you a very brief summary and a very fast walkthrough of what happens over there. So actually you will be having some sort of a thing called as an x-ray tube and this is the source from where an x-ray generate is generated. Now since it is not basically optical photon, so unlike your LED lights or your tungsten filament lamps where you just pass down electricity and it is directly generated, over here it is through an indirect process.

So over here what is happening is, you basically have a cathode over there which is heated up and then you have lot of electrons which are just flowing out. Now as these will be going and moving towards this anode which is the positively charged surface, they would be striking anode. Now as it strikes over there, it will be emitting x-rays and generally these x-rays come down in a part which is different from the part in which it is coming. And to make it even simpler that I want exit path coming over there, because I wanted to focus at some different part of the body.

So there is basically a 45 degree inclination angle over there. So now if you have something striking at 45 degree, you will get a reflected and the angle between this reflected and the incidence ray is 90 degree. So that is how you get an x-ray coming out over there, but the other difference is that unlike say lasers they are not a very concentrated beam, but it is a very diversion beam which is coming out over there which actually has an advantage to our case, because now we can actually create very easy magnifications coming out of it and again since it is emitted from one single point, so it behaves as if a point source of light which is emitting isotropically in all directions.

So you put down an object over there, you get a magnified view on a film which is placed behind it. So an x-ray is generally generated when say this electron either gets absorbed within the nucleus, so that is a highest energy of x-ray which is possible to be generated or the other one is that it just slows down while passing down through this atom. So necessarily this electron will get slowed down only when it strikes a heavy metal, otherwise it is not going to get slowed down, it has to passed down to multiple number of balanced electrons layers over there and only then it can slow down.

Now since its kinetic energy is slowing down, so this difference of the energy has to be dissipated in some way and that is basically a unit of x-ray photon which is dissipated over there. Now what happens is how it is guided is through this ones. So say that you have a electric so say basically the electron is travelling at a velocity of v and that is again due to the electric field U over here. So the total kinetic energy of this one is basically the electric field times the charge of the electron over there, okay.

Now, it is if it is perfectly absorbed over there, then everything is supposed to be emitted in terms of a particular x-ray photon which has a wavelength of lambda minimum, okay or which will specify which will actually correspond to the maximum energy of the photon which is, so if you have the minimum wavelength, you have the maximum energy, that is how it is related because it is inversely proportional. So this will have the highest frequency, minimum wavelength and the highest energy coming out if say everything is absorbed over there.

Or if it is partially absorbed then the value of the actual x-ray photon energy is much lower than e minimum e maximum over here. So the wavelength over here will be something more than the lambda minimum wavelength over there, okay. So as a result what you will have is say that it strikes at any point of time, so over here we have something with tungsten body over here.

So this anode if this is made out of complete tungsten and if this one strikes over here the electrons strikes and the x-rays are generated, then you will get a relative photon flux curve which looks something like this. Now this y axis over here is basically an arbitrary unit curve, so say that you are counting over a long duration of time the energy of the photons and the number of photons coming out for that specific energy, you will get down a curve something looking like this.

Now over here you would see that there is some sort of a model maxima over here and there are two peaks which are greater than the modal maxima over here. Now they correspond to very interesting facts, the first one is basically the k shell energy peaks, so this is about that say that this electron goes over here and just sits in the kth shell. If it just sits in this kth shell, then it is going to release all of the energy which it was having over there and that corresponds to this energy of 69.5 kilo electron volt, okay.

Now, the other one corresponds to a transition between k to the k minus 1th shell over there, okay. So say it goes down and sits in this particular one, then it is going to release this much amount of energy and that is And these two bounding shells over there are the ones in which you have the maximum amount of electrons transitions happening at a point of time.

And for that specific reason, you get these two peaks over there, okay. Now there is another interesting thing which actually happens because you have two different peaks at two different energies, so they when we come down later on to the next module which is on photo matter interaction and then look into what happens with biological tissues, you would find out that just because of corresponding to different energy bands over there, you can actually very easily discriminate different types of matter much easily over here.

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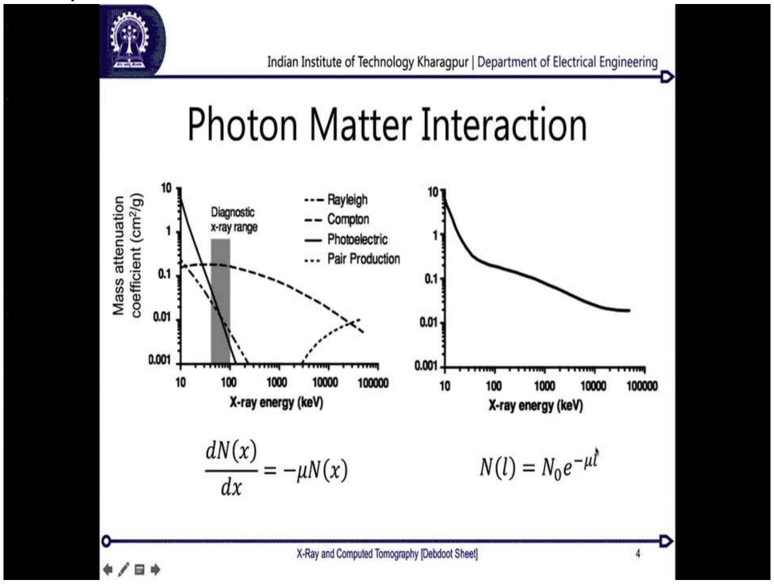

