Power System Analysis
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Lecture - 37
Power System Stability – V

Welcome to lesson 37 in Power System Analysis. In this lesson, we will continue our discussion on Power System Stability. In the last few lessons, we were discussing rotor angle stability, we had discussed about the small signal stability in the last lesson. We discussed about large signal stability, which we call as transient stability of power system. And there, we introduced the concept of equal area criterion, for trying to analyze this transient stability of a single machine connected to infinite bus bar system.

In this lesson, we will continue with the same discussion. That is, we will start with the equal area criterion. We will look into different kinds of problems and how to solve or analyze the stability of power system, under different conditions, using the equal area criterion. We will take a few examples and then, we will discuss this problem.

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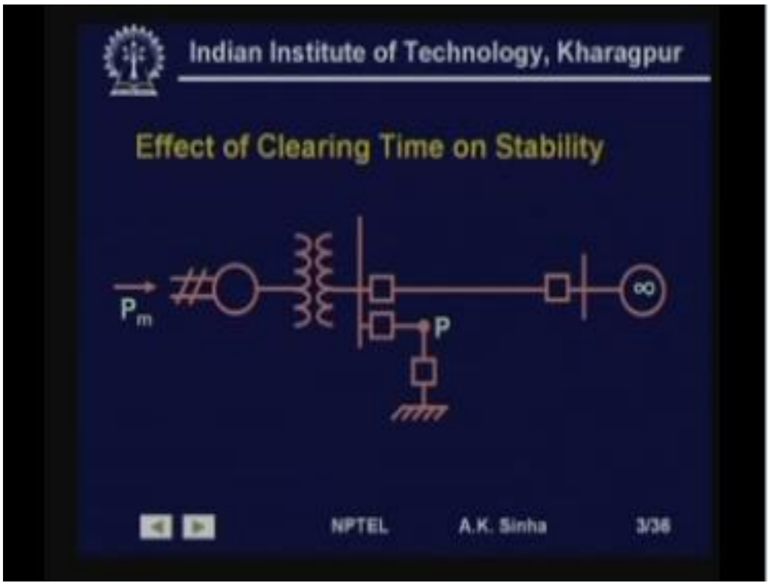

We will start with the problem of effect of clearing time on stability. In fact, in the last lesson, we talked about, how the magnitude of a disturbance can affect the stability of the system. In this, we will talk about, how clearing time can affect the stability. That is, when a fault occurs in the system, in that case, if we clear the fault much faster, that is much very quickly and then, the system may remain stable.

Whereas, if we take more time to clear the fault, then the disturbance, that is the accelerating power or decelerating power will be remaining for a much longer period of time. And the system may become unstable, under such situation. So, let us take a simple example, where we have a synchronous machine, connected to an infinite bus, shown here. Through, a transformer and a transmission line, P m is the mechanical input to the synchronous machine.

That is turbine synchronous machine system and we have a load, which is connected here. And suppose, we have a fault which occurs at this point P. Now, this point P is adjacent to this bus here. So, we can assume this fault to be almost same as to be on this bus. Now, what is going to happen, when a three phase short circuit takes place? In fact, for most of the faults or most of the disturbance, that we try to analyze for in transient stability analysis is the three phase fault.

Because, it is the most severe fault on the system, therefore we will always be considering three phase fault here. In fact, the other faults like line to ground faults or double line to ground fault or line to line fault. All these are somewhat less severe than three phase fault. And therefore, if we want to analyze the stability of the system we normally consider the most severe condition on the system.

And if we find, that the system is stable for this most severe disturbance, then we expect it to be stable for lesser disturbance. So, here we are considering a three phase fault at this point P, which occurs. And if a three phase fault occurs at this point P, which is same as this point or this bus bar. So, the voltage at the bus bar is go down to 0, which means, that the power flow electrical power output from the generator to the infinite bus will drop down to 0.

That is the electrical power output from this synchronous machine will be 0. Because, here the voltage is 0 and this is a purely reactive circuit, we have a transformer; we have the transient reactance of the machine. And this is a fault, three phase fault here. So, the current, which is flowing in this into this fault, will be a purely reactive current. And there will be no real power, which will be coming out from the machine. So, the real power output from the machine, during the period of the fault is going to be 0.

