

**Power System Analysis**  
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**Lecture - 5**  
**Transmission Line Capacitance**

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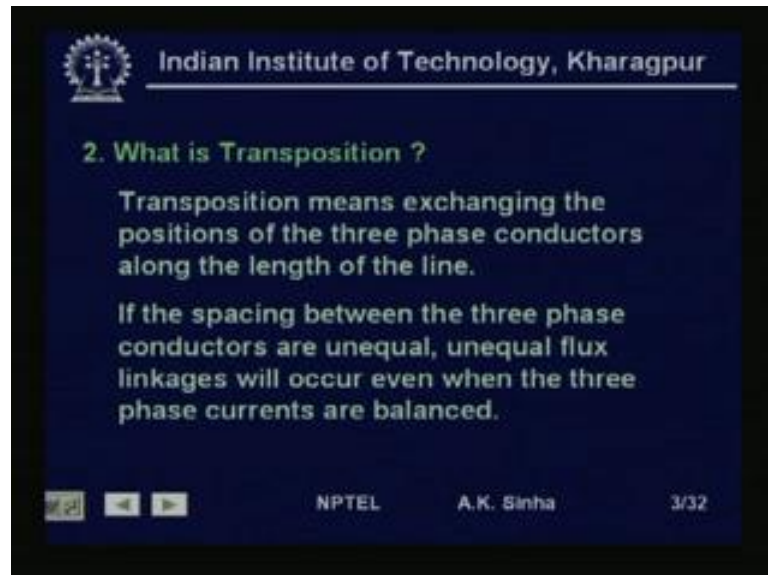
The slide is a dark blue presentation slide with white and yellow text. At the top left is the IIT Kharagpur logo. The title 'Indian Institute of Technology, Kharagpur' is at the top center. Below it, 'Questions from Lesson 4' is written in yellow. The main question is '1. Why bundled conductors are used in EHV lines?'. The answer is split into two parts: 'Bundling → Reduces Electric Field Strength on conductor surface → Reduces Corona' and 'Bundling → Increases Effective Radius (GMR) → Reduces Inductance'. A mathematical formula for the effective radius is shown:  $D_S = \sqrt[9]{(r' \times d \times d)^3} = \sqrt[3]{r' d^2}$ . To the right of the formula is a diagram of a three-conductor bundle with three circles representing conductors arranged in a triangle, with 'd' labels indicating the distance between each conductor. At the bottom, there are navigation icons, 'NPTEL', 'A.K. Sinha', and '2/32'.

Welcome to lesson 5 on Power System Analysis course. In this course we will talk about the Transmission Line Capacitance, before we get into the calculation of transmission line capacitance. I would like to answer those questions that I asked in lesson 4. First question was why bundled conductors are used in EHV lines? Well the answer to this question is bundling of conductors that is instead of using one single conductor; use of a number of conductor connected by conducting frames, reduces electric field strength on conductor surface, which in effect reduces the corona losses, which result in power loss as well as radio interference and audible noise in the system. Bundling also increases the effective radius of the conductor. And there by reduces the inductance of the transmission line. This in effect well improve the regulation of the transmission line.

As seen from here, the effective radius for a three conductor bundle which are spaced at a distance  $d$  from the center or  $d$  from each other with a radius  $r$  is given by 9th root of  $r$  dash into  $d$  into  $d$  whole cube, which is equal to cube root of  $r$  dash  $d$  square. This is much larger than  $r$  dash, which is used when a single conductor is used. And therefore,

bundling helps in reducing the inductance as it increases the effective radius. Second question was, what is transposition?

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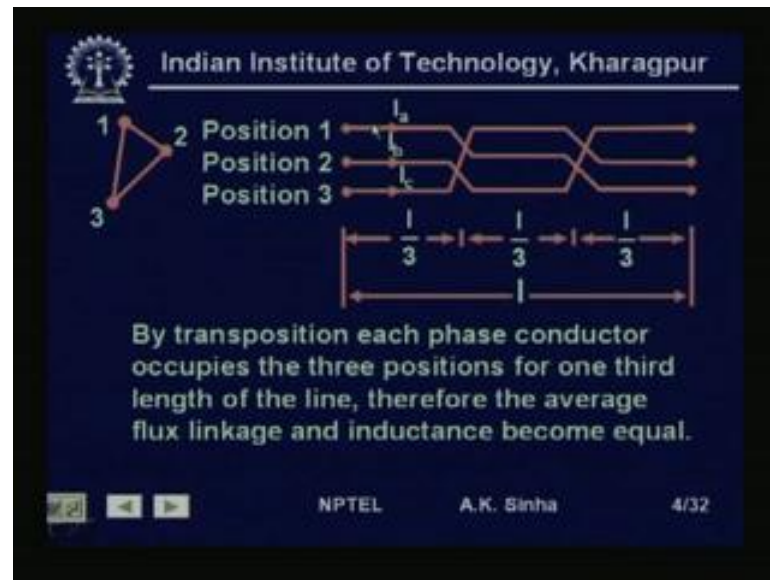
Well transposition means, exchanging the position of the three phase conductors along the length of the line. That is when we have three phase conductors, which are not equilaterally spaced. That is equal spacing between them is not possible due to physical constraints of that transmission line construction. There in such a case we will get unequal flux linkages with the three conductors.

Most of the time if you have seen the power line transmission towers. You will find that the three phase conductors are either placed in a horizontal configuration, if it is a single circuit line. Or they may be placed in a vertical configuration if it is a double circuit line. Therefore the equilateral spacing is not there and the distance between the conductors phase conductors are not equal. This results in different difference in flux linkages with the three conductors, Resulting in difference in inductances; in order to have a balance three phase system.

What we do is we make these conductors a phase conductors go through all the three positions. That is what we do in transposition. We exchange the position of the three conductors along the length of the line. So, that each conductor or each phase conductor is in all the three positions for equal length of the line, which means that the average flux linkage of all the conductors or for all the three phase conductors has same.

So, if the spacing between three conductors are in unequal. Unequal flux linkages will occur when the three phase conductors are even when the three phase currents has balanced.

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Here we show the transposition the phase conductor a is in position 1 for  $1/3$ rd length of the line. It occupies position 2 for the next  $1/3$ rd length of the line. And position 3 for the last  $1/3$ rd portion of the line. Similarly, phase b conductor is in position 2 for  $1/3$ rd of length of the line next  $1/3$ rd length of the line it is in position 3.

And the last  $1/3$ rd length of the line it is in position 1. In this way each phase conductor is occupying all the three positions, which results in average flux linkages to be equal. So, we say that by transposition each phase conductor occupies the three positions for  $1/3$ rd length of the line. Therefore the average flux linkage and inductance become equal.

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3. How the effect of earth return current is taken into account in inductance calculation?

J. R. Carson (1923) → ground currents → earth return conductors having same GMR and located directly under the overhead conductors at a distance  $D_e$ . Where

$$D_e = 658.5 \sqrt{\rho/f}; \rho = \text{Earth's resistivity}$$

$f = \text{system frequency}$

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The third question was how the effect of earth return current is taken into account in inductance calculation? Well J R Carson in his paper a 1923 said that the earth return currents can be taken into account by means of considering earth return conductance. That is we assume imaginary conductors, which are placed below the ground. These conductors are suppose to have the same radius are the GMR as the overhead conductors and they are placed directly below them in the ground.

