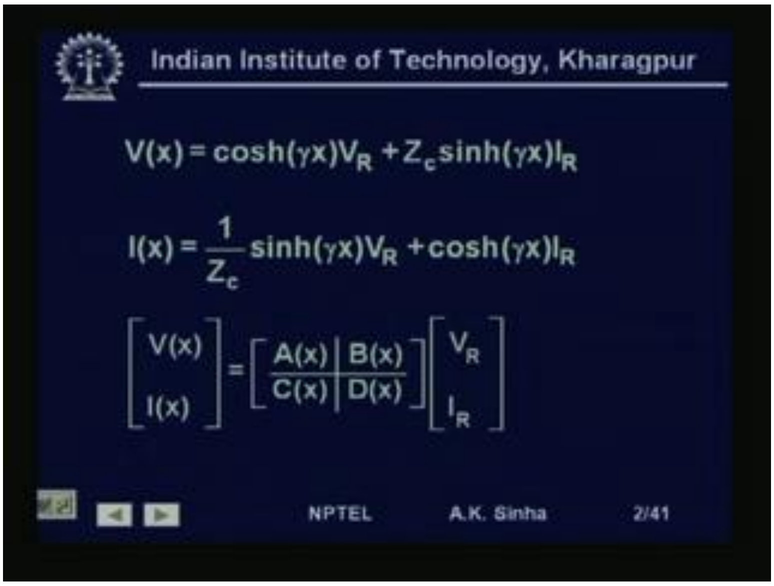
Power System Analysis

Prof. A. K. Sinha
Department of Electrical Engineering
Indian Institute of Technology, Kharagpur

Lecture - 08
Transmission Line Modeling Long Line (Contd)

Welcome to lesson 8 on Power System Analysis. This lesson is a continuation on Transmission Line Modeling, especially modeling of a long line. Now, if you remember in the previous lesson, that is lesson 7, we talked about distributed parameter model for long transmission lines. That is lines which are longer than 250 kilometers. For these line we said that the line voltage V x.

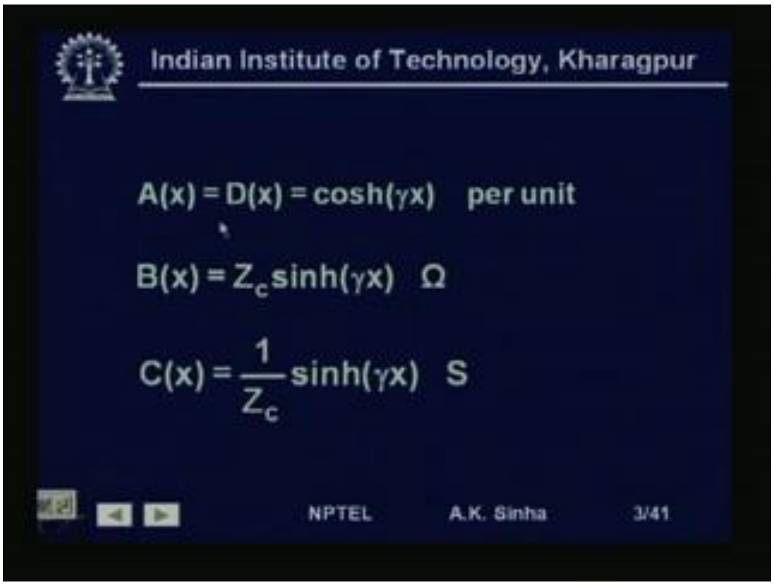
(Refer Slide Time: 00:51)



And at distance x from the receiving end is given by cos hyperbolic gamma x into V R plus Z c sin hyperbolic gamma x into I R, where gamma is called the propagation constant, x is the distance of the point from the receiving end. V R and I R are the voltages and currents at the receiving end. Similarly, the current at a distance x from the receiving end I x is equal to 1 by Z c sin hyperbolic gamma x into V R plus cos hyperbolic gamma x into I R.

Where, Z c is the characteristic impedance of the transmission line. And it is given by square root of Z pi y, where Z is the series impedance of the transmission line, per unit length and y is the shunt admittance of the transmission line per unit length. As we see these models, these equations involved hyperbolic functions. Now, since we have been writing all these transmission line equations, in terms of ABCD parameters. We can write this equation also in those terms. So, in matrix form, we can use this as V x I x is equal to A x B x C x D x and V R I R. These form where this relationship is in terms of voltage and current at any distance x from the receiving end.

(Refer Slide Time: 02:56)

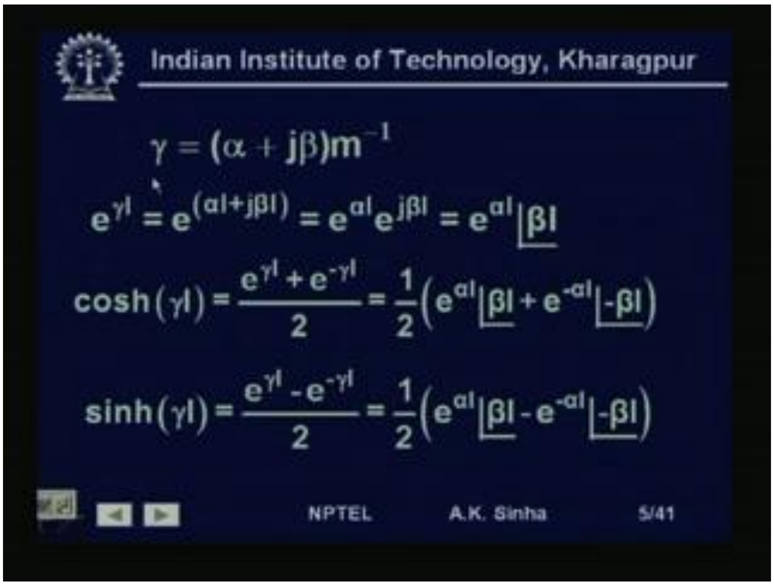


Now, here if you see A x is equal to D x and that is equal to cos hyperbolic gamma x, in per unit. That is if you see this relationship this is your A x and this is your B x, this is C x and this is D x. So, A x is equal to D x is equal to cos hyperbolic gamma x, V x is equal to Z c sin hyperbolic gamma x, V x is equal to Z c sin hyperbolic gamma x. C x is equal to 1 by Z c sin hyperbolic gamma x and D x as we have already seen is equal to A x.

So, this is the model where A and D are basically dimensionless, B has a dimension of impedance and C has a dimension of Siemens, that is admittance. Now, normally we are interested only in the terminal conditions. That is the sending end voltages and currents, and the receiving end voltages and currents rather than the voltage and current at any intermediate point on the transmission line. Therefore, we can find out the voltage and currents at the sending end, in terms of voltage and current at the receiving end by substituting x is equal to l, where l is the total line length.

In that case we will get the equation V S I S is equal to ABCD. Now, we are not writing x because, we are now dealing with the terminal conditions only their fore we will have A, B, C and D parameter into V R I R. Here, A is equal to D is equal to cos hyperbolic gamma l, instead of x we are substitute l. This is the dimensional less quantity, it will be in per unit if voltage and currents are all in per unit. B is equal to Z c sin hyperbolic gamma l, C is equal to 1 by Z c sin hyperbolic gamma l. This B has a dimension of impedance, C has a dimension of admittance. Now, here the term gamma is what we call, propagation constant of the line.

(Refer Slide Time: 05:32)

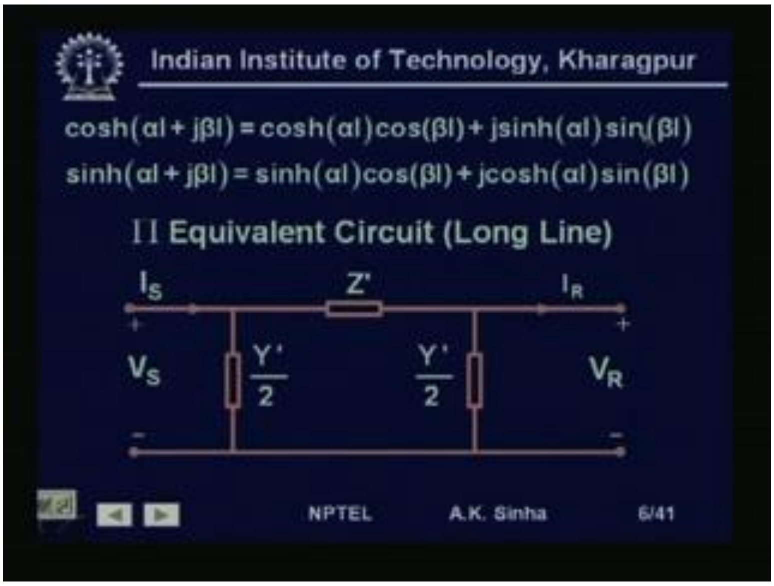


And gamma in general will a complex quantity. That is gamma is equal to alpha plus j beta, it is dimension will be per meter. In fact, if you see gamma l, the dimension of gamma l will be dimensions. So, gamma is equal to alpha plus j beta per meter. Now, if we write e to the power gamma l, this will be equal to e to the power alpha l plus j beta l. And this will be equal to e to the power alpha l into e to the power j beta l, this we can write as e to the power alpha l angle of beta l.

