Power System Analysis
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Lecture - 38
Power System Stability – VI

Welcome to lesson 38 in Power System Analysis. In this lesson we will continue our discussion on Stability Analysis of Power System. In the last few lessons, we have been discussing about the rotor angle and stability of the power system. And we had started with the swing equation. Then, we talked about small signal rotor stability analysis by line arising the swing equation.

After that we talked about large signal. That is, when we have large disturbance within the power system, how does the system behave. So, we talked about the swing equation and we need to solve the swing equation. Since, this is a second order non-linear differential equation. We try to solve this using a graphical method, which we can apply for a single machine infinite bus system. And in the last lesson we talked about a method called equal area criterion for studying the rotor angle stability analysis or the transient stability analysis of a single machine connected to infinite bus system.

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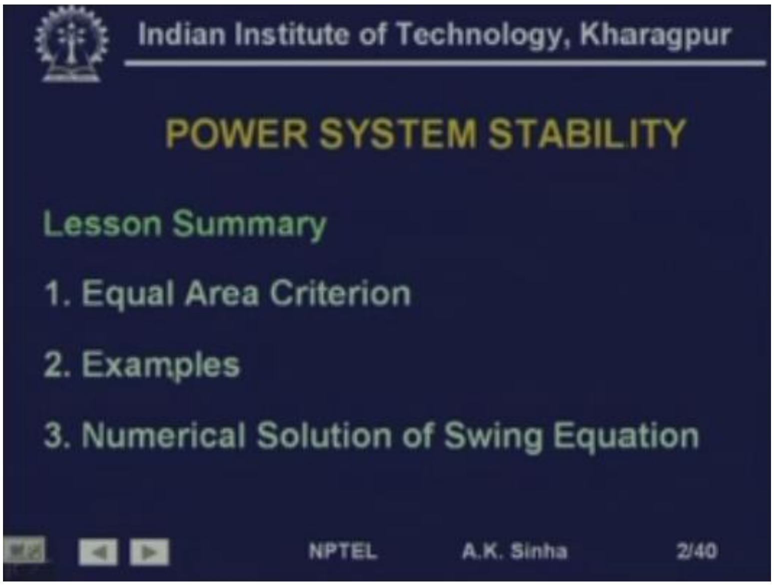

In this lesson we will continue with the discussion. And we will start with the equal area criterion method of analysis for single machine connected to infinite bus system. Here, we will be talking mostly about a single machine; which is connected to an infinite bus by means of double circuit line. And what happens under different situations.

We will take up an example on this. And if the time permits we will talk about the numerical solution of the swing equation itself. So, instead of using the graphical solution, that we can work out for a single machine infinite bus system. We will talk about the analysis, where we have number of machines in the system. That is multi machine stability analysis. There, we certainly need to do the numerical integration of the swing equation to get the solution for the swing of the various machines.

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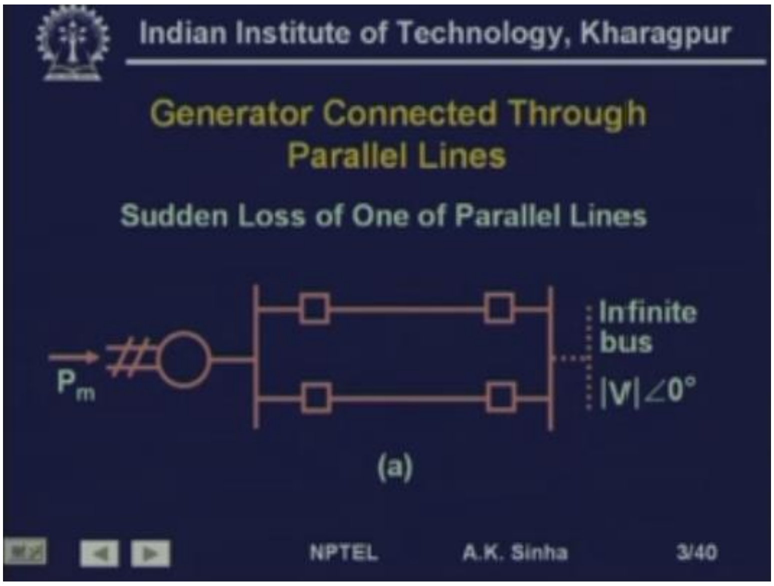

So, we will start with a generator connected to the infinite bus by means of a parallel lines. That is, we have two lines which is connecting this generator to the infinite bus. And in this case, first we will take a case of where one of the transmission line is removed. That is, sudden loss of one of the parallel lines, this line can be removed for maintenance purpose or may be when a fault occurred or some such situation occurred, where we wanted to take this line out. So, suddenly if we remove this line, how the system is going to behave, because when we remove this line. The total reactance which is or the total impedance between the synchronous machine and the infinite bus will change. And therefore, the power flow pattern will also change, electrical power output will change because of this situation.

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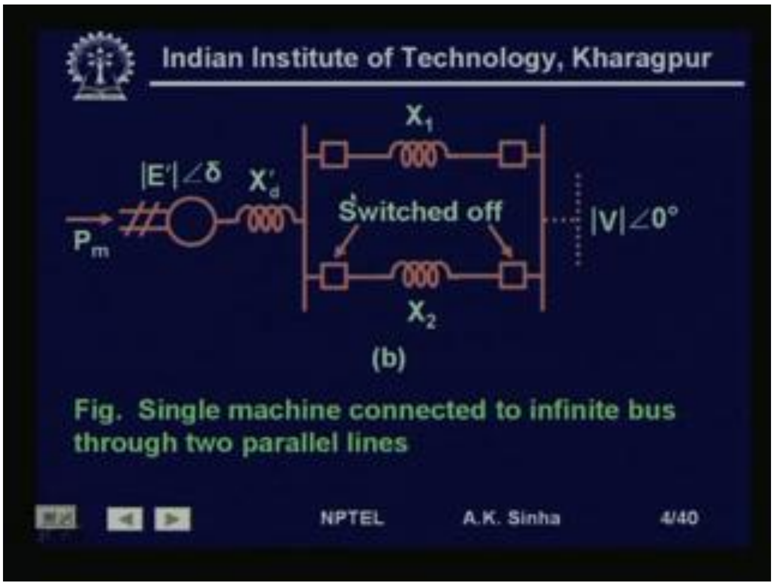

So, this is what we have, we have mechanical input P m which we have said earlier we will consider it to be constant for the period over, which we are doing this analysis, the internal voltage of the machine. That is voltage behind the transient reactance of the machine is E dash angle delta X d dash is the transient reactance of the machine. We have two transmission lines, the reactance of which are X 1 and X 2. This in general will be equal.

But, we can take a case where there can be unequal also. So, we have X 1 and X 2 as the reactance of these two lines. And suppose, we want to switch off these lines by opening the circuit breakers at these two ends. The voltage of the infinite bus is V angle 0 degree, as we have said earlier. We assume the angle of the infinite bus to be the reference angle. And the voltage magnitude also most of the time is considered as 1.0 per unit. But, anyway it can be any value, so we have taken here as V.

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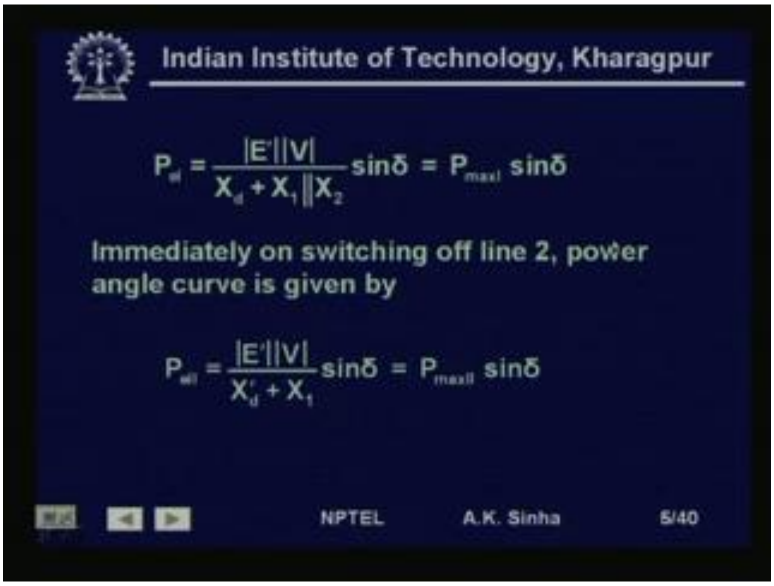

Now, in this system the power flow when ((Refer Time: 05:43)) both the lines are working will be given by the voltage E dash into V divided by the total reactance, which is between these two voltage sources. So, into sin of delta angle between these two voltage sources, this is 0 and this is delta. So, the angle difference is delta.

So, we have got electrical power output is given by E dash V by X d plus X 1 and X 2 in parallel ((Refer Time: 06:20)) X 1 and X 2 are in parallel. And the X d is in series with that, that gives the total reactance between the machine internal voltage and the infinite bus voltage.

So, we have the electrical power output given by this relationship E dash V by X d plus X 1 and X 2 in parallel sin of delta, where delta is the angle difference between E dash and V. This we can write as P max one sin delta. So, this where P max 1 is equal to E dash V by X d plus X 1 X 2 in parallel. Now, even the line is switched off what happens ((Refer Time: 07:05)).