The distance between the overhead conductor and these imaginary conductors is given by the distance  $D_e$ . Where  $D_e$  or is given by an empirical relation  $658.5 \sqrt{\rho/f}$ , where  $\rho$  is the earth resistivity normally if earth resistivity is not known. Then we assume it to be around 100 ohm meter. And  $f$  is the system frequency. So, having answer these questions, now we will going to the main part of the lesson 5.

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**Lesson 5**  
**TRANSMISSION LINE CAPACITANCE**  
**Lesson Summary**

1. Electric Field and Voltage Calculation
2. Transmission Line Capacitance for:
  - a) Single phase line (Solid Conductor)
  - b) Three phase line with equal spacing
  - c) Three phase line bundled conductor and unequal spacing

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Now, this lessons as I said earlier we will be talking about transmission line capacitance and it is calculation. So, here we will first start with electric field and voltage calculation. Next we will take up the transmission line capacitance calculation for a single phase line with solid conductors. Then we will take up three phase line with equal spacing. And finally, we will talk about three phase line with bundle conductors and unequal spacing.

Well as we have learnt in the lessons 3 and 4 about calculating the resistance and inductance of the transmission line. The transmission line also has capacitance, this is mainly because the line conductors have a voltage difference between the phases. That is phase conductors have voltage difference and there is a difference of voltage between the phase conductor and the ground.

And these conductors are separated by a dielectric medium for overhead line this is air. And in case of cables, this can be some kind of a dielectric, which can be impregnated paper or it can be XLPE that is cross linked polyethylene insulation or some other kind of insulation. Therefore we have two conductors at different voltages and there is a dielectric between them, which results into a capacitance between the conductors or between the conductors and the ground.

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## TRANSMISSION LINE CAPACITANCE CALCULATIONS

Gauss's Law → Electric Field Strength (E)  
Voltage between conductors  
Capacitance → ( $C = q / V$ )

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Now, for calculating the transmission line, capacitance we have to go through first using Gauss's law to calculate the electric field strength. Once we are calculated the electric field strength, we find out the voltage between the conductors and then we find out the capacitance by relationship  $C$  equal  $q$  by  $V$ . So, will start with the Gauss's law and calculation of electric field strength.

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## Electric Field and Voltage Calculation

### Gauss's Law

Total electric Flux leaving a closed surface  
= Total charge within the volume enclosed  
by the closed surface → Normal Electric  
Flux density integrated over the closed  
surface = charge enclosed

$$\oiint \mathbf{D}_\perp \cdot d\mathbf{s} = \oiint \epsilon \mathbf{E}_\perp \cdot d\mathbf{s} = Q_{\text{enclosed}}$$

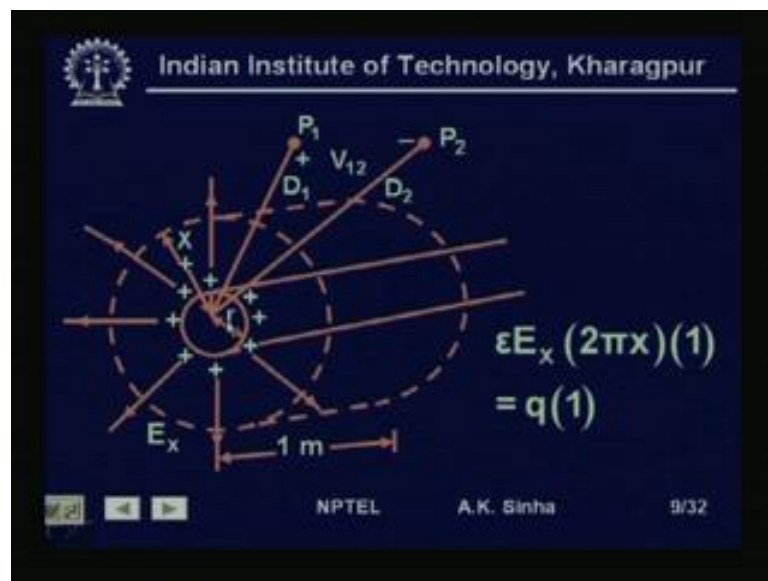
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Well Gauss's law states that, total electric flux leaving a close surface is equal to total charge within the volume enclose by the close surface, which basically leads to that

statement that normal electric flux density integrated over the closed surface will be equal to the charge enclosed by thus closed surface. Mathematically we can write this as the surface integral over the closed surface of electric flux density  $D$  or the norm the electric flux density which is normal to the surface into  $d s$  will be equal to or integrated over the surface. Will be equal to the surface integral of epsilon  $E$ , which is again normal to the surface into  $d s$  that is over the whole surface.

Now, this epsilon is the permittivity of the medium and  $E$  is the electric field strength at the surface. So, that is what we have done is electric flux density  $D$  is replaced by epsilon  $E$ , because  $D$  equal epsilon  $E$ , that we know from electric field theory. So, this is equal to the total charge enclosed by the volume, enclose by this close surface, this we can see in this figure very clearly.

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Let us say we have a conductor, which is of very long conductor of radius  $r$  and this is charge with  $q$  Coulomb's per meter. Here we have assume  $q$  to be a positive charge it can be positive or negative a does not matter. So, here we have assume there is conductor of radius  $r$  is having a charge of  $q$  Coulomb's per meter and it is very long conductor.

Now, since this is perfect conductor electric field inside is going to be zero. Now, for finding out the electric field, outside the conductor what we need to do is we take up concentric cylindrical volume of 1 meter long length. So, this volume shown here this is

a concentric cylinder of 1 meter length at with a radius of X. Now since the electric field lines will be radial to this surface, which is of this concentric cylinder.

Therefore, there will be no tangential component of the electric field. Now, we can find out with this, what is the fields strength at the surface of the this concentric cylinder. Now, using Gauss's law, we will use the integration surface integration of epsilon E d s. So, epsilon E x that is field strength at this surface of this concentric cylinder integrated over this close surface. Closed surface area is to going be equal to twice pi X into 1 meter length of this cylinder. So, epsilon Ex into twice pi X into 1 is equal to the charge enclosed which is q into 1 meter length of the line. Because q coulomb per meter is the charge on the conductor from this we can calculate the electric field strength, at the surface of the concentric cylinder of radius X.

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$$E_x = \frac{q}{2\pi\epsilon X} \text{ V/m}$$

$$V_{12} = \int_{D_1}^{D_2} E_x dx$$

$$V_{12} = \int_{D_1}^{D_2} \frac{q}{2\pi\epsilon X} dx = \frac{q}{2\pi\epsilon} \ln \frac{D_2}{D_1} \text{ volts}$$

$$\epsilon = \epsilon_r \epsilon_0$$

$$\epsilon_0 = 8.854 \times 10^{-12} \text{ F/m}$$

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So, E x equal to q by twice pi epsilon X volt per meter. Now, using this Ex or the relationship for the field strength at any point we can always find out the voltage difference between two points. So, if you want to find out the voltage difference between 2 points P 1 and P 2, which are at a distance D 1 and D 2. From the center of the conductor, what we need to do is integrate from D 1 to D 2 of Ex d x over D 1 to D 2.

So, E x the integral Ex to E x d x to from D 1 to D 2 will give me a the voltage difference V 1 2. So, V 1 2 equal to integral D 1 integral of q by twice pi epsilon X d x from D 1 to D 2 this is equal to q by twice pi epsilon log n D 2 by D 1 volts. Here epsilon is as I said



earlier is the permittivity of the medium and this equal  $\epsilon_r$  into  $\epsilon_0$ , where  $\epsilon_0$  is the permittivity of the free space and for here also it is almost the same,  $\epsilon_0$  equal to  $8.854 \times 10^{-12}$  Farad per meter. Now, real take since we have now found out the electric field strength and the voltage between any 2 points for any charge conductor. We can extended to a multi conductor system.