So keeping this in mind, let us move to the next part over there. So say that there was a unit of photon which got generated and then what can happen to this photon at different energy levels. Now as you see over here, we have this x axis which is basically the axis of x-ray energy photon, so say there was an energy photon at this particular one and then we look into what is the total mask attenuation happening at that particular energy level. And now for different levels, you have a different attenuation coefficient going down.

So if you look carefully the diagnostic x-ray imaging range which lies around this 100 kilo electron volt, so this is 5200 kilo electron volt it is a standard range for diagnostic x-ray imaging in which you would see that the Rayleigh catering affect is actually quite nominal and it keeps on decreasing, you would see that (photoelas) photoelectric effect is also quite nominal and keeps on decreasing and the maximum effect which takes place is actually a Compton scattering, okay.

Now interestingly, since you have this Compton scattering and no effect of the pair production coming down over there, so this is something which will be used in molecular imaging which we will be studying later on as to what happens with this pair production, that is also another interesting way of doing it. So here it is just a Compton scattering which dominates over here and most of your effect is due to the Compton scattering within the tissue energy over there.

And the whole stuff builds up something like this that if your photon gets an absorbed heavily, so that is due to some heavy metal compounds over there, so you can look into them. Now we will come down to what in the body is sort of a heavy material which will have a higher Compton scattering and what is not a heavy material which does not have that much of a higher Compton scattering. So cumulatively, including all of these average together this is the kind of a graph you will be getting down for energy absorption at different energy levels over there.

And as you see that if you are much higher energy, your absorption coefficient actually go much lesser than one and they are tending towards 0.1 or even lesser than that. That is why a very high energy x-rays are not something which is fruitful for diagnostic imaging. You need very low energy x-rays over there, and the other problem is that if it is say very low energy x ray something around over here when it is 10 kilo electron volt, then you will just be having dominance of this photoelectric affect over there.

And so it is not actually going to emit out any photons which go out. So most of it is just going to produce electric fields over there. So that is why we cannot use very low energy x-rays in any way for diagnostic imaging and this is the ideal zone where we would like to have it. So typically, as it passes down through matter you will see that it is guided by this sort of a decay equation.

So how what it matters is say that at a particular point x, if there is a small volume over there which is called as dx, okay in space then the amount of photons which are absorbed over there, so this differential amount of x-ray photons will be absorbed over there and that is proportional to this absorption coefficient mu times the actual density of photons which are arriving over there. Now if you solve out this equation, what would end up is that you get that say there is a small length which photons have traversed and say this is called as called as L.

Then the total amount of photon energy which was absorbed in this zone L is actually equal to the product of the number of photons which are incident onto this length L over here and then exponential times of negative of this log decay and there is and this mu over here is basically the decay coefficient. So as it passes through a longer length for a shorter decay coefficient it would have the same amount of attenuation as say for a shorter length for a higher decay coefficient. So mu L together is something which guides, the whole attenuation which comes over there.

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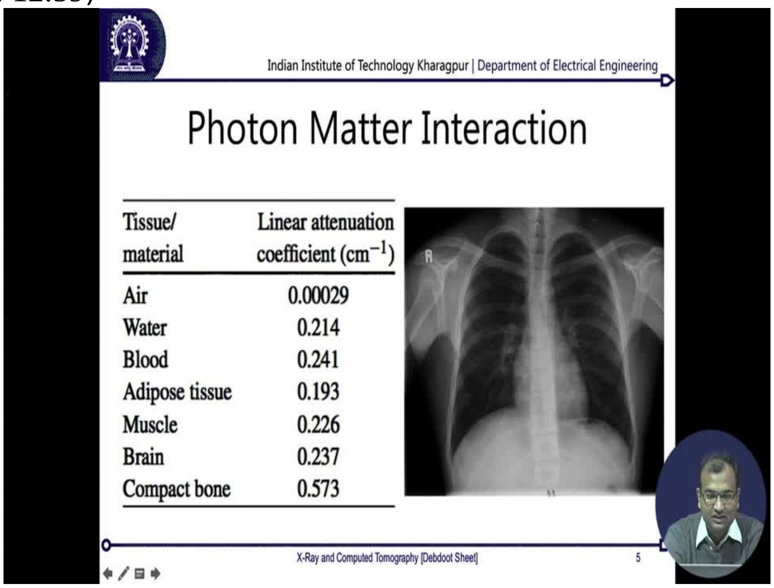

So if we look into for certain very standard materials you would see that in air it basically has the least attenuation coefficient and that is why basically for over here you would see that when you are going to do an x-ray imaging to basically have the source you are standing in front of the source and behind you is the detector and nothing is replaced in any kind of a closed proximity, you can have the good amount of segregation and separation between them.