When, the line is switched off we have the reactance between this voltage generator internal voltage. And the infinite bus that will be equal to X d dash plus X 1 only. So, we will get the electrical power output will be given by the relationship E dash V by X d dash plus X 1 into sin delta. Where, again delta will be the angle between the internal voltage of the synchronous machine and the infinite bus.

We call this as P max 2 into sin delta. Because, this is the maximum power, which can be transferred during this situation where one of the line is switched off. So, we have P max 2 into sin delta. So, what we can do is?

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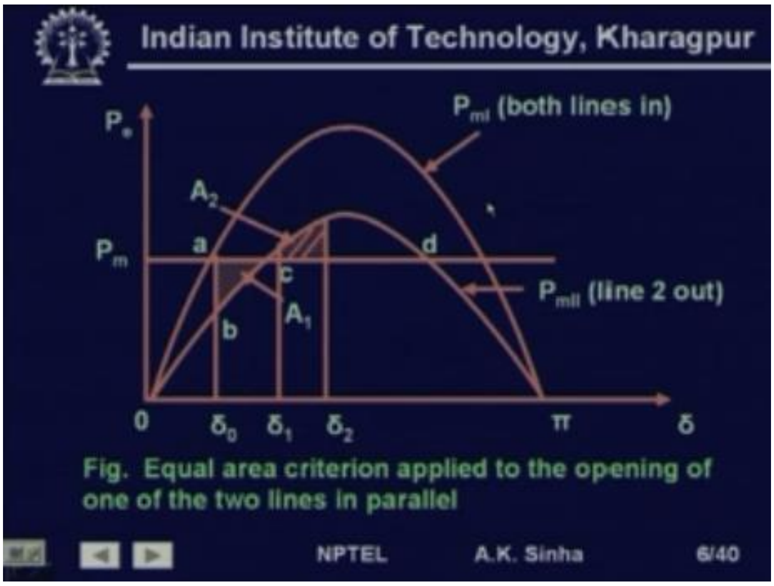

We can draw the power angle characteristics for both the situation. First situation when both the lines are n. Then, we have got the power angle characteristics P delta characteristics of the machine given by this relationship. And when the one of the line is out, we get the power angle characteristics which will be like this.

Now, P max 2, P max 2 is going to be smaller than P max 1. Because, here ((Refer Time: 08:26)) we have X d plus X 1 and X 2 in parallel which is smaller than X 1 or X 2. And here we have X d dash plus X 1. Therefore, we have in fact, here we should have X d dash not X d. So, we will find that this when both the lines are on the power angle characteristics or the maximum power that can be transmitted will be much more.

So, we have these characteristics here, whereas in the other case we have this characteristics. Now, initially the machine is working with both the lines on in that case P m is the mechanical power input to the system ((Refer Time: 09:07)). And the electrical power output under that situation will be equal to the mechanical input, because we have assumed no losses in the system.

So, we will have P m is equal to P e and therefore, the operating point that the system will be working in the initial condition will be this point a. Where, we have delta 0 as the angle between the internal voltage of the machine and the infinite bus. Since, infinite bus voltage we have take as 0. So, delta 0 is the power angle or the angle of the synchronous machine.

And this is the operating point. Now, suddenly when one of the line that is line with reactance X 2 is dripped. Then, the operating point will shift from this point to this point b. Because, now the power angle characteristics change the power flow the electrical power output is going to change, because the transfer reactance is now different. So, suddenly what we find is, the operating point shifts from a to b.

And in this case what happens the electrical power output is much less, than the mechanical input. And therefore, we have an accelerating power available. And so delta angle will keep on moving as when it comes to this point P m and P e will be equal. But, the speed will be somewhat higher than the synchronous speed. So, it will keep on moving like this.

So, what we are going to see is, that this will keep on moving on this line, till the speed becomes equal to synchronous speed at this point. And then at that time we find that since there is more electrical power, then the mechanical input. So, there is a retardation which will be taking place. And because of, which again delta will start going down. In fact, this line should be at this point.

So, finally, what we will find is there is going to be an oscillation of delta angle about this point c, c should be this point. So, this point c it should be oscillation. And finally, the oscillations will be damped out and it will start working at this point c. And we will see that this area A 1 which is basically the area during, which period the rotor is accelerating.

And this area A 2 which will be between this c and this fall area. Where, during which period the rotor will be decelerating or the decelerating area, these will be equal when we reach this point delta 2. So, this is what we get when one of the line is suddenly removed. Now, suppose instead of working at this point, if we are working at a much higher point.

That is the electrical output was more. And initially the P m also would have been more like instead of working here, if we are working at this point then what happens. Then, we will find that suddenly it from a it jumps to b. And then it will start moving now we have much larger acceleration available or the accelerating area available is much larger.

And therefore, the decelerating area must be equal to this or the decelerating area available must be equal to this or more. Now, if we move it in such a way, that this a, b, c area is equal to c, d. And this line, that is this hatched area. Then, what we say is at this point we will have critical stability or the system will be just stable.

Because, if we work at any point above this. Then, that decelerating area available will become less than the accelerating area. That is, we will have more accelerating area and decelerating area available will be less. And therefore, the acceleration of the rotor will continue. That is, we will go beyond this point and the system will lose stability, that is the rotor will be under a runaway condition.

So, this is what we say that the point at which we can operate the maximum power at which we can work will be given by this. And the angle d at this point d, which is delta 2 is the maximum angle delta max. And this delta max will be equal to how much, this will be equal to this delta 0 pi minus this will be equal to pi minus delta 1. This is the delta 1 and this is pi minus delta 1 will be this.

So, we can see what is the maximum angle that can be reached. That is delta 2 under this case. In case we work at a power higher than this we are going to lose stability. So, we need to work always lower than this power. And this will be the critical stability point for this situation.

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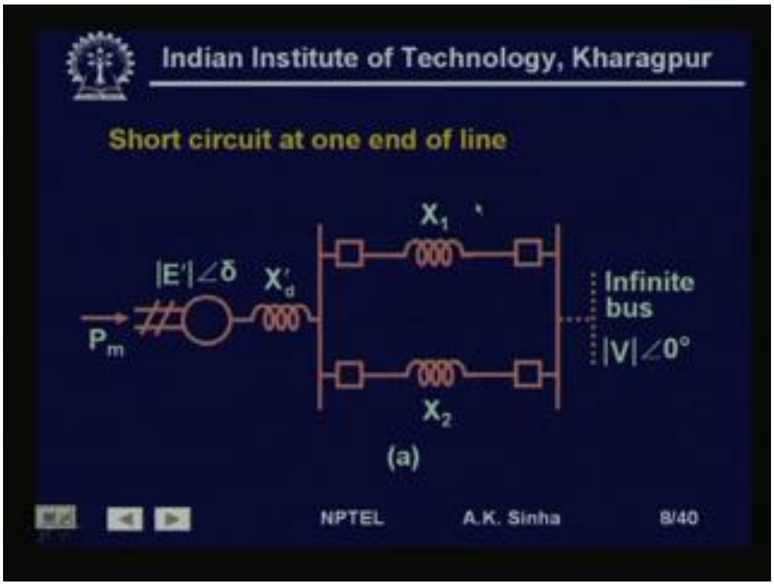

Now, let us take another case. Where, we have the same system, again P m is the input E dash angle delta is the voltage behind the transient reactance. X d dash is the transient reactance X 1, X 2 are the line reactance's. And infinite bus voltage is V angle 0 degree. Now, we are assuming a short circuit at one end of the line. Say, at this end it does not matter it can be this end also.

In both the cases, when a short circuit occurs on the line which is very near to this bus. Then, what happens is the voltage of this bus is also equal to 0. And under such a situation, there can be no power transfer. Same case, if the fault occurs very near to this bus at this end of the line. Then, also the voltage of this bus becomes 0 and there will be no power transfer which will take place under such a situation.

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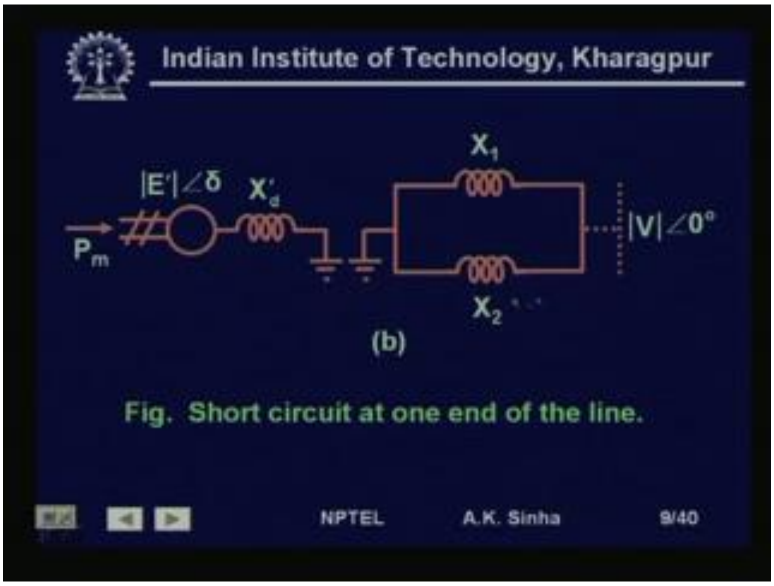

That is, we are shown the situation ((Refer Time: 15:59)) when we have a fault at this end of the line. So, on anyone line if the fault occurs, just near the bus. Then, the voltage of this bus is 0, this is what I shown. So, the voltage at this bus is shown as 0, it is grounded. And therefore, there will be no power transfer which can take place under such situation.