Now, let us a that we have a system of  $n$  conductors with any conductor  $k$  having radius  $r_k$  and has a charge  $q_k$  per meter length of the conductor. Then we can find out the voltage between 2 conductors  $i$  and  $j$  due to charge on this conductor  $k$ . We write this as  $V_{ij}^k$ . That is voltage between conductor  $i$  and  $j$  due to charge on conductor  $k$ . This will be equal to  $q_k$  by twice  $\pi \epsilon_0 \log n$  the distance  $d$  of  $k$  to  $j$ ,  $k$  to  $j$  and divided by distance of  $k$  to  $i$ .

As we can see using the old relationship, that we have found here;  $q$  by twice  $\pi \epsilon_0 \log n$   $D_2$  by  $D_1$ . Now, if you want to find out the voltage difference due to charge from all the conductors, which are in this system. Then what we need to do is use the superposition theorem. That is add the voltages due to the charge from each of this conductors. So,  $V_{ij}$  in that case due to charges on all the conductor will be equal to summation  $k$  equal to 1 2  $n$  of  $q_k$  by twice  $\pi \epsilon_0 \log n$   $D_{jk}$  by  $D_{ik}$  volts. So, this way we can find out the voltage between any two conductors or any two points due to charge on various conductors in a multi conductor system.

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Single phase Line

Diagram showing two conductors with radii  $r_x$  and  $r_y$ , and charges  $q$  C/m and  $-q$  C/m. The distance between the conductors is  $D$ . The voltage  $V_{xy}$  is given by:

$$V_{xy} = \frac{1}{2\pi\epsilon} \left[ q \ln \frac{D_{yx}}{D_{xx}} - q \ln \frac{D_{yy}}{D_{xy}} \right]$$

$$= \frac{q}{2\pi\epsilon} \ln \frac{D_{yx} D_{xy}}{D_{xx} D_{yy}}$$

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Now, let us try to apply this to find out the voltage and then the capacitance of a single phase line. Let us say we have a single phase line with two conductors are with conductor x and conductor y. Conductor x has a radius  $r_x$  and has a charge  $q$  Coulomb per meter. Conductor y has a radius  $r_y$  with a charge of minus  $q$  Coulomb per meter because conductor y will be the written conductor.

So, it will be carrying the written currents. So, the charge will be negative after charge on the conductor x. So, now we using the same formula we can find out  $V_{xy}$  will be given by one twice pi epsilon q log n D y x by D x x. That is because of the charge on this conductor the voltage difference between these two will be given by this relationship 1 by twice pi epsilon q log n D y x by D x x.

And the voltage between these two conductors that is  $V_x$  to  $V_y$ . So,  $V_{xy}$  will be equal to minus 1 by twice pi epsilon minus q log n D y y by D x y. So, this can be reduce to q by twice pi epsilon log n D y x into D x y divided by D x x into D y y. That is this minus is converted to plus by changing reversing these D x y and D y y; that is taking the inverse of this. So, we get D y x into D x y divided by log n D x x by D y y. So, finally, we get the voltage between to conductors by the relation q by twice pi epsilon log n D y x into D x y divided by D x x into D y y.

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$$V_{xy} = \frac{q}{\pi\epsilon} \ln \frac{D}{\sqrt{r_x r_y}} \text{ volts}$$

$$C_{xy} = \frac{q}{V_{xy}} = \frac{\pi\epsilon}{\ln \left( \frac{D}{\sqrt{r_x r_y}} \right)} \text{ F/m line-to-line}$$

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Which if we substitute the values we get  $V_{xy}$  equal to q by pi epsilon log n D by square root of  $r_x, r_y$ . Because, this would have become D square and this would be  $r_x r_y$ . So,

this square is taken out that becomes 2 into q and this 2 pi epsilon. So, this 2 will cancel out and we get finally, q by pi epsilon log n D by square root of r x into r y.

Now, once we have found out the voltage, we can find out the capacitance very easily. Capacitance C x y that is capacitance between the two conductors x and y is equal to q by voltage between the two conductor. So, this will be equal to pi epsilon divided by log n D by square root r x r y. That is what we have done is divided q by this term. And then we get this C x y is pi epsilon divided by log n D by square root of r x r y.

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$$C_{xy} = \frac{\pi\epsilon}{\ln(D/r)} \text{ F/m line-to-line}$$

$$V_{xn} = V_{yn} = V_{xy} / 2$$

$$C_n = C_{xn} = C_{yn} = \frac{q}{V_{xn}} = 2C_{xy}$$

$$= \frac{2\pi\epsilon}{\ln(D/r)} \text{ F/m line-to-neutral}$$

Diagram 1: Two conductors x and y with capacitance C<sub>xy</sub>.

Diagram 2: Two conductors x and y with a neutral point n in the center. Capacitances are C<sub>xn</sub> = 2C<sub>xy</sub> and C<sub>yn</sub> = 2C<sub>xy</sub>.

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So, this if we consider the radius of the two conductors to be equal then r x will be equal to r y; and therefore, square root of r x into r y equal r. Therefore we can write C x y equal to pi epsilon divided log n D by r Farad per meter this is a line capacitance between line to line. Now, if we see the system, in the center of this, we will get 0 voltage because, this will be positive potential, this will be equal negative potential.

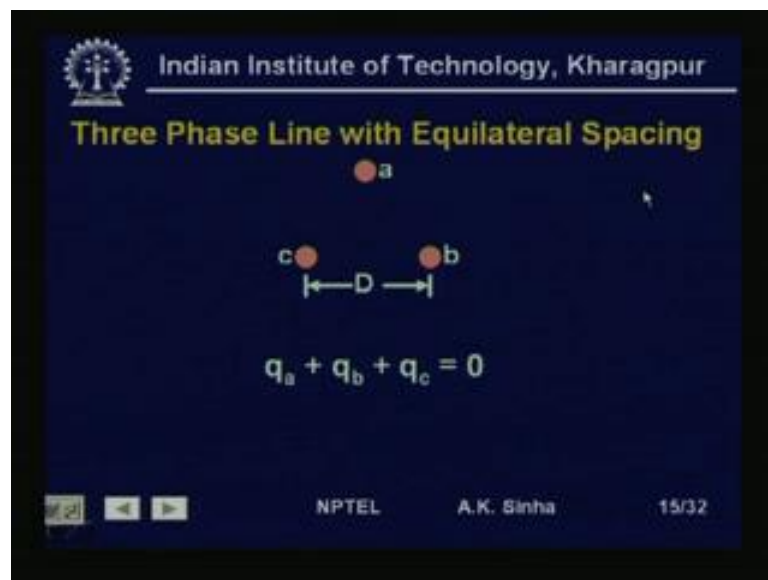
So, by symmetry at the center we will get a 0 potential or the potential of the ground or the neutral. Therefore we can find out the voltage to the neutral as V x n equal to V y n equal to V x y divided by 2. This will be half the voltage because the voltage of one conductor is positive another conductor is negative. So, the zero potential line will be in between at the half, so V x n equal to V y n equal to V x y divided 2. Therefore we can find out the capacitance to the neutral or capacitance to ground as C n which will be

equal to  $C \times n$ . And which will be all again equal to  $C \times y \times n$  that is capacitance from to ground for conductor x will be same as capacitance to ground for conductor y.

