

**INDIAN INSTITUTE OF TECHNOLOGY
KHARAGPUR**

**NPTEL
ONLINE CERTIFICATION COURSE**

**On Industrial Automation and
Control**

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**Topic Lecture – 11
P-I-D Control**

Good morning and welcome to lesson 12 of this course on PID control.

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So as usual before starting the course we will review the instructional objectives.

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Instructional Objectives

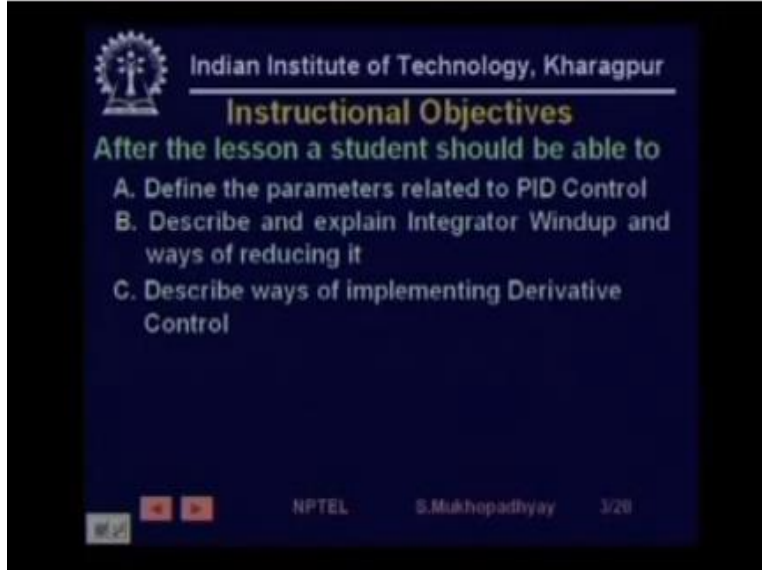
After the lesson a student should be able to

A. Define the parameters related to PID Control

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And these are firstly that we will be related will be will learn how to define the related parameters of PID control in an industrial context.

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Instructional Objectives

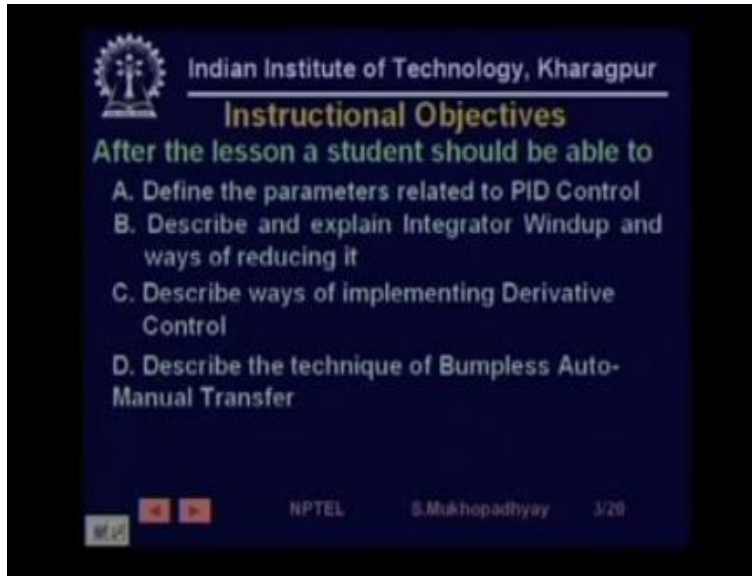
After the lesson a student should be able to

- A. Define the parameters related to PID Control
- B. Describe and explain Integrator Windup and ways of reducing it
- C. Describe ways of implementing Derivative Control

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Secondly we will describe and explain in detail about a phenomenon which many times occurs with PID control known as integrator wind up and the ways of reducing that we will describe various ways of implementing the derivative control part we will also describe the --

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Instructional Objectives

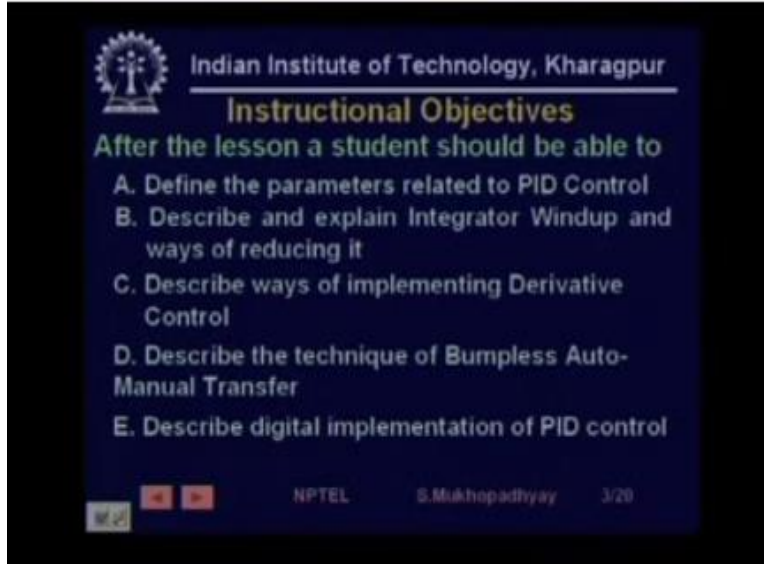
After the lesson a student should be able to

- A. Define the parameters related to PID Control
- B. Describe and explain Integrator Windup and ways of reducing it
- C. Describe ways of implementing Derivative Control
- D. Describe the technique of Bumpless Auto-Manual Transfer

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One technique of you know bump less auto manual transfer that is when the control is transferred from auto to manual or manual to or to how so that it can happen without any shock to the process and finally we will describe digital implementations.

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The slide is a presentation slide from NPTEL. It features the Indian Institute of Technology, Kharagpur logo in the top left corner. The title 'Instructional Objectives' is centered at the top in a yellow font. Below the title, the text 'After the lesson a student should be able to' is written in a light blue font. A list of five objectives (A through E) follows, each preceded by a letter and a period. The objectives are: A. Define the parameters related to PID Control; B. Describe and explain Integrator Windup and ways of reducing it; C. Describe ways of implementing Derivative Control; D. Describe the technique of Bumpless Auto-Manual Transfer; and E. Describe digital implementation of PID control. At the bottom of the slide, there are navigation icons (back, forward, search, etc.) on the left, and the text 'NPTEL S. Mukhopadhyay 3/20' on the right.