Now, here alpha if you see is called the attenuation constant of the line, and beta gives the phase angle. So, it is called the phase constant of the line. So, propagation constant has two terms alpha, the attenuation constant and beta the phase constant. Now, if we write cos hyperbolic gamma l, this will be equal to e to the power gamma l plus e to the power minus gamma l by 2. This is equal to half e to the power alpha l angle beta l plus e to the power minus alpha l angle minus beta l.

Similarly, sin hyperbolic gamma l is equal to e to the power gamma l minus e to the power minus gamma l divided by 2. This is equal to half into e to the power alpha l angle beta l minus e to the power minus alpha l angle beta l. So, in this way, if we know the alpha and beta, we can calculate the terms sin hyperbolic gamma l and cos hyperbolic gamma l. And therefore, we can calculate V S I S, in terms of V R I R.

(Refer Slide Time: 07:43)



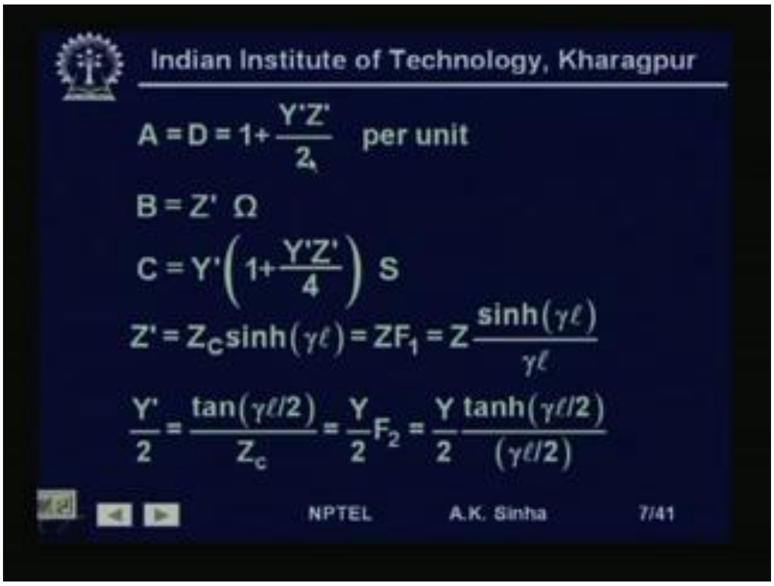
Or we can use the other identity, that is cos hyperbolic alpha l plus j beta l. That is cos hyperbolic gamma l is equal to cos hyperbolic alpha l into cos beta l plus j sin hyperbolic alpha l into sign beta l. Or sin hyperbolic gamma l, that is sin hyperbolic alpha l plus j beta l is equal to sin hyperbolic alpha l into cos beta l plus j cos hyperbolic alpha l into sin beta l. So, knowing the Z and Y, we can calculate the propagation constant gamma and therefore, alpha and beta.

And we can also calculate the characteristic impedance Z c. So, knowing these values, we can evaluate the sending end voltage and current, in terms of receiving in voltage and current. That is using these relationship, we can evaluate the terminal condition of the line. Now, in most of the analysis that we do, especially using computers most of the analysis that we do, we need circuit parameter model rather than the ABCD model of the transmission line.

As we have said earlier we had created a pi model or a nominal pi model for a medium length line. Similarly, we can find out or we can represent a long line, in terms of a equivalent pi model. The difference between nominal pi and equivalent pi is only that, instead of using Z, which is the series impedance of the line. And Y the shunt impedance of the line, we have to use here Z dash which is a not equal to Z, but modified value of Z.

And Y dash which is again a modified value of Y. So, some modification into on series impedance and shunt admittance of the transmission line, will have to be used here in the long line model. Now, this is the pi model, where we have V S, the sending end voltage Is the sending end current V R the receiving end voltage I R the receiving end current. Z dash is the series impedance of the line and the total shunt admittance, Y dash is divided into 2 parts Y dash by 2 at the sending end and Y dash by 2 at the receiving end. This is the pi equivalent model for the long line. Now, we need to find out these values of Z dash and Y dash for making this model complete.

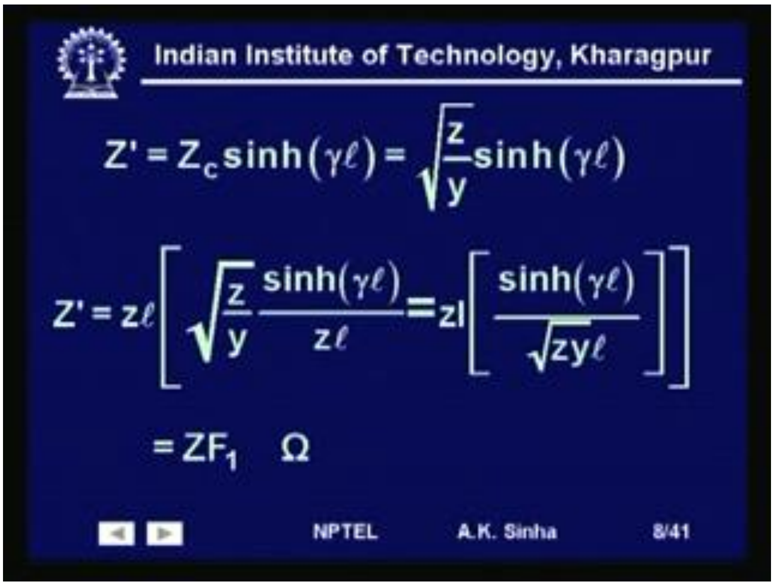
(Refer Slide Time: 10:45)



Now, we know that for the pi model A is equal to D is equal to 1 plus Y dash Z dash by 2. This is the same thing that we had done in lesson 7, where we had found the ABCD parameters. And transmission line nominal pi model parameters. There we had seen A is equal to D is equal to 1 plus Y dash Z dash by 2 in per unit. This is the dimension less quantity and B is equal to Z dash, this is ohms, that is a impedance units. And C is equal to Y dash into 1 plus Y dash Z dash by 4 Siemens, that is the unit for admittance.

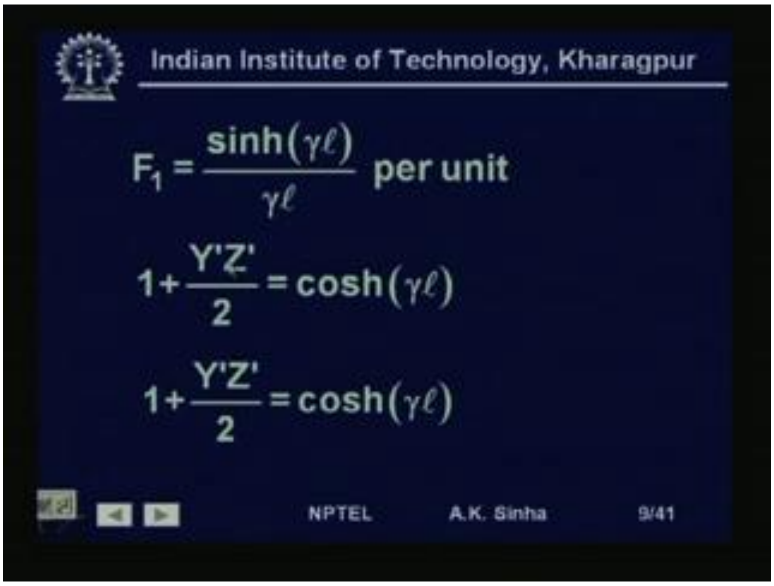
Now, using this here we have Z dash is equal to Z c sin hyperbolic gamma l, which we can write as Z, which is the series impedance of the line. Total series impedance of the line into F 1 or this is equal to Z into sin hyperbolic gamma l by gamma l. Similarly, Y dash by 2 will be equal to tan hyperbolic gamma l by 2 divided by Z c. This is equal to Y by 2 into F 2, this is equal to Y by 2 into tan hyperbolic gamma l by 2 divided by gamma l by 2. We will see how we get these relationships.

(Refer Slide Time: 12:24)



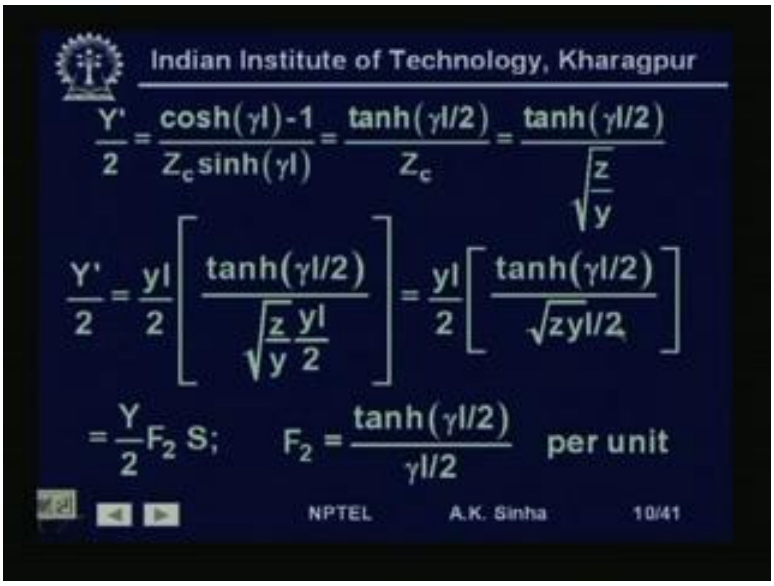
Z dash is equal to Z c sin hyperbolic gamma l. This is equal to square root of Z by Y, that is Z c is Z by Y, square root of Z by Y into sin hyperbolic gamma l. Therefore, Z dash is equal to Z l into square root of Z by Y sin hyperbolic gamma l by Z into l, this is equal to Z into l sin hyperbolic gamma l by square root of Z y l.