Now, if we clear this fault by opening the circuit breaker here, then the system will again be restored. So, that means, once we open this circuit breaker, the fault will get isolated. This function is done by means of relays, which will sense this short circuit current flowing, through this part of the circuit. And will give a command to the circuit breaker to open and once the circuit breaker opens, this fault is isolated from the system and the voltage at this bus, will recover back to it is normal value.

And there is going to be a power transfer from the machine to the infinite bus of the system. So, let us see, how we can analyze the situation by means of equal area criterion.

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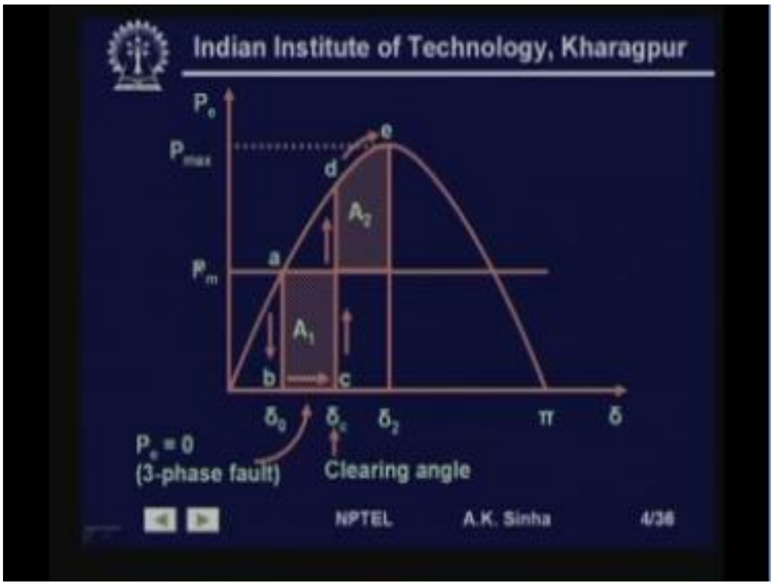

So, let us assume initially the system is working at this point P m. That is the mechanical input to the system is P m and the electrical output also is equal to P m. That is P e is equal to P m, just before the fault has occurred. And the operating point of the system is at this point a. The power angle of the machine, under this situation is delta 0. Now, once, the fault occurs on the system, that is, when it is operating at this point a.

The speed of the synchronous machine is synchronous speed; this system is working under normal conditions, with power angle delta 0. Now, once this fault occurs, what happens is, the electrical power output goes to 0. That is the electrical characteristics, if you see shifts from point a to point b here, that is the electrical power output becomes 0. And what we have instantaneously is a large amount of accelerating power.

That is, accelerating power which is equal to P m, because P m minus P e is the accelerating power and P e is equal to 0, in this case. So, the accelerating power available in this time is P m here. Because, of this large amount of accelerating power, there is going to be acceleration of the rotor and therefore, delta angle will start increasing. But, the electrical power output will remain 0. Because, the voltage at that bus is 0 and only reactive power is flowing.

So, real power output from the generator will be 0 and the characteristics or the operating point will start moving on this X axis, like this. So, it will keep moving like this. That is delta angle will keep on increasing like this. And once, it comes to say point c, where we clear this fault by operating or opening the circuit breaker as shown here. That is the circuit breaker we open, so the fault gets cleared.

Now, if the fault gets cleared, once we reach this angle delta c, which we will call the clearing angle, then what happens? The electrical power output, now will suddenly jump from c to d. Because, with angle delta c, the electrical power output, which is P m or P max sin delta. Now, delta is equal to delta c. So, it is P max sin delta c and the value for this is coming up to this point d. That is the operating point, now suddenly shifts, from point c to point d, once the circuit breaker is opened.

Now, what is happening, now we have electrical power output, which is much larger than the mechanical input and so there is going to be decelerating power available. That is P m minus P e is negative, so the rotor will start decelerating. However, when it started from this point and reached to this point. This speed has also increased, that is speed is now super synchronous. So, at this point, the speed is super synchronous.

And the delta angle will keep increasing, though the rate of increase is going to be much lower. Because, now we have deceleration acting on the system, so the speed of the rotor will start slowing down, but still it will be above the synchronous speed. Now, suppose, when it reaches this point e, this speed has become equal to the synchronous speed, then what is going to happen?