Because air is not going to do any sort of an attenuation. For water it is about 0.214, for blood it is 0.41 which very close down to water as well, adipose tissue of the subcuticular fat which you have, so just the fat below your skin that has the least attenuation amongst everything in the present in the body. And the maximum attenuation is something which is caused by compact bones, okay. So bones of major bones within your body and if you look through them, you would see that the compact bones cause almost twice or even more attenuation than blood and water and brain or muscles together, okay.

And that is the reason when you have this sort of a chest x-ray coming down, your bones are always brighter, okay. Now the point is, I also see this part bright, now this part over here which is my lower abdominal which is my upper abdominal part over there, so this is just below the diaphragm of my chest. So these are my lungs, this is where my heart is present and this is where my whole gastric cavity is present.

Now most of our gastric cavity is made out of adipose tissues and muscles together over there and although you have a lower attenuation coefficient but the total length which the x-ray has to traverse which is the width of your body, so if you are somebody who is a very healthy person, then you have a much wider width which is to be covered.

So your total attenuation over there is high, that is why you would see that whatever is the attenuation for the bones, say for your rib cage and for your arm bones over here for your collar bones, the same kind of an attenuation is also reflected over here, so from one projection x-ray you can definitely not never make out that whether this is a bone or this is some other structure until and unless you have a very good understanding of the anatomy. Just looking at the grey scale intensities it is not possible to do that.

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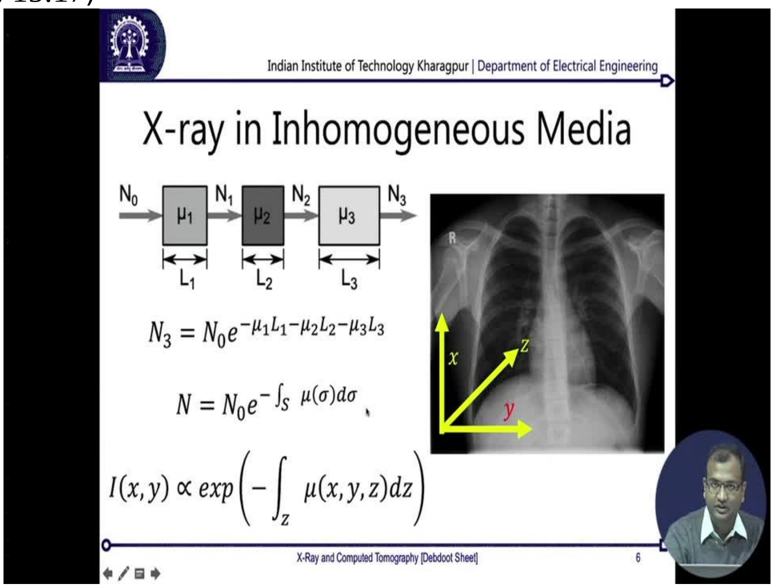

Now from there we enter into a something like very interesting. So I have been speaking out about all of this homogenous media till now. Now say that it is passing down through some heterogeneous media, then how will you do it? So say there are so let us take this particular equation, over here what you have is basically there are three different blocks which have three different attenuation coefficients mu1, mu2, mu3 and each of them have three different lengths as well, L1, L2 and L3.

Now if we look through that your total attenuated x-ray which is n3 which comes out over here is something which is guided by this sort of an equation from, okay. This is from a linear understanding over there. Now the question is that if I do not have this sort of a thing because in our body we cannot divide it into piece wise blocks in any why. What I have is just a continuous function over space, okay. If that is so then I can write down my continuous function of attenuation on space as say mu of sigma and any point in space called as with sigma with a volume of d sigma, okay.

And a particular line which the x-ray follows over there is given by this line equation s, okay. If that is the situation, then this is what will be my total attenuated value of x-ray which is coming out at the detector. Now given that I know N0 which is my x-ray intensity originally which is coming out from my x-ray tube and I know that because I know what is the kilo electron volt of x-ray so what is the total kilo volt of potential difference I am creating between the cathode and the anode that will give me my total energy over there.