(Refer Slide Time: 13:27)



So, this is sin hyperbolic gamma l, Z l into sin hyperbolic gamma l divided by square root of Z y into l, this is equal to Z into F 1, where F 1 is sin hyperbolic gamma l divided by gamma l in per unit. And 1 plus Y dash Z dash by 2 that is A is equal to cos hyperbolic gamma l.

(Refer Slide Time: 13:47)



Therefore, 1 plus Y dash Z dash by 2, it is the same equation or Y dash by 2 is equal to cos hyperbolic gamma l minus 1 divided by Z c sin hyperbolic gamma l. This is equal to tan hyperbolic gamma l by 2 divided by Z c, this is equal to tan hyperbolic gamma l by 2 divided by square root of z by y, that is Z c. Therefore Y dash by 2 is equal to y into l by 2 into tan hyperbolic gamma l by 2 this term. Now, here we have multiplied it by y l by 2, so in the numerator and therefore, we have divided in the denominator.

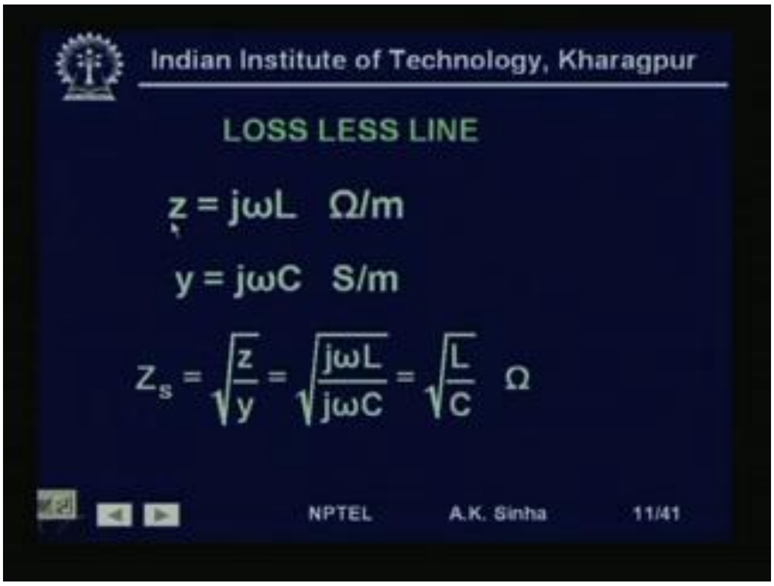
So, square root of z by y into y l by 2, therefore we will get this equal to y l by 2 into tan hyperbolic gamma l by 2 divided by square root of z into y into l by 2, which is equal to Y by 2 in to y l will give the Y, capital Y that is total shunt admittance of the line. So, this is Y by 2 into F 2 Siemens, where F 2 is equal to tan hyperbolic gamma l by 2 and root Y Z is nothing but, gamma, so this is gamma l by 2 in per unit.

Now, these relationships give us the model for finding out the voltage and current at any point on the line or the terminal conditions of the line. That is sending end voltage and current of the line. These are the accurate ABCD parameters of the transmission line or the accurate parameter for the pi equivalent circuit of the line. Now, most of the time since, we see this relationship that we have seen earlier, these relationships are involving hyperbolic functions.

So, doing computation with these is going to be some more tedious. Also many times what we need to do is try to get some approximate idea, of what is happening in the transmission line. This is very important, especially when we are designing a transmission line, because we will have to work out large number of options. So, for this purpose what we do is, we try to introduce the concept of a loss less line.

That is a line where the series impedance consists of only the inductance part, or the reactance of the line and the resistance is negligible. Similarly, the shunt admittance consists of only the capacitance and the shunt conductance is neglected. One of the advantages of this kind of a modeling is also that, it gives us a very good idea about some of the important concepts of power flow, on the transmission line another aspects.

(Refer Slide Time: 17:22)

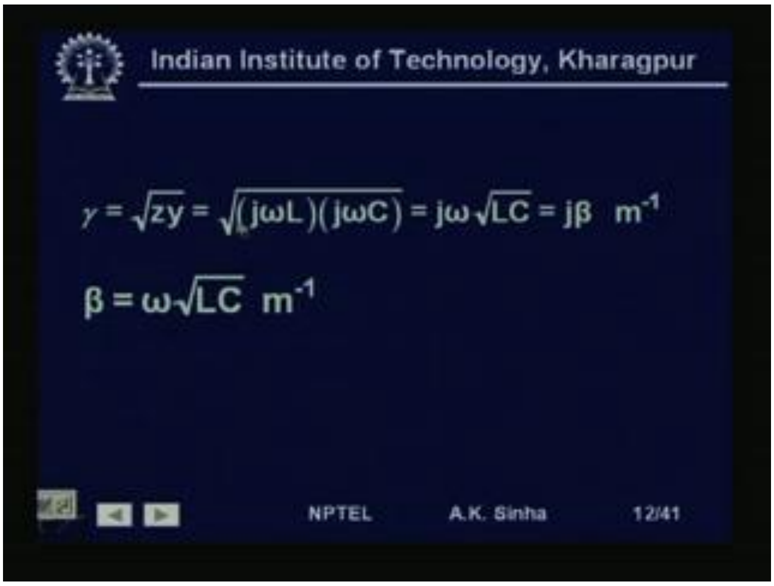


Now, in case of the loss less line, we have the series impedance per meter length, is equal to j omega L, where L is the inductance of the line per meter length. And shunt admittance y per meter length will be equal to j omega C, where C is the shunt capacitance of the line per meter length. So, this is y is in terms of Siemens per meter, z is in terms of ohms per meter.

Now, here if we take the characteristic impedance of this line. Then it will be z by y, since this line is loss less, we call this characteristic impedance for a loss less line as the surge impedance of a line. So, for this line, the surge impedance Z s is equal to square root of z by y which is equal to square root of j omega L by C j omega C, which is equal to square root of L by C ohms.

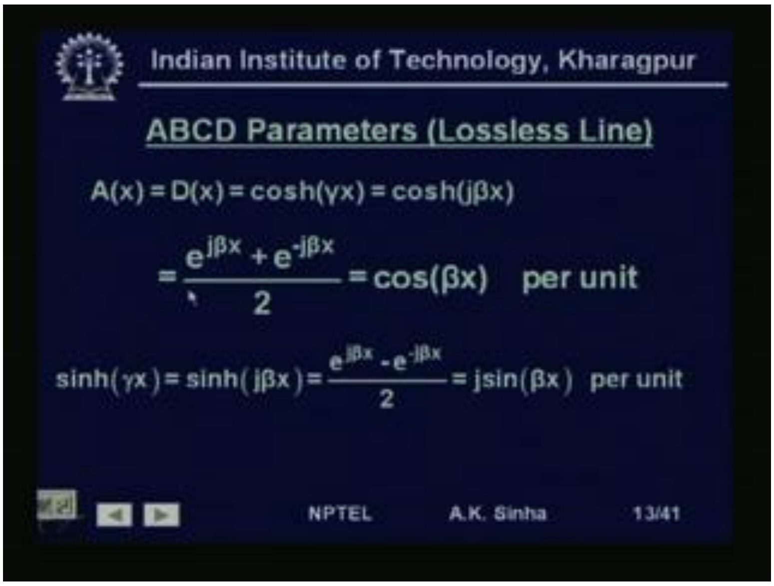
Now, if you look at this Z s, you find that this quantity Z s is a pure real number. That is Z is or surge impedance is pure resistive in nature, it is not a complex number. As we had seen earlier, Z C is a complex number.

(Refer Slide Time: 19:04)



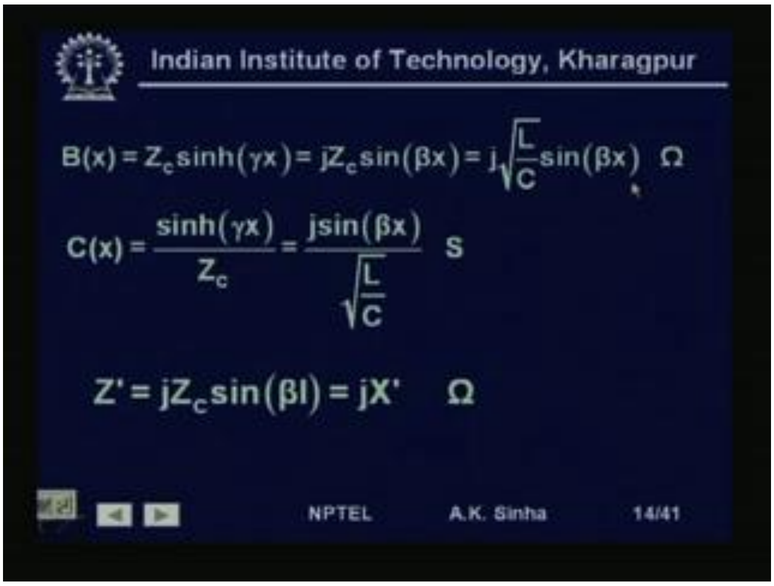
Similarly, the propagation constant gamma is equal to square root of z y, which will be equal to square root of j omega L into j omega C. This comes out to be j omega square root of root L C, which we write as j beta. Because, we find that this term, now is purely imaginary. And therefore, we have only imaginary term for the propagation constant, or it is in terms of only the phase constant. The attenuation constant is 0 which is natural because there are no losses in this line. So, gamma is equal to j beta per meter. And if you are talking in terms of the whole length, then gamma l will be dimensionless. Here, beta is equal to omega into root L C per meter, which is the phase constant. Now, we will try to develop the ABCD parameters for this loss less transmission line.