And there; that means, the during fault period, the electrical power output of the machine will be 0. And after this, ((Refer Time: 16:35)) if we switch off this line. That is this fault which has occurred on this line at this end will be isolated by opening of these two circuit breakers, because this fault will be sensed by the relay. And the relay will send signal to the circuit breakers at these two ends to open.

And therefore, the fault will be isolated by opening these two circuit breakers. In that case, again we have a system where we will have X d dash and only one of the line working instead of both of the lines working in this situation.

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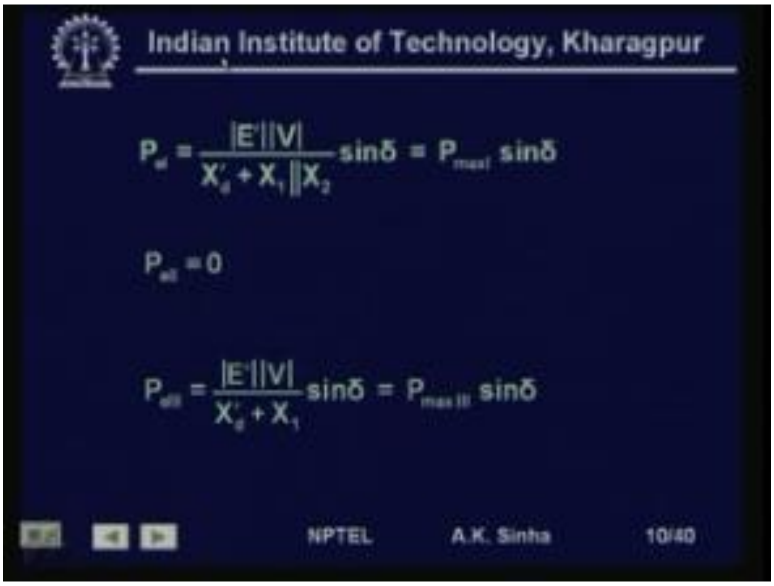

So, we have initial case we have both the lines working. So, the electrical power is given by the relationship E dash V by X d dash plus X 1 and X 2 in parallel into sin delta. That is what we write as P max one sin delta. In the case when the fault is on, that is during short circuit period we have electrical power output is equal to 0. That is P electrical in the second situation. That is, when the fault is on in that condition is equal to 0.

When the fault is cleared, then the electrical power output is given by the relationship P e 3. That is after fault clearance or post fault situation is given by E dash V by X d dash plus X 1, because the other line has now been tripped. So, this into sin delta which we call as P max 3 sin delta. So, we can again draw the power angle characteristics under all the three situations.

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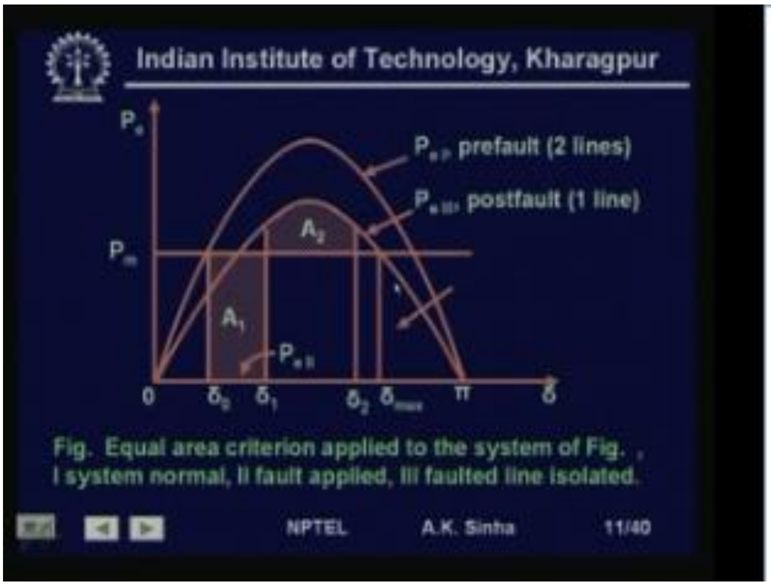

So, the first one is the pre fault condition, the initial operating condition, which is given by this P delta characteristics. Second condition, when the fault is on is electrical power output is 0. So, it is this horizontal line which defines that situation. The third one is, when one of the line is out and that is given by this situation.

So, we have ((Refer Time: 18:43)) here P max 1 which is pre fault. This is during fault and this is P max 3, which is defined in the post fault condition. So, pre fault, during fault and this is the post fault condition. Now, initial operating point we have P m and P equal. So, the angle at that point we say is delta 0 ((Refer Time: 18:43)). That is here P electrical will be equal to P mechanical input. So, it will be given by this operating point, where the angle is delta 0.

When the fault occurs the electrical power output becomes 0. So, the rotor angle will start traversing on this, because electrical power output now is 0. And suppose it has reached point delta 1, when the fault is cleared. When, the fault is cleared by opening the circuit breakers, now we have only one line acting or one line working on the system.

So, suddenly the operating condition will now jump from this point to this point. And we will have again delta moving along this characteristics, which is the characteristics for post fault condition. So, what we have during this period, we have acceleration. And the accelerating area all through this period, the accelerating power is equal to P m, because P e is equal to 0.

So, P m minus P a is equal to P m in this case. Suddenly, it will jump to this point, where the electrical power output will be governed by these characteristics. So, at delta 1 it has jumped here. So, in this case we have deceleration. So, the rotor will start decelerating, but since it is speed is more than synchronous speed it will still keep moving. When, it reaches this point delta 2. Then, the speed becomes equal to the synchronous speed.

But, since the mechanical power input is less than the electrical power output. So, delta will keep on moving along this. And what we will find is finally, that it will start operating at this point, where P m is equal to P e. So, at this delta angle it will start working finally, because the oscillations will die out. Now, when we see this whole area is the area during, which there is acceleration of the rotor.

This area that is between P m and P e, this curve that we have is giving us the decelerating area. And we will get this when A 1 is equal to A 2 at whatever point. That is at point delta 2, then we will have reached the synchronous speed at this point. So, what we find is the accelerating area and decelerating area must be equal.

Now, suppose instead of working at this point, if you would have worked at a little higher point. Here, that is electrical output being a little more. Then, what it was earlier. So, P m also would have matched that point and we would have been working here. Then, in that case we would find that accelerating area will increase and the decelerating area will reduce somewhat.

But, the maximum decelerating area is available is above this line and below this curve. So, that is up to this point delta max, which again will be equal to pi minus this angle that we have. So, pi minus this angle will give me this delta max. And that is the maximum point up to which we can work. So, that will give me the critical angle. With the same thing we can see instead of tripping this line, when we have reached delta 1.

If we tripped it a little later that is if it is delta one was here then again what we find is, the accelerating area will increase up to this point. And decelerating area available will be only up to this much. That is, if this area becomes equal to the area here, then we will have a critical stability.

And then that angle at which we have cleared this, where this A 1 is equal to the maximum decelerating area available will be called the critical clearing angle for this situation. So, we can have this critical stability depends on two things, that is the initial operating point and when we are clearing the fault.

So, the faster we clear the fault smaller is the accelerating area available and more is the chance of system remaining stable, because you have larger decelerating area available. If you clear your fault later, suppose we clear it up to at this point. Then, the accelerating area available will be coming up to this point. And decelerating area available may not be enough, that is A 2 may become the maximum A 2 which is available, may become less than the accelerating area which is there and system may become unstable.

So, if we keep on increasing it slowly, slowly we will come to a point. Where, the accelerating area A 1 is just equal to the maximum available decelerating area, under that condition. And that point or that angle we call as the critical clearing angle.

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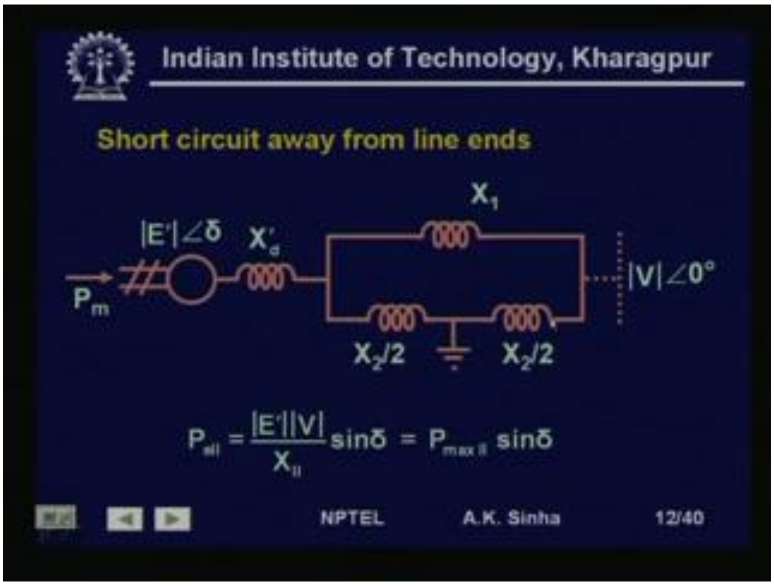

Now, let us take another situation, where the fault occurs somewhere in between in the line. In this case we have considered a situation, where the fault has occurred at the middle of the line. But, it can be anywhere in the line. Only thing which is there is, initially when there is no fault both these lines are working. And the system has the electrical power characteristics given by E V by X d dash plus X 1 and X 2 in parallel into sin delta.