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Instructional Objectives

After the lesson a student should be able to

- A. Define the parameters related to PID Control
- B. Describe and explain Integrator Windup and ways of reducing it
- C. Describe ways of implementing Derivative Control
- D. Describe the technique of Bumpless Auto-Manual Transfer
- E. Describe digital implementation of PID control

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Of PID control so in other words we are going to look at various practical aspects of PID control today so let us begin with the PID equation.

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PID Control


$$u(t) = K_p e(t) + \frac{K_p}{T_i} \int e(t) dt + K_p T_d \frac{de(t)}{dt}$$

K_p : Proportional Band
 T_i : Reset time (Mins/repeat)

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This is the PID equation which we have seen in the last lesson also where K_p is the proportional gain or sometimes we this is not proportional band as written but it is proportional gain we will but a very similar parameter called proportional band is also used in the context of PID controllers we will see soon how it is related to the proportional gain next is the parameter TI the parameter TI here which is called the reset time and expressed in a peculiar sounding unit called minutes per repeat ext is the derivative time.

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PID Control

$$u(t) = K_p e(t) + \frac{K_p}{T_i} \int e(t) dt + K_p T_d \frac{de(t)}{dt}$$

K_p : Proportional Band
 T_i : Reset time (Mins/repeat)
 T_d : Derivative time (Min)

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The derivative time here note the time units of minutes these are rather unusual it may seem rather unusual but remember that typical chemical processes have time constant of the order of minutes so these times are often expressed in minutes.

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PID Control

$$u(t) = K_p e(t) + \frac{K_p}{T_i} \int e(t) dt + K_p T_d \frac{de(t)}{dt}$$

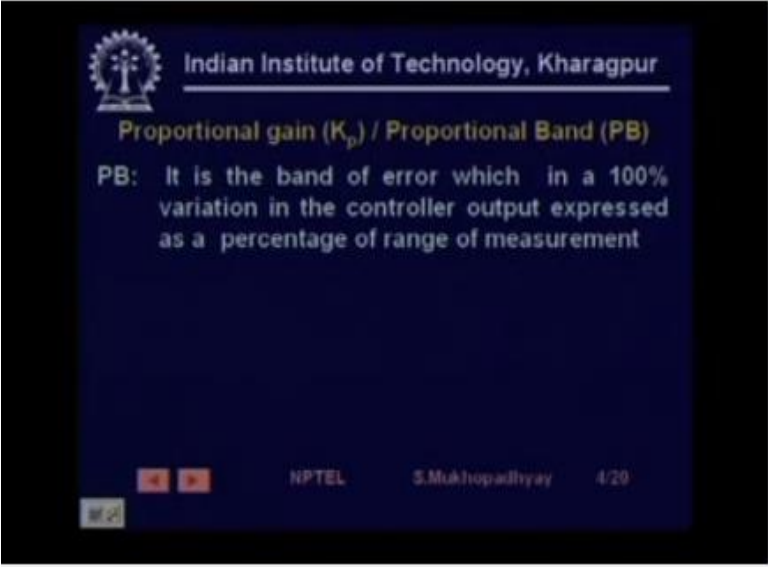
K_p : Proportional Band
 T_i : Reset time (Mins/repeat)
 T_d : Derivative time (Min)

"Text Book Version" : Aström

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This is as very well-known control scientist Karl a Astrom says a so-called textbook version of the PID control equation as we will see in this equation this lesson that there are various modifications that you have to do to this equation before it can be implemented so let us first go about defining the various terms so we first define proportional gain.

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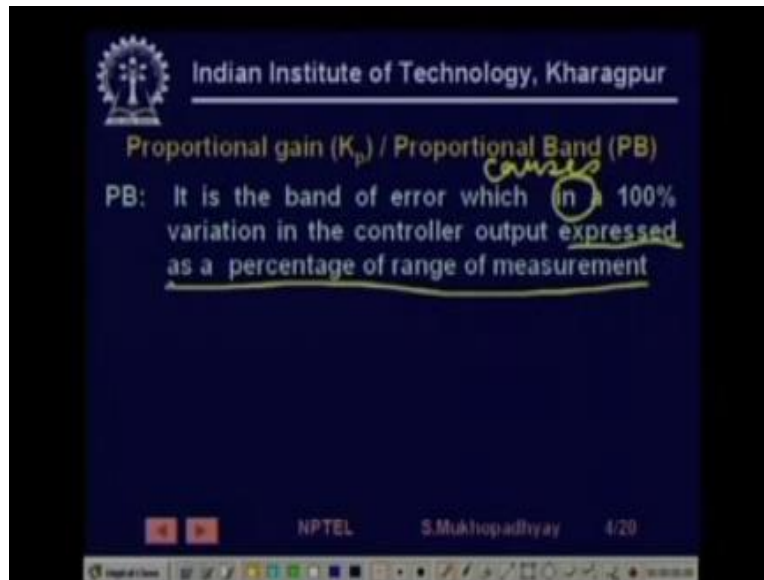
Proportional gain (K_p) / Proportional Band (PB)

PB: It is the band of error which in a 100% variation in the controller output expressed as a percentage of range of measurement

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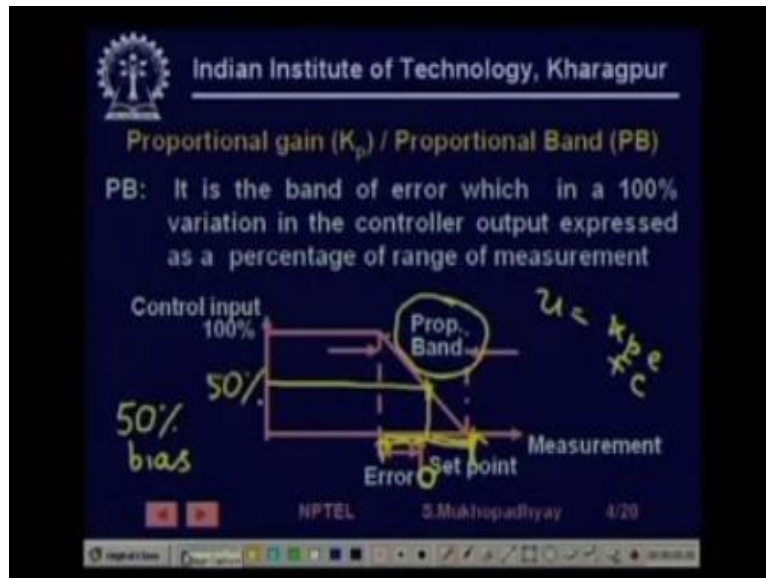
Or proportional band proportional gain is well known it is K_p which is $\Delta u / \Delta e$ or u/e while proportional band this term is new.