(Refer Slide Time: 20:23)



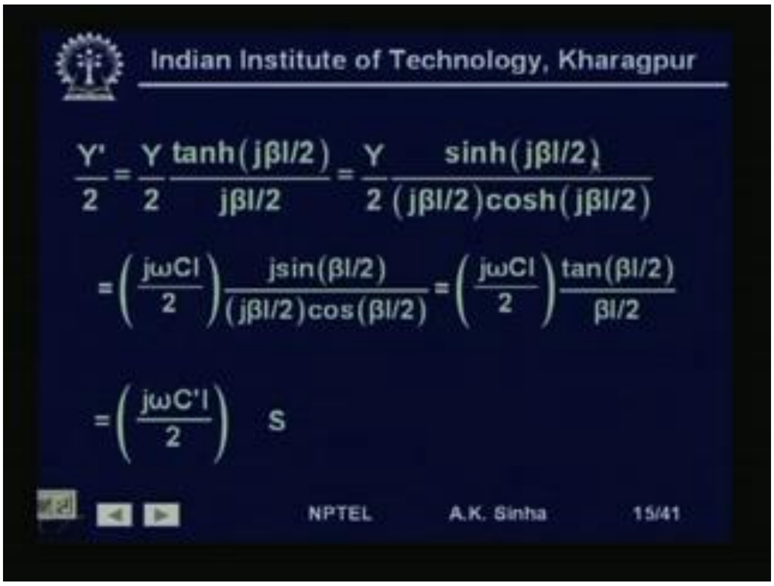
Now, again we had seen earlier A x is equal to D x is equal to cos hyperbolic gamma x. Now, gamma is equal to j beta, therefore this is equal to cos hyperbolic j beta x, which will be equal to e to the power j beta x plus e to the power minus j beta x divided by 2, which is equal to cos beta x in per unit, this is a dimensionless quantity. Now, here we see that A and D are in terms of cos beta x, not hyperbolic functions. Similarly, sin hyperbolic gamma x is equal to sin hyperbolic j beta x, which is equal to e to the power j beta x minus e to the power minus j beta x divided by 2, which again comes out to be equal to j sin beta x. Again this is a pure trigonometric function, it is not a hyperbolic function.

(Refer Slide Time: 21:25)



Therefore, B x will be equal to Z c sin hyperbolic gamma x, this is equal to j times Z c sin beta x. This is equal to j times square root of L by C Z c is root over L by C into sin beta x. Now, again this, if we see is a purely imaginary quantity or the quantity for reactance, this has a unit of ohm. Similarly, C x will be equal to sin hyperbolic gamma x by Z c, that is j sin beta x divided by root L by C. So, this is again an imaginary quantity, which is very similar to capacitance. So, which Z dash will be equal to j Z c sin beta l, which we can write as j X dash.

(Refer Slide Time: 22:34)

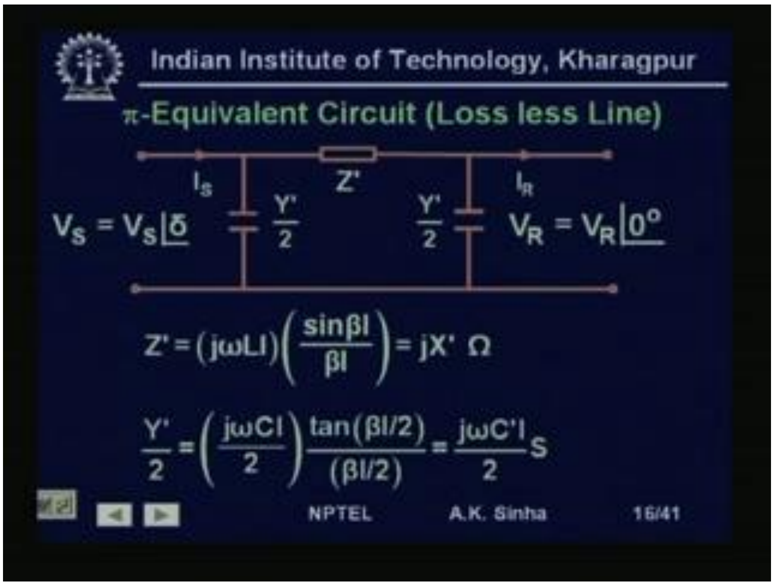


Similarly, Y dash by 2, we can write as equal to Y by 2 sin hyperbolic gamma l by 2 was there, so this is j beta l by 2 divided by gamma l by 2 was their earlier. Now, gamma is equal to j beta only, so it is j beta l by 2, this is equal to Y by 2 sin hyperbolic j beta l by 2 divided by j beta l by 2 into cos hyperbolic j beta l by 2. That is tan hyperbolic j beta l by 2, we have put it as sin hyperbolic j beta l by 2 divided by cos hyperbolic j beta l by 2.

This is equal to j omega C into l, this is for Y total line capacitance divided by 2, so Y by 2 into j sin beta l by 2. Because, sin hyperbolic j beta l by 2 we have seen is equal to j sin beta l by 2 divided by j beta l by 2 this term, into cos hyperbolic j beta l by 2. We have seen is equal to cos beta l by 2. So, this terms comes out to be j omega C into l by 2 into tan beta l by 2 divided by beta l by 2.

This we can write as j omega C dash l by 2, where C dash is now the modified capacitance of the transmission line or j omega C dash l is the modified admittance of the line. And that C dash is equal to C times tan beta l by 2 divided by beta l by 2. So, for a loss less line, we see that we reduced the relationship from hyperbolic functions, for a normal loss long line to a pure trigonometric function relationship for a lossless line. This reduces our computational burden, considerably and one can do hand calculations with this kind of relationships.

(Refer Slide Time: 25:05)

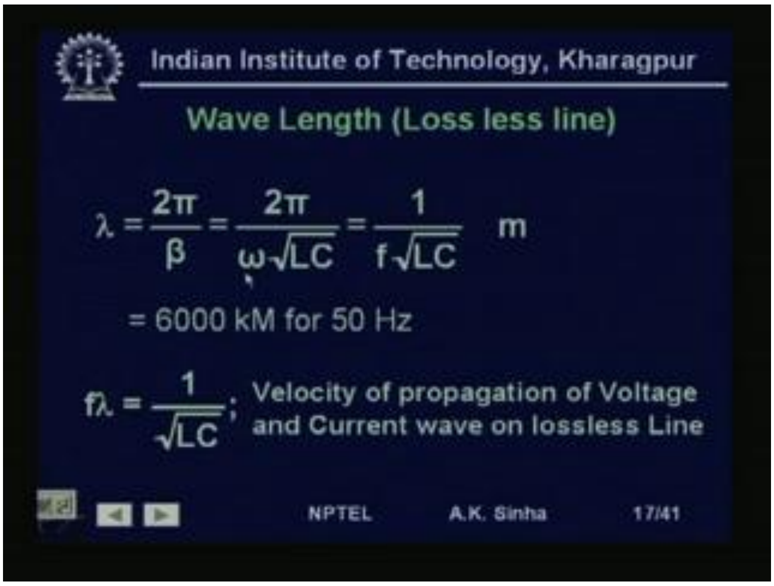


Therefore, the pi equivalent circuit for a loss less line will look like this, where V S is equal to V S angle delta, I S is the current flowing Y dash by 2 is the half of the total modified admittance of the line. Z dash is the total series impedance of the line, that is the modified series impedance of the line. Y dash by 2 the other half of the total shunt modified, total shunt admittance of the line is placed at the receiving end, I R is the receiving end.

Current and V R is equal to V R angle 0 degrees which is the receiving end voltage, we have chosen the receiving end voltage as a reference voltage, where we have as already seen Z dash is equal to j omega L into l is the length of the line, the sin beta l divided by beta l which is a pure imaginary quantity, or the reactance j X dash. And similarly Y dash by 2 can be written as j omega C dash l by 2 Siemens, which is purely capacitance.

Now, for lossless lines, we can also introduce the concept of wave length, that is the length of the voltage and current wave on the line as it moves by 360 degrees. That is a total change in face of voltage and current wave, on the transmission line over a 360 degree displacement.

