Power System Analysis

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Lecture - 7
Transmission Line Modeling

Welcome to lesson 7 in Power System Analysis. In this lesson, we will talk about Transmission Line Modeling.

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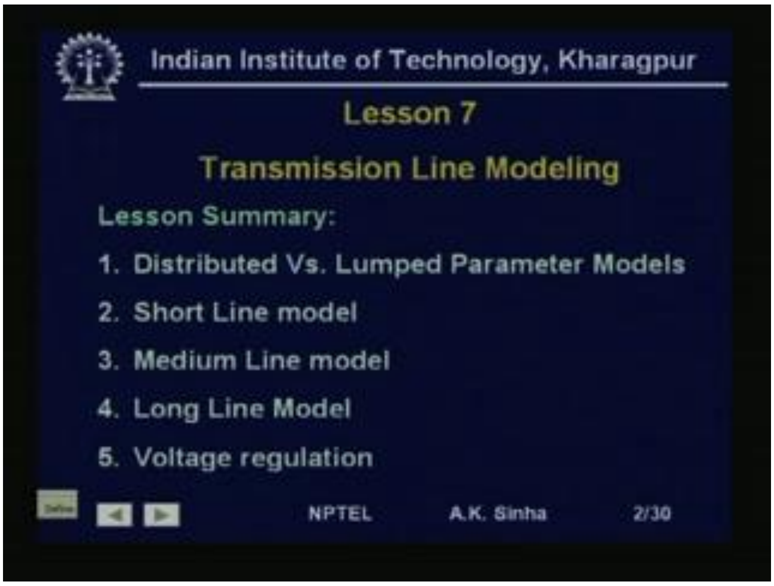

In this first we will talk about distributed versus lumped parameter models. Then, we will talk about short line model, medium line model and long line model. We will also discuss voltage regulation for transmission lines. Well, as we have seen in our previous lessons, when we discussed the calculation for transmission line parameters. We found that transmission line have in general three parameters, the line resistance, the line inductance and a capacitance of the line to neutral or ground. Now, as we had seen in the previous lessons, these parameters are distributed all along the line. That is if we see a line it will be something like this, where I have taken a small section of a line.

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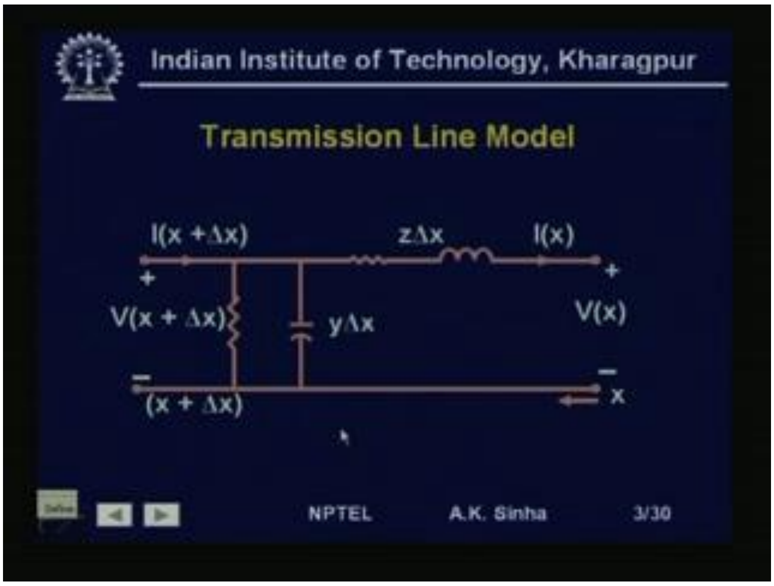

For this line of a length delta x, we have the transmission line impedance. Or the series impedance of the transmission line, which is given by z into delta x. It consists of the resistance of the line and the inductance of the line. These will be per unit length or per meter multiplied by delta x will give us the value of series impedance, z delta x for the delta x length of the line.

Similarly, we will have a capacitance to ground for the line which will be y; and for delta x of the line this shunt admittance is going to be y into delta x. There is capacitance will provide us an admittance j omega c, which will be equal to y. And for that will be per unit length of the line multiplying it by delta x the length of the line we get y delta x. Here I had also shown a conductance part for the line normally for 60 Hertz line or 50 Hertz line the power frequency line.

This conductance is negligible and most of the time we neglect this conductance. So, we have resistance and inductance per unit length of the line. This will form the series impedance per unit length of the line. And we have the capacitance per unit length of the line, which forms the shunt admittance of the line. Now, when we talk about modeling of transmission lines, what we have to see is. If we use distributed parameter model of the line, then the modeling of the transmission line becomes very complex. And it will result into differential equations.

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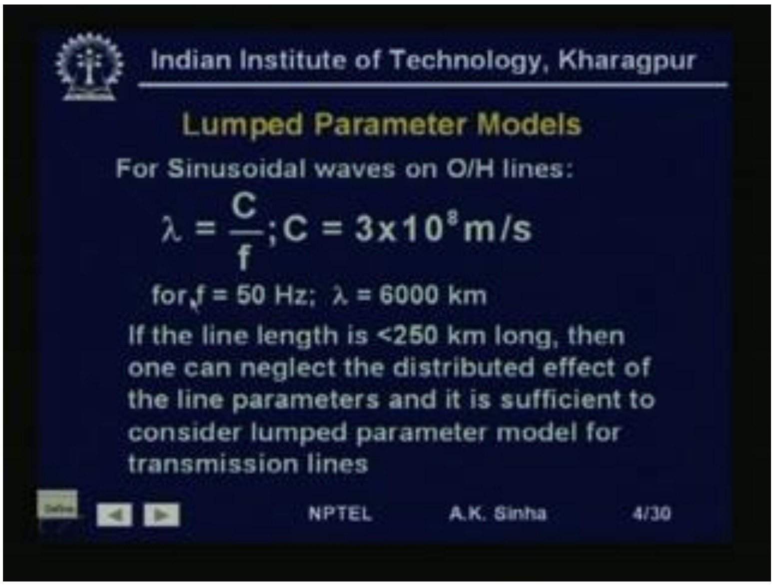

What we will see is that for sinusoidal voltage and current waves on overhead line. The wavelength of the line is given by this lambda; wavelength of the line is given by C by f, where, C is the velocity of light or the velocity of propagation of the waves, current or voltage waves on the transmission line. For overhead transmission line, this will be very nearly equal to the velocity of light.

Therefore, wavelength lambda is equal to C by f, where C is 3 into 10 to the power 8 meters per second. If we substitute this value of C and for frequency of 50 hertz, we get lambda is equal to 6000 kilometers. That is the wavelength of a transmission line at 50 Hertz is of the order of 6000 kilometers. Since most of the transmission lines that we have, are only a of a few 100 kilometer length.

Therefore, in most of the cases, we can neglect the distributed parameter model of the line. That is the distributed effect can be neglected and it is sufficient to model the line as lumped parameter model. That is if line length is less than about 250 kilometers, then one can neglect the distributed effect of the line parameters. And it is sufficient to consider lumped parameter model for transmission lines. Now, once we have used a lumped parameter model for the transmission line. Then, we can consider this transmission line as a simple two-port network.

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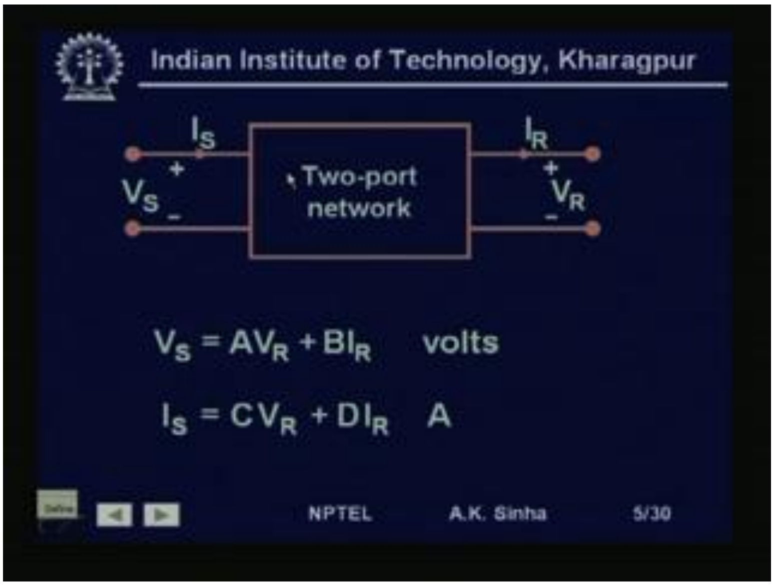

Where we have an input port, where we have the voltage V S and I S. That is the sending end port, V S is the sending end voltage, I S is the sending end current. And V R is the receiving end voltage and I R is receiving end current. So, transmission line can be visualized as a two-port network. For this two-port network, we can write down the relationship between the sending and voltage and current. In terms of receiving and voltage and current as V S is equal to A times V R plus B times I R volts.