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And it is defined in just the inverse way so proportional band is defined as it is the band of error which in a hundred which causes which causes a hundred percent variation in the controller output and generally expressed as a percentage of the range of measurement so that is the definition so it is in an inverse way where gain is you by E here we are defining PV as the band of error which causes a hundred percent variation in the in the controller output or the manipulated input to the plant right so in that sense it is a it is the inverse of K_p .

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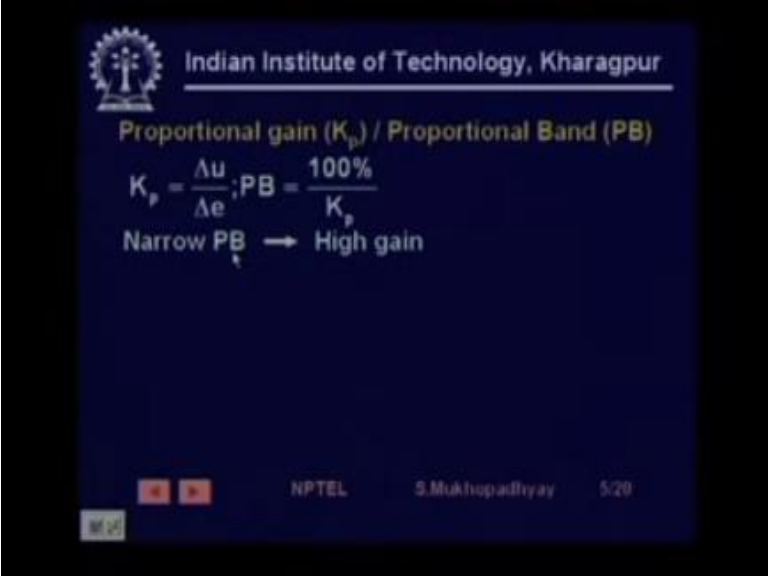


So look at this diagram will clarify matters further so here look at the controller input and suppose this is the set point currently the set point is set here okay so if the measurement or the output is the measurement of the output could be anywhere in this zone so for very so it will cause various kinds of error and if you use a proportional band then as the error will increase the output will increase in this case the proportional controller actually has a 50% bias which means that.

When the error is 0 there is still a 50% output of the controller sorry this line is getting so this is typically set at 50% so there is a so the controller output equation is actually given as $U = K_P$ into $e + +$ a constant term so plus a constant term C and this constant term is actually 50% so when the e is 0 still you get 50% output because otherwise they will always be a steady-state error as we have seen in the last lesson.

So what happens is that as the error changes to this side or to this side the output decreases or increases and if the error changes from here to here the input to the plan increases from 0 % 200% so this is the band of error which causes the causes and output a variation in the controller output from 0% to 100% and this is the proportional band right so look at let us look at an example.

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Proportional gain (K_p) / Proportional Band (PB)


$$K_p = \frac{\Delta u}{\Delta e}; PB = \frac{100\%}{K_p}$$

Narrow PB \rightarrow High gain

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So while K_P is $\Delta u / \Delta e$ proportional band is defined as one hundred percent by K_P so you can easily find out that this gives the error in percentage which will cause a 100% input change obviously an arrow K_P means low value of K_P implies a high value of get an arrow.

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Proportional gain (K_p) / Proportional Band (PB)

$$K_p = \frac{\Delta u}{\Delta e}; PB = \frac{100\%}{K_p}$$

Narrow PB \rightarrow High gain


Example

Full scale measurement = 50°C

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PB or a low value of the proportional band implies a high proportional gain right so let us look at an example suppose the, we are talking about a temperature control loop where the full scale measurement is 50°C degree centigrade.

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Proportional gain (K_p) / Proportional Band (PB)

$$K_p = \frac{\Delta u}{\Delta e}; PB = \frac{100\%}{K_p}$$

Narrow PB \rightarrow High gain


Example

Full scale measurement = 50°C
Error change of 2°C = 4%
Input change by 100% for 2°C error change

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Right suppose an error of 2°C which is 4% of 50°C causes an input change by 100% so maybe there is a heater whose output will change from 0 watt to 5000 watts or 1000 watts or whatever so if the error changes by 2°C then the heater output will change from 0% to 100% so in such a case 4% change in error causes a 100% change in input.

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Proportional gain (K_p) / Proportional Band (PB)

$$K_p = \frac{\Delta u}{\Delta e}; PB = \frac{100\%}{K_p}$$

Narrow PB \rightarrow High gain

Example

Full scale measurement = 50°C
Error change of 2°C = 4%
Input change by 100% for 2°C error change
PB = 4%

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So the proportional band in this case is 4% so this is the meaning of the proportional band.

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Now let us look at the integral gain which is again expressed in terms of the integral time and the proportional band now here I would you might think.

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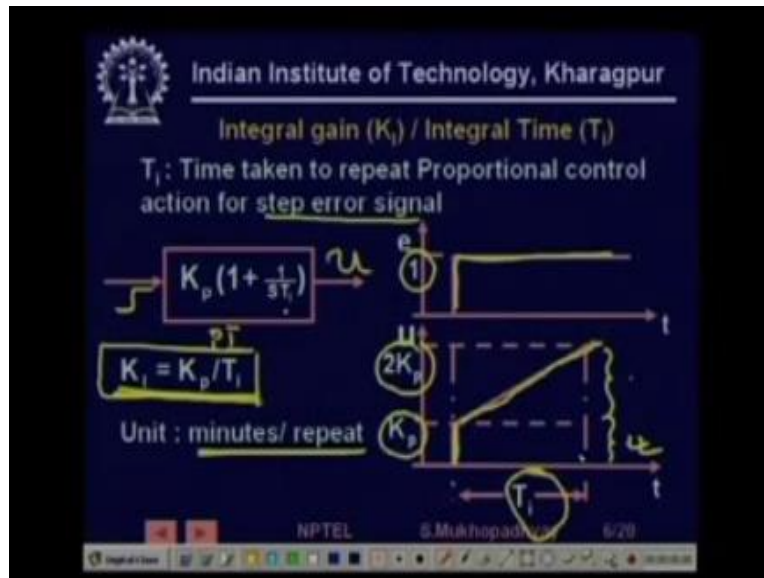


That why is it that rather than expressing the integral gain rather than expressing the integral gain as K_I why I am expressing it as K_P into T_I why the derivative gain which I could call K_D I am I am expressing as K_P into T_D what is the reason the reason is actually embedded in history it turns out that in the older you know hydraulic and pneumatic PID controllers the construction of the controller was such that one part of the device used to control K_P another part of the device used to control T_I another part of the device used to control T_D .