(Refer Slide Time: 26:54)



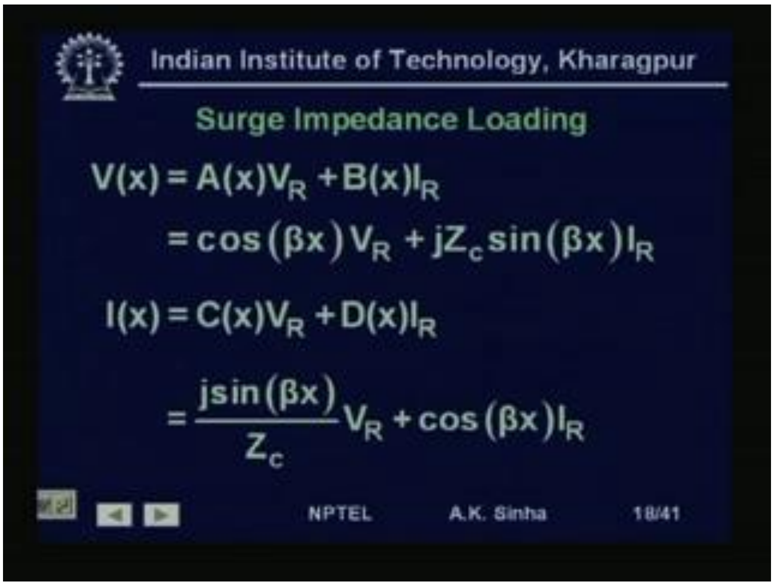
Now, if lambda is the wavelength of the line, then lambda will be equal to 2 pi, which is 360 degrees divided by beta the phase constant of the line. Therefore, we can write lambda the wavelength of the line is equal to 2 pi by omega root L C, beta is omega root L C which is equal to 1 by f root L C, because omega is twice pi f. So, if we write here twice pi f twice pi will cancel out, this is 1 by f root L C which comes out to be about 6000 kilometer for 50 Hertz line.

Now, this will be approximately 6000 kilometer of the line. Because, if you see the relationship for L and C. And since, we can take R dash the effective radius in calculating the inductance to be all most same as that of R. Because, D is much larger compare to R. Therefore, in that case if you substitute the relationship for L and C here, then we will find that this comes out to be this much.

Similarly, we will get f lambda, that is if we multiply it here 1 by root L C, which is the velocity of propagation of voltage and current wave on a lossless line. And this for a lossless line comes out to be same as that of speed of light. So, the velocity of propagation of voltage and current wave on a lossless long transmission line is the same as that of light. This relationship, again you can see by simply substituting the relationship for L and C.

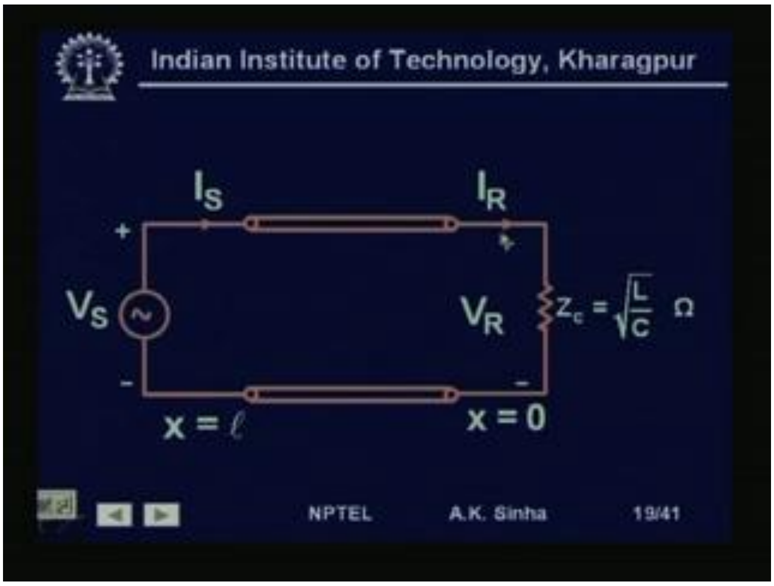
Here, this will then come out to be 1 by mu 0 into root over mu 0 into epsilon 0, which comes out to be 3 into 10 to the power 8 meters per second, which will be same as the velocity of the light. Now, for lossless line, we can introduce one more concept which is very important, especially when we are designing the transmission lines or transmission systems. This concept is of surge impedance loading.

(Refer Slide Time: 29:30)



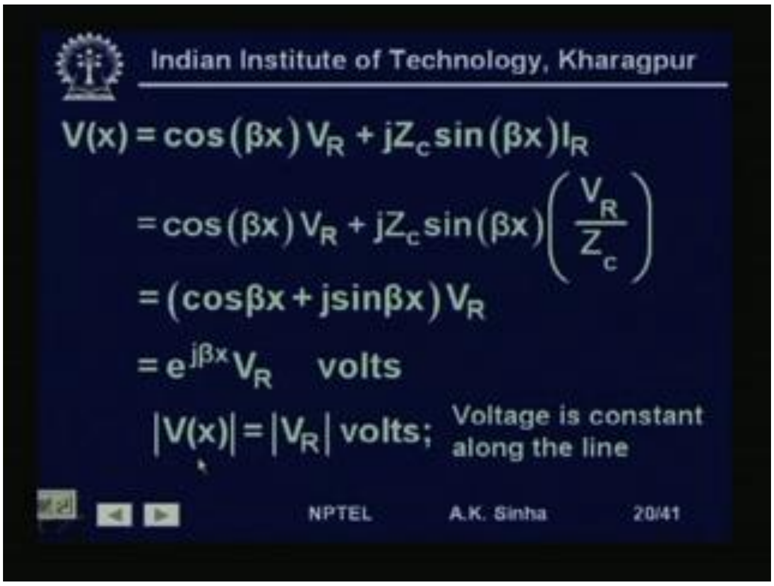
As we had seen earlier V X is equal to A X into V R plus B X into I R, where A X is cos beta x in case of lossless line. So, and V X is equal to Z c sin beta x, therefore V X is equal to cos beta x into V R plus j Z c sin beta x into I R. And similarly I X will be equal to C X V R plus D X into I R, C X is equal to j sin beta x by Z c and D X is equal to A X which is equal to cos beta x. Therefore I X is equal to j sin beta x by Z c into V R plus cos beta x into I R.

(Refer Slide Time: 30:18)



Now, if we terminate this transmission line into an impedance Z c. Now, here this figure is showing you the transmission line, or a single phase transmission line, we can consider it also as a single phase to neutral for this transmission line. Now, if we look at this transmission line and terminate it into Z c. That is if we load this line with an impedance equal to the characteristic impedance or the surge impedance for a lossless line, that is Z c is equal to root L by C.

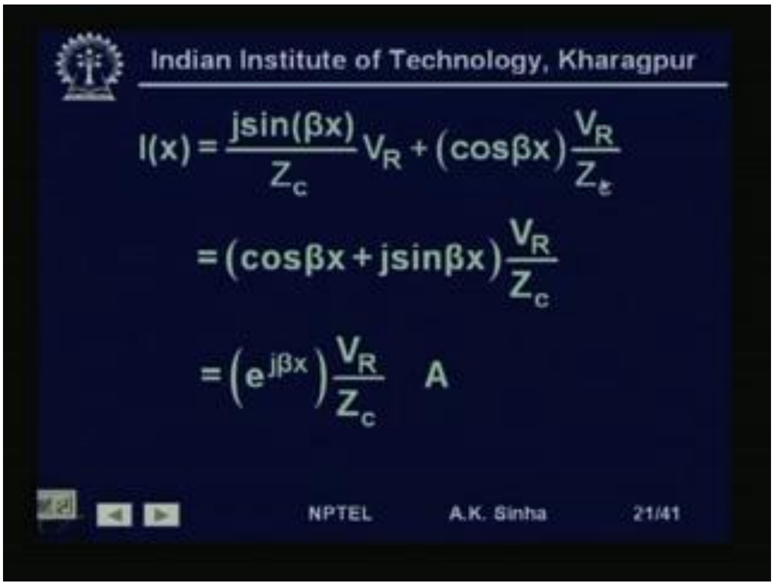
(Refer Slide Time: 31:05)



Then we get V X is equal to cos beta x into V R plus j Z c sin beta x into I R. Now, I R will be equal to, if you look at this circuit, then I R will be equal to V R, the voltage at this point divided by Z c, the impedance connected here. So, we replace this I R by V R by Z c, therefore we will get this as cos beta x plus j sin beta x into V R. This Z c, this Z c cancel out, so we get this as equal to e to the power j beta x into V R volts.

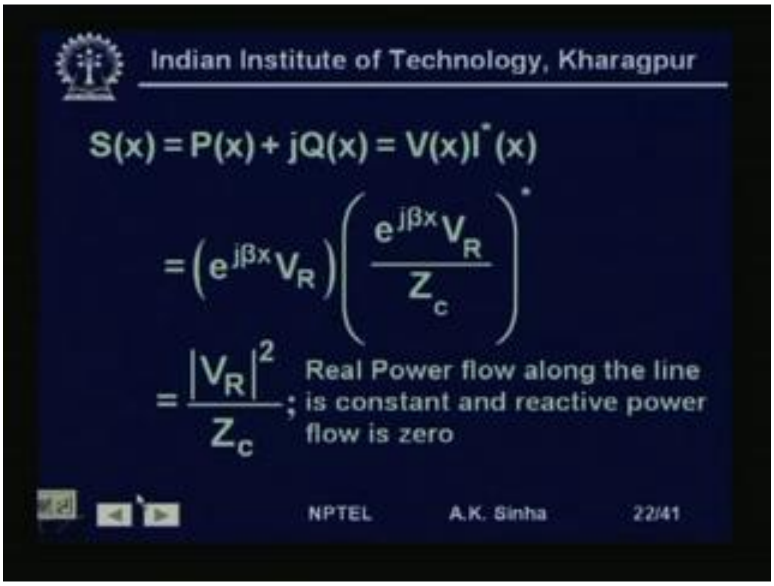
Now, here if we take the magnitude of this, then what we get is that V X is equal to, magnitude of V X is equal to magnitude of V R. That is the voltage at any point on the line is constant, that is it is equal to the receiving end voltage. So, from sending end to receiving end at all point the voltage is same, that is the magnitude of the voltage is same. This is one great advantage in using the loading on the line, which is equal to the surge impedance. And this is a particularly important characteristic of lossless line. That is if you terminate this line into surge impedance, then the voltage all along the length of the line is going to remain constant.