And I S the sending and current is equals to C times V R, that is receiving an voltage. Plus D times I R the receiving in a current. Of course the unit will be amplix. Now, this is a general relationship for two-port network. It is valid for any two passive bilateral linear two-port network. And one of the properties of this from network theory, we get one of the properties for this network as A D minus B C is equal to 1. That is if we are putting this relationship in a matrix form, we get V S, I S is equal to ABCD into V R I R. This is the general transmission line ABCD parameter model for any transmission line with the relationship that A D minus B C is equal to 1.

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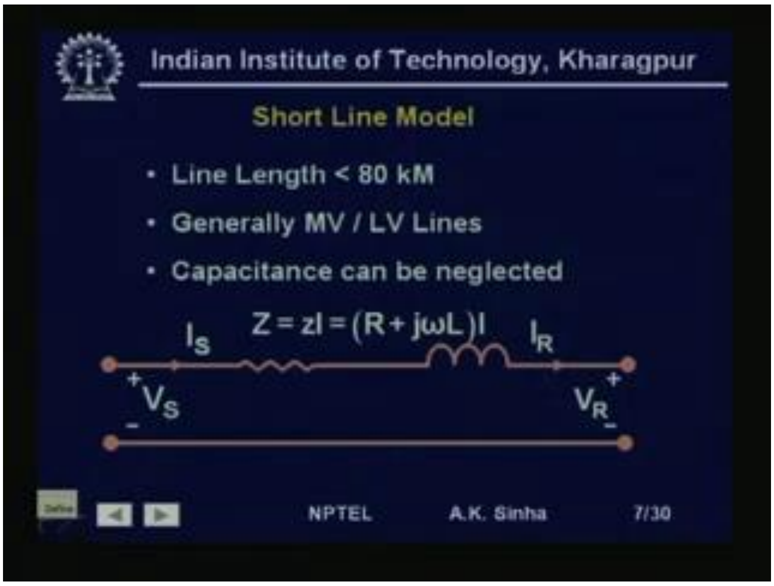

Now, transmission lines which are less than 80 kilometers long or about 50 miles long, we neglect the effect of capacitance. Because, the capacitive effect or the charging current is not very large and it is effect can be neglected. Also these lines are generally medium voltage and low voltage lines, because these are very short lines. And therefore, also the charging current, because the voltage being small will also be much smaller.

So, generally for line lengths less than 80 kilometer, which are generally medium voltage or low voltage lines, sometimes even high voltage lines can be there. The capacitance for these lines can be neglected, in that case the lumped parameter model for the transmission line, will consist of only the resistance and inductance. That is the series impedance of the line, the series resistance of the line and the series inductance of the line.

Here, the series impedance of the line Z is equal to z into l, small z into l, where small z is the per unit length series impedance. That is R plus j omega l per unit length, that is resistance per unit length and inductance per unit length, as we have seen we can calculate it for any line. So, Z is equal to z into l, this is equal to R plus j omega l into l, where R is the resistance per unit length of the line and j omega l is the reactance per unit length of the line.

So, from this model we have V S the sending end voltage, I S the sending end current only series impedance, that is here R and this j omega l or X. And I R is the receiving end current and V R is the receiving end voltage. From this circuit it is very clear I S and IR will be equal. Therefore, we can write down the circuit equation for this transmission line model as V S is equal to V R plus Z into I R.

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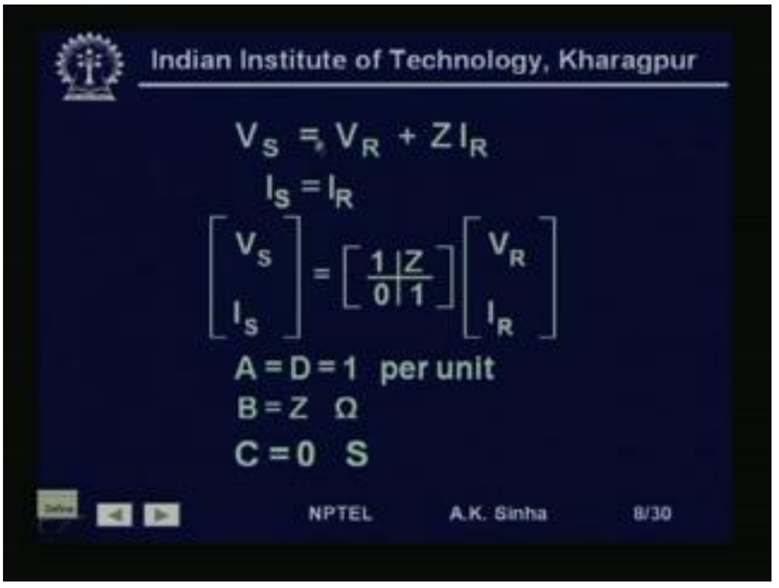

V S is going to be equal to this voltage plus the drop in this transmission line impedance. Series impedance of the transmission line, this will be equal to Z into I R. So, we have V S is equal to V R plus Z into I R. And as we have all ready seen I S will be equal to I R, therefore writing it in terms of matrix equation for ABCD parameters. We have V S I S is equal to V R plus Z into I R and I S is equal to 0 V R into plus 1 into I R. That is ABCD parameters from here, if you see are A is equal to D is equal to 1 per unit.

And B is equal to Z ohms and C is equal to 0 Siemens, that is the unit of A and D is basically dimensionless, it will be in terms of per unit. Whereas, the unit of B is the unit of impedance or ohms and unit of C is the unit of admittance or Siemens. So, from this relationship for a short line, we have A and D is equal to 1 B is equal to Z and C is equal to 0.

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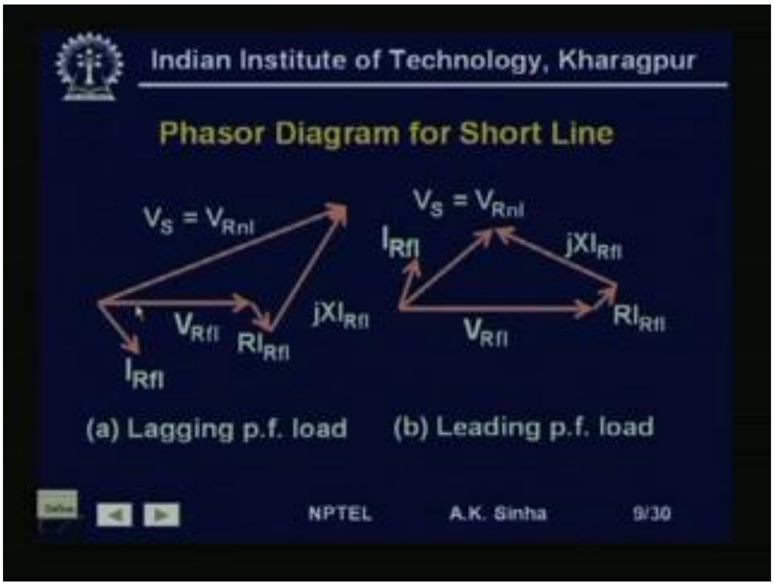

The Phasor diagram for short line can be developed, we can start with the voltage at the receiving end at full load. So, V R f l as the reference voltage and we have a current I R at full load, which is in this direction, that is lagging the voltage by some angle theta. Now, with this we can find out using the circuit model, voltage at the sending end, which will be equal to V R f l plus R into I R f l.

That is resistance, the drop in the resistance due to the current, receiving end current at full load. Plus the drop in the reactance, that is j X into I R f l, so the drop in the reactance. So, when these drops are added, we get the voltage V S. That is what we see is for lagging power factor load, where I R f l is lagging V R f l by an angle theta. We get V S which will generally be larger than V R, that is there is going to be a positive drop of voltage for the system.

Similarly, if the current is of leading power factor, that is again choosing V R full load as a reference voltage. And since, the current I R full load is a leading power factor load. So, it leads this voltage by some angle theta. Then, again by adding the voltage drops, we will get R times I R f l, which is the drop in the resistance plus j X times I R f l, which is the drop in the reactance. Or the inductive reactance this will give us the value V S, and this V S as we see will be many times smaller than V R.

That is in case of leading power factor load, the sending end voltage magnitude can be lowered than the receiving end voltage magnitude. Of course, in case of short lines the V

S is also equal to V R at no load. Because, if there is no current flowing in this, if there is no current flowing in this ((Refer Time: 15:29)), then the voltage V R and V S will be equal. So, V R at no load is equal to V S, that is what we have shown here. That is V S is equal to V R at no load V R n l, this we required for calculating regulation as we will see later.

Now, for line lengths or lines having a length between 80 to 250 kilometers, we call these lines as medium Lang lines. These lines are generally high voltage or extra high voltage lines. And for these lines, we cannot neglect the shunt capacitance or the capacitive admittance of the line. The shunt admittance needs to be taken care of.