So there are certain distinct parts of the controller which is to realize these terms K_P t_i and T so that an average so that an overall integral gain of K_P by T_I and an overall derivative gain of K_P into T_D is realized so it is for from that principle that the integral time and the derivative time terms are continuing but if you have a microprocessor-based controller then all these terms need not be considered and you could equivalently work with you know K_I and K_D but still let since I mean see since the terminology still continues.

So let us see the meanings because K_I and K_D we understand very well they are just simply the K in terms so let us see what is integral time.

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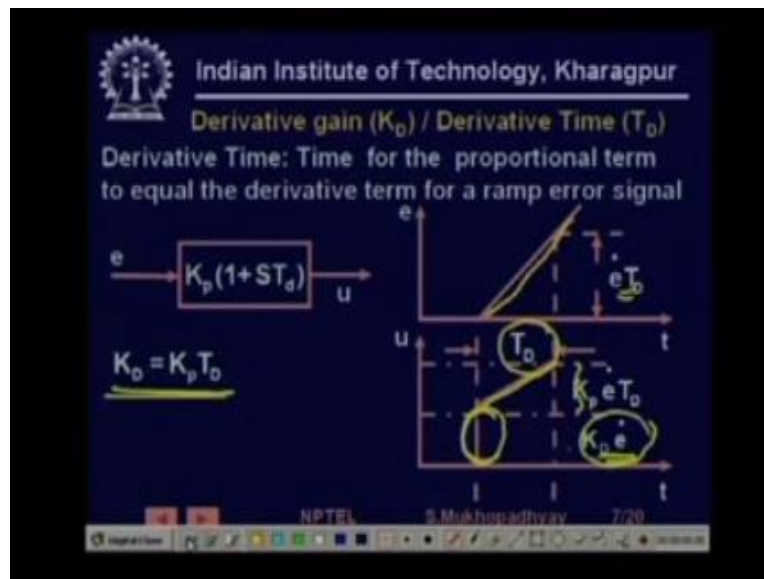
So integral time is the time taken to repeat the proportional control effort or action for a step error signal so what happens is that let us this probably not so clear so let us look at the scenario so now suppose take this as a PI controller right this is a PI control controller suppose we are giving a step input to it so here we have i am sorry so here we have a step input if you give a step input like this to the controller then how will they you output vary the U output will vary like this.

So immediately the KP part will rise so this will be KP into E and then the integral term will start integrating there so it will go up right and after sometime this integral part of the input will equal the proportional part of the input so the so it turns out that after exactly after T_i amount of time the input will become if the proportional control input is K_p because i have taken the error as unity then after T_i amount of time the total input will become $2 K_p$ or the integral part will repeat the proportional part.

So in that sense this definition is now explained that is time taken this time taken to repeat the proportional control effort action for a step error signal okay and it is given as we all know it is given as K_p by T_i D proportional the integral gain is expressed as K_p by T_i now so from this definition perhaps it is now clear why it is expressed as minutes per repeat so for if this continues then every T_i minutes the integral term will produce another K_p times input right.

So these are the proportional control it will continuously repeat every TI minutes in that sense the unit is minutes called B so that is the that explains the integral term now we go to the derivative term again we have a derivative gain.

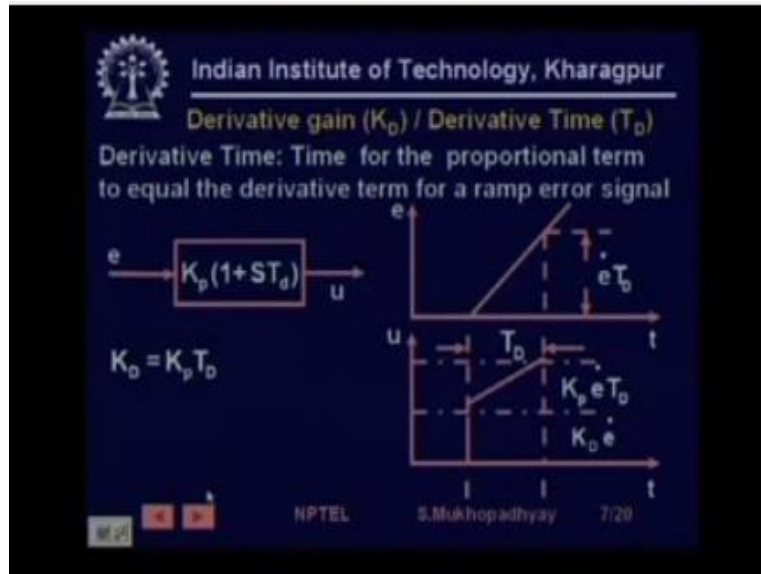
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And we have a derivative time so again now the derivative time is the time taken for the proportional term to equal the derivative term for a ramp error signal so again a similar thing so let us look at the diagram here so here now we have a PD controller right so in the PD controller if you feed it a ramp signal now let us say a ramp signal of some slope \dot{e} some slow \dot{e} dot then immediately what will be now that now the derivative term will jump because there is a constant \dot{e} dot.