From the total energy I can fit it out at actually onto my curve over there and find out which are the two basic bands of x-rays which are coming down or what will be the relative intensities between them. So once I know this one, I will be able to solve this one out. So on a 2d space, say this is the whole space which is divided into x, y and z coordinates. So this z is basically coming out of the plane, it is outward projected. In that case, my image the intensity of my image which is I(x, y) is basically proportional to the exponential of negative of this whole quantity over there, okay.

So that is how you will be able to create this sort of an image coming down over, okay. Now one curious question which a lot of people ask is that if this is the situation, then mu or sigma (x, y, z) dz so this mu is much higher for my bones, okay. So e to the power of minus of something high value is actually a much lower. So your bone should actually be appearing darker whereas your air should actually be appearing in white or brighter shade, but that does not happen because this image is basically a negative image.

So what happens classically is that you expose a film and once this film is exposed so all the regions where which were heavily exposed, so they correspond down to this black regions over there, because there the silver collide basically solidifies over there and not much of light can pass down when you are seeing into the image and that is why you again get your bones in bright which is what is desired on the clinical side as well.

Now from this, though we having known this one and the total attenuation along a line, we take all of this information together in order to get our new modality which is called as computed tomography.

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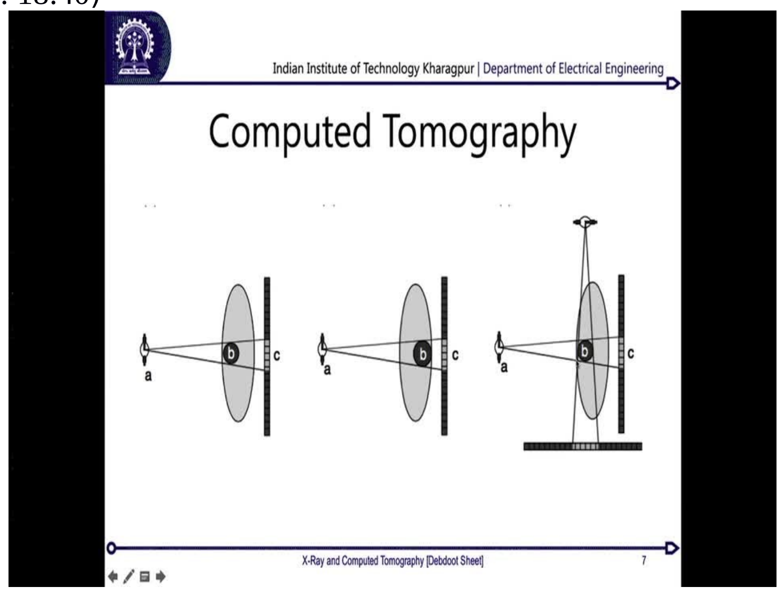

Now look into one thing that say you had this object over here and you had a bony structure called as b over here and then I had my x-ray source and I was projecting it onto this film where it has casted this small amount of white region which is c, okay. Now in the other scenarios say this b is moved down and b is made even bigger, okay so the radius of this b is much smaller than this b over there. But still I would see that it is going to cast down the same amount of shadow over here.

And for that reason, just by looking at c you will never be able to discriminate as to what is the total thickness of b and where is b located in space. You just know that it is you can just resolve along this direction, but you cannot resolve along this direction in any why and now computed tomography comes out with that particular idea that I need to be able to resolve along both the directions over there together. Now say this since this is a film, this c is a film so that is obviously a 2d structure which is orthogonal to the plane of the screen present over here.

So you can discriminate anything which lies on the plane of the film, so anything on this one is easily discriminate able, but anything which lies just in this particular plane which is the line along with x-ray is passing you are not able to do until you get down two different projections over there. Now if I have two different projection I should be able to solve it out, right? It is a very straight forward problem that I can project this one out, this one out and then I will be getting down an area over here which will be doing.

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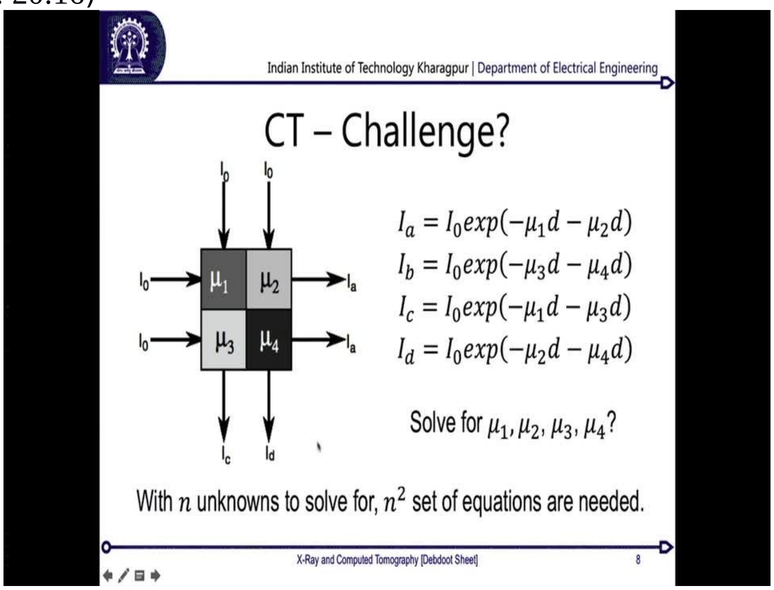