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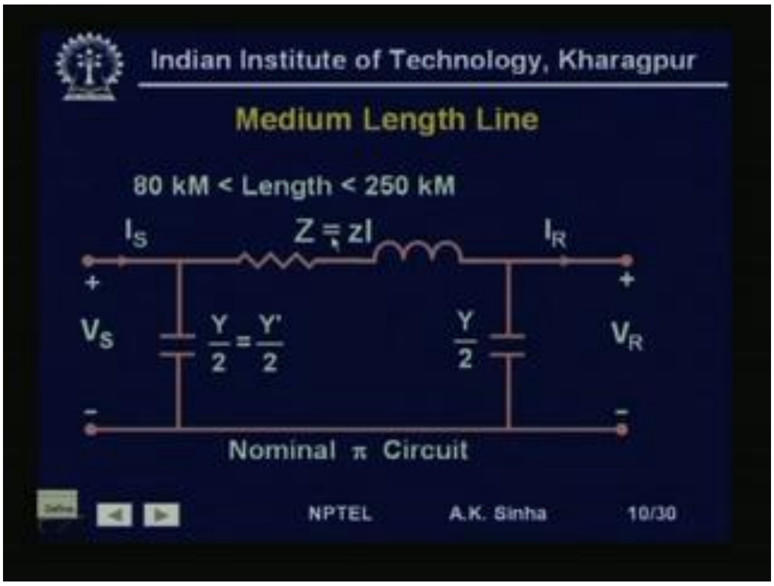

What we do here is, again we are use a lumped parameter model where the complete series impedance that it is total resistance of the line, is lumped as one resistance. And the total inductance of the line is again lumped together. So, we get this R plus j omega l as the series impedance of the line. So, Z the total series impedance is equal to z into l, where small z is the series impedance of the line per unit length.

Now, the total shunt admittance of the line will be Y, which will be equal to small y the shunt admittance of the line per unit length into l. Now, what we do is normally, we divide this total shunt admittance into two parts and put them at the two ends. So, we have shunt admittance Y by 2, put at the sending end and another Y by 2 put at the receiving end.

So, this model it looks like a pi model pi and therefore, we call the model as a nominal pi circuit for medium length line. Now, again here we have this as a two-port network with V S, I S the sending end voltage end current and V R, I R the receiving end voltage end current.

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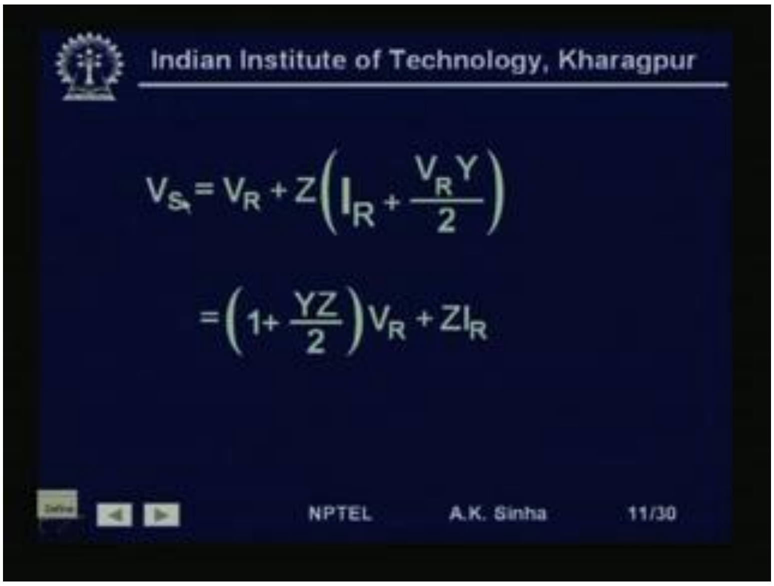

And we can use the ABCD parameter for this as V S, from here we can see is equal to V R plus Z into I R plus V R into Y by 2. Now, if you look at this circuit, what we are finding is, V S this voltage will be equal to the drop in this. Now, what will be the drop in this, this drop will be equal to how much current is flowing in this part IR plus VR into Y by 2.

So, I R plus V R into Y by 2 is the current this multiplied by Z will give us the drop in this. So, that is what we are writing V R is the receiving end voltage plus the drop, drop is Z into I R plus V R Y by 2. So, this is the current which is flowing in the charging admittance of the line placed at the receiving end. So, V S is equal to V R plus Z into I R V R Y by 2. If we rearrange this by combining all the terms of V R together, we get this as 1 plus Y Z by 2 into V R plus Z into I R. Similarly, we can write down the relationship for the sending end current. Now, sending end current sending end current I S is going to be equal to this I R plus the current here. That is V R into Y by 2 plus the current here which is V S into Y by 2.

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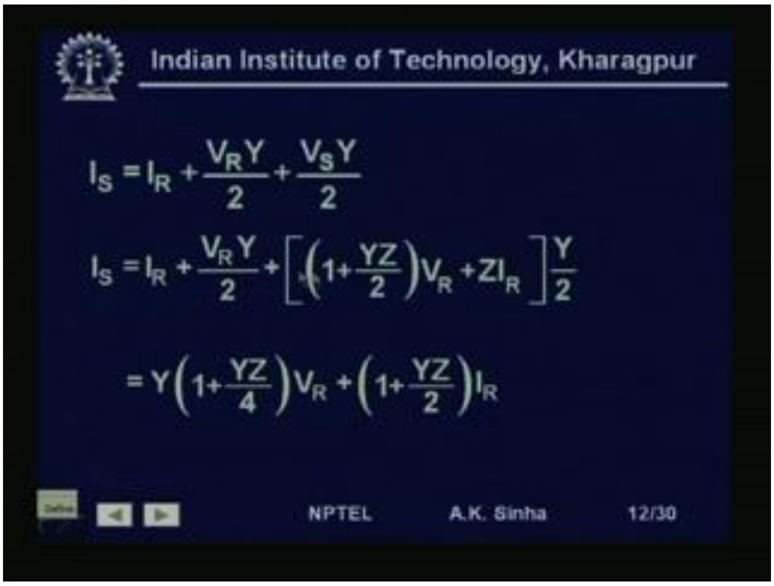

So, we are writing I S is equal to I R plus V R into Y by 2 plus V S into Y by 2. Now, again rearranging this by combining terms, we write this as I R plus V R into Y by 2 plus V S. We are writing the relationship for V S from the previous equation here, V S is equal to 1 plus Y Z by 2 into V R plus Z I R. So, substituting this value for V S, we are putting it here, so V S into Y by 2 we write here.

So, this is 1 plus Y Z by 2 V R plus Z I R for V S into Y by 2. Now, again rearranging it, we will get this as Y into 1 plus Y Z by 4 into V R plus 1 plus Y Z by 2 into I R. This is the relationship for the current I S, in terms of V R and I R. So, now using these relationships, we can write down this.

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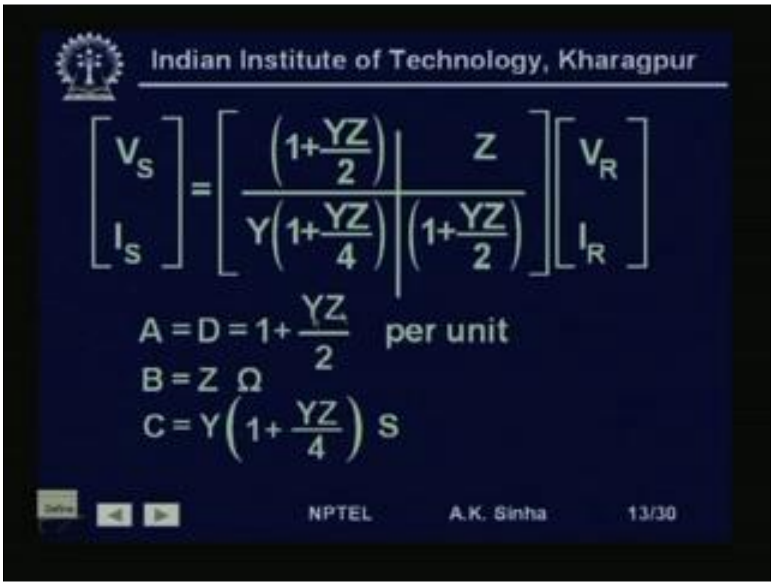

In terms of ABCD parameters as V S is equal to 1 plus Y Z by 2 V R plus Z I R, I S is equal to 1 Y into 1 plus Y Z by 4 VR plus 1 plus Y Z by 2 I R, which shows that A is equal to D, that is A is equal to D is equal to 1 plus Y Z by 2 this will be in pi unit. B will be equal to Z ohms and C will be equal to Y into 1 plus Y Z by 4 Siemens. So, for a medium length line, when we are using a nominal pi equivalent for this medium length line, we have the ABCD parameters.

Given as A is equal to D is equal to 1 plus Y Z by 2, B is equal to Z ohms and C is equal to Y into 1 plus Y Z by 4 Siemens. Now, we will talk about voltage regulation. Now, voltage regulation, what we mean by voltage regulation is basically in when there is no load on the line, the voltage on the line will be somewhat higher. When we load the line, generally the loads will be of a lagging power factor loads.

So, when we load these lines, then the voltage at the receiving end drops. And voltage regulation is basically telling us how much this voltage drop is, in terms of the rated voltage or the sending end voltage. Therefore, we define this voltage regulation as a percentage voltage regulation.