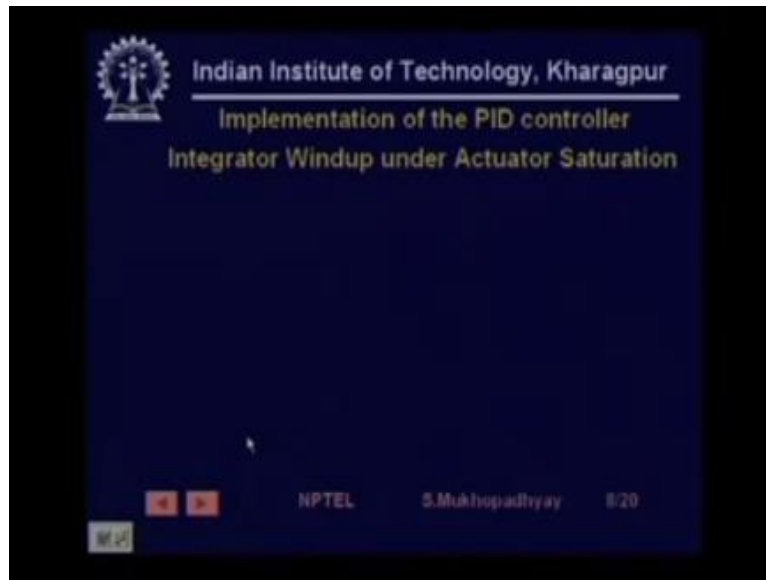
So there will be immediately a K_D into \dot{e} term and the K_P term will now start going up because he is going up so after T_D time this is going to be $K_P \dot{e}$ into T_D so if $K_P \dot{e}$ into T_D has to equal K_D into \dot{e} which is the derivative term output then $K_D = K_P$ into T_D so this is the time or this is the derivative time after which the proportional action will repeat the derivative action so that explains.

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What is the meaning of the term derivative time now we come to the implementation of the PID controller.

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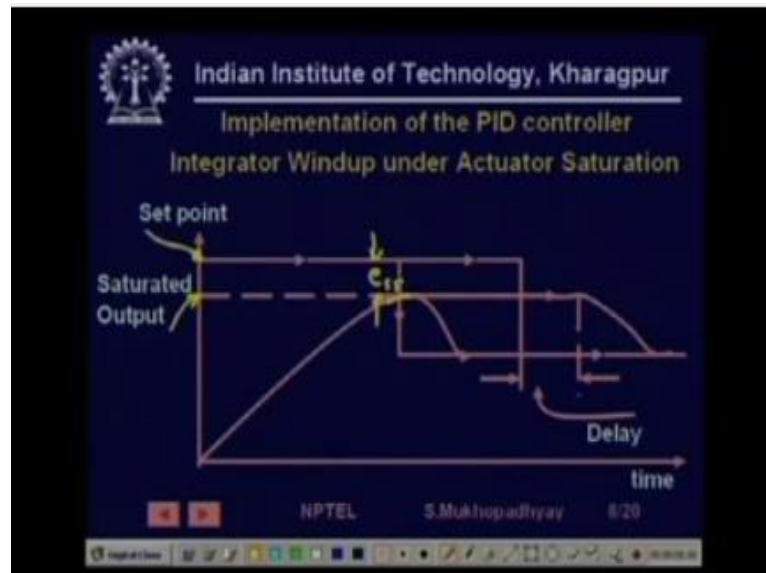


And we basically we are trying to see that what are the problems that might occur if you just simply implement the term as a proportional as an integral plus derivative so first we look at the integral term in detail and see what happens when there is actuator saturation now you see actuator saturation is actually very common in the sense that only in certain cases see the set point keeps on varying so suppose the set point stays suppose set point stays 80% of the time it stays in about 60% of its maximum value and probably a 5% of the time it reaches.

Something like it reaches 100% if you have to make an in an actuator which can really deliver full output even for 100% control input completely proportional then the actuator has to be very large and the actuator setting has to be I mean the actuator power rating has to be very large so often it is it is very common that okay we will we will choose an actuator which can deliver input proportional to the control input for about 70, 75% and then it will saturate so the cases where you will get you will get.

You are going to give very rare cases sometimes very exceptional cases maybe you will give more than 75 or 80% and that time there will be some error and you are willing to tolerate it so this happens in many cases now so we want to see what happens to the PID controller in such cases of actuator saturation.

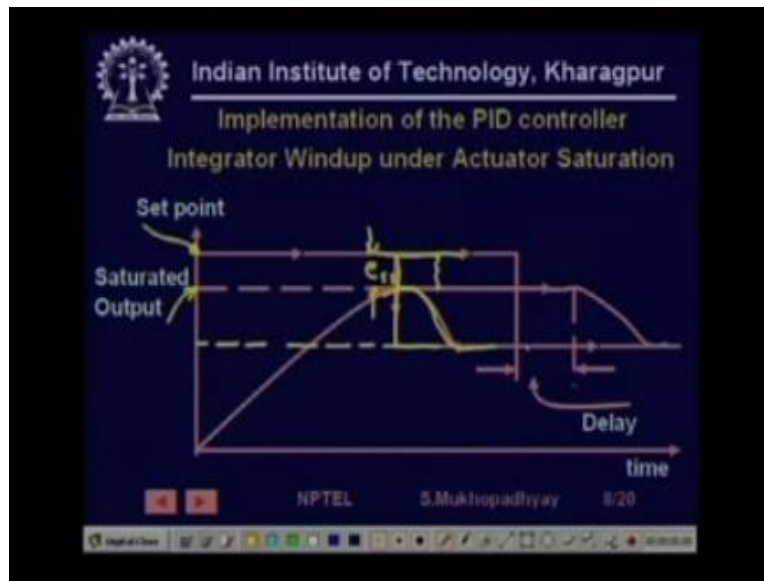
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So let us look at this case very carefully so you see that suppose the maximum possible output that the actuator can produce is this so here it saturates it cannot produce any further output but a set point is given which is higher than that so the actuator naturally cannot give enough input corresponding to this set point so what will happen is that the output will rise and then here it will saturate it cannot so the output cannot increase beyond this point and this amount of error this amount of error will exist this is the steady-state error which will exist one cannot do anything about it simply because.

Whatever control you apply the actuator will not be able to give input so the plant input will not increase be on this that is fine.