And this will be equal to  $q$  by  $V \times n$ , which is equal to 2 times  $C \times y$ . As we can see here between x and y we have a capacitance  $C \times y$ . And if we have taken the neutral at the center, then we have now capacitance  $C \times n$ , between x and n. And capacitance  $C \times y \times n$  between n and y. And these capacitance is will be equal to 2 times the capacitance between x and y, because they are in series. So,  $2 \times y$  and  $2 \times x$ , the  $2 C \times y$  and  $2 C \times x$  in series will give me  $C \times y$ .

So,  $C \times y \times n$  or  $C \times n$  which is equal to  $C \times n$  capacitance to ground for any conductor, after for any of the 2 phase conductor will be equal 2 times  $C \times y$ . This equal to twice  $\pi \epsilon$  divided by  $\log n D$  by  $r$  Farad per meter for line to neutral capacitors. Now, we will take up the case of 3 phase system, because we know that post of the power system that we have to the specially transmission systems are 3 phase systems. Therefore will take the case of a three phase system.

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Will start with the three phase line with equilateral spacing, because as we have seen for inductance calculation. We can always convert, if the line is transposed, we can always convert any system into the equivalent 3 phase equivalent equilateral spacing of the conductors by finding out the equivalent distance  $d_{eq}$ .

So, here we have equilateral spacing conductor a b and c each with a distance d from each other. We also assume that this system consist of only three conductors. So, the some of the total charges will be equal to 0. That is q a plus q b plus q c equal 0. Again as earlier we can find out the voltage between any two conductors.

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$$V_{ab} = \frac{1}{2\pi\epsilon} \left[ q_a \ln \frac{D_{ba}}{D_{aa}} + q_b \ln \frac{D_{bb}}{D_{ab}} + q_c \ln \frac{D_{bc}}{D_{ac}} \right]$$

$$V_{ab} = \frac{1}{2\pi\epsilon} \left[ q_a \ln \frac{D}{r} + q_b \ln \frac{r}{D} + q_c \ln \frac{D}{D} \right]$$

$$= \frac{1}{2\pi\epsilon} \left[ q_a \ln \frac{D}{r} + q_b \ln \frac{r}{D} \right] \quad \text{volts}$$

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So, let us find out the voltage between a and b. So, V a b will be equal to again using the same relationship that we are used earlier. It will be equal to 1 by twice pi epsilon into the q to charge on conductor a q a log n D b a by D a a. Due to charge q b it will be plus q b log n D b b by D a b that distance of conductor b to conductor b divided by distance of the conductor b to conductor a. And due to charge q c log n D b c by D a c, that is distance from conductor c to conductor b and distance from conductor c to conductor a. Now, this if we substitute the values of D b a D a b or D b a and D a a D b b and so on. This can be written as V a b equal to one twice pi epsilon into q a log n D by r D b a is equal to D a b, which is equal to D. And D a a is its self distance that is r the radius of the conductor plus q b into log n D b b again.

Since the conductor radius we have assume to be same that is 3 conductors are same radius or diameter. Therefore, D b b is equal to r Dab we will be equal to D plus qc log n D b c which is equal to D and D a c, which is equal to D. So, this is q c log and D by D which will be q c log and 1; therefore, this term will go to 0. Therefore we have V a b is equal to 1 by twice be epsilon into q a log n D by r plus q b log n r by D.

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$$V_{ac} = \frac{1}{2\pi\epsilon} \left[ q_a \ln \frac{D_{ca}}{D_{aa}} + q_b \ln \frac{D_{cb}}{D_{ab}} + q_c \ln \frac{D_{cc}}{D_{ac}} \right]$$

$$= \frac{1}{2\pi\epsilon} \left[ q_a \ln \frac{D}{r} + q_b \ln \frac{D}{D} + q_c \ln \frac{r}{D} \right]$$

$$= \frac{1}{2\pi\epsilon} \left[ q_a \ln \frac{D}{r} + q_c \ln \frac{r}{D} \right]$$

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Similarly, we can find out the voltage between conductor a and conductor c. So,  $V_{ac}$  will be equal to  $\frac{1}{2\pi\epsilon}$  times due to charge  $q_a$ ,  $q_a \ln$  the distance between a and c  $\frac{D_{ca}}{D_{aa}}$  the distance between a to a. So,  $D_{ca}$  plus  $q_b \ln$  distance of b to c that is  $\frac{D_{cb}}{D_{ab}}$  distance of b to a. So, that is  $D_{cb}$  plus  $q_c \ln$  distance of c to c divide by distance of c to a, that is equal to  $\frac{D_{cc}}{D_{ac}}$ .

This will be equal to  $\frac{1}{2\pi\epsilon}$  times  $q_a \ln \frac{D}{r}$  that substituting the distances  $\frac{D}{r}$  plus  $q_b \ln \frac{D}{D}$  plus  $q_c \ln \frac{r}{D}$ , again this term since it is  $\frac{D}{D} \ln 1$  one. So, this term goes to 0, so we have  $V_{ac}$  is equal to  $\frac{1}{2\pi\epsilon}$  times  $q_a \ln \frac{D}{r}$  plus  $q_c \ln \frac{r}{D}$ .

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$$V_{ab} = \sqrt{3}V_{an} \angle +30^\circ = \sqrt{3}V_{an} \left[ \frac{\sqrt{3}}{2} + j\frac{1}{2} \right]$$

$$V_{ac} = -V_{ca} = \sqrt{3}V_{an} \angle -30^\circ = \sqrt{3}V_{an} \left[ \frac{\sqrt{3}}{2} - j\frac{1}{2} \right]$$

The diagram shows an equilateral triangle with vertices a, b, and c. A neutral point 'n' is at the center. The distance from 'n' to each vertex is labeled as  $\frac{1}{\sqrt{3}} \times \text{side length}$ . The side length is labeled as  $3a_n$ .

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Now, if we look at this equilaterally spaced conductors, here the voltage  $V_{ab}$  is equal to root 3 times  $V_{an}$  at an angle of plus 30 degrees from the x axis. So, this is equal to root 3 times  $V_{an}$  into root 3 by 2 plus j half. That is we have converted this from polar coordinate to rectangular. Similarly  $V_{ac}$  if you see is equal to minus of  $V_{ca}$ . That is negative of  $V_{ca}$ , this is equal to again root 3 times  $V_{an}$ . And this if you see  $V_{ac}$ 's at minus 30 degree from the x axis. So, root 3  $V_{an}$  with an angle of minus 30 degrees. This is equal to root 3  $V_{an}$  into root 3 by 2 minus j half.

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$$V_{ab} + V_{ac} = 3V_{an} \text{ volts}$$

$$V_{an} = \frac{1}{3} \left( \frac{1}{2\pi\epsilon} \right) \left[ 2q_a \ln \frac{D}{r} + (q_b + q_c) \ln \frac{r}{D} \right]$$

$$V_{an} = \frac{1}{2\pi\epsilon} q_a \ln \frac{D}{r}$$

$$C_{an} = \frac{q_a}{V_{an}} = \frac{2\pi\epsilon}{\ln(D/r)} \text{ F/m line-to-neutral}$$

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So, now if we add these two voltages  $V_{ab}$  plus  $V_{ac}$  we get this as equal to  $\sqrt{3}$  times  $V_{an}$ . That is if we see if we add these two this  $j$  plus  $j$  half and minus  $j$  half will cancel out and we have got two times  $\sqrt{3}$  by 2 into times  $V_{an}$ . So, that two will cancel out and  $\sqrt{3}$  into  $\sqrt{3}$  will give us 3. So, this will be equal to  $3 V_{an}$ . Therefore, we will get  $V_{an}$  that is voltage of phase a to the neutral will be equal to  $\frac{1}{3}$ rd of the  $V_{ab}$  plus  $V_{ac}$ .