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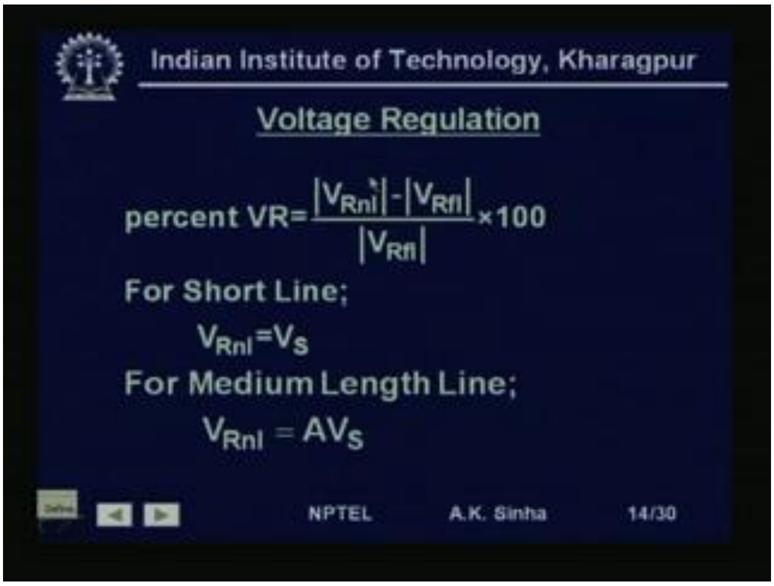

So, percent voltage regulation is defined as V R no load minus V R full load divided by V R full load into 100 percent. So, this is what we define, basically what we are trying to say here is that suppose we have a line which is loaded to it is full load value. And if the load is suddenly thrown off what will be the voltage at the receiving end. How much rise of voltage is there at the receiving end.

So, this V R n l as showing what will be the voltage of the receiving end? When the load is thrown off and V R f l is the voltage when the full load is there on the transmission line. So, this percentage voltage regulation gives us a very good parameter. Because, when we are designing a transmission line, we cannot allow more than about 8 to 10 percent voltage drop.

So, voltage regulation will tell us what is going to be the voltage drop, when the transmission line is fully loaded, because, if the voltage drop is more, then we need some compensation. We will also we had seen that when the load is capacitive in nature or of leading power factor, then the receiving end voltage can go higher than the sending end voltage. And this also needs to be checked for the transmission line.

Because, if the receiving end voltage becomes very high, then it may endanger the transmission line insulators, because the design voltage will be based on the nominal value or the rated value for the transmission line. Therefore, if you see here for the short line, when we want to calculate the voltage regulation we need to calculate the voltage at the receiving end at no load. And as we have seen earlier the voltage at the receiving end at no load, for the short the line is nothing but, equal to V S.

That is when I R is equal to 0, that is I R and I S both are equal to 0, because I R there is only I S and I R are equal. So, when the load current is 0, that is the system is at no load, then this drop is also equal to 0, so V R is equal to V S. For a medium length line, that is the pi model that we have used, we have seen that V S is equal to A V R plus Z into I R. Now, if we see this then when the I R is 0, that is at no load we have V S is equal to A V R plus Z into 0. So, V S will be equal to A V R only.

And therefore, we have the no load voltage at the receiving end, that is V R at no load will be equal to A into V S. And therefore, we can calculate V R at no load from this relationship. Now, we will take up the ABCD matrix for various kinds of line configurations. That is various kinds of network models or parameters that we use.

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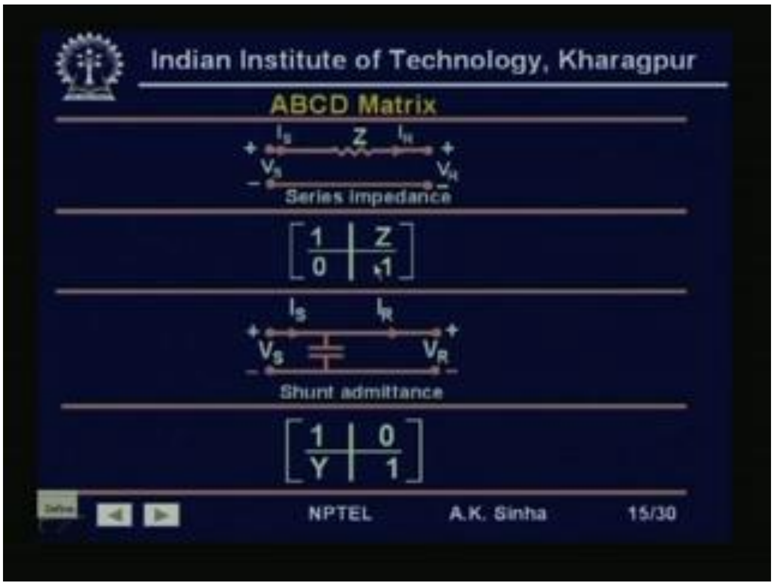

So, if we have a simple series impedance which is the case for a short line. Then we have the ABCD parameters as A is equal to 1, B is equal to Z, C is equal to 0 and D is equal to 1. If we have a simple capacitance, this is what we many times use when we have to compensate for the reactive power drawn by the load. So, many times when the load which is highly inductive, draws a large amount of reactive power. We need to compensate this reactive power by putting capacitance at the receiving end.

The shunt capacitance at the receiving ends, will then provide some reactive power. And this will compensate for the large amount of reactive power drawn by the load or consumed by the load. So, capacitance or the capacitor will generate reactive power at the receiving end itself. And compensate for the large amount of reactive power drawn by the load. For simple shunt capacitance the ABCD parameters will be A is equal to 1, B is equal to 0, C is Y and D is equal to Y.

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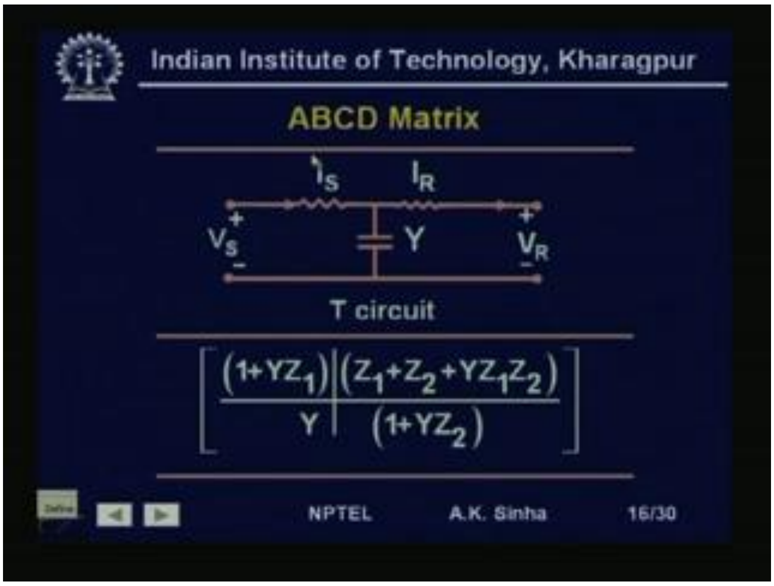

Similarly, instead of using a pi equivalent model, if you use a T equivalent model, where what we do is we divide the total series impedance into two equal parts. And lump the total capacitance as one capacitance and put it at the centre of the line. Then in this model we have Z by 2 and Z by 2 here, and total Y is here, in this kind of a model which is a T equivalent circuit for the transmission line. The ABCD parameters will be given by A is equal to 1 plus Y Z 1, where this is Z 1 and this is Z 2.

So, 1 plus Y Z 1 and B will be equal to Z 1 plus Z 2 plus Y Z 1 into Z 2. So, and C will be equal to Y and D will be equal to 1 plus Y Z 2. Again for this if you see A D minus B C will be equal to 1, for the pi equivalent circuit, we have all ready seen. Now, here for this pi circuit we have Y 1 and Y 2 which may not be equal, whereas for the nominal pi circuit we had both Y 1 Y 2 where equal.

That is the total shunt capacitance was divided equally and placed half of it was placed at the sending end and half was placed at the receiving end. Now, for this general pi circuit, if we calculate the ABCD parameters, we will get A is equal to Y 1 plus Y 2 Z, B will be equal to Z, C will be equal to Y 1 plus Y 2 plus Y 1 Y 2 into Z and D will be equal to 1 plus Y 1 Z. Again here if you see A D minus B C will be equal to 1.

Now, as I said many times we use compensation like adding a capacitance at the sending end. And then the line model will consist of two parts, one will be the transmission line model, another will be for the compensating equipment.

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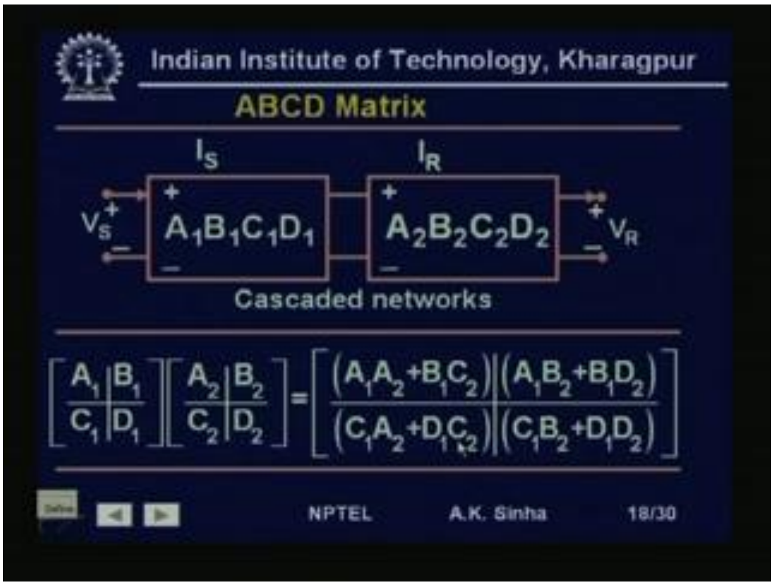

And in that case we have two cascade ABCD systems. And for this complete system also we can find out the ABCD parameters, which will be nothing but, this A B, A 1 B 1 C 1 D 1 multiplied by A 2 B 2 C 2 D 2 will give the composite ABCD of this. That is A for the composite system or cascaded network will be A 1 A 2 plus B 1 C 2. B will be equal to A 1 B 2 plus B 1 D 2, C will be equal to C 1 A 2 plus D 1 C 2 and D will be equal to C 1 B 2 plus D 1 D 2.