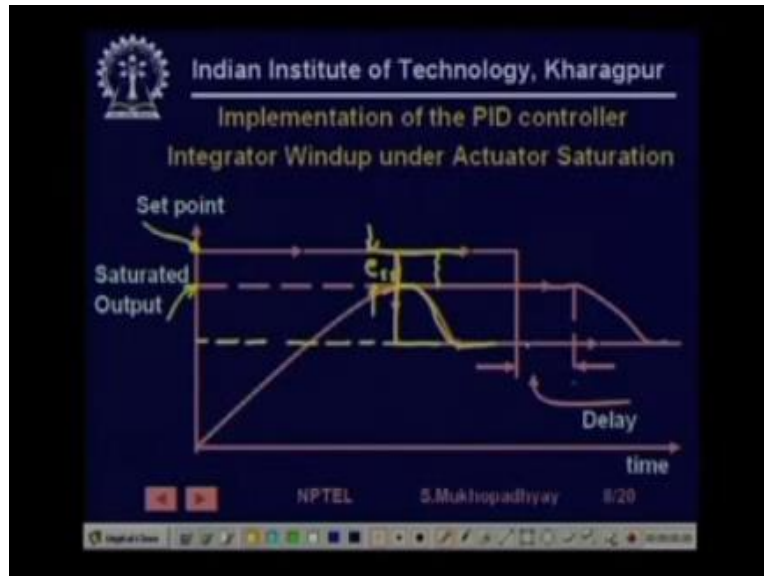
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Now suppose the set point is reduced here it is you have realized that it cannot reach that set point so it is reduced so immediately now this output level is very much reachable by the actuator so what is desirable is that the actuator will immediately because by control action the output will come down and we will reach h_0 steady-state error point as is common under integral control but exactly that does not happen why it does not happen now suppose that you have held this error you have not immediately reduced it.

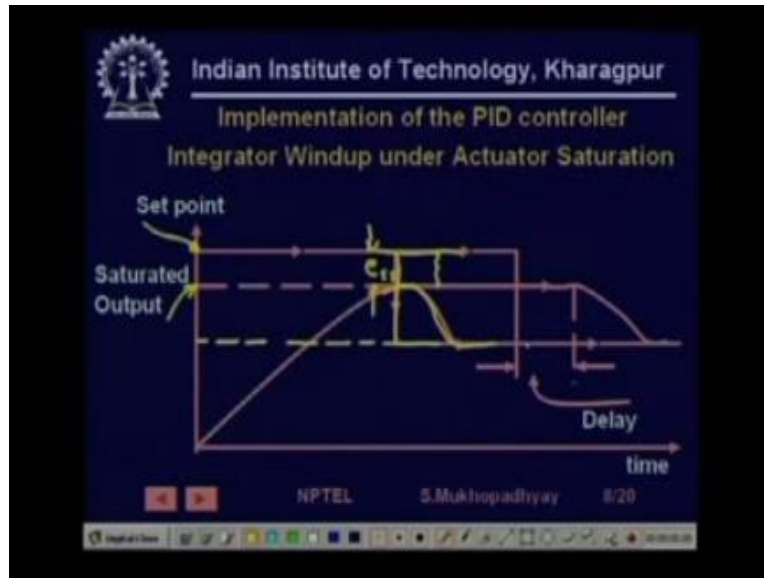
But you have continued with it for some time so now what is happening here during this time the prop the error is constant so the proportional term part of the controlling put remains constant but the integral term of the control input goes on increasing.

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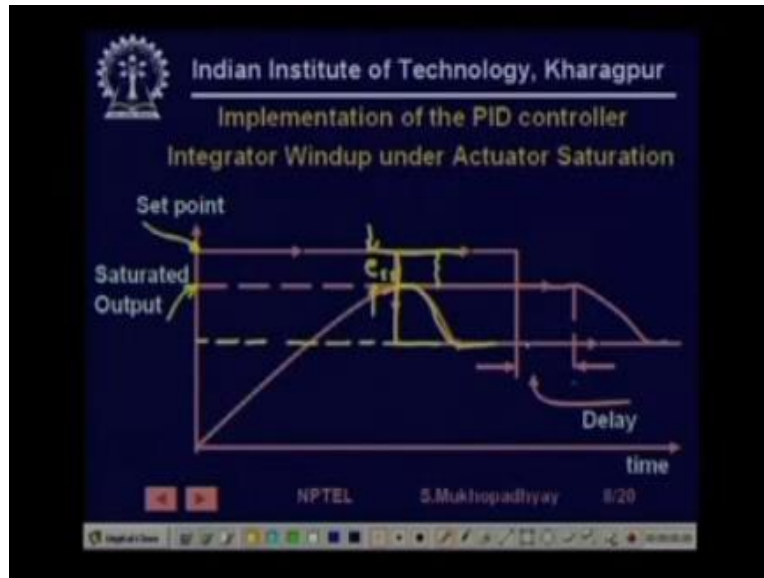
So the integral in the PID controller goes on integrating the error however it cannot produce a control input because they actually that input is given so the as if the PID controller output is the controller output is continuously increasing it is also coming to the actuator input but the actuator is not able to give that output because we already saturated now suppose that after some time.

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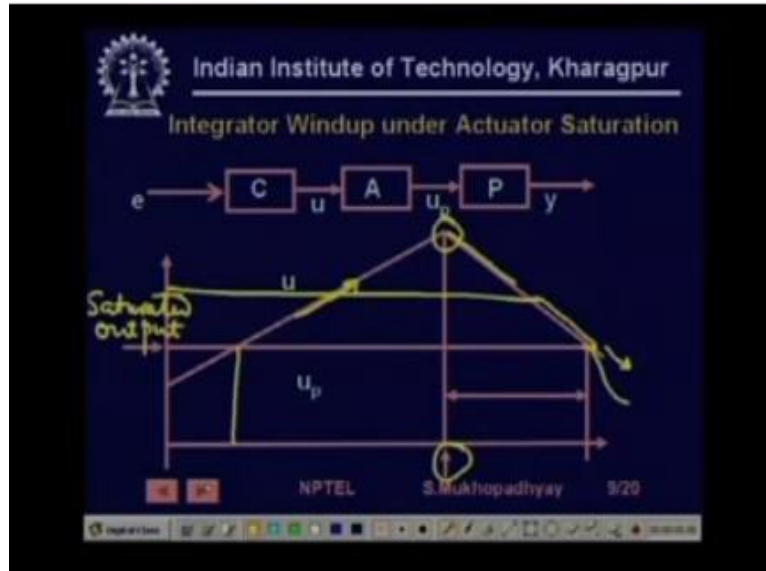
The control input is now reduced now what is going to happen what will be observed is that the while the why it would have been desirable that the control input immediately falls down and reaches as reaches the desired steady state point it does not do that rather it continues at the same level we ignoring that the set point has now been reduced and overly after some time only after some time does the actuator does the.