So, substituting the values from  $V_{ab}$  for  $V_{ab}$  and  $V_{ac}$  as we have calculated earlier. We will get this as  $\frac{1}{3}$ rd of  $1$  by twice  $\pi$  epsilon into  $2$  times  $q_a \log \frac{n}{D}$  by  $r$  plus  $q_b$  plus  $q_c \log \frac{n}{r}$  by  $D$ . That is if we see here  $V_{ac}$  is  $q_a \log \frac{n}{D}$  by  $r$  plus  $q_c \log \frac{n}{r}$  by  $D$  and  $V_{ab}$  is  $1$  by twice by epsilon  $q_a \log \frac{n}{D}$  by  $r$  plus  $q_b \log \frac{n}{r}$  by  $D$ . So, if we add we will get twice  $q_a \log \frac{n}{D}$  by  $r$  and  $q_b \log \frac{n}{r}$  by  $D$  plus  $q_c \log \frac{n}{r}$  by  $D$  that is what we have got here.

So, we have two times  $q_a \log \frac{n}{r}$  by  $D$  plus  $q_b$  plus  $q_c \log \frac{n}{r}$  by  $D$ . So, from this relationship, we can find out now  $V_{an}$  is equal to  $1$  by twice  $\pi$  epsilon into; now  $q_b$  plus  $q_c$  is equal to minus  $q_a$ , because  $q_a$  plus  $q_b$  plus  $q_c$  equal to 0. So, this becomes twice  $q_a \log \frac{n}{r}$  by  $r$  minus  $q_a \log \frac{n}{r}$  by  $D$ , which can be put as plus  $q_a \log \frac{n}{r}$  by  $D$  by  $r$ . And therefore, we get  $3 q_a \log \frac{n}{D}$  by  $r$ .

And this three will cancel with this  $1$  by 3. So, we have got  $1$  by twice  $\pi$  epsilon  $q_a \log \frac{n}{D}$  by  $r$ . So, once we have calculated the voltage between phase conductor a and the neutral, which is at zero potential. Then we have got  $C_{an}$  or the voltage or the capacitance between conductor a and the neutral or the ground will be equal to  $q_a$  by  $V_{an}$ , which comes out to be equal to twice  $\pi$  epsilon divided by  $\log \frac{n}{D}$  by  $r$  Farad per meter. This is line to neutral capacitance or line to ground capacitance.



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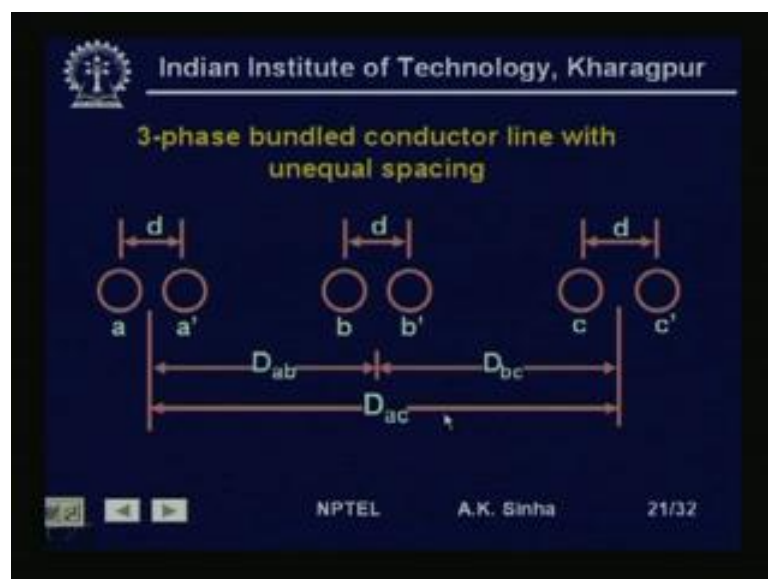
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$$C_{an} = \frac{2\pi\epsilon}{\ln(D_{eq}/r)} \text{ F/m}$$
$$D_{eq} = \sqrt[3]{D_{ab}D_{bc}D_{ac}}$$

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In general, we instead of having  $D$  as we have seen earlier, we if the conductor spacing are not equal. That is we do not have a equilateral configuration of conductors, then other configurations can always be converted into an equivalent equilateral configuration there the distance  $D$  equal to  $D_{eq}$  that is the equivalent equilateral configuration distance. So,  $D_{eq}$  is equal to cube root of  $D_{ab}$  into  $D_{bc}$  into  $D_{ac}$ ; that is what we had seen in earlier lecture on 3 phase line that is in lessons 4.

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Now, let us take a more general case, where we have considered a 3 phase bundle conductor line with unequal spacing. We have here a 3 phase line with 2 conductors in a bundle for each phase. And this configuration for the three phase system is now horizontal configuration. That is distance from a to b that is centre of conductor bundle a to conductor bundle b is  $D_{ab}$ .

The distance between centre of conductor bundle b and conductor bundle c is  $D_{bc}$ . And distance from the centre of the conductor bundle a to centre of conductor bundle c is  $D_{ac}$ . Whereas each conductor has a radius  $r$ , here we are assuming that all the conductors have the same radius. And the bundle distance that is distance between the centre of the two conductors is  $d$ .

As we had said earlier this  $d$  is generally much larger than  $r$ , it is normally 10 times or more  $r$ . In general we have around 30 centimeter or 40 centimeter distance between the conductors for bundle conductor lines. So,  $d$  can be of the order of 30 to 40 centimeters where as the conductor radius will be of the order of 2 to 3 centimeter. So, now again we need to find out the voltage difference between the two phase conductors.

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$$V_{ab} = \frac{1}{2\pi\epsilon} \left[ \frac{q_a}{2} \ln \frac{D_{ba}}{D_{aa}} + \frac{q_a}{2} \ln \frac{D_{ba'}}{D_{aa'}} + \frac{q_b}{2} \ln \frac{D_{bb}}{D_{ab}} + \frac{q_b}{2} \ln \frac{D_{bb'}}{D_{ab'}} + \frac{q_c}{2} \ln \frac{D_{bc}}{D_{ac}} + \frac{q_c}{2} \ln \frac{D_{bc'}}{D_{ac'}} \right]$$

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So, we find out  $V_{ab}$ , now here what we have assumed ((Refer Time: 39:41)) is that conductor a is having a total charge of  $q$ . And since we have two conductor bundle. So, each conductor or the sub conductor in the bundle will be having a charge  $q$  by 2. So, it will be having  $q$  a by 2, this will be having  $q$  a by 2. This conductor will be having

charge  $q_b/2$ , this will be having charge  $q_b/2$ , because all of them are of the same kind and same radius.

So, c will be having charge  $q_c/2$  this sub conductor and this subsequently conductor will be having charge  $q_c/2$ . So, now in order to find out  $V_{ab}$  again what we need to do is find out the voltage between a and b due to the charges on each of these conductors or sub conductors.