So, in this way we had seen how we can model the transmission line, for short lengths and medium lengths. But, when transmission line lengths are very long, which is the case in extra high voltage lines. Or some of the lines which are connecting the remote power generating sources, such as hydel plants to the load centered. Then in that case it is no longer possible to use a lumped parameter model.

And we will have to use the distributed parameter model, or the more accurate model otherwise. The errors which will come, because of using the lumped parameter model may not justify the simplification that we have used.

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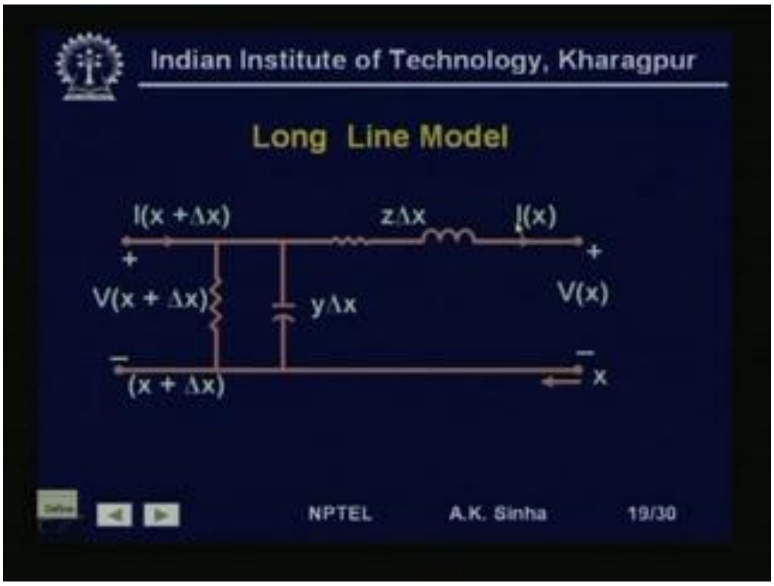

So, here again we come back to the distributed parameter model. So, for long line as I said, we will use this distributed parameter model. We have taken a section of line which is delta x in length a small length delta x. The series impedance of the line is z into delta x, where z is the series impedance of the line the resistance plus the inductive reactance of the line R plus j omega l per unit length. So, that is represented here as z into delta x.

And the total shunt admittance is y into delta x, for this delta x part of the line where small y is the shunt admittance of the line per unit length. As I said for the transmission line at power frequency 50 Hertz or 60 Hertz, this part the conductance is negligible and this is generally neglected and only capacitance is there. Now, we have taken the parameter x or the distance parameter from the receiving end side.

So, the receiving end voltage or the voltage at a point at a distance x from the receiving end we write this voltage as V at distance V x. And the current at this point as I at a distance x form the receiving end that is I x. Similarly, the voltage of the line delta x meter in this positive direction, that is from receiving end towards the sending end will be given by V into x plus delta x. That is voltage at x plus delta x from the receiving end; current will be given by I at x plus delta x, that is at current at x plus delta x distance from the receiving end. Now, with this these values now we will write the Kirchhoff's voltage law.

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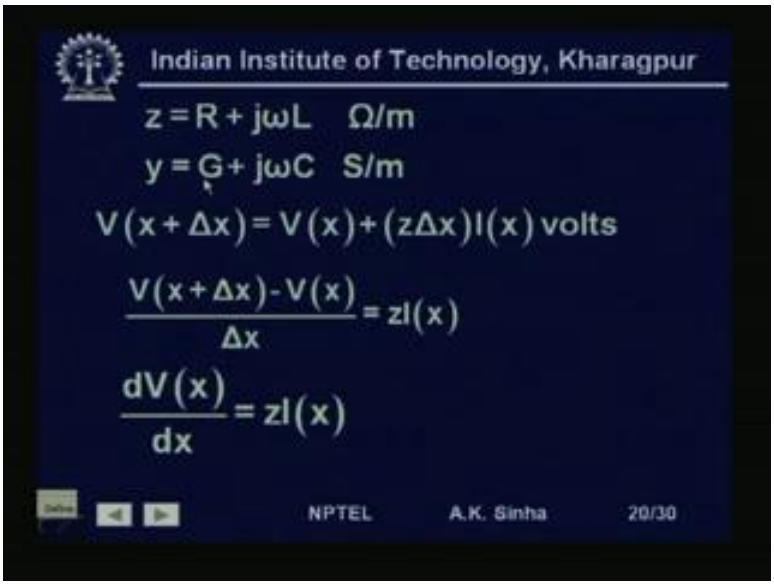

Here, we have V plus V at x plus delta x will be equal to V x. That is the voltage at distance x plus the voltage drop, that is the voltage drop here will be I x into z delta x. So, the voltage at this end, that is V at x plus delta x is going to be equal to V at x plus the voltage drop, which is I at x into z delta x. That is what we have written here, V at x plus delta x is equal to V at x plus z into delta x into I x I at x.

Now, here as I said earlier z is the series impedance per unit length, which will be R plus j omega L ohms per meter. And y is G plus j omega C Siemens per meter, where G for power frequency or transmission lines we have is almost 0 or negligible. Now, from this relationship of voltage at distance x plus delta x, in terms of voltage at distance x, if you rearrange, this we will get as V at x plus delta x minus V at x divided by delta x.

This delta x term we have taken on this side and this V x, we have taken on this side, so this becomes negative. And this delta x comes in the denominator. So, V at x plus delta x minus V at x divided by delta x is equal to z into I at x. Now, if we take a limit with delta x tending to 0, then we can write this as d V x by d x is equal to z I x. That is derivative of voltage at x will be equal to z into I at x.

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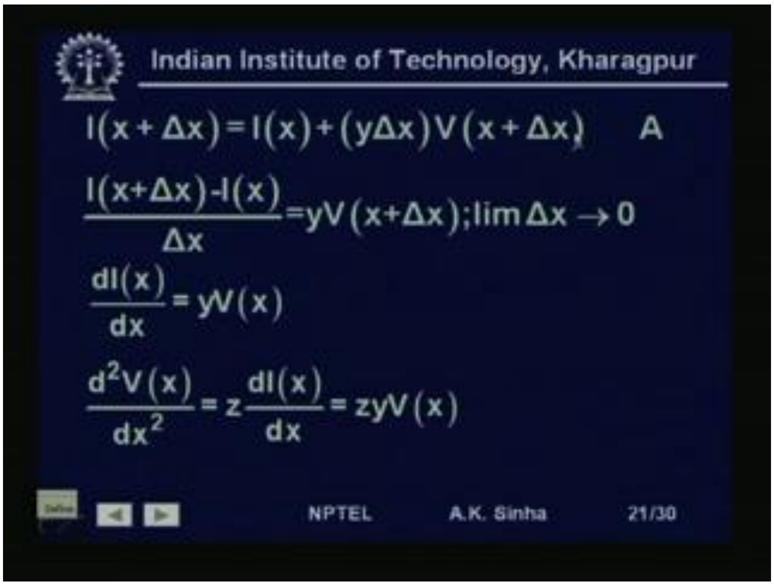

Similarly, writing the Kirchhoff's current law. We will have I at x plus delta x will be equal to I at x plus y into delta x into V at x plus delta x. That is if we look at this ((Refer Time: 38:27)) I at x plus delta x is going to be equal to I at x. Plus the current which is flowing in this admittance which will be equal to y into delta x multiplied by voltage at this point which is V at x plus delta x.

That is why we have got I at x plus delta x is equal to I x, the current at x plus y into delta x the admittance multiplied by V at x plus delta x. Again rearranging by taking this I x on this side and dividing by delta x, we have I at x plus delta x minus I at x divided by delta x is equal to y into V at x plus delta x. Now, taking limit delta x tending to 0, we can write this as equal to d I x by diagnosis, that is first derivative of current at x with respect to x is equal to y into V at x.

Now, what we have is if we see our previous equation for V. If we differentiate it again with respect to x, then what we will get is d 2 V x by d x 2 is equal to z d I x by d x, d 2 V x by d x 2 is equal to z d I x by d x. Now, substituting the value of d I x by d x from this point or this equation, we get this is equal to z into y into V x. We can write this as d 2 V x by d x 2 minus z y V x is equal to 0. That is what we have done is taken this term on this side. So, d 2 V x by d x 2 minus z y V x is equal to 0.