(Refer Slide Time: 32:57)



Similarly, we can find out the current at any point X from the receiving end. So, I X is equal to j sin beta x by Z c into V R plus cos beta x into V R by Z c, that is I R V R replacing by V R by Z c. So, this comes out to be cos beta x plus j sin beta x into V R by Z c, which again is equal to e to the power j beta x into V R by Z c. Again the magnitude if you take is going to remain constant all along the line. Now, if we talk in terms of the power, which is flowing in the line when we terminate this lossless transmission line through a surge impedance.

(Refer Slide Time: 33:51)



Then we have S x the power the complex power flowing, at any point in the line at a distance x from the receiving end is equal to P x plus j Q x, where P x is the real part or the real power flowing at a distance x from the receiving end. And Q x is the reactive power which is flowing at a distance x from the receiving end. This is equal to V x into I x conjugate.

Now, if we do this then we write for V x as e to the power j beta x into V R and for I x, we write e to the power j beta x into V R by Z c, we take the conjugate of that. Then, if we do this calculation e to the power j beta x will cancel out. And we get V R square, that is the mod of V R square and divided by Z c, that is the real power flow along the line is constant. That is all along the line the real power flow is constant. This is natural, because there is no losses which are taking place in the line.

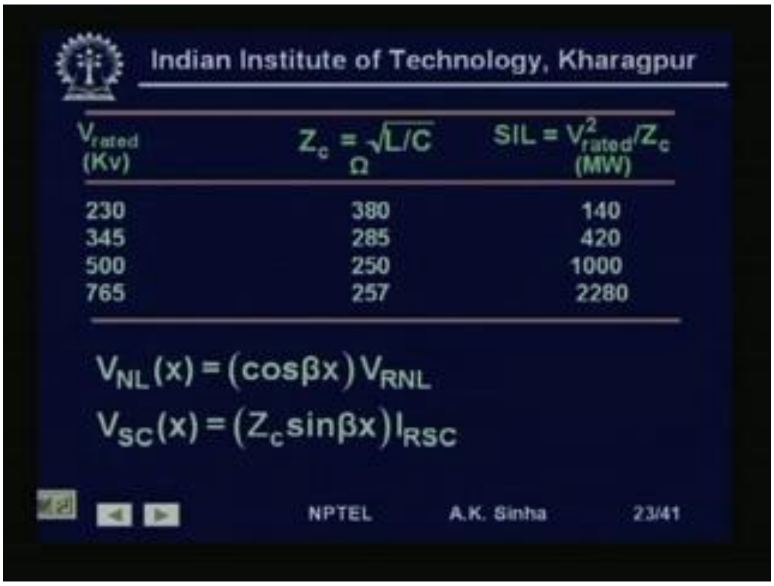
So, whatever power is being sent from the sending end, we are receiving the same power at the receiving end. And real power all along the line is constant. And also a very important characteristic of this lossless transmission line being terminated into it is characteristic impedance, that reactive power flow is 0. That is whatever reactive power losses which take place in the line, because of the series reactance of the line is being produced by the shunt capacitance of the line.

That is whatever is the reactive power generated by the shunt capacitance of the line, that is being consumed by the series impedance of the line. And therefore, the reactive power flow on along the line is 0. So, the surge impedance and surge impedance loading gives us very important characteristic. That is what we find from this is that, if we load the transmission line at it surge impedance. Or the surge impedance loading of the line will provide us a constant voltage all along the line.

That is both V S and V R will be same, this is a very great characteristic. That is the regulation of the transmission line is going to be 0 percent there is no voltage drop which takes place. Similarly, we do not have to provide any reactive power support for the reactive power consumed by the series impedance of the line. Because, reactive power consumed by the series impedance of the line is being provided by the shunt impedance of the line itself.

And there is not reactive power flow, so we can transmit the full capacity of the line in this case. Now, this surge impedance value for different voltage levels of the line is given in this table.

(Refer Slide Time: 37:37)

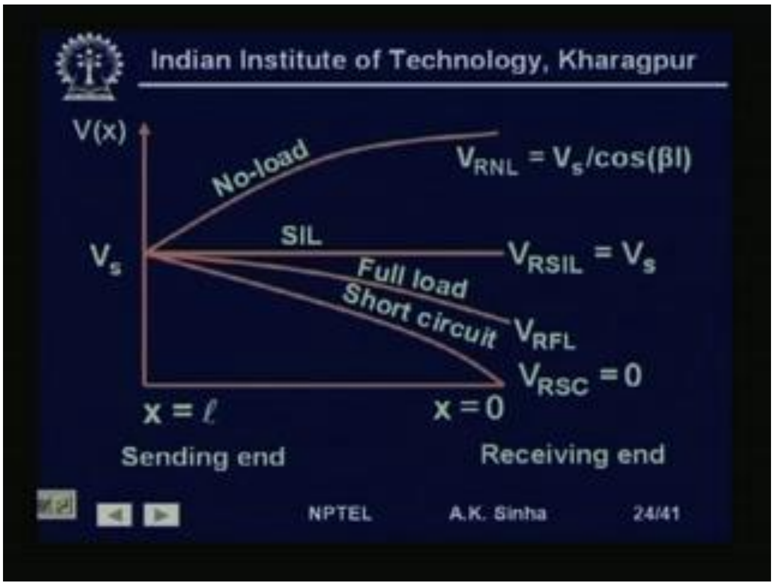


These are typical values, these may not be the exact values, but their approximately the values which we have. So, for a rated 230 KV line to line voltage three phase system, Z c is about 380 ohms. And the surge impedance loading in that case is going to be above 140 mega watt. Similarly, same for 345 KV, it is about 285 ohms and the surge impedance loading is four 120 mega watts.

For 500 KV this Z c, that is the characteristic impedance or surge impedance is equal to 250 ohms, and the surge impedance loading of the line is 1000 mega watt. And if we go to 765 KV, then the value of Z c is again approximately around 257 ohms. This can vary from 250 to 260 ohms and the surge impedance loading will be about 2280 or 2300 mega watt. Now, what we see from here is if we go for higher and higher voltages, our surge impedance loading keeps on increasing.

And that is why when we need to transmit more and more power, we go for higher and higher voltages. Because, if you see this relationship the voltage, that is the power flowing is proportional to square of the voltage. So, if you are doubling the voltage, you are able to transmit 4 times the power and so on. Now, I will show you the characteristics of this long lossless transmission line.

(Refer Slide Time: 39:49)



When we have a surge impedance loading, the voltage across the line all along from the sending end to receiving end is going to be same. If the line is unloaded, then the sending end voltage V S or V no load at any distance x, which we can write as at distance L is equal to cos beta x or cos beta L for sending end voltage into V RNL. Now, in this case we find that the sending end voltage is going to be less than the receiving end voltage

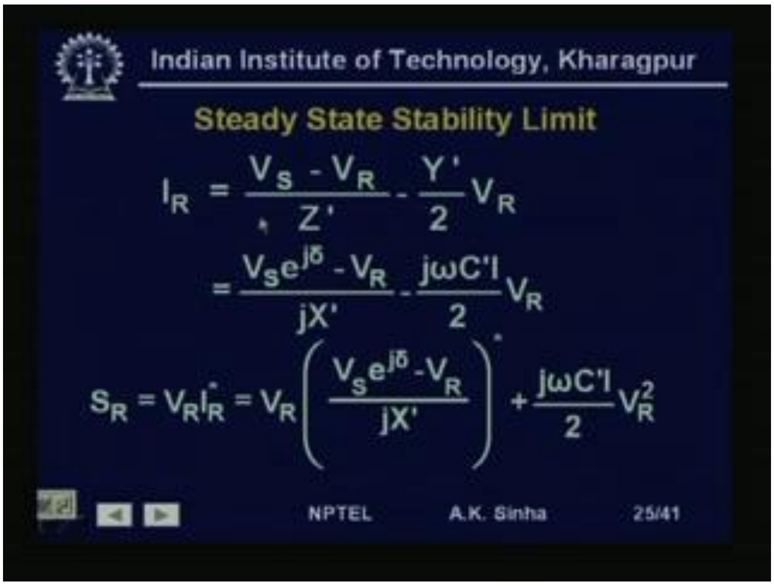
And if we keep the sending end voltage as 1 per unit, then what we find at receiving end voltage is going to be much higher. Similarly, if we do a short circuit of the transmission line at the receiving end, then the receiving end voltage is going to be 0 and the sending end voltage being 1 per unit. We will get the short circuit voltage at any point x is equal to Z c sin beta x into I R short circuit.

So, in this case again the voltage, that we get will be following this profile coming down to 0 at the receiving end, where the short circuit has occurred and it will be V s at the sending end. When the line is loaded at some other loading, then the surge impedance loading, generally lines which are not very long can be loaded more at loads, which are more than the surge impedance loading. That is full load is in general for not very long lines will be larger than the surge impedance loading.