So,  $V_{ab}$  will be equal to  $1/2\pi\epsilon_0$ . Now  $q_2$  charge on conductor sub conductor one of phase a it will be  $q_a/2$ , that is the charge on that sub conductor into  $\ln(D_{ba}/r)$  that is the distance between a to b divided by distance of a to a that will be again the radius of the conductor. Plus  $q_a/2$  due to the sub conductor the second sub conductor in phase a that is, this conductor this sub conductor it will be  $q_a/2$  into  $\ln(D_{ba}/D_{aa})$ .

Now, due to this sub conductor the voltage  $V_{ab}$  will be given by  $q_b/2 \ln(D_{bb}/D_{ab})$  divided by  $D_{ab}$  again for the other sub conductor of phase b this will be plus  $q_b/2 \ln(D_{bb}/D_{ba})$ . Similarly, for the two sub conductors charge on the two sub conductors of phase c. We will have components  $q_c/2 \ln(D_{bc}/D_{ac})$  plus  $q_c/2 \ln(D_{bc}/D_{cb})$ .

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$$= \frac{1}{2\pi\epsilon} \left[ \frac{q_a}{2} \left( \ln \frac{D_{ab}}{r} + \ln \frac{D_{ab}}{d} \right) + \frac{q_b}{2} \left( \ln \frac{r}{D_{ab}} + \ln \frac{d}{D_{ab}} \right) + \frac{q_c}{2} \left( \ln \frac{D_{bc}}{D_{ac}} + \ln \frac{D_{bc}}{D_{ac}} \right) \right]$$

$$= \frac{1}{2\pi\epsilon} \left[ q_a \ln \frac{D_{ab}}{\sqrt{rd}} + q_b \ln \frac{\sqrt{rd}}{D_{ab}} + q_c \ln \frac{D_{bc}}{D_{ac}} \right]$$

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This after substituting the values will be given by  $q_a \cdot 2 \log_n D_{ab} / r$ . That is that  $a$  plus  $\log_n D_{ab} / r$ , which is the distance  $D_{ab}$ . Now, here what we are saying that  $D_{ba}$  and  $D_{ab}$  are almost same, because the distance between the two phase conductors is much larger compared to the distance between two sub conductors of a bundle.

So, this small approximation we have made, that is  $D_{ab}$  is the distance between the centre of the two conductors. So,  $D_{ab}$  this is  $q_a \cdot 2 \log_n D_{ab} / r$  plus  $\log_n D_{ab} / r$ . Similarly for conductor  $b$  or the charge due to conductor  $b$  will be  $q_b \cdot 2 \log_n D_{bc} / r$  plus  $\log_n D_{bc} / r$  plus due to charge on conductor  $c$   $q_c \cdot 2 \log_n D_{bc} / r$  plus  $\log_n D_{bc} / r$ .

Now, this when we add these terms finally, can be written as  $1 / (2\pi\epsilon) \cdot q_a \cdot \log_n D_{ab} / r$  square root of  $r/d$  plus  $q_b \cdot \log_n D_{bc} / r$  square root of  $r/d$  plus  $q_c \cdot \log_n D_{bc} / r$  square root of  $r/d$ . Now, this if we remember the inductance relationship is very similar to that relationship.

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$$C_{an} = \frac{2\pi\epsilon}{\ln(D_{eq}/D_s)} \text{ F/m}$$

$D_s = \sqrt{rd}$  for a two-conductor bundle

$D_s = \sqrt[3]{rd^2}$  for a three-conductor bundle

$D_s = 1.091 \sqrt[4]{rd^3}$  for a four-conductor bundle

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And therefore this can be again written in terms of capacitance is equal to, capacitance between neutral can be written as  $2\pi\epsilon \log_n D_{eq} / D_s$ , where we have already defined  $D_{eq}$ , as cube root of  $D_{ab}$  into  $D_{bc}$  into  $D_{ca}$ .

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$$C_{an} = \frac{2\pi\epsilon}{\ln(D_{eq}/r)} \text{ F/m}$$

$$D_{eq} = \sqrt[3]{D_{ab}D_{bc}D_{ac}}$$

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Therefore, capacitance to neutral is given by the same relationship except that we have now replaced  $r$  by  $D_s$ , Where  $D_s$  is the effective radius of the bundle conductor and  $D_{eq}$  is the equivalent equilateral spacing for that conductor. So, for two conductor bundle as we had see earlier or in this case the  $D_s$  is equal to the self distance.  $D_s$  is equal to square root of  $r$  into  $D$  for a 3 conductor bundle the self  $D_s$  is equal to cube root of  $r$  into  $D$  square and for a 4 conductor bundle.

It is equal to 1.0491 4th root of  $rd$  cube this we had seen in the last lesson. Now, once we have calculated the capacitance. Now, we can find out the charging current, because there is voltage and the there is capacitance between the conductor and the ground. So, there will be some current, which will be flowing in the system and this current, which is flowing in the capacitance is called charging current for the system.

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Line Charging Current and Reactive var Generation

$$I_{\text{chg}} = Y_{xy} V_{xy} = j\omega C_{xy} V_{xy} \text{ A}$$
$$Q_C = \frac{V_{xy}^2}{X_C} = Y_{xy} V_{xy}^2 = \omega C_{xy} V_{xy}^2 \text{ var}$$
$$I_{\text{chg}} = Y V_{an} = j\omega C_{an} V_{LN} \text{ A}$$

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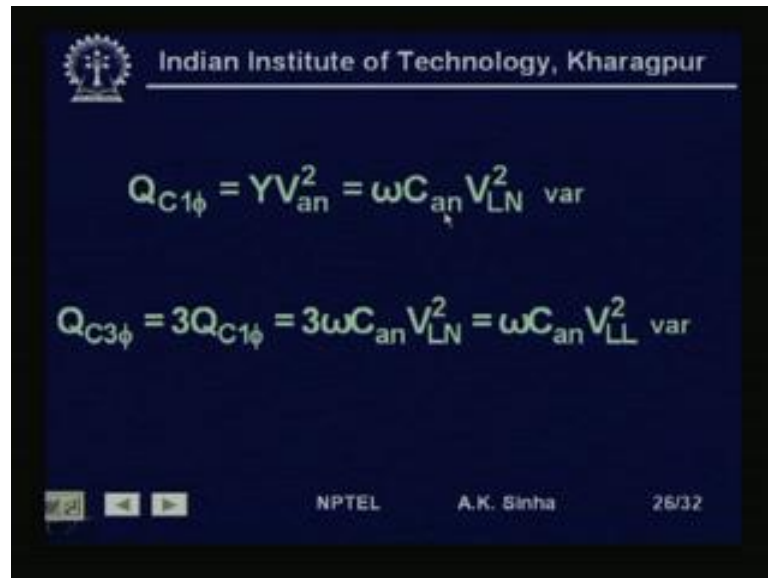
Now, this transmission line has a capacitance  $C_{an}$  for phase a  $C_{bn}$  for phase b  $C_{cn}$  for phase c. Now, this capacitance is we can find out that admittance, admittance will be equal to twice pi f into c. That is we will have y will be equal to minus j omega c.

So, we can find out the admittance between the phase conductor and neutral or the ground. So, if we are finding between two conductors x and y. So, then we will find out the admittance for  $C_{xy}$  as  $Y_{xy}$  and the charging current will be  $Y_{xy}$  into  $V_{xy}$ . This is equal to j omega  $C_{xy}$  into  $V_{xy}$  Amperes. So, once we have calculated the charging current, we can also find out how much is the volt Ampere reactive, which is supplied by this charging capacitance of the transmission line.