So let us look into this as a linear algebra problem, so say that I have a whole in which I have a four block over there mu1, mu2, mu3 and mu4 are the coefficients of each of these blocks and I pass down the same intensity of x-ray through it and I whatever I get down I call it as Ia, Ib, Ic and Id, okay. In that case these are related down by these kind of equations to the other. Now once I have this one, the question is solve out for mu1, mu2, mu3 and mu4. Now easily looking at it basically there are four unknowns over here and four equations, so you should be able to solve it out, it comes out that easy.

But unfortunately, these are all dependent sort of equations, so if you (tig) if you take arrange this one into a rank order matrix over there you would see that the determinant of this one will never be coming out as non-zero, you will get a zero determinant equation which makes this as a unsolvable set of equation. So two equations are basically dependent on other two equations, that is what comes down. So if you do Ia plus Ib you will get the same as Ic plus Id, that is interestingly what happens over here.

Now the problem is that we basically have n number of unknowns to solve over here and we will need some n square number of equations instead of two n number of equations which we are getting over there, since this is just a two plus two block so it becomes hard to realize. Now if you increase this one to say make it a two plus three block over there, so then we will be having basically six such unknowns and we will need 36 sets of equations, whereas if we are taking just two projections we are going to get down only 12 set of such equations coming down over. Now from that we will never be able to make out the whole problem.

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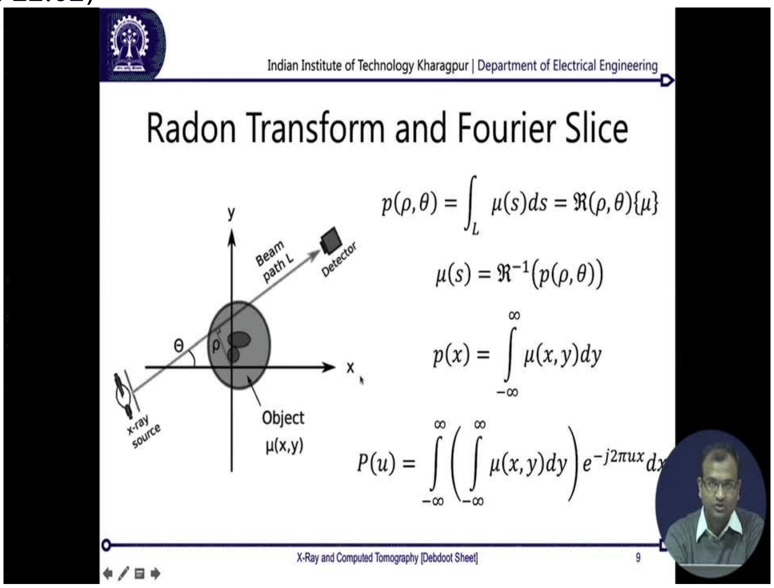

So instead of that, what we want to do is something like this. So say that I have my whole body over here which has different attenuation coefficients at different spots and I have my x ray source such that I can actually align my x-ray source in some way, okay. So that can be a theta alignment which goes down along this x-ray source and I have my detector over here. Now as such what I can write down is if this is the whole line equation which passes down from my source to the detector at an angle theta, then my line equation can be written down in terms of L, such that my total attenuation along this line will be something which is guided by this whole thing.

So or that is basically the total attenuation which encounters along this line, okay. That thing interestingly is also called as radon transform according to the classical notation of projection geometries as well. Now what this radon transfer does is basically it integrates all values which come along a particular line, it is just a whole line integral. Now since this line is defined by row and theta, so row is basically the distance from this point to this point and theta is sorry the row is the distance from the origin of the coordinate system to my line and theta is basically the orientation at which my line is.

So if I draw another line say this one that is also at a distance row but then my theta is different for that particular line. Or say that I have a line which is casted parallel to this line, I have a different row but the same value of theta coming down and there will be different entries over there. Now somehow if we have this equation it should definitely be possible that if I am able to invert my radon transform I will be getting down my mu s. That is what it says.

So if that is the scenario, then let us look into something like this say that I have a line which is parallel to the y axis, okay. So this p(x) over here is just a line which is parallel to the y axis so your row can be any value but if theta is always going to be 90 degree as such. And then let us look into what is the total integration along this one and this is what it is going to stand over there. And that is the scenario let us look into the Fourier transform of this u of this quantity called as p(x).