That is what we will we get. When, a fault occurs here and this is cleared by opening of the circuit breakers at the two ends. Then, again we have the power transfer given between the machine and the infinite bus is again by E dash V divide by X d dash plus X

1 into sin delta that will be the post fault condition. During fault in this case, when the fault occurs not at either end of the line. But, somewhere in between we are going to get some amount of power transfer, which is depended on what is the transfer reactance, under the faulted condition. That is with fault on here what is the transfer reactance between this point and this point, this we can find out.

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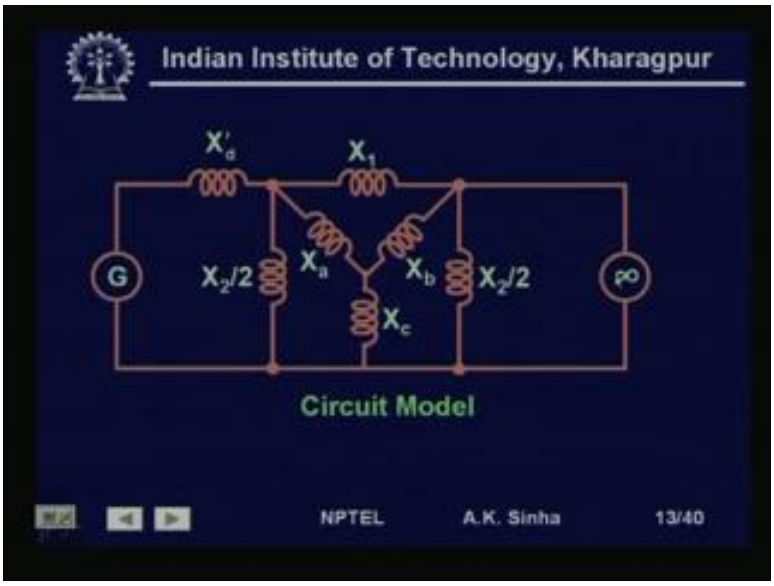

That is ((Refer Time: 26:20)), if we see look at this circuit condition. We have X d dash and this X 2 by 2 connected to the ground. And this X 2 by 2 connected to the ground and this X 1 between these two points. So, this is the situation X 1 between these two points X 2 by 2 connected to the ground X 2 by 2 connected to the ground here.

So, this forms a delta X 1, X 2 by 2 and X 2 by 2. And we can always use star delta transformation to transform it into a star. Where, we have the star impedances given as X a, X b and X c. That is for this delta that we have, we can convert it into a star equivalent star and the impedances for that equivalent star will be X a, X b and X c.

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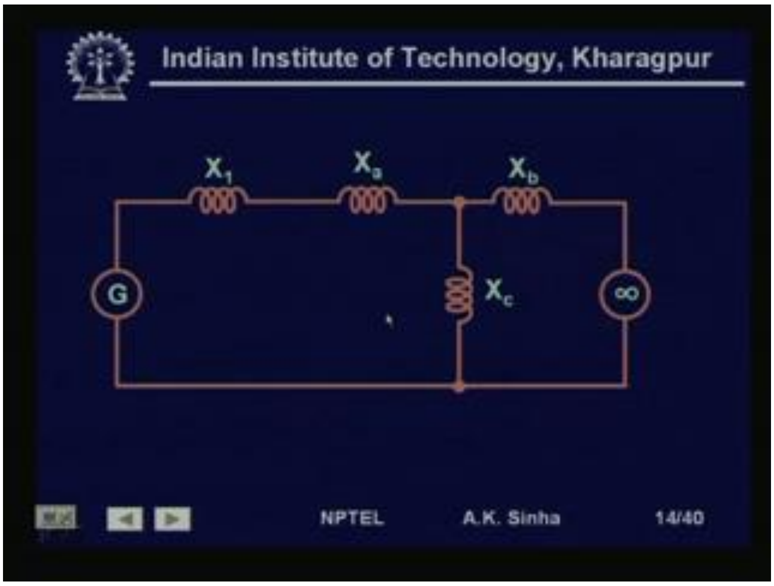

So, under this situation the circuit will look like this X a, X b and X c here. And we have now X 1 and X a in series here X b on this side and X c here. Now, again this looks like a star. And this star can now be converted into another delta using star delta transformation.

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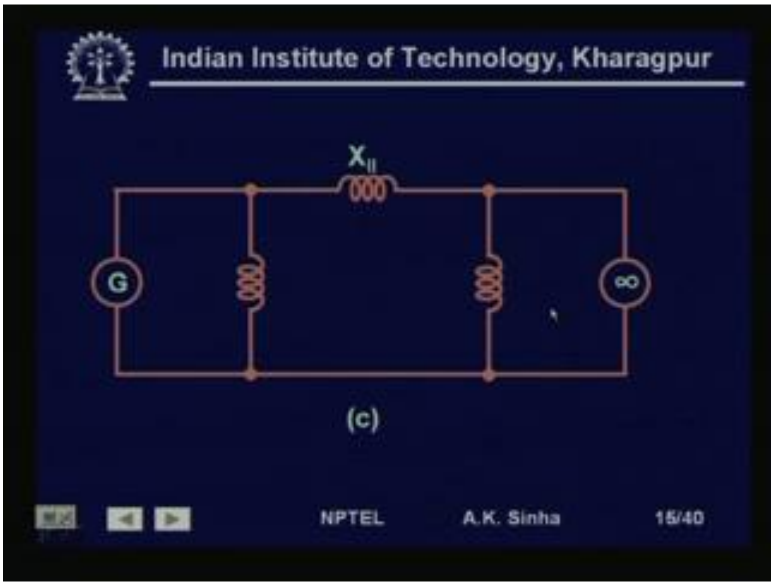

And then we will get this as X 2 and this is an impedance here. This is another impedance here. Now, these two impedances since they are across the voltage sources, they do not have any effect on the power transfer, which takes place between this and this. So, this is what we get as the transfer reactance for the system.

So, what we need to do is find out this X 2. And this case we will get the value of power transfer will be given by E dash V divided by X 2 into sin delta.

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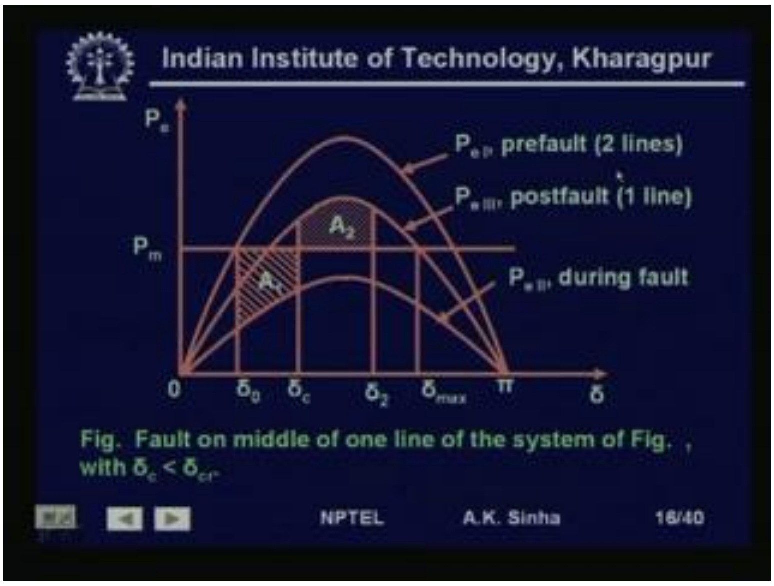

So, now we have three parts, first part is pre fault which is E dash V by X d dash plus X 1 and x 2 in parallel. So, that is the pre fault power angle characteristics that we have. Then, we have during fault characteristics which is given by E dash V by X 2 that is ((Refer Time: 28:47)) this divide by X 2 into sin delta.

So, this is the during fault characteristics and when one of the line is removed. Then, we have E dash V by X d dash plus X 1 into sin delta, these characteristics is given by this post fault characteristics here. So, we have P 1, P 2 characteristics that is during fault and P 3 characteristics post which is post fault. Now, initially the system is operating at this point, where P m is equal to P e.

A fault occurs, then suddenly the operating point will shift to the during fault operating characteristics at this point. Since, we have more mechanical power input and less electrical output the generator rotor angle will start accelerating. So, rotor angle will keep on increasing. Let us say, at this point we delta c we clear the fault. Then, once we clear the fault at this point, what happens the operating point will jump to the post fault characteristics that is up to this point.

And during this period the accelerating power available at each of these points will be between this P m. And this characteristics, that is the during fault characteristics. So, this is the accelerating area which is available. And once, we have gone to this post fault condition, we are finding that the electrical power output is greater than the mechanical input.

So, we have deceleration of course, rotor angle will keep on increasing. Till the speed is because of deceleration reaches back to the synchronous speed. So, which happens say at this point delta 2. So, we will get the decelerating area, which will be between this P m and under the post fault curve here A 2. So, when these two are equal we will get the final operating point here. That is, delta will move from here to this go up to this point that is start from here, jumps here, moves like this goes here goes up to here.

And then it will keep on oscillating about this point. And finally, it will work on this point. So, this is again if either we clear it a little later. Then, what happens the area A 1 will increase and the area A 2, the maximum available will be between this curve and this P m that is up to this point. So, this can shift only up to this point. So, if we clear this fault a little later A 1 will increase and A 2 will keep on reducing. That is the available A 2 will keep on reducing.

So, when this A 1 is just equal to A 2 available A 2 we have the critical clearing of the fault. That is, if we clear our fault before that delta clearing, that is critical clearing angle. Then, the system is stable if we clear the fault after the critical clearing angle, then the system becomes unstable. So, that is the situation that we get. Now, we can always find out what is the critical clearing angle. Let us see how we do that.