So, the capacitance of the transmission line will. In fact, be supplying reactive power to the system. And this will be given by  $Q_c$ , which is equal to  $V_{xy}$  square divided by  $X_c$  or  $Y_{xy}$  into  $V$  square  $x y$  or  $V_{xy}$  square. So, we can find out the charging volt Ampere reactive as omega  $C_{xy}$  into  $V_{xy}$  square. So, this will provide as the volt Ampere reactive which is generated by the transmission line.

And if we are looking for the charging current to the ground, then this will be equal to y into  $V_{an}$ . This will be equal to j omega  $C_{an}$  into the voltage between line and the neutral. So, once we know this current we can find out by multiplying this by another  $V$  that is j omega  $C_{an}$  into  $V_{ln}$  square will give us the reactive power generated by this line.

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$$Q_{C1\phi} = YV_{an}^2 = \omega C_{an} V_{LN}^2 \text{ var}$$
$$Q_{C3\phi} = 3Q_{C1\phi} = 3\omega C_{an} V_{LN}^2 = \omega C_{an} V_{LL}^2 \text{ var}$$

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So, for a single phase system we have the charging reactive power supplied by the line given by  $YV_{an}^2$ . This is equal to  $\omega C_{an} V_{LN}^2$  var. reactive of the 3 phases system this will be equal to 3 times the charging volt ampere reactive generated by a single phase line or between each phase and neutral of the line.

So, this will be equal to three times  $\omega C_{an} V_{LN}^2$ , which is equal to  $\omega C_{an} V_{LL}^2$  that is if you take line to line voltage or the system voltage for a 3 phase system. Then charging reactive power generating by the line will be given by  $\omega C_{an} V_{LL}^2$ . Now, that we have seen we can calculate the capacitance and also find out the charging current and the reactive power generated by the line. Let us take one example to see how we calculate these quantities for a Transmission system.

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**Example:**

A three phase, 400kV, 50Hz, 350km overhead transmission line has flat horizontal spacing with three identical conductors. The conductors have an outside diameter of 3.28 cm with 12 m between adjacent conductors.

Determine the capacitive reactance-to-neutral in ohms/m/phase and the capacitive reactance for the line in ohms/phase.

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So, let us this example we have a three phase 400 kV 50 Hertz 350 kilometer overhead transmission line that has flat horizontal spacing with three identical conductors. That is we have three identical conductors placed in a flat horizontal spacing. The conductors have on outside diameter of 3.28 centimeter that is the diameter of the conductor is 3.28 centimeter and the distance between the adjacent conductors is 12 meter. Now, for this system determine the capacitive reactance to neutral in Ohms per meter per phase and the capacitive reactance for the line in Ohms per phase.

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**Solution:** For a fully transposed three phase line, we have the line voltage given by

$$V_{ab} = 1/(2\pi k) \{ q_a \times \ln(D_{eq}/r) + q_b \times \ln(r/D_{eq}) \}$$

Where  $D_{eq} = (D_{12} \cdot D_{23} \cdot D_{31})^{1/3}$   
 $= (12 \times 12 \times 24)^{1/3}$   
 $= 15.119 \text{ m.}$

Similarly,

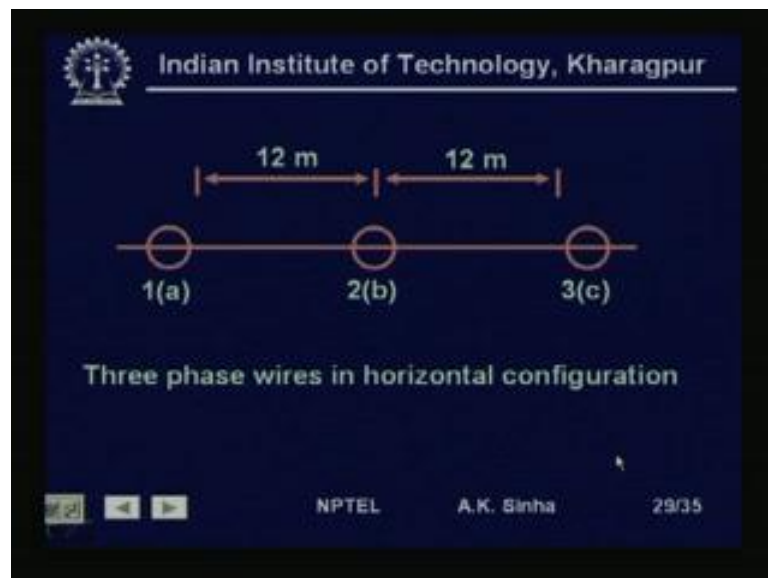
$$V_{ac} = 1/(2\pi k) \{ q_a \times \ln(D_{eq}/r) + q_c \times \ln(r/D_{eq}) \}$$

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So, for solving this, we will take case of a fully transpose 3 phase line. So, we have the fully transpose 3 phase line. The voltage is given by the relationship  $V_{ab} = \frac{1}{2\pi\epsilon} \left[ q_a \ln \frac{D_{e,q}}{r} + q_b \ln \frac{D_{e,q}}{r} \right]$ , where  $D_{e,q}$  is equal to  $D_{12} D_{23} D_{31}$  into the power  $1/3$  that is cube root of  $D_{12} D_{23} D_{31}$ . Now, substituting this values we get this as equal to 12 meters into 12 meters into 24 meters. That is the distance between the conductor a and c, the cube root of this will give us equal to 15.119 meter. Similarly, we can write the relationship for  $V_{ac}$  and  $V_{ac}$  will be equal to  $\frac{1}{2\pi\epsilon} \left[ q_a \ln \frac{D_{e,q}}{r} + q_c \ln \frac{D_{e,q}}{r} \right]$ .

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So, substituting again the values for the system.

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Also, we have

$$V_{ab} + V_{ac} = 3V_{an}$$
$$\text{And } q_b + q_c = -q_a$$

Hence, adding the values of  $V_{ab}$  and  $V_{ac}$  we get

$$V_{an} = q_a / (2\pi\epsilon) \ln(D_{eq}/r)$$

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We have substituting the values we will get  $V_{ab}$  and  $V_{ac}$  and we know  $V_{ab} + V_{ac}$  equal to  $3V_{an}$ . And also we know that  $q_b + q_c$  equal to minus  $q_a$  therefore, adding  $V_{ab}$  and  $V_{ac}$  we will get  $V_{an}$  equal to  $q_a$  divided by twice pi epsilon log  $n D_{eq}$  by  $r$ .

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Hence,  $C_n = q_a / V_{an}$

$$= 2\pi\epsilon / \ln(D_{eq}/r)$$
$$= 8.163 \times 10^{-6} \mu\text{F/m}$$
$$Y_n = (2\pi \times 50 \times C_n)$$
$$= 2.565 \times 10^{-9} \text{ S/m per phase}$$

Given length of the line = 350 km.

Hence,  $Y_n = 8.978 \times 10^{-4} \text{ S per phase}$

$$X_n = 1/Y_n = 1.1138 \times 10^3 \Omega \text{ per phase}$$

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Therefore the capacitance to neutral will be equal to  $q_a$  by  $V_{an}$ , which will be equal to twice pi epsilon log  $n D_{eq}$  by  $r$ . And this will after substituting value comes out to be 8.163 into 10 to the power minus 6 micro Farad per meter. And we can find out the

admittance the charging admittance of the line to neutral, which will be equal to twice  $\pi f$  into  $C_n$ .