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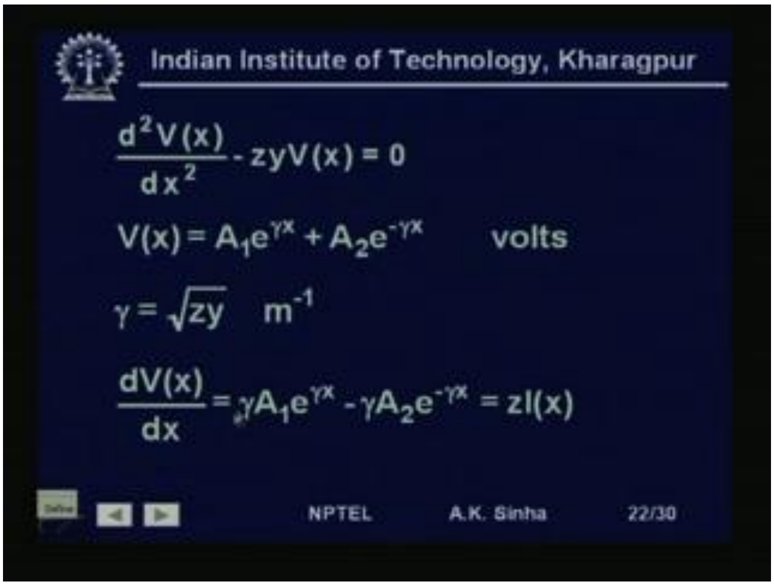

Now, this a second order homogeneous equation. And we know the solution for this equation can be obtained in this form, where V x will be given by A 1 e to the power gamma x plus A 2 e to the power minus gamma x volts. So, this will be a general solution for this second order differential equation, where gamma is equal to root z y and the unit for this will be per meter.

Now, again differentiating this equation, we can write d V x by d x differentiating this equation with respect to x. We have got d V x by d x is equal to gamma A 1 e to the power gamma x minus gamma A 2 e to the power minus gamma x. And this will be equal to z into I x, because d V x by d x is z into I x. Therefore, we can write that I x is equal to the this term divided by z.

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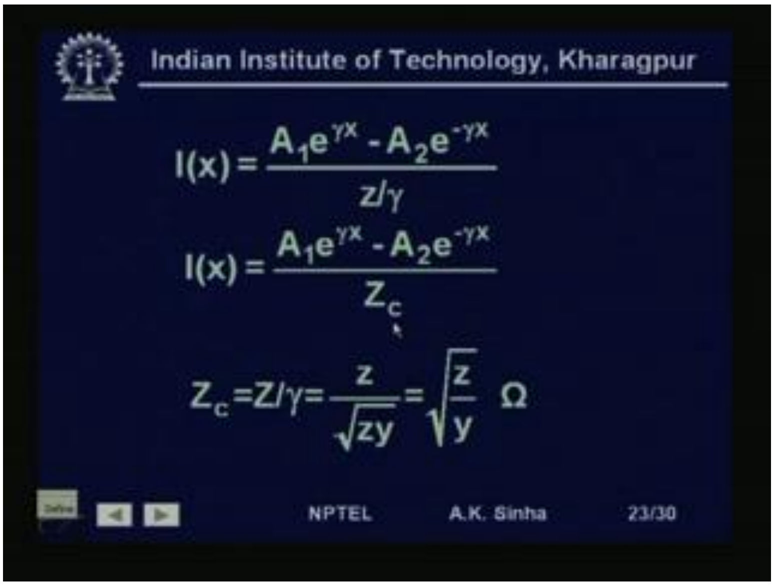

So, I x is equal to A 1 e to the power gamma x minus A 2 e to the power minus gamma x divided by z. And the gamma which was here multiplied with A 1 and A 2 that we have taken on in the denominator, so this is z divided by gamma. Therefore, we can write I at x is equal to A 1 e to the power gamma x minus A 2 e to the power minus gamma x divided by Z c. Now, this term z by gamma we are writing this as Z c, Z c is called a characteristic impedance of the transmission line.

Now, if you see Z c as we have put here is z by gamma. This is equal to z divided by gamma is square root of z y and therefore, this is equal to square root of z by y. That is square root of series impedance by shunt admittance. And the unit for this is ohms and therefore, this Z c is termed as the characteristic impedance of a transmission line. Now, we need to find out the constants of integration that is this A 1 and A 2. So, for this what we do is, we use the boundary conditions.

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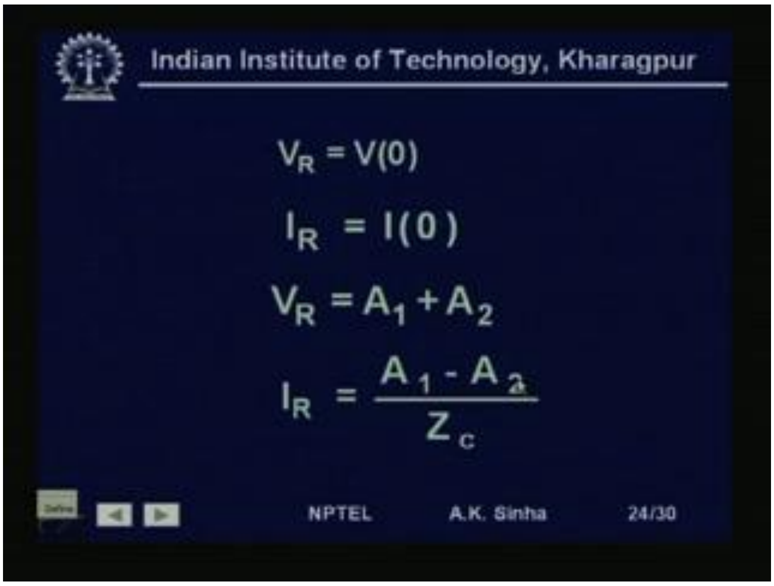

So, here using the boundary, that is at x is equal to 0, V x is equal to V R. So, at 0 V at 0 is equal to V R and I at 0 is equal to I R, therefore V R will be equal to A 1 plus A 2. That is if we substitute in this relationship x is equal to 0, then we get V 0 which is equal to V R is equal to A 1 plus A 2 as these terms will become 1. So, V R is equal to A 1 plus A 2 and again substituting it in this ((Refer Time: 44:36)) I x here x is equal to 0 in these we will get I x is equal to A 1 minus A 2 by Z c, A 1 minus I R is equal to A 1 minus A 2 by Z c. From these two relationships A 1 plus A 2 is equal to V R and A 1 minus A 2 by Z c is equal to I R, we can calculate A 1 and A 2.

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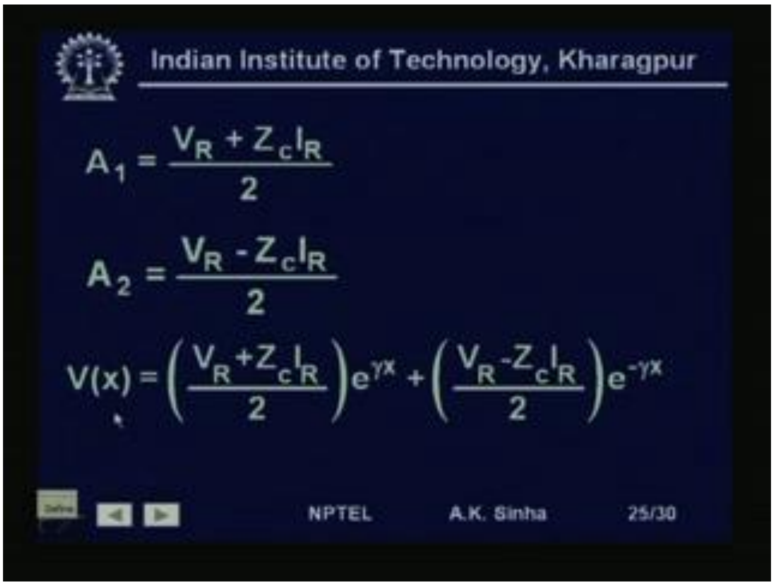

And A 1 comes out to be equal to VR plus Z c into I R by 2 and A 2 will come out to be equal to V R minus Z c into I R by 2. Now, substituting the values of this A 1 and A 2, in the relationship for V x, we can get V x is equal to V R plus Z c into I R by 2. That is A 1 into e to the power gamma x plus V R minus Z c into I R by 2, this is A 2 into e to the power minus gamma x.