And therefore, the voltage for that case is going to be somewhere, in between these two conditions which is the short circuit and the surge impedance loading. There is going to be some voltage drop and this is what we will find there is going to be some kind of a voltage regulation for this system. So, again from this we find one thing, that if we load the line to surge impedance, then the voltage remains constant. And that is why when we are designing the transmission line, we try to work on the bases of surge impedance loading.

Because that is the ideal condition, that we would like to have, that is voltage remains constant all along the transmission line. Now, for these lossless lines, there is another very important concept that we would like to discuss. And that is steady state stability limit for a transmission line.

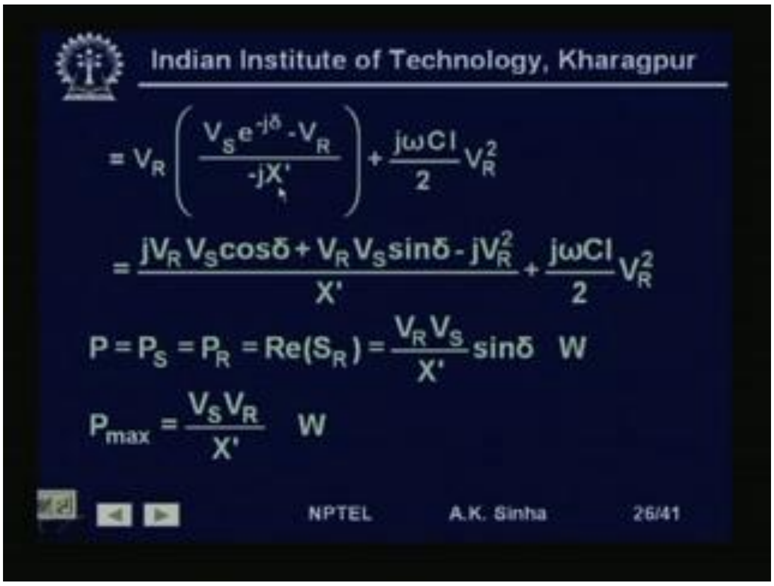
(Refer Slide Time: 43:25)



Now, if you look at the pi equivalent circuit, then we can calculate this I R will be equal to V S minus V R divided by this Z, which will be the current flowing in this, minus the current flowing here which will be V R into Y dash by 2. So, if you look at that, then we have I R is equal to V S minus V R by Z dash minus Y dash by 2 into V R. Now, substituting the value of V S and V R, V S we said is the sending end voltage with a magnitude V S and angle delta.

And V R is the magnitude of V R at an angle 0 degree, because that is chosen as reference. So, and Z dash in this case is only j X dash, because we are considering an lossless line. So, V S minus V R by Z dash, can be written as V S e j delta minus V R divided by j X dash and Y dash by 2 can be written as j omega C dash l by 2 into V R. Now, the receiving end power as we had talked earlier will be given by this S R is equal to V R into I R conjugate, so V R into I R conjugate is this terms. So, we take the conjugate of that. So, V S e j delta minus V R by j X dash conjugate and the conjugate of this will be negative of this. So, plus j omega C dash l divided by 2 into V R square, because with this V R gets multiplied on this side also.

(Refer Slide Time: 45:46)



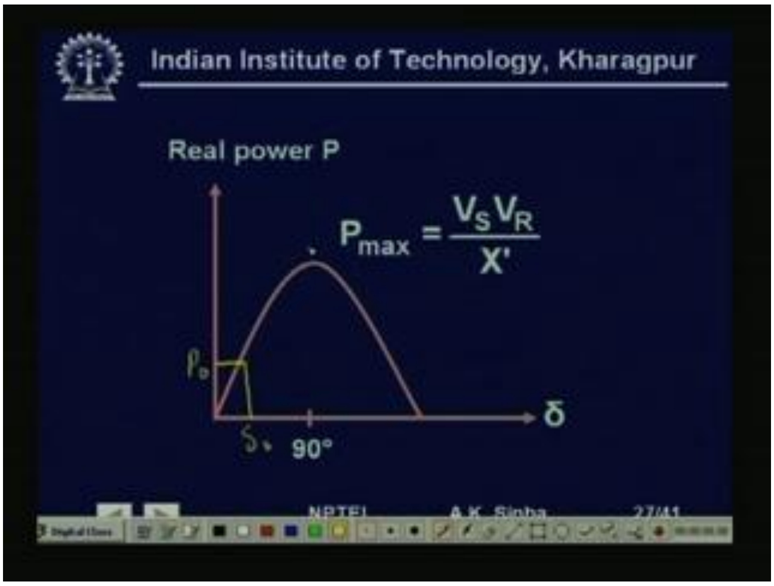
So, this is equal to V R into V S e to the power minus j delta minus V R minus j X dash, because we are taken the conjugates. So, this minus of angle comes and minus j X dash for that is shifting of the angle will come. So, this minus j X dash plus j omega C l divided by 2 into V R square. Now, if we expand this, then we will get this as j V R into V S cos delta plus V R into V S sin delta minus j V R square by X dash plus j omega C l by 2 into V R square.

There is a slight mistake here ((Refer Time: 46:36)), this should be C dash and this should be C dash. Now, if we look at the real power only, that is we take the real part of this when P is equal to P S is equal to P R, which is real part of the complex power S R. Then, this will be equal to V S V R sin delta by X dash all other terms are having j, that is their having imaginary, their imaginary components. That is their terms for the reactive power.

So, P the real power at the sending end is equal to the real power at the receiving end. And is given by V S into V R by X dash into sin delta, this will be watts or mega watts depending on whether we are using volts or kilo volts. Therefore, P max is equal to V S

V R by X dash, that is the maximum power that can be transmitted over this transmission line will be equal to V S V R by X dash. Because, this terms comes, this will be maximum when delta is 90 degree, that is sin delta is equal to 1.

(Refer Slide Time: 47:56)



The relationship is plotted here and it shows how with change in angle delta the real power varies. Now, this is a very, very important relationship, this tells us one very important thing, that the maximum power transmitted over the transmission line depends on the square of the voltage, because normally V S and V R are will be very close to 1 per unit. So, we will be talking in terms of square of the voltage only, the rated voltage of the transmission line.

So, P max is proportional to the square of the voltage. So, when we double voltage, system voltage we are able to transmit 4 times the power. Also the maximum power which can be transmitted over the line is inversely proportional to the reactance of the line. Now, this is again very important. And we have seen earlier that, when we use bundle conductors, then the inductance of the line gets reduced, because the effective radius of the line gets increase considerably. And that produces this reactance and therefore, use of bundle conductor will allow you to transmit more power on the transmission line. There is another very important aspect is that, suppose we have a synchronous machine which is connected to the system by means of a transmission line.

If this synchronous machine is supplying certain amount of power to this system, let us say it is working at this point somewhere.

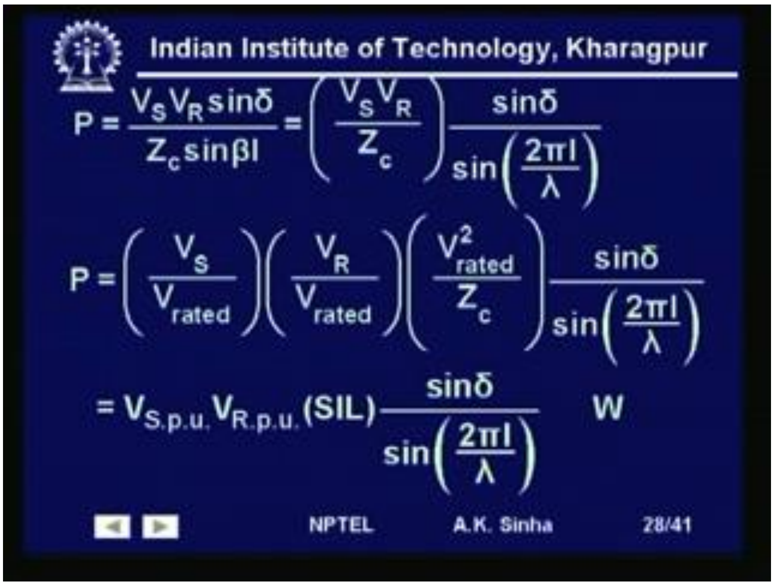
So, the delta angle for this will be there, the power which is being transmitted. Suppose, here the power which is being transmitted here, and the angle power angle or the voltage angle at the machine is delta 0. Now, if we slowly start increasing the input to the machine, what will happen is the machine output is remaining same, then input has been increase. So, it will experience some acceleration, since it is synchronized it cannot accelerate as such as speed will not change much.

But, what will happen is the delta angle will start increasing. If we keep doing this, what will happen is delta angle will keep increasing and as delta angle increases, the electrical power output also increases. So, the mechanical power and electrical power are becoming equal and the machine will be operating in stable region. Suppose, we keep doing it very slowly, we keep on moving on this, when we reach this point, when delta is equal to 90 degrees.

If we now increase the mechanical input to the synchronous machine, what is going to happen is delta angle will increase, but electrical power output is going to get reduced, which means that, there is going to be some accelerating power available mechanical input is more electrical output is less. So, delta angle will keep increasing, as delta angle increases the accelerating power also increases. And which will further increase this delta angle and machine will become unstable.