So p(x) will be something which varies along x. So you have a 1d signal sort of acquisition along x and then I want to take the Fourier transform of this 1d signal acquisition over there. So the Fourier transform will again be another integral with e to the power of minus j2 pi ux where u is the frequency along the x direction, okay. So this is what comes out by talking down the Fourier transform along that one.

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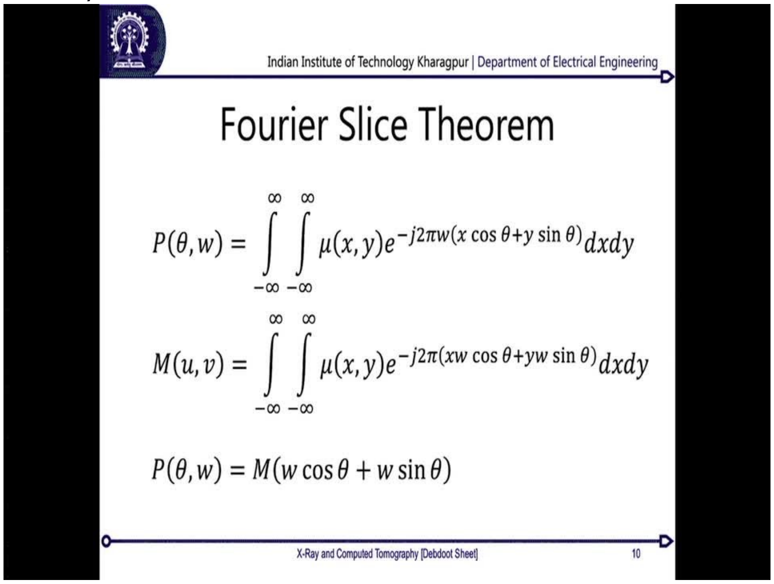

Now from there we can actually generalize and come out to an idea that say that line is rotated at any particular angle theta so my Fourier transform will appropriately be rotated over there as well. And then it is translated by a factor called as w on the frequency domain itself, okay so if that is the scenario then in total I get something like this one which is the Fourier transform of my projection at an angle theta and w. So w corresponds to that row factor which we have over there on the actual projection like equation, okay.

Now on the other side of it say that I have this mu xy together given down and I want to take only the Fourier transform of this whole image together over there. Now for that one I would (get) interestingly you would see that both the Fourier transform look amazingly equal to each other and this actually allows us to some sort of inevitability which says that if I am able to just do a inverse Fourier transform of my radon transform over all the of the Fourier transform of radon transform I should be able to get down these mu s xy as well in 2d space.

And that is why just doing a inverse of the Fourier transform of my recordings I should be able to get down my CT as well and that is what happens in a CT reconstruction, basically.

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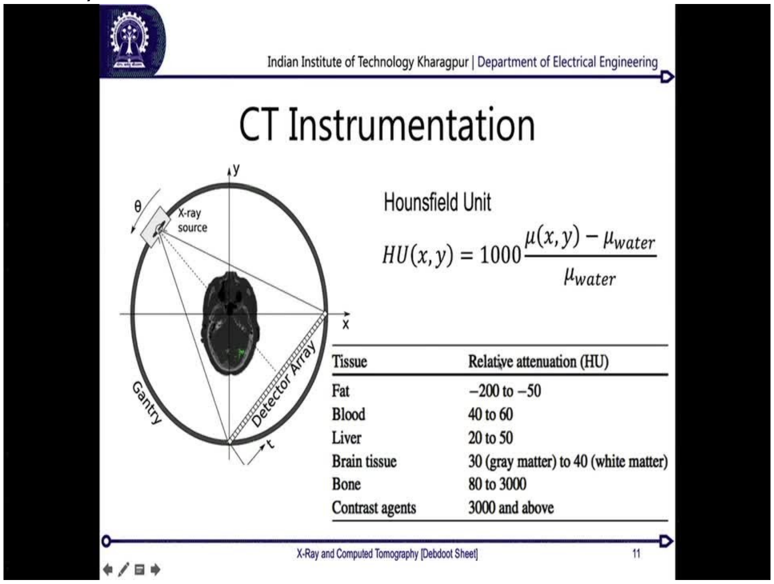

So now that we have known the math, the next part where we need to understand is the instrumentation of how to acquire these images. Now one particular point is we were solving all of these in terms of that there is one line which is joining the source to the detector, but then in just the previous slides we had seen that for an x-ray it produces a very divergent beam, it is not a very focused single beam which comes out.

Now if it is divergent obviously it is not going to be one single one were you place one detector array and do it, but the best way is basically you can place an array of detectors and each has a different line equation which is being solved out so that you get the whole projection coming down to you, okay. This you can do it on a 2d space. And now what you will have to do is basically keep on rotating this pair of source and detector array.