Now, at this point, we have so much of electrical power output available and the mechanical input is only, this much. So, this much amount of decelerating power is available. So, now, the speed is going to go down and as speed goes down delta angle will also reduce. Because, we are measuring delta as, with respect to synchronously rotating reference frame, so speed is going below synchronous speed. So, delta angle will show a reduction. So, delta will keep on reducing like this.

When, it reaches this point a, we will find that P m is equal to P e, but the speed is lower than the synchronous speed. And therefore, the delta angle will keep on reducing and we will find that, as it reduces, we have accelerating power available. So, speed will try to increase and we will get some point, where we will again get this speed coming back to the synchronous speed. But, since during that time, we have accelerating power available.

So, again, we will have an oscillation taking place and because of damping available these oscillations will die out. And finally, the system will come back to an operating state, which will be the same point, operating point a. So, this is what the situation takes place, now as we have seen, while we discuss the equal area criterion. So, that, the area under acceleration must be equal to the area under deceleration. So, this area under acceleration is given by this rectangle a 1.

That is from a, the operating point shifts to b, this is the amount of accelerating power available. And this power remains constant up to point c, where the circuit breaker opens to clear the fault. So, the accelerating power is given by a 1. Once the fault is clear, the operating point comes to d and it moves from d to e along this curve. So, decelerating and P m, all the time, we have assumed it to remain constant.

So, P m remains here. So, d, decelerating power available at different time will be like this. So, the total decelerating area available will be given by this a 2. So, what we find is, this point delta 2 will be reached, when a 1 is equal to a 2. That is accelerating area. These are is equal to decelerating area. So, we will use this criterion to find out the angle delta 2 or delta c.

Now, there is a question, which comes is, now suppose, we do not open the circuit breaker at this point delta c, but we allow it to move further. That is the action of the relay and circuit breaker takes longer time. Then, what happens, this point c will be shifting further on this side, which means the accelerating area will be becoming larger and larger.

And if that happens, then we will find that, what is the maximum decelerating area, which we can get, will be up to this point only. That is above this line and up to this point is, what we will get as decelerating area. And we find that, if delta 2 can be shifted maximum up to this point. That is this angle at this point, only if the movement goes beyond this. That is delta 2 goes beyond this angle.

Then again have the accelerating power available. So, accelerating area is further going to increase. That means, we say that, we will get a limiting condition, when we have this delta 2 equal to this point. That is, this delta 2 will be equal at this point, which is same as pi minus delta 0. Because, here we have a delta 0 and here, we have pi. So, this angle will be pi minus delta 0.

So, till pi minus delta 0 is the maximum angle, which delta 2 can take. If delta 2 goes beyond that, then the system becomes unstable, because in that case, the accelerating area is going to become more than the decelerating area. Because, the maximum decelerating area can be available, only up to this point.

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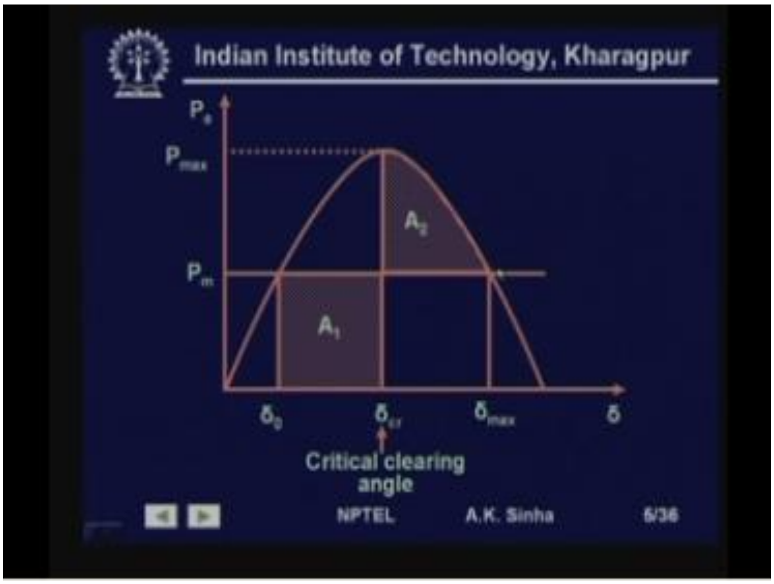

The same case is shown in this figure, that is, what we have done is, we have allowed or we have taken more time for the circuit breaker to open. That is fault to be cleared. So, if we clear this fault around this point. That is delta clearing time is this. Then, we will get the decelerating area, maximum up to this point. And if this, a 1 is equal to a 2 and then we have a limiting condition for transient stability.