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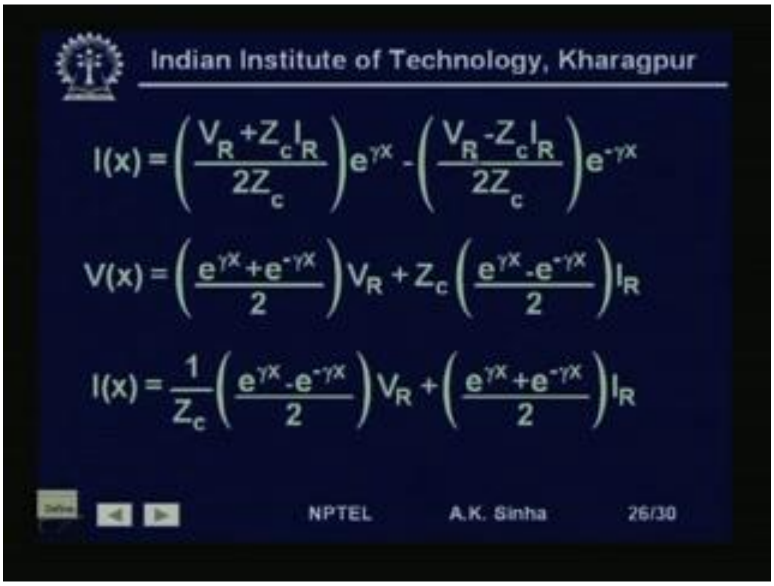

And similarly putting it for the value of A 1 A 2 for I x relationship, we get I x is equal to V R plus Z c into I R by 2 into Z c e to the power gamma x minus V R minus Z c I R divided by 2 into Z c into e to the power minus gamma x. Now, if we again rearrange the equations for V x and I x, that is in terms of V R and I R, then combining all the terms of V R together and terms of I R together. Then we can write from this equation take all the terms which are having V R together and take all the terms which are having I R together.

Then, we can write V x is equal to e to the power gamma x plus e to the power minus gamma x divided by 2 into V R plus Z c e to the power gamma x minus e to the power minus gamma x by 2 into I R. And I x will be equal to 1 by Z c into e to the power gamma x minus e to the power minus gamma x by 2 V R plus e to the power gamma x plus e to the power minus gamma x by 2 into I R, this is the relationship that we will get.

Now, here we have got a relationship of V at a distance x, in terms of receiving end voltage and current. Similarly, current at any distance x, in terms of receiving end voltage and current.

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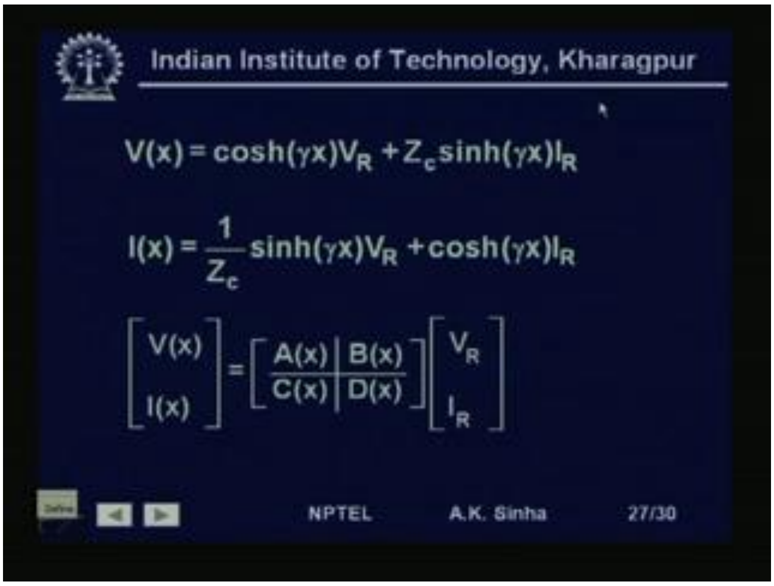

Now, from this we can ((Refer Time: 47:46)) this term e to the power gamma x plus e to the power minus gamma x by 2 is equal to cos hyperbolic gamma x. And e to the power gamma x minus e to the power gamma x by 2 will be equal to sin hyperbolic gamma x. Therefore, we have this cos hyperbolic gamma x into V R plus Z c into sin hyperbolic gamma x into I R, that is what we get here.

That is voltage at any point in the transmission line can be found, in terms of the hyperbolic functions and the receiving end voltage and currents. Similarly, the current at any point in the transmission line any distance x from the receiving end, can be found out. Similarly, using the relationship I x is equal to 1 by Z c sin hyperbolic gamma x into V R plus cos hyperbolic gamma x into I R.

Now, if you look at this, this is giving us a relationship of voltage and current, at any point x, in terms of receiving end voltage and current. Therefore, we can write this in terms of ABCD parameters as V x I x is equal to A x B x C x D x V R I R, where ABCD, A is equal to cos hyperbolic gamma x, B is equal to Z c sin hyperbolic gamma x, C is equal to 1 by Z c sin hyperbolic gamma x. And D is equal to cos hyperbolic gamma x, again you will see A D minus B C is equal to 1.

Now, since most of the time we are interested only in the terminal conditions. That is conditions at the sending end and the receiving end, not anywhere in between the lines. So, we need to just substitute l instead of x and then we will get the sending end voltage and sending end current.

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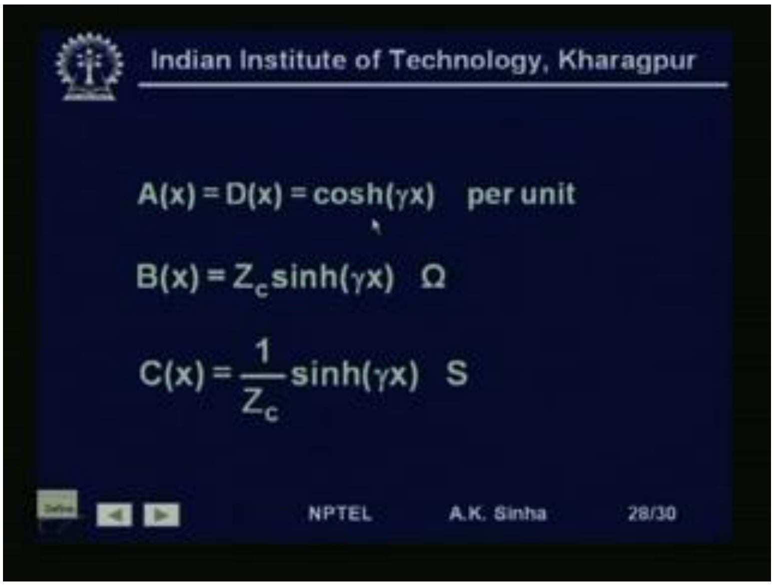

So, this relationship can also be put in the form of sending end and receiving end voltage end current, where x is replaced by l.

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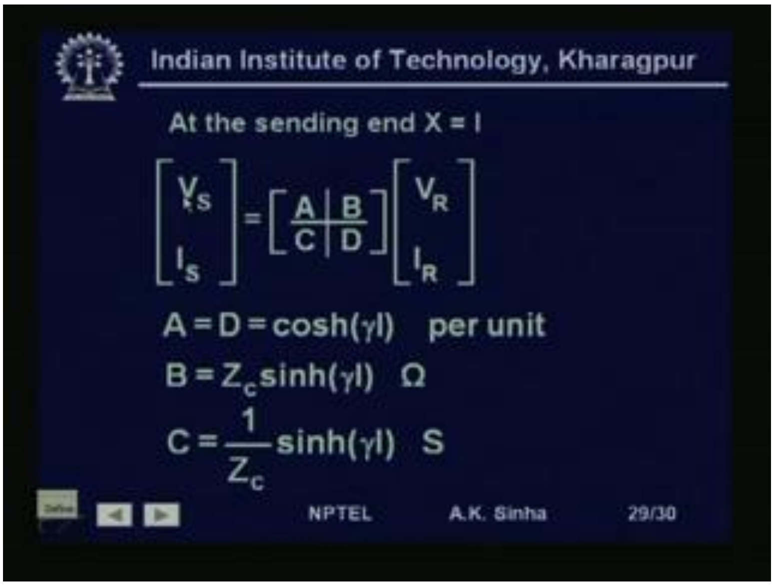

That is at sending end x is equal to l, therefore we can write this relationship as V S I S is equal to ABCD V R I R. And where A is equal to D is cos hyperbolic gamma l per unit, B is equal to Z c sin hyperbolic gamma l, C is equal to 1 by Z c sin hyperbolic gamma l in Siemens. So, again we have got the ABCD parameters for the transmission line and with for a long transmission line, the only thing here, what we are seeing is we have to use hyperbolic trigonometric functions.

That is ABCD parameters are in terms of hyperbolic trigonometric functions, not just Y and Z as we had for lumped parameter models. So, with this we stop today and we will be continuing this line long transmission line model. In the next class, where we will talk about the velocity of propagation the speed, the phase angles and another things. Also we will talk about regulation and efficiency of transmission lines.

Thank you.

Preview of Next Lecture

Lecture - 08

Transmission Line Modeling Long Line (Contd.)

Welcome to lesson 8 on Power System Analysis. This lesson is a continuation on Transmission Line Modeling, specially Modeling of the Long Line. Now, if you remember in the previous lesson that is lesson 7, we talked about distributed parameter model for long transmission lines. That is lines which are longer than 250 kilometers.