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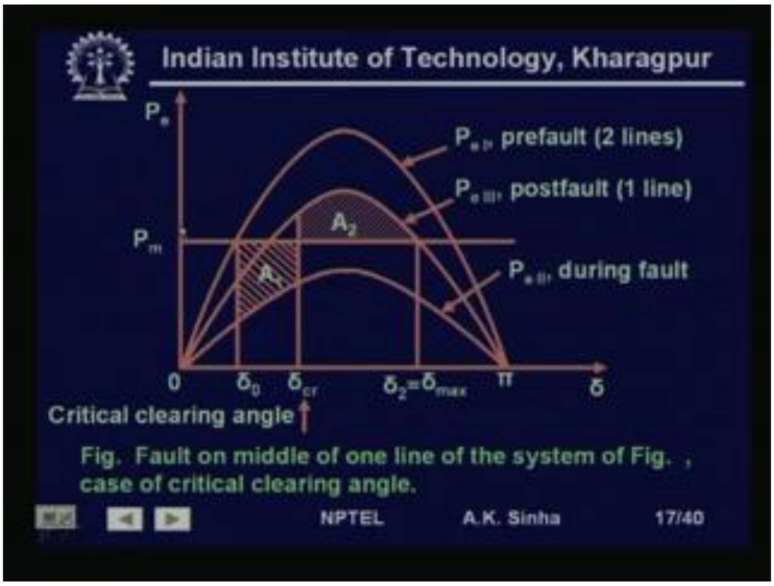

So, this is what is depicting the situation. This area A 1 increases also if our initial operating system is higher or if our clearing angle is larger. Now, in this case we see this A 1 is just equal to the maximum available A 2. And therefore, we call this as critical clearing angle. Now, how we find this critical clearing angle.

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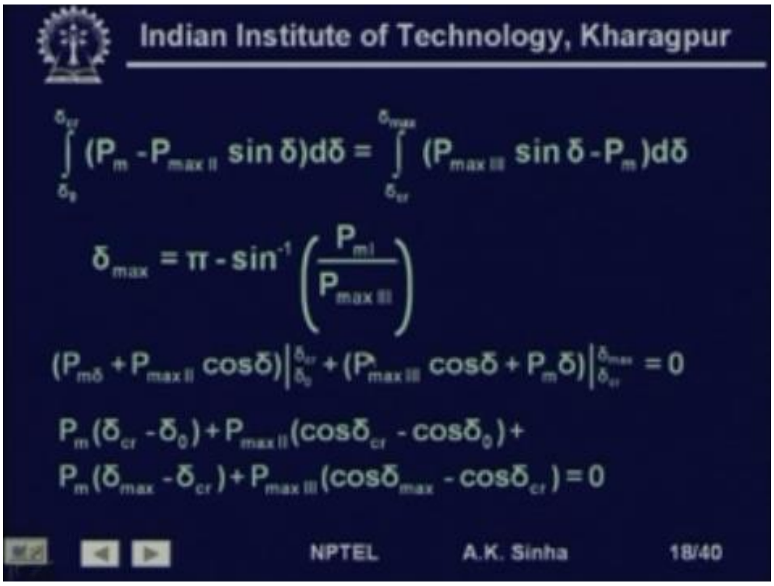

Again, what we need to do is calculate these two areas. That is the maximum available decelerating area and the accelerating area up to the clearing time. And when we get this equate these we will get the value of the clearing angle. And that clearing angle we call as the critical clearing angle. So, let us see ((Refer Time: 33:20)) this situation that is initial starting point is delta 0, the fault is cleared at delta c r and we have delta 2 which is equal to delta max in this situation.

So, what we have is the accelerating area is equal to P m minus P max 2 sin delta ((Refer Time: 33:43)). That is P m minus P max 2 sin delta, this is this characteristic. So, above this all this area, that we are talking about… So, P m minus P max 2 sin delta d delta integrated from which point delta 0 from this angle up to this angle delta c r.

So, delta 0 to delta c r. So, this is the integral that we have and this must be equal to how much? The decelerating area which is available ((Refer Time: 34:16)). So, decelerating area which is available is given by integral from delta c r to delta max of P e 3 that is this P e 3 sin delta minus P m. That is this area will be area under this curve. And this line, that is area below this curve and above this line.

So, this is what we are writing here from integral from delta c r to delta max P max 3 sin delta minus P m d delta ((Refer Time: 34:55)). That is P max 3 sin delta is giving this characteristics minus P m. So, we have got this as the decelerating area maximum amount of decelerating area available. Because, delta max we have used delta max is equal to pi minus sin inverse P m 1 by P max 3 ((Refer Time: 35:22)).

That is, delta max is going to be equal to pi minus this angle is not it. This is the angle where it is going to work. And this angle is equal to how much? This is electrical output in this case is going to be equal to P e 3 sin delta. And we have the mechanical input, which is equal to P m 1 into sin delta 0 that is this point.

So, using this we get delta max is equal pi minus sin inverse P m 1 by P max 3 or P max 1 by P max 3. So, this is what we get as delta max. Because, we have pi minus delta max is equal to sin inverse P max 1 by P max 3. So, putting for this we integrate this we get P m which is a function of delta P m into delta this delta should not be the subscript mode.

This is P m into delta, because we are integrating this plus P max into cos delta sin delta, when we integrate this becomes minus cos delta this is minus. So, plus P max to cos delta from delta 0 to delta critical delta 0 to delta critical. Plus we take this on this side, then we will have this as minus term. So, minus when we differentiate integrate this. This sin delta will become minus cos delta.

So, this is plus P max 3 cos delta plus this is minus, when we have taken on this side, this becomes plus, so plus P m into delta. So, this also should be P m into delta from delta critical to delta max. So, we substitute these values. So, we will get P m delta critical minus delta 0. That is, P m into delta we are writing P m into delta critical minus delta 0.

When, we substitute the limits plus P max 2 cos delta critical minus cos delta 0 plus p max 3 cos delta max minus delta critical P max 3 cos delta max minus cos delta critical and plus P m delta which will be P m into delta max minus delta critical P m into delta max minus delta critical. This will be equal to 0.

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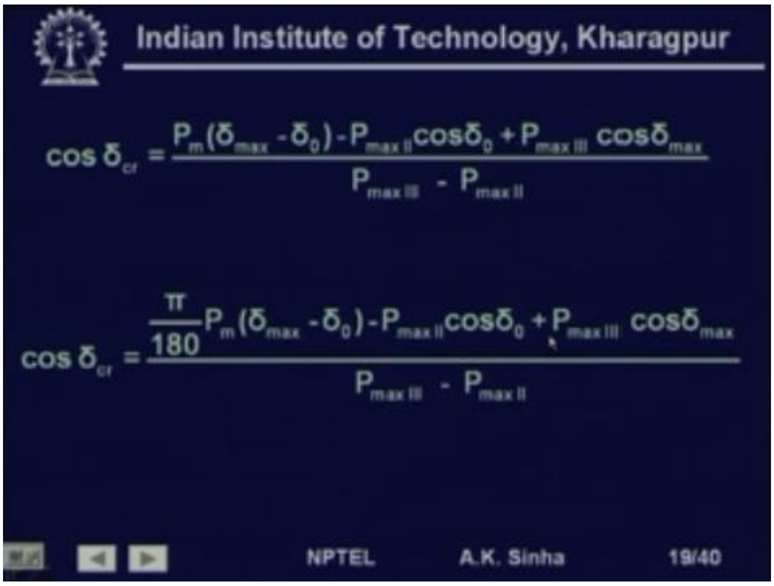

So, now substituting the values we can finally, calculate cos delta critical is equal to P m delta max minus delta 0 minus P max 2 cos delta 0 plus P max 3 cos delta max divided by P max 3 minus P max 2. So, this is after simplifying ((Refer Time: 38:36) from this, we can get cos delta critical as this expression. Now, this angle that we will get from here will be in terms of radiance.

If we are using degrees, then we will have to multiply it with 180 by pi. So, cos delta critical in that case, if we are using delta max and delta 0 all these has degrees. Then, we have to multiply them convert them into radiance. So, it will be pi by 180 we multiply them and we get this value. So, when we are using radiance, we are using like this, when we are using degrees. Then, we have to multiply or convert them into radiance. So, this is what how we can calculate the critical clearing angle.

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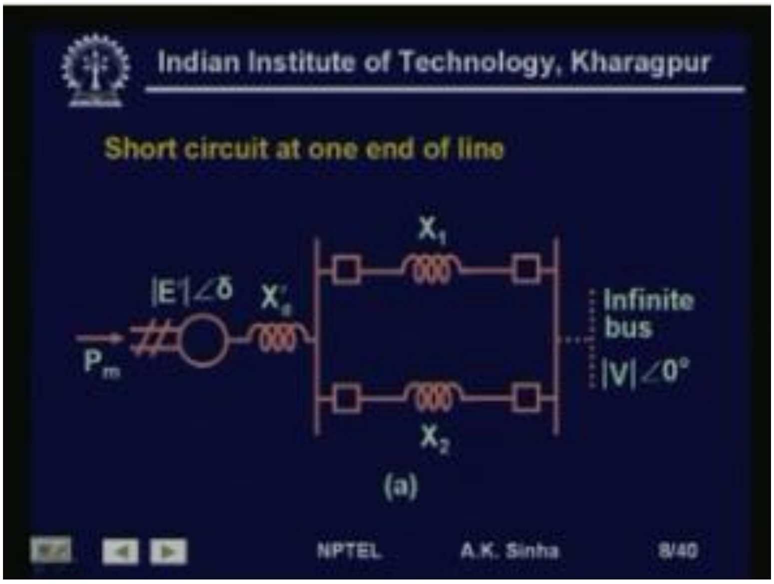

Now, here is another situation which comes Mainer times ((Refer Time: 39:38)) is the circuit breakers, that we have for ((Refer Time: 39:48)) the circuit breakers that we have in the system Mainer times have what we call re closer or automatic re closer. That is, these circuit breakers because most of the time the faults which occur on the power system are transient in nature. That is, once you clear them if you again close the line things will become normal.