That is the delta 2 is now equal to the maximum delta, which is possible, under this situation. And therefore, this is a limiting situation and we call this angle for clearing the fault has critical clearing angle. Because, if we have the fault clearing, after this critical clearing angle. Then, the system becomes unstable and we must clear the fault, before this delta c r, for the system to remain stable.

So, this is what we see that, this is how, we are seeing that fault clearing time or the fault clearing angle is affecting the stability. That is, if we make our clearing angle greater than delta c r, then the system becomes unstable. If we make it, less than delta c r, then system will remain stable. And at delta c r, we have limiting stability, it is, just stable at that condition.

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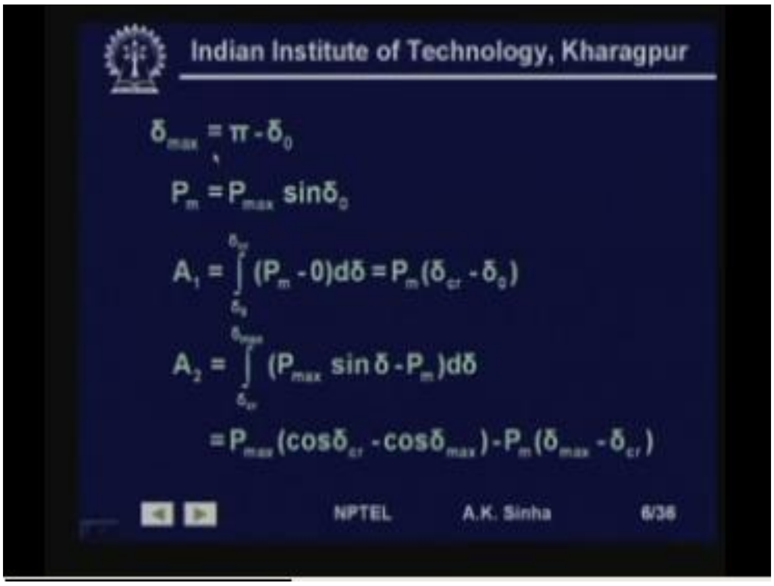

So, now, we need to find out, what is this critical clearing angle for this system. So, let us say from this system, we have seen this delta max is equal to pi's angle here and this is delta max. So, this angle is equal to delta 0. So, delta max is equal to pi minus delta 0. So, delta max is pi minus delta 0, P m, we know is the mechanical input and initial operating point, before the fault. The system is having P m equal to P e, which is same as P max sin delta, where P max is E v by X.

So, we have this P m is equal to P max sin delta 0. Therefore, the area a 1, accelerating area a 1, if we look at this, this area a 1 will be equal to how much P m minus P e is 0 in this case. So, P m into delta c r minus delta 0, P m minus P e, which is 0. So, integral from delta 0 to delta c r, P m minus P e d delta. That is what we have as the accelerating area and this in this case comes out to be since P a is equal to 0. This comes out to be P m, delta c r minus delta 0, because P m is a constant, it is not a function of delta.

So, we get accelerating area, which is rectangle in this case, which is P m in into delta c r minus delta 0. Mind it, when we are doing this, we must use these delta angles in radiance. Otherwise, if you use them in degrees, there will be problem. P m is in per unit, delta values will be in radiance. Now, what is the decelerating area available, this will be from delta c r to delta max integral P max sin delta, which is the electrical power output? That is, if we see here, this is P max sin delta from delta c r to delta max.

So, this is this curve that we have minus P m. So, this area is integral form delta c r to delta max minus P m d delta. So, this what we have got, A 2 is equal to integral from delta c r to delta max P max sin delta, which is the electrical power angle characteristics minus P m d delta. This is equal to P max cos delta r delta c r minus cos delta max minus P m delta max minus delta c r. So, we have just integrated this, sin delta will become cos delta and so on. So, I am putting the limit delta max and delta c r, we will get this relationship. Here, we have got cos delta c r minus cos delta max, because the integral of sin delta will be minus cos. So, here we have got P max cos delta c r minus cos delta max minus P m delta max minus delta c r.

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