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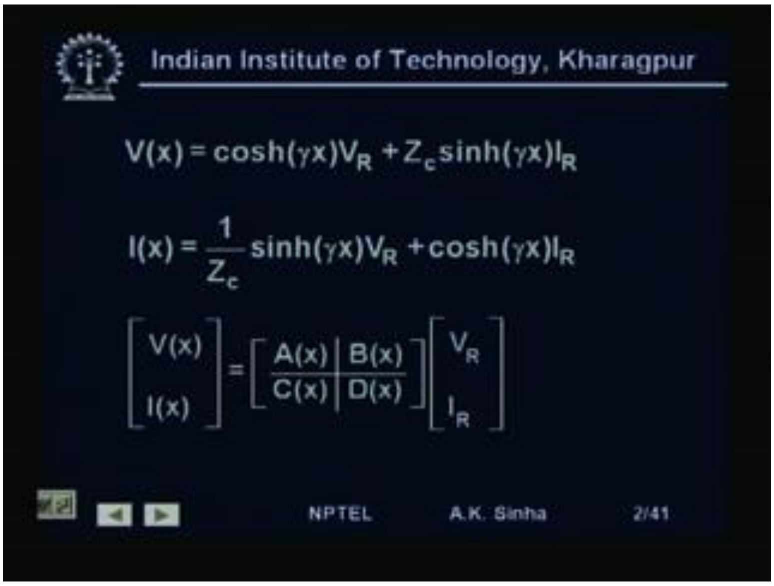

For these lines we said that the line voltage V x, at a distance x from the receiving end is given by cos hyperbolic gamma x into V R plus Z c sin hyperbolic gamma x into I R, where gamma is called the propagation constant, x is the distance of the point from the receiving end, V R and I R are the voltages and currents at the receiving end. Similarly, the current at a distance x from the receiving end, I x is equal to 1 by Z C sin hyperbolic gamma x into V R plus cos hyperbolic gamma x into I R, where Z c is the characteristic impedance of the transmission line.

And it is given by square root of Z by Y, where Z is the series impedance of the transmission line per unit length. And Y is the shunt admittance of the transmission line per unit length. As we see these models these equations involve hyperbolic functions. Now, since we have been writing all these transmission line equations, in terms of ABCD parameters, we can write this equation also in those terms. So, in matrix form we can use this as V x, I x is equal to A x B x C x D x and V R and I R this form, where this relationship is in terms of voltage and current at any distance x from the receiving end.

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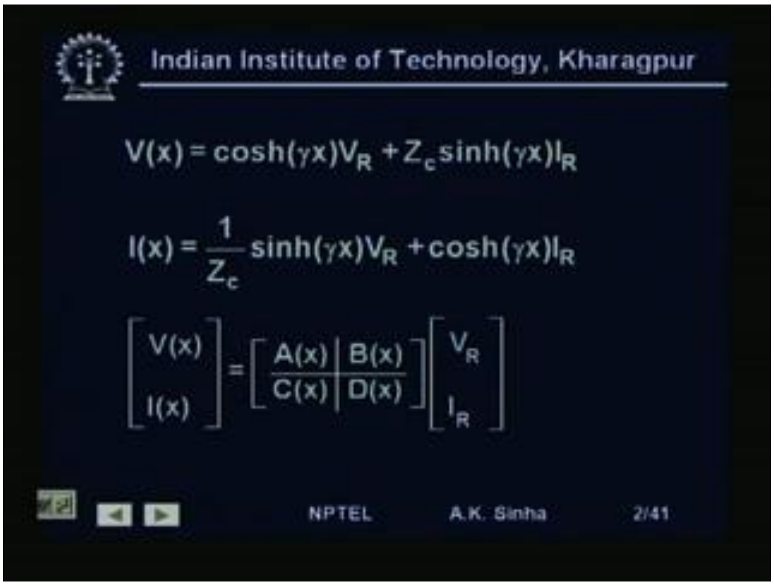

Now, here if you see A x is equal to D x and that is equal to cos hyperbolic gamma x in per unit. That is if you see this relationship this is your A x and this is your B x, this is C x and this is D x. So, A x is equal to D x is equal to cos hyperbolic gamma x B x is equal to Z c sin hyperbolic gamma x, V x is equal to Z c sin hyperbolic gamma x. C x is equal to 1 by Z c sin hyperbolic gamma x and D x as we have all ready seen is equal to A x.

So, this is the model, where A and D are basically dimensionless, B has a dimension of impedance and C has a dimension of Siemens that is admittance. Now, normally we are interested only in the terminal conditions. That is the sending end voltages end currents and the receiving end voltages end current. Rather than, the voltage end current at any intermediate point on the transmission line. Therefore, we can find out the voltage end currents at the sending end, in terms of voltage end current at the receiving end by substituting x is equal to l, where l is total line length.

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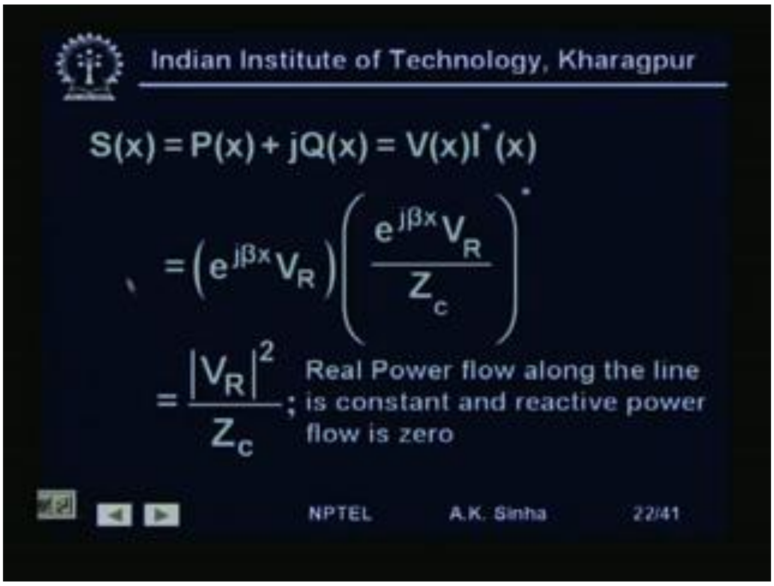

See this relationship, the voltage that is the power flowing is proportional to square of the voltage. So, if you are doubling the voltage, you are able to transmit 4 times the power and so on. Now, I will show you the characteristics of this long lose less transmission line.

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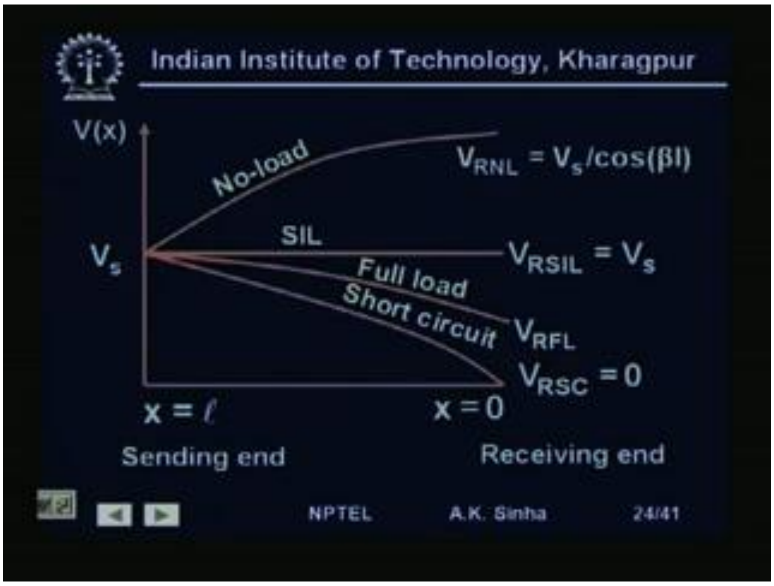

When we have a surge impedance loading. The voltage across the line all along from the sending end to receiving end is going to be same. If the line is unloaded, then the sending end voltage.

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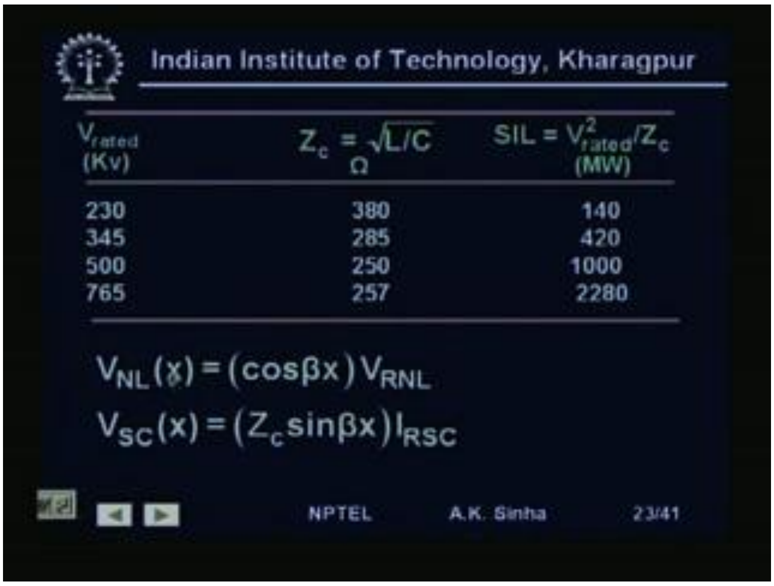

V S or V no load at any distance x which we can write as distance l is equal to cos beta x or cos beta l, for sending end voltage into V RNL. Now, in this case we find that the sending end voltage is going to be less than the receiving end voltage. And if we keep the sending end voltage as 1 per unit, then what we find at receiving end voltage is going to be much higher.

Thank you and in the next class, we will take up some problems on transmission lines.