That is, so a fault has occurred and if we clear the fault. And after that, if we reclose the circuit after a short time. Then, the circuit will be in general normal, this operation we call as the re closer of the circuit breaker. Now, if that is the situation, that is a fault has occurred on this line. Let us say and we have opened the circuit breaker. Then, we have only one line left in that case.

But, after some time if we reclose and the fault has been cleared. Then, the both the circuit will be on and so the power angle characteristic will change to pre fault condition. However, this re closer has certain disadvantages also. Under certain situations this can create more problems, because if the fault was not a transient fault that is it was a persistent fault. Then, if we reclose the circuit breaker we will be closing it on the fault.

So, instead of going to the pre fault for angle characteristics, that is the electrical power output will now depend in this situation for the power angle characteristics, which given by the faulted condition. That is, if we reclose on the fault. Then, again we are going to the faulted conditions. So, insisted of going to the pre fault condition, we go back to the faulted condition from the post fault condition.

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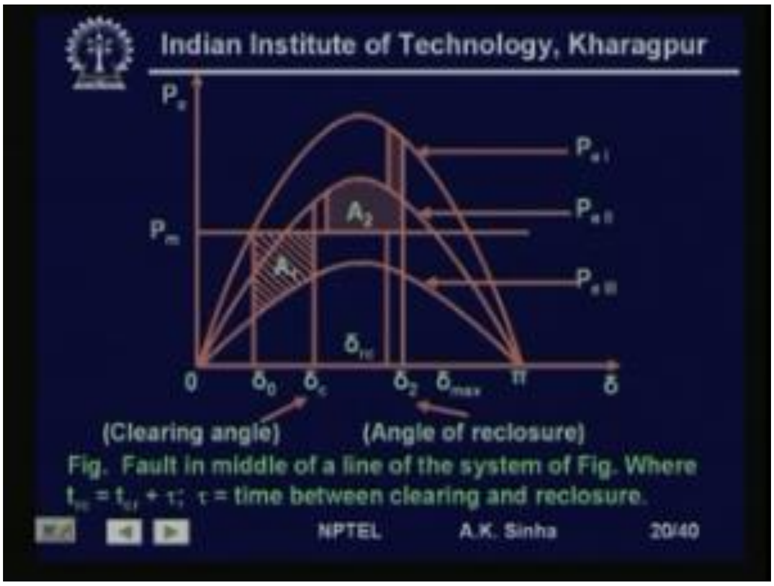

So, this situation is shown here. That is, we are initially operating at this point, which is the pre fault power angle characteristics with P m is equal to P and the angle is given by delta that is delta 0. Because, P m is equal to E dash v by X d dash plus X 1 and X 2 in parallel into sin delta 0. So, this is delta 0 at this point.

Suddenly the fault occurs, then say in the middle of the line or somewhere. Then, the based on the transfer reactance between the infinite bus and the generator it drops down to the post fault condition, so post fault characteristics. So, suddenly it will drop down here, delta angle will keep on increasing, because of accelerating power available.

We clear the fault at this point. Then, suddenly it will be jump to this point. And we will have this moving, this whole area should be like this not from here. But, it should be from here. So, this A 2 that is this point should be here. So, this whole area from this point up to this should be hatched.

So, here we have going moving on this point. And suddenly at when it has reached this point, if we reclose the circuit breaker. Then, if the fault has been cleared the operating point will jump to this. This point and it will keep moving like this. Now, in this situation what we find, if the fault was a transient fault. Then, what we are seeing that decelerating area available will be much larger it will be like this.

This area which will be a available, because now the operating point will be on this characteristics. But, in case the fault was a persistent fault, then from this point operating point it will suddenly drop to this point. Rather than, going up it will drop here we will have more acceleration available. And the system may become unstable also in such situation.

So, normally and most of the situations we allow one or two recloses. And after that the circuit breaker is locked with it is contact open. Otherwise, what happens that is it may lead to instability in the system. So, again in this case, if we are assuming it to be a transient fault, which is cleared which is most of the time. Then, we can find out the accelerating area available. That is, the accelerating area up to the clearing time and the decelerating area available will now be given by this. So, if re closer occurs at delta c r. Then, certainly the operating point jumps to this point and their available area decelerating area available will be given by this.

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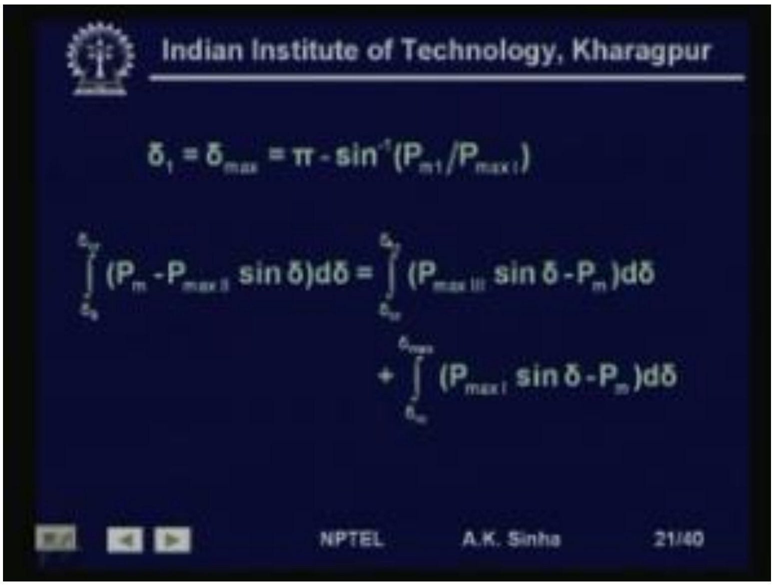

So, again we can calculate that, in this case delta 1 is equal to delta max that is pi minus delta max. Here, this delta max which is available will be here now and so in this case this will be equal to pi minus delta 0 as such. So, if we look at the accelerating area, it is from delta 0 to delta clearing P m minus P max 2 sin delta d delta ((Refer Time: 45:28)). That is P m minus P max 2 sin delta this area from delta 0 to delta c. This delta clearing this is what we get.

And then the decelerating area available from delta c to delta c r this should be delta c and this should be delta c to delta c r. Here, this is from delta c to delta re closer delta r c naught c r. So, delta c to delta r c P max 3 sin delta minus P m. This is the part, that we get up to this point that is this area. And from delta r c it jumps to this point. So, we have plus delta r c to delta max P max 1 sin delta minus P m.

That is now the operating characteristic is given by this. So, P max 1 into sin delta is coming minus P m minus this. So, from here it this area will be available for deceleration. So, this area is what we are talking about here delta c r to delta max. Now, again using this we have this should be delta clearing delta c or delta c r it does not matter.

So, here we have delta 0 to delta c r P m minus P max 2 sin delta that is here I should write delta c r instead of delta c. So, this what we are trying to do is trying to equate the accelerating area with the available decelerating area. And using that, we can calculate the critical clearing angle or the re closer time required.

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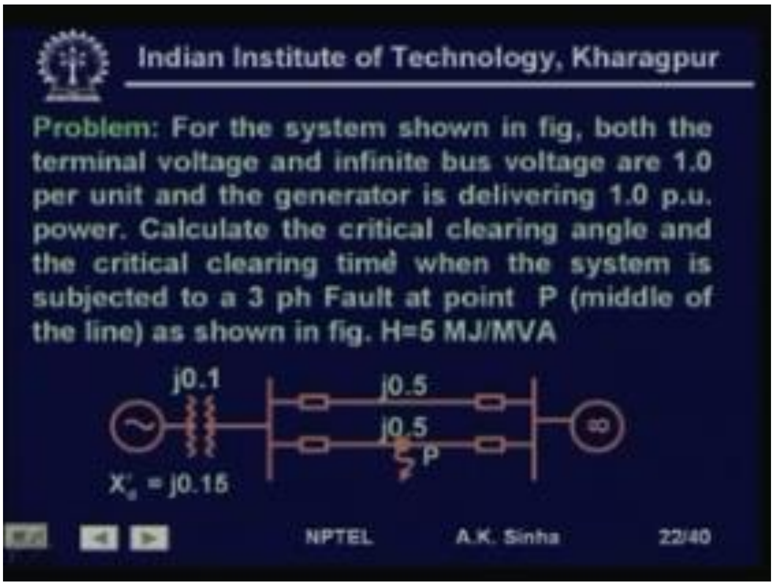

Now, let us take one small example for this. Again here what we have a single machine connected to an infinite bus by means of a double circuit line. The fault occurs at the middle of the line. For the system shown in figure, both the terminal voltages and infinite bus voltages are 1 voltage are 1 per unit. And the generator is delivering 1 per unit power.