So, after substituting the value, we will get this as equal to  $2.565 \times 10^{-9}$  Siemens per meter per phase. Now, we given the length of the line as 350 kilometers therefore,  $Y_n$  equal to  $8.978 \times 10^{-4}$  Siemens per phase. That is multiplying this by 350 kilometer which is  $350 \times 10^3$  meters.

So, this gives as this much and the reactance will be equal to  $1/Y_n$ , which comes out be equal to  $1.1138 \times 10^3$  Ohms per phase. That is 1.1138 kilo Ohms per phase. That is all per this lecture, we will continue with capacitance calculation in the next lessons. Where will talk about how we take into account the effect of earth for capacitance calculation.

Thank you.

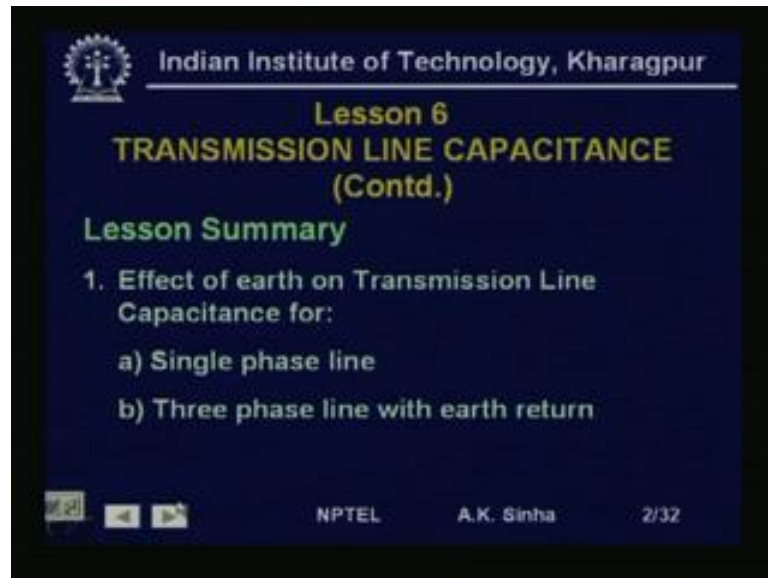
Preview of next lecture

Lecture No. #06

Transmission Line Capacitance (Contd.)

Welcome to lessons 6 on Power System Analysis. In this lesson we are going to discuss Transmission Line Capacitance, which we were discussing in lessons 5, we are continuing with it.

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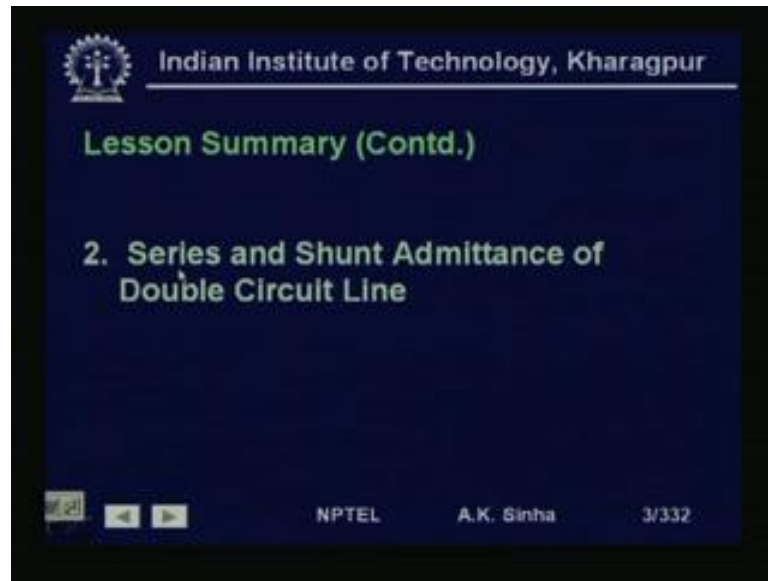


The slide is a presentation slide from NPTEL. It features the Indian Institute of Technology, Kharagpur logo in the top left corner. The title is 'Lesson 6 TRANSMISSION LINE CAPACITANCE (Contd.)' in yellow and white text. Below the title is a 'Lesson Summary' section in green text. The summary lists '1. Effect of earth on Transmission Line Capacitance for:' followed by two sub-points: 'a) Single phase line' and 'b) Three phase line with earth return'. At the bottom, there are navigation icons, the text 'NPTEL A.K. Sinha', and the slide number '2/32'.

Here what we will do is we will consider the effect of earth on transmission line capacitance. Actually when we have this transmission lines, the phase conductors are in above the ground in over transmission system and the distance between the phase conductors. And the distance between phase conductor and ground are of the same magnitude. And therefore, the earth which acts as equi-potential surface thus affect the electric field lines and there by the capacitance.

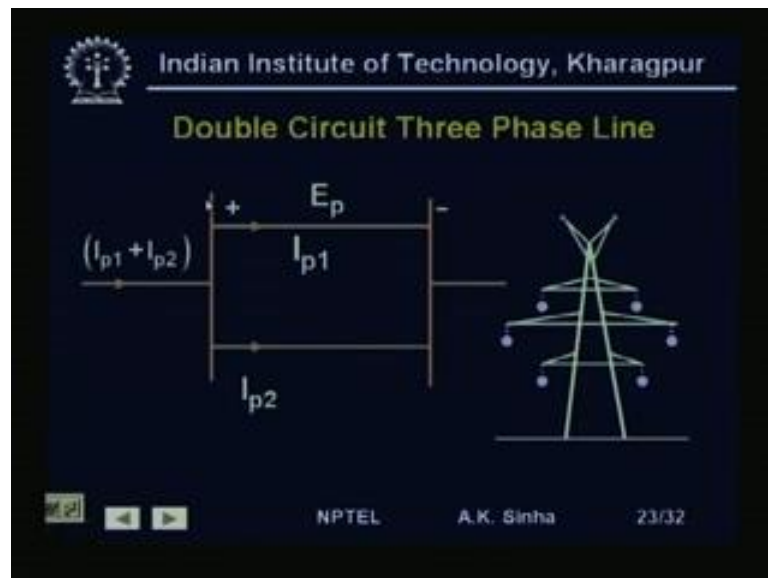
So, we need to consider the effect of earth and calculating the capacitance for transmission lines. So, in these lessons we will discuss how the earth or the ground affects the capacitance of a single phase transmission line and how it affects the 3 phase transmission line.

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Then we will talk about a double circuit line and we will talk about, how we calculate the series and shunt impedance of the double circuit line.

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Such a system we are showing here, here we have a three phase line. This is one circuit and this another circuit which is placed here. So, this is a double circuit system, here we are showing this double circuit line. This is one circuit, this is another circuit connecting two bus bars or two substations.

Now, the current flowing in one circuit is  $I_{p1}$ , current flowing in the other circuit is  $I_{p2}$  where  $I_p$  is basically a three element vector, which consists of  $I_a$ ,  $I_b$  and  $I_c$ . So, we have  $I_{a1}$ ,  $I_{b1}$ ,  $I_{c1}$  after circuit 1 and  $I_{a2}$ ,  $I_{b2}$ ,  $I_{c2}$  for the circuit 2. Now, this current flowing here from one circuit will be  $I_{p1}$  plus  $I_{p2}$ , which gets divide into  $I_{p1}$   $I_{p2}$  and may be  $x$  getting connected on this side. This is the system that we are showing we may we have learnt about we will do a lessons 7.

Thank you.