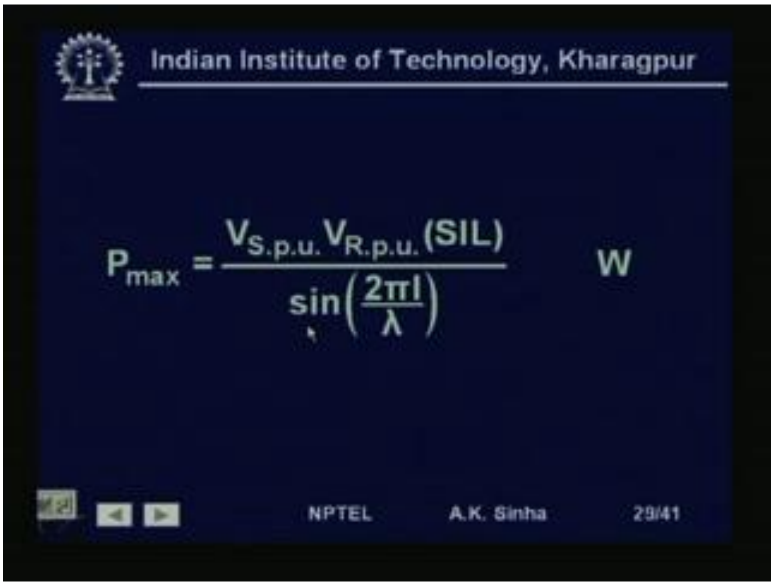
That is what we see is, if we gradually increase the input to the machine, the machine will adjust to that. And will increase it is electrical power output till delta is equal to 90 degree. Beyond that, it will become unstable, because the mechanical input increase is not going to bad accompanied by similar amount of increase in electrical output. And that is why this P max at delta is equal to 90 degree, which is given by V S into V R by X dash is also called as steady state stability limit.

(Refer Slide Time: 53:38)



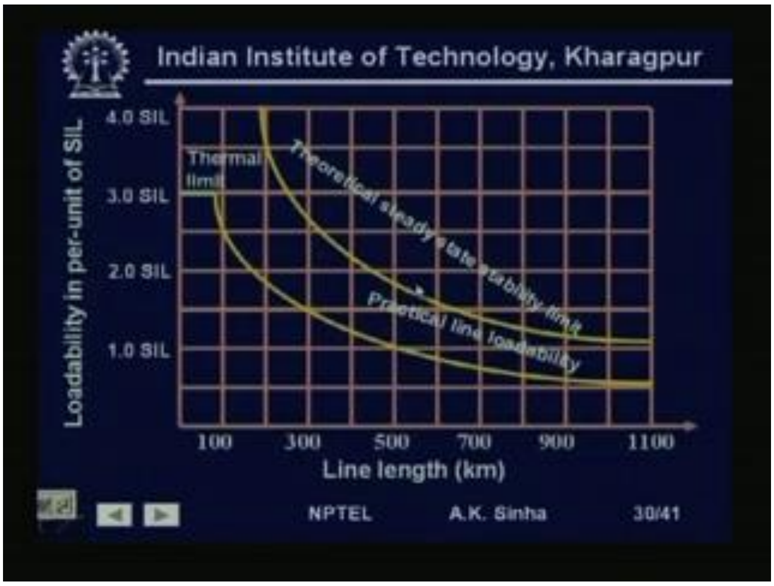
Now, we can write this P of the real power for a lossless line, P is equal to V S V R into sin delta by Z c sin beta l, which is equal to V S V R by Z c into sin delta by sin twice pi l by lambda; which, that is we are trying to write the real power in terms of surge impedance loading. So, P is equal to V S by V rated, we want to put everything in terms of per unit system. Therefore, V S by V rated into V R by V rated into V square rated by Z c, that is this V rated V rated we will get canceled here. So, V square rated by Z c into sin delta divided by sin twice pi l by lambda, which is equal to V S per unit into V R per unit into this term is surge impedance loading. So, into SIL into sin delta by sin twice pi l by lambda, in terms of watts or mega watts.

(Refer Slide Time: 55:02)



Or P max is equal to V S per unit into V R per unit into surge impedance loading divided by sin twice pi l by lambda, where lambda is the wavelength of the line and l is the length of the line. That is we can see that, the transmission line value of P max is going to depend on V S V R. And it is going to, that is if you increase V S and V R, then your maximum power transfer will increase. And if you increase l your maximum power which can be transmitted will reduce.

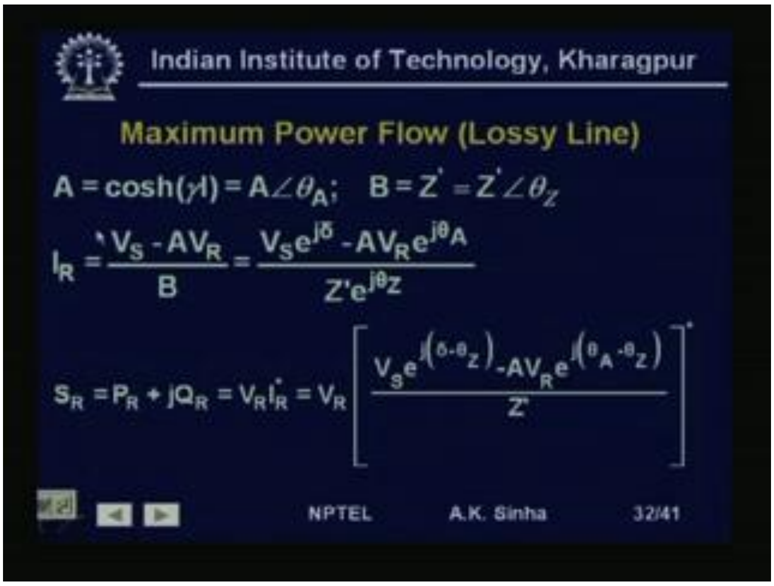
(Refer Slide Time: 55:57)



And if we plot this curve, we will get this curve like this, this is the theoretical one, where as the practical one will be much lower, because we normally work at an angle of delta around 30, 35 degrees only not beyond that. Normally between 20 to 30 degrees. And here we see that for short lines, that is lines up to 80 kilo meters also, it is the thermal limit which is the limiting factor for the transmission line load ability.

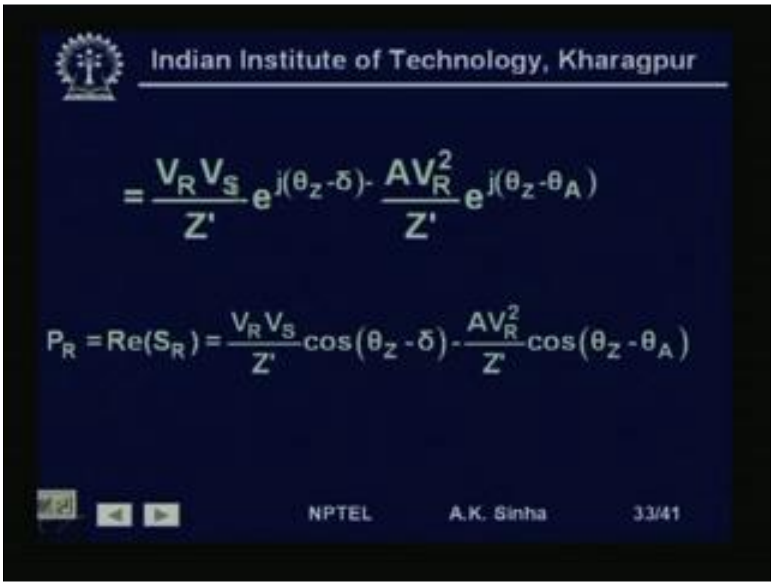
And this can be as high as 3 times as higher, but as we go beyond this 80 kilo meter length. Then the line load ability keeps on increasing with the practical line characteristics with delta around 30 degrees. We will get this kind of a characteristics which shows that beyond 500 kilo meters we have to load the line at a limit, which is lower than surge impedance loading. So, we see that if the line length increases our power transfer capability also reduces. This is the typical surge impedance loading at different voltages. And this is the thermal rating of the line. So, we see that for the lines, the thermal rating for the most of the cases is much higher than the surge impedance loading.

(Refer Slide Time: 57:44)



The same concept we can derive for the lossy lines, where instead of using the lossless line that is gamma being equal to beta, we will use the gamma as a complex number. Then, we can write this A is equal the cos hyperbolic gamma l, which is equal to A angle theta A, B is equal to Z dash is equal to Z dash theta Z. Then, we can write I R in the same way as V S minus A V R divided by B. And substituting these values we will get S R, that is equal to V R I R conjugate which is equal to V R into this I R conjugate.

(Refer Slide Time: 58:41)



This terms, this will come out to be V R V S Z dash e to the power j theta Z minus delta minus A V R square by Z dash e to the power j theta Z minus theta A. And P R which is the real part of this complex power at the receiving end will be equal to V R V S by Z dash into cos theta Z minus delta minus A V R square by Z dash cos theta Z minus theta A. That is what we are seeing is this is the term, which is getting subtracted from the earlier term that we had for a lossless line. That is for a lossy line, the real power is transfer is going to be less. And the maximum power that we can get, for a lossy transmission line will be when theta Z is equal to delta.

Thank you. And in the next class, we will take up some problems on transmission lines.