So it keeps on rotating in this way and you will get down multiple projections in which you are going to get the whole thing and then from each of these projections you can actually backtrack and iteratively reconstruct your whole object. So that comes down into the CT reconstruction which you are eventually going to do. Now, for any kind of a CT image we do not generally represent it in terms of the mu xy's but it is normalized by whatever is the attenuation for water and that is defined as a Hounsfield unit.

Now this is the unit in which all CT images whichever you are going to have a deal with will be shown down. So, now commonly for a certain of these tissues you have something like this. So fat has a Hounsfield unit somewhere in minus 200 to minus 50 and that is for the reason that fat has a attenuation which is lower than water. That is why you get a negative number over there.

And high and contrast agents or bones (the) contrast agents have a value which is higher than 3,000, bones have a value between 80 and 3,000. So contrast agent has something like metallic suspended particles which offer attenuation which is much higher than the bones. So for blood you have between 40 and 60, for liver you have between 20 and 50. So from this one you can easily make out that on a CT image the brightest of the structures will be bone and contrast agent and the darkest of the structures will be fat which have a negative.

And unlike to your 8-bit integer images where you have just positive numbers these in 16-bit representations are basically signed number representations. So you have negative numbers, you have positive numbers as well because Hounsfield unit varies between negative and positive in together. So for brain around the range of 30 is grey matter, around the range of 40 is white matter and that is for that reason it is very hard to discriminate between grey matter and white matter so easily in the CT itself because you have a very low contrast window coming down between grey and white matter.

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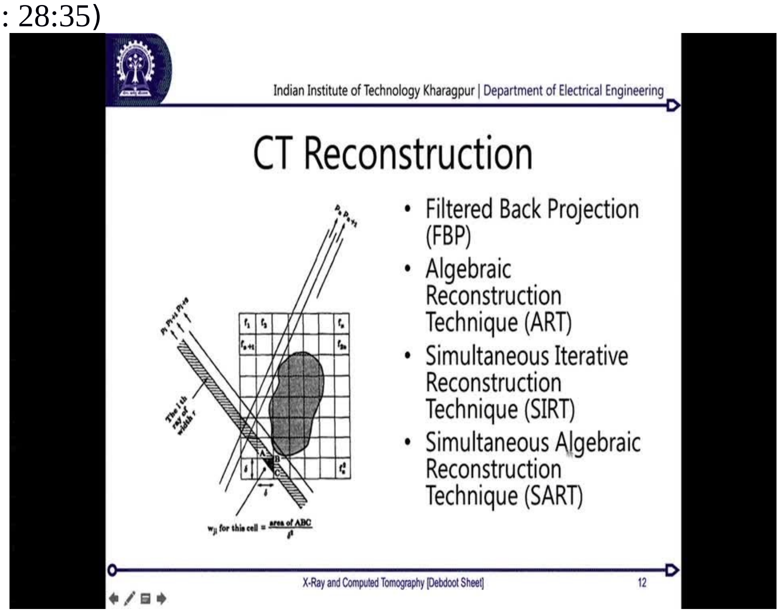

So now that this is your instrumentation the next part is obviously to look into how this reconstruction framework works out. So what we have is we assume that your whole object which is being imaged is divided into some sort of a grid of different points and each grid point over there has its own attenuation, okay. Now, say that there is a whole beam which is passing through this one we will be looking into what is the projection over there and what is the total radon transform.

And now using all of this, it is basically a set of linear equations which just have to be inverted and filled up within this grid and then you can eventually solve out. So one way of doing it by is by the Fourier transform way which is also called as filtered back projection so what it does is at one particular projection you back track this whole thing.

So you basically get down the inverse of the Fourier transform of this one, now what you have to do is you have to smear it back into this length of the line and then you take all of the multiple projections over there and keep on subsequently smearing them back and together at the end of reconstructing using all of these projections you will get down your whole image which would be looking something similar to the actual attenuation map which was supposed to be there at every different place.

Now the down side is obviously since it is in the Fourier domain, so you have a higher computational complexity associated with Fourier domain processing. Other down sides are that you have since your Fouriers are again not infinity Fouriers over here. Just doing a fast Fourier transform or a discrete Fourier transform over there. So you are basically limiting the number of frequencies you are taking, so with the limited of frequencies you are losing down high frequencies and so you will have the same affects with ringing or missing frequency problems which are there present in images as well.

Now, in order to get rid of that and make techniques even simpler there are techniques called as Algebraic Reconstruction Techniques which relay on iterative solvers for inversion of linear equations or the set of equations which we have seen initially and how to solve it out. So they are being solved out using these kind of techniques called as ART. Now the next one is Simultaneous Iterative Reconstruction. So what it does is simultaneously it keeps on iteratively reconstructing and then again using the reconstructed map as some sort of a prior information for reconstructing in the next one.