Calculate the critical clearing angle and the critical clearing time of course, critical clearing time in this case we cannot calculate, when the system is subjected to 3 phase fault at point P. That is, middle of the line as shown in figure. H for this synchronous machine is given as 5 mega joules per MVA.

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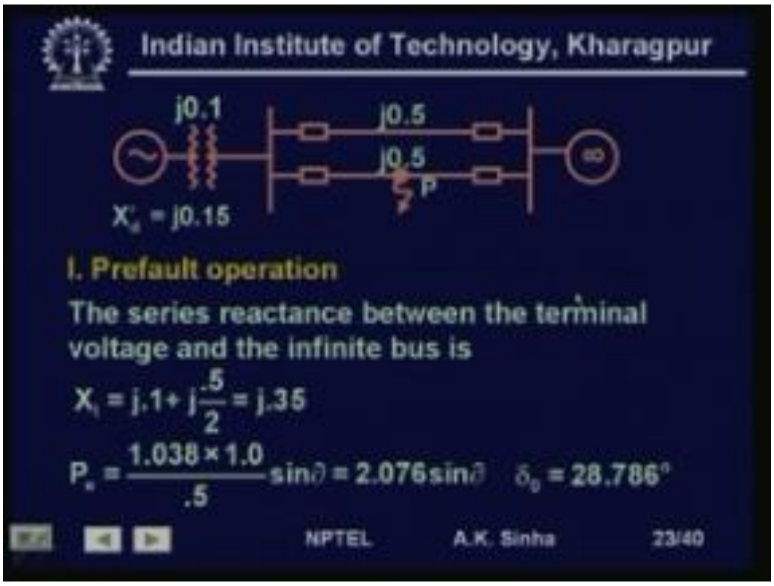

Now, here if we see the X d dash for the generator is given as 0.15. The transformer has a reactance of 0.1 per unit. The lines have reactance of 0.5 per unit. So, pre fault operation, what we have we have been given here ((Refer Time: 48:43)) the terminal voltage and infinite bus voltage are 1 per unit. That is, the voltage at this point is given as 1 per unit.

The voltage behind transient reactance we do not know. So, we need to find that out. So, first thing is what we will do is, we will try to find out the transfer reactance between this point and the infinite bus. So, the transfer reactance between this point and the infinite bus is j 0.1plus these two in parallel. So, this becomes j 0.25, so it is j 0.1 plus j 0.5 divided by 2 two lines in parallel this is j 0.35.

So, what is the power angle characteristic that we will get P e is equal to using this relationship we will be able to find out. What is the value of the power which is flowing. And therefore, after that we will be able to find out the voltage behind the transient reactance for the machine. If we calculate that it comes out to be 1.038. And if we use this relationship, then Pa is equal to 1.038 into 1.0.

And the total reactance that we will get, here will be this 0.15 plus j 0.1 with these two in parallel. So, that is going to be equal to 0.5. So, 1.038 into 1 divide by 0.5 into sin delta is equal to 2.076 into sin delta. Now, since we have been given that P e that electrical power transferred is 1 per unit. So, using this as 1 per unit, we will get delta 0 as 28.786 degrees.

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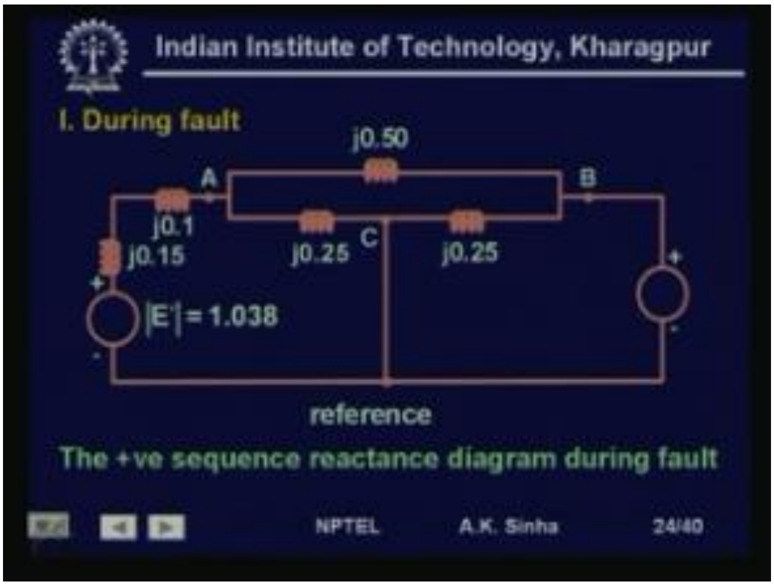

Now, when the fault has occurred at the middle of the line again, we have the circuit relationship like this E is given as 1.038. This the transient reactance. This is the transformer reactance. And this is the line one and this is the other line, where the fault has occurred at the middle. Now, what we have is we have this delta from which we can convert into a star.

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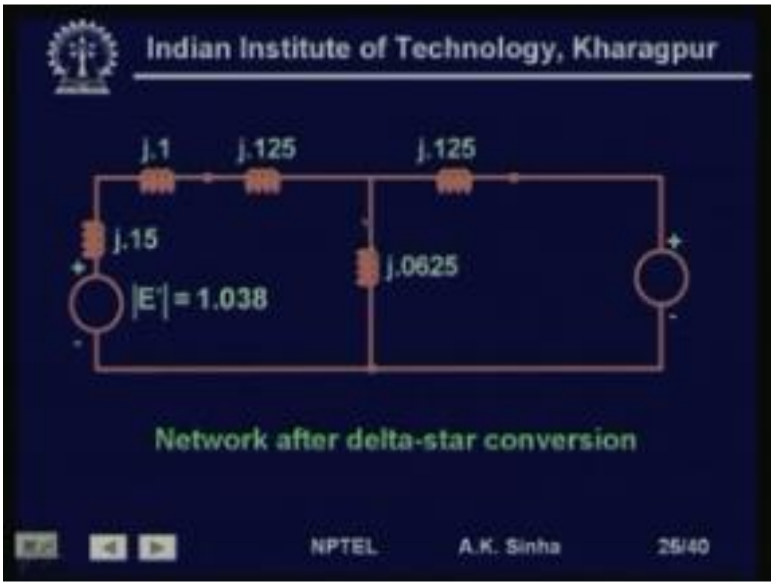

So, if we do that, then we will get this as 0.125, 0.125 and j 0.0625. So, this is the star for this delta. So, this j 0.1 is the transforner reacatance and this j 0.15 is the transient reactance of the machine. So, now this is the circuit that we have. So, we add this up 0.25 pluse 0.125. So, 0.375 J 0.15 and j 0.625 this forms now a star circuit, which we can convert to a delta.

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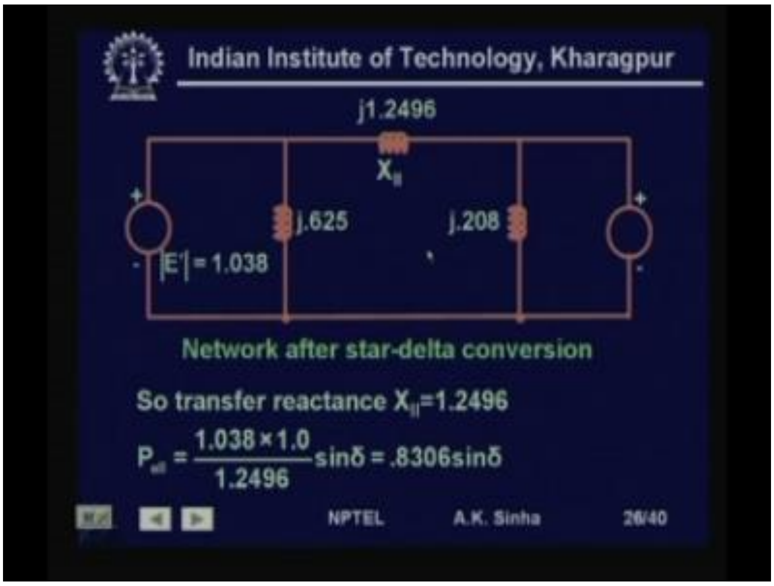

If we do that, then we have got the transfer reactance here as j 1.2496. So, this is now the transfer reactance between this voltage behind. The transient reactance of the machine and the infinite bus. And therefore, we will have the power angle charastristics for the faulted period give by P e 2 is equal 1.038 into 1 divided by 1.2496 this is the reactance. This is the E dash and this is V into sin delta. So, this comes out to be 0.8306 into sin delta.

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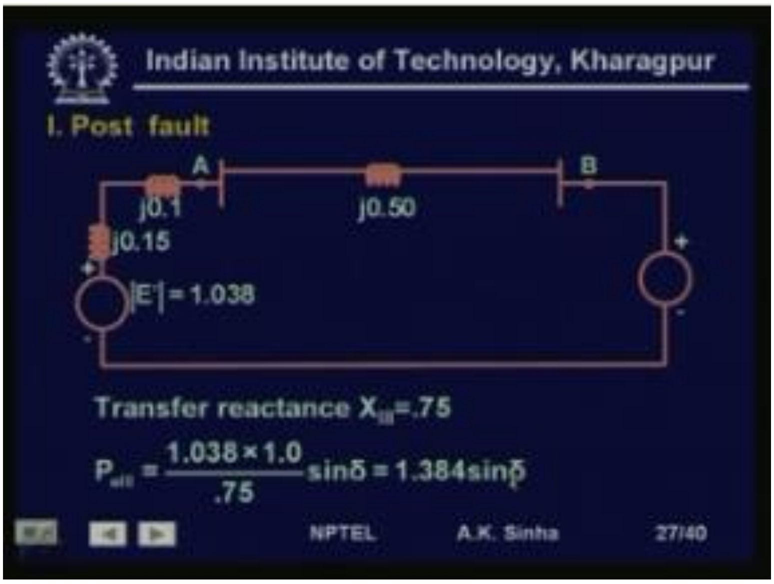

When, the fault is cleared by opening the circuit breakers of the two or the two ends of the faulted line. Then, we have the system like this, where this is the transient reactance, this is the transformer reactance, this is only one line now j 0.5 and this is the infinite bus. So, transfer reactance in this case comes out to be 0.5 plus 0.1 plus 0.15, so j 0.75. Therefore, P 2 is equal to 1.038 into 1, this is E dash V divided by 0.75 into sin delta this comes out to be 1.384 sin delta.