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Control start respond to the stage to the set point now this phenomenon is called integrator windup basically what has happened is that the integrator has become bloated or floated so it has not realized that that the plant cannot reach this output so the error is will persist so it is unnecessarily trying to give more and more control input and getting blown up right so that is integrated windup and it happens essentially because of the fact.

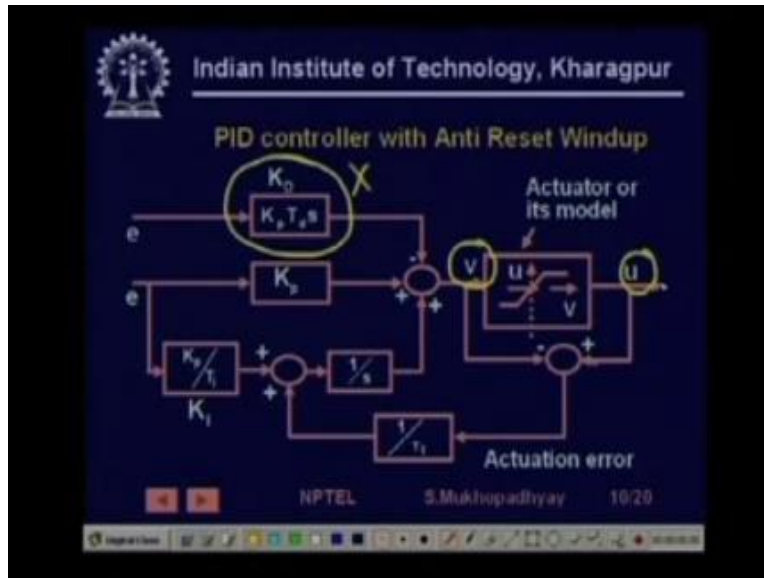
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That see during the time when the error is persisting say from this point the error is persisting so the proportional input is remaining constant but the integral input is growing so suppose at this time it has reached this value now the set point is reduced so it is at this point that the set point is reduced now what is going to happen and this is the saturated output so the integral is way beyond the saturated level so now that it is the set point is reduced now the error has become negative so the integral value is now reducing but still it is positive see at this point at this point the control input is still greater than the saturated level.

So what goes is actually this level and therefore the output persist so only at this time after so much time does it come to the statute below the saturated input level and then it goes further below so there so from this point onward the output will start reducing so this is what happens so this is so the whole idea is that the integral should not be allowed to blow up and continuously blow up with time if the.

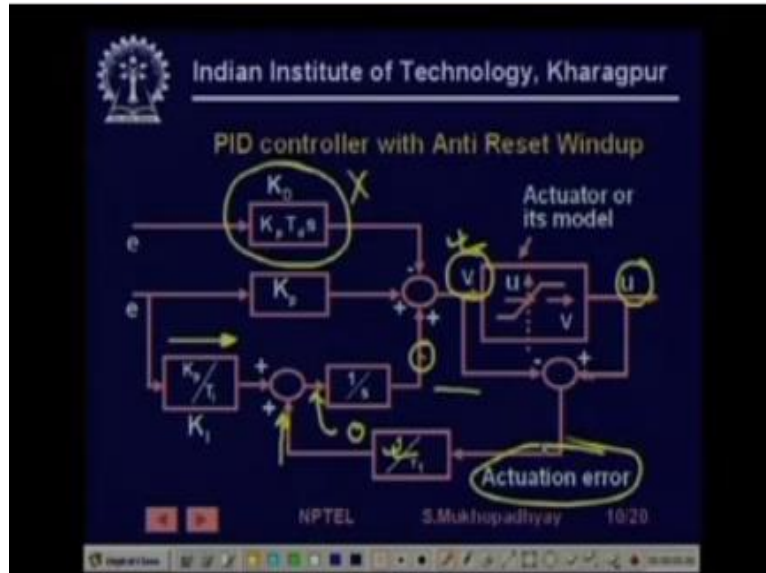
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Sorry if the error persists due to a phenomenon like actuator saturation so that is precisely that can be done in many ways and here is the scheme once one of many possible schemes which will realize that so how do that so look at this controller simple controller we have we have the usual PD that the derivative term which you can ignore for the time being is not concerned so we have a proportional term which is coming we also have a derivative term which is coming which we did not consider any in this slide so what is happening here is that here actually it is here suppose this is the actuator right.

So the actuator has a saturation characteristic so even if this is the controller output and this is the plant input in between sit statue it so what you are doing is you are actually sensing the physical plant input you could either do that or you could have an model of the actuator in the controller itself and check before giving the input check whether this is really going to cross the actuator limit so you can either do it in software or you can use a sensor to again see what input is going now when C becomes larger.

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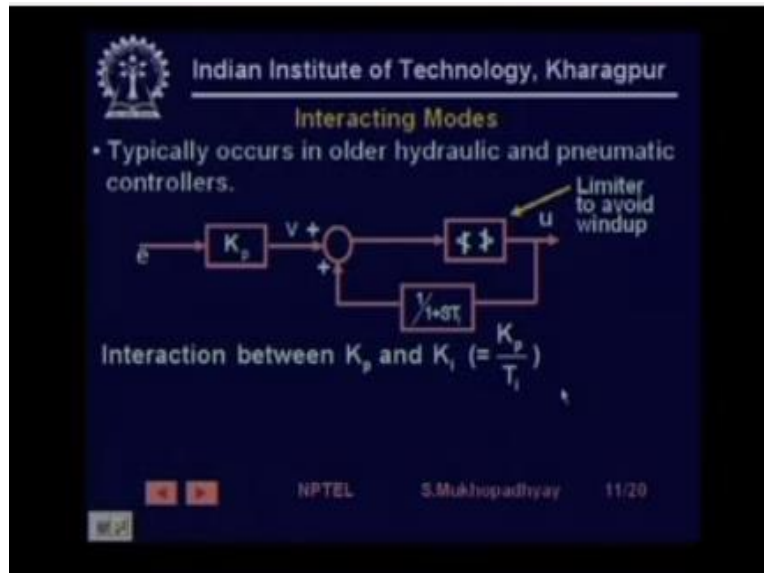


Then you then your sub and then this actuation error becomes negative now what you want to do is now you take this actuation error and you feed it so here a negative term is coming and here error is positive so through the PID integral term a positive term is coming so you have to define this gain in a suitable manner such that whenever V this becomes negative this signal becomes 0 this signal becomes 0 so when this signal becomes 0 this integrator does not build up so this integrator output remains at constant.

Value so you see that whenever you are giving an input which is going to cause an actuator saturation the this but special path which we have added to the PID controller will now present will now present the integral term from blowing up so that when the set point is reduced the plant output will follow very smoothly right so this is the scheme which can be used for anti reset wind up sometimes integral wind-up is called so-called reset windup.

So coming to the next one now as I was telling that PID controllers where historically many of them were made if using hydraulic and pneumatic devices so they used to have you know certain realization structures.

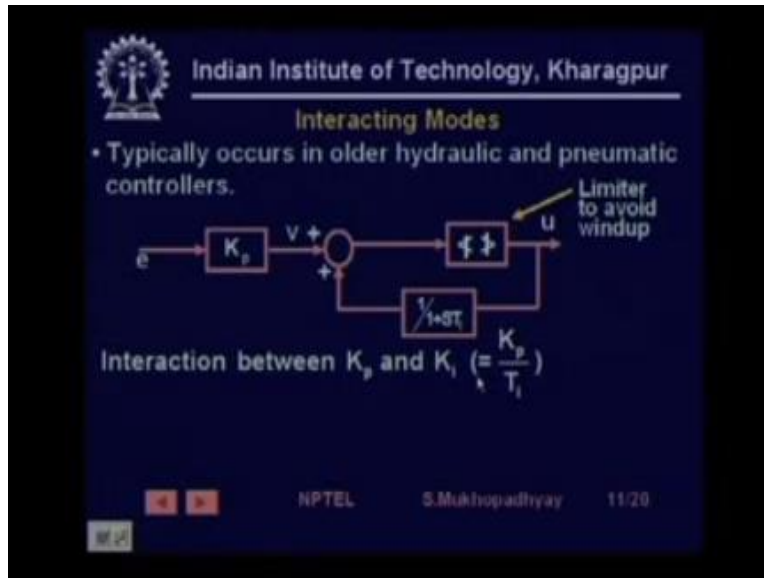
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So this is atypical structure where you know as I said that you know one part of the controller used to be real I used to realize the gain typically you know devices like flapper nozzles which we will see and there were there were various kinds we know bellows or orifices constrictions which are used to realize these time constants so the controller structure look at this structure so if you realize this for the time being let us forget about this one assume that it is it goes directly it is one.

So if you take this structure then you will find that you will find that what is the if you compute the transfer function between U and V if you compute the transfer function between U and V first of all note that there is that is K_I is realized as K_p / T_I so there is interaction this is called an interactive mode because if you change the proportional gain K_p .

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Then the integral gain K_I also changes so whenever you change K_P if you want to keep the integral gain constant you have to also change T_I so the various parameters cannot be varied in a non interactive mode but they must they will be interacting okay.

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Interacting Modes

- Typically occurs in older hydraulic and pneumatic controllers.

Interaction between K_p and $K_i (= \frac{K_p}{T_i})$

Transfer function (ignoring limiter)

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And the transfer function between V and you ignoring the limiter this is a limiter that is if the value goes beyond a certain value it will limit it if it goes below a certain value it will also limit it if you ignore the limiter for the time being then you will find the transfer function between U.

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Interacting Modes

- Typically occurs in older hydraulic and pneumatic controllers.

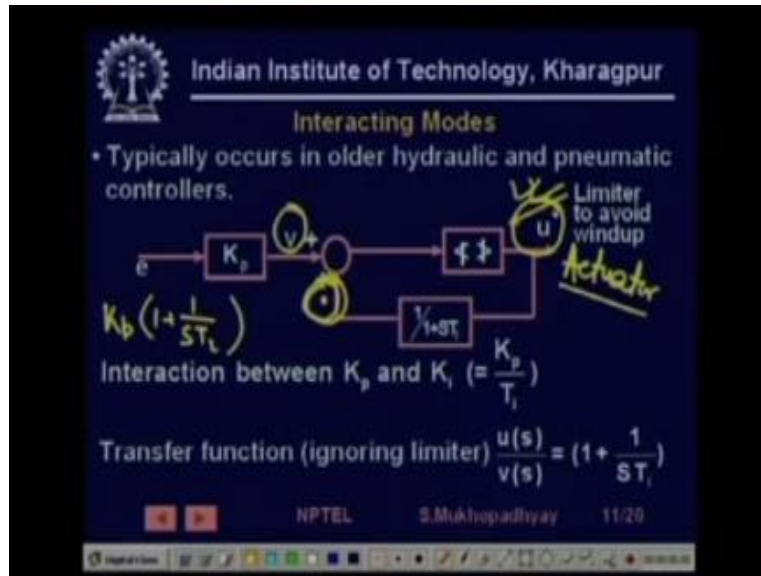
Interaction between K_p and K_i ($= \frac{K_p}{T_i}$)

Transfer function (ignoring limiter) $\frac{u(s)}{v(s)} = (1 + \frac{1}{sT_i})$

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Is given as $1 + 1 / sT_i$ so when you multiply it by K_p you get the transfer function of a PI controller okay so you can also see so basically what you have realized is the same.

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Transfer function that is $K_p \times 1 + 1 / ST_i$ but you have realized it in this way now if you now let us look at the role of the limiter which is also used in this structure to avoid integral wind-up so you see that if there is if you goes to high this is actually going to the actuator this is the controller output which is going to the actuator so if this you goes to goes very high then what is going to happen is that this limiter which is inside the controller itself is going to limit this so this you will become constant so when this u becomes constant you can see these are the simple first order transfer function.

So at that point of time this input will also be constant so now what is happening is that the error is the error is constant so therefore this V is constant and because this you has gone to a high level so even at some level depending on the where you have set the limiter this is also constant so therefore U becomes constant so the output of the PI controller does not build up in definitely but gets limited so this is another way by which an anti reset windup scheme can be implemented typically in hydraulic and pneumatic controls.