And then the most advanced one is SART or the Simultaneous Algebraic Reconstruction Technique. So it has an iterative always as well as a component of the algebraic technique together in order to solve this whole thing out. So that is how we reconstruct down CT images and we are almost at the end.

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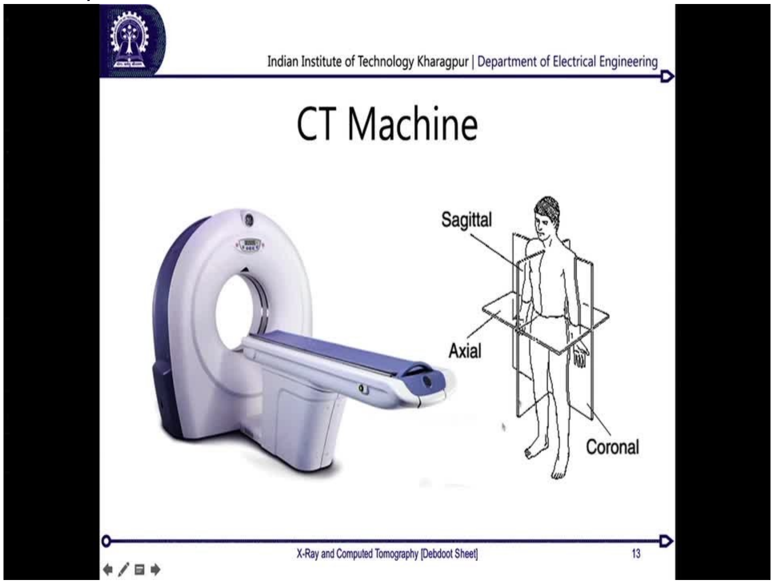

So finally this is what a CT machine actually looks like and now in general for imaging we follow down certain conventions. So a human body is basically divided into 3 planes as in for real life object you have 3 axis x, y and z. So in case of medical images it is basically 3 orthogonal planes in which it is defined and that is how because on when you are looking at those images you will be seeing one particular plane over there of 2d slice. So that is why this particular plane which crosses from your left hand side to the right hand side is called as Coronal and you can move the Coronal plane from front to back or back to front to look into as if you are lying flat over there.

The next one which is Sagittal plane is a front to back plane, so this divides your body into left part and right part and you can move it between left and write. The other one is called as Axial plane which is something which divides your body into top part and lower part, the upper part and lower part and then you can move it up and down basically.

Now whenever there is an image being taken down this portion is going lie flat over here, so obviously you are Coronal plane is something which is aligned to the bed over here your Sagittal plane is something which is orthogonal to the bed over here in this direction and your actual plane is something which is aligned along with this ring of the detector.

So the rest of the x-ray source and detector everything is located within this ring over here and how it works is this keeps on rotating and this body keeps on this bed keeps on moving front and back so that you get down different slices over there and now you start down all the slices along the Axial so you get down basically Axial slices you stack all of them and you get your 3d volume. That is how a CT image is basically constructed.

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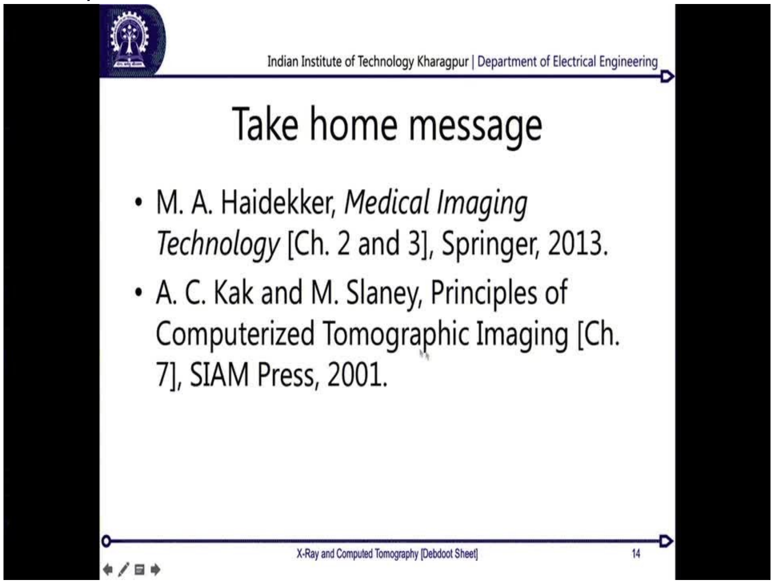

If you want to read more about them, you can follow this particular book on medical imaging technology by Haidekker and chapter 2 is on x-ray imaging, chapter 3 is on CT and CT reconstruction. The there is another very famous book on CT reconstruction by Kak and Malcolm Slaney and so the book is called as principles of Computerized Tomographic Imaging and chapter 3 is what deals with algebraic reconstruction techniques as a whole. So with that we come to an end for x-rays and CT imaging and so thanks.