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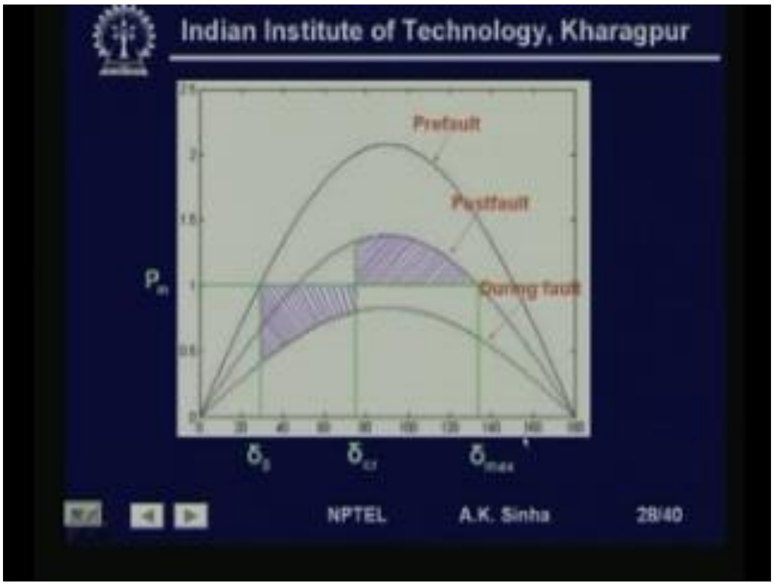

So, we can draw the three characteristics pre fault characteristics which is given by ((Refer Time: 53:32)) 2.076 into sin delta ((Refer Time: 53:37)) during fault characteristics, which is given by 0.8306 into sin delta ((Refer Time: 53:41)). And post fault characteristics which is given by 1.384 into sin delta. So, this is P max is 0.836, this is 1.384 and the initial one is 2.076. So, 2.076 is the maximum power here.

So, we get these three characteristics. Now, we have of initial operating point here, which is 27 degrees. And the fault occurs, the operating point jumps down to this point. And it accelerates around this when the fault is cleared, at this point it will jump here and it will move around this.

Now, if this point of clearance is the critical clearing time or critical clearing angle this delta c r. Then, this total area which is the decelerating area must be equal to this area, which is the accelerating area.

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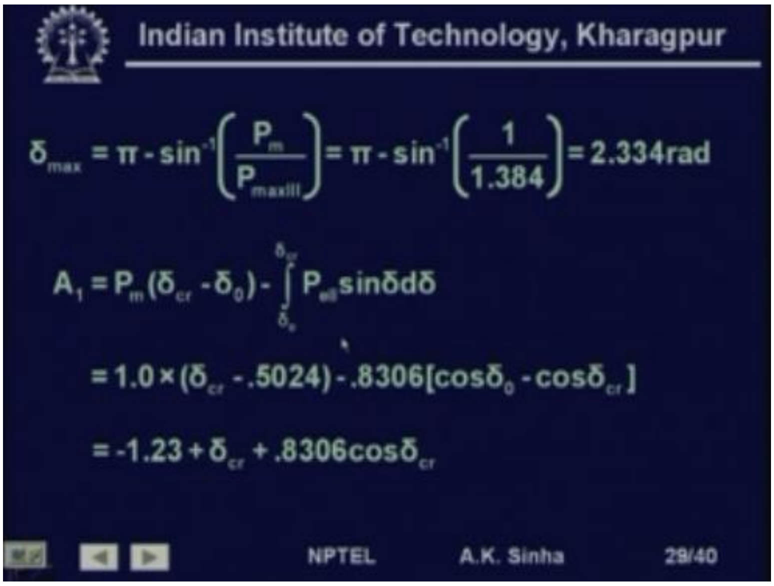

So, again using the same relationship that we had developed earlier delta max is equal to pi minus sin inverse P m 1 by P max 3. So, this is pi minus sin inverse 1 by 1.384 which is equal to 2.334 radiance. That is here delta max is 2.334 for radiance. That is what we have got? And the area A 1 is equal to P m into delta c r minus delta 0 minus integral delta 0 to delta c r P e 2 sin delta d delta.

That is ((Refer Time: 55:30)) we are say this rectangle which is P m into delta 0 delta c r minus delta 0. Subtract from this, this area below this curve which is again from integral of delta from delta 0 to delta critical of this characteristics. So, we are writing this P e 2 sin delta d delta. So, this is the accelerating area that we have...

So, we substitute the values here, then we will get this as 1 into P m is 1 delta critical minus delta 0 is 27 degrees. So, this is minus 0.5024. And this P e 2 is given as 0.8306 and with this sin delta d delta, when we do with the substitution of the limit. This is cos delta zero minus cos delta c r. So, we substitute these values then we get this as the area, which is the accelerating area.

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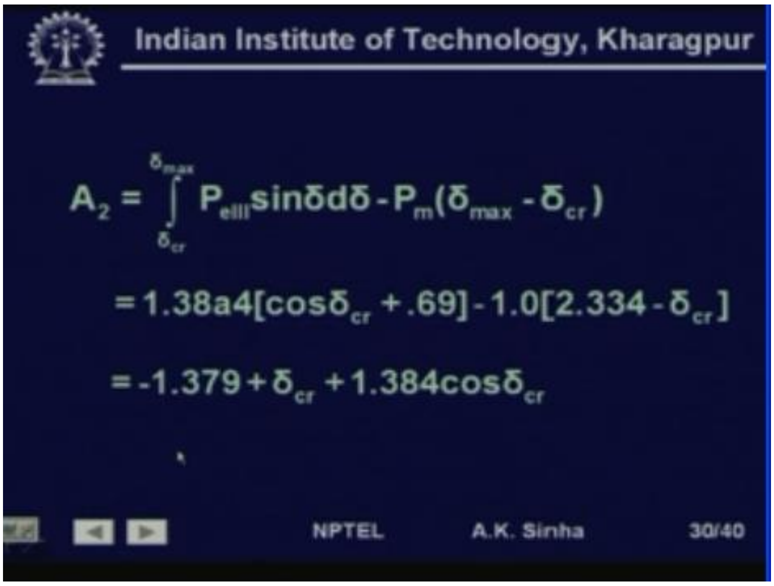

Similarly, we will get the decelerating area A 2 as delta critical to delta max P e 3 sin delta d delta minus P m delta max to delta critical ((Refer Time: 57:01)). That is from this point we are saying the area under this curve from this point to this point minus the area rectangular area here. So, that is what we are writing here. So, P e 3 sin delta d delta from delta critical to delta max minus P m delta max minus delta critical. So, substituting the value this comes out to be minus 1.379 plus delta critical plus 1.384 cos delta critical.

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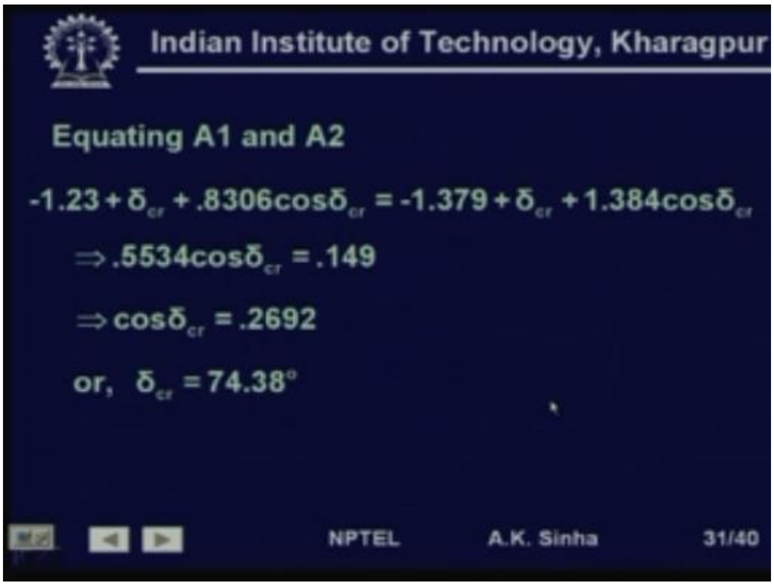

So, equating these two A 1 and A 2 we get this relationship delta critical delta critical will cancelled out from which we will get 0.5534 cos delta critical is equal to 0.149 from which we will get cos delta critical is ((Refer Time: 57:50)) 0.2692. And from which we will get delta critical is equal to 74.38 degrees. So, this way we can calculate the critical clearing angle. So, this is how we use the equal area criterion for transient stability analysis of the system. In the next lesson, we will talk about the numerical integration of swing equations, and how we use it for multi machine system stability analysis.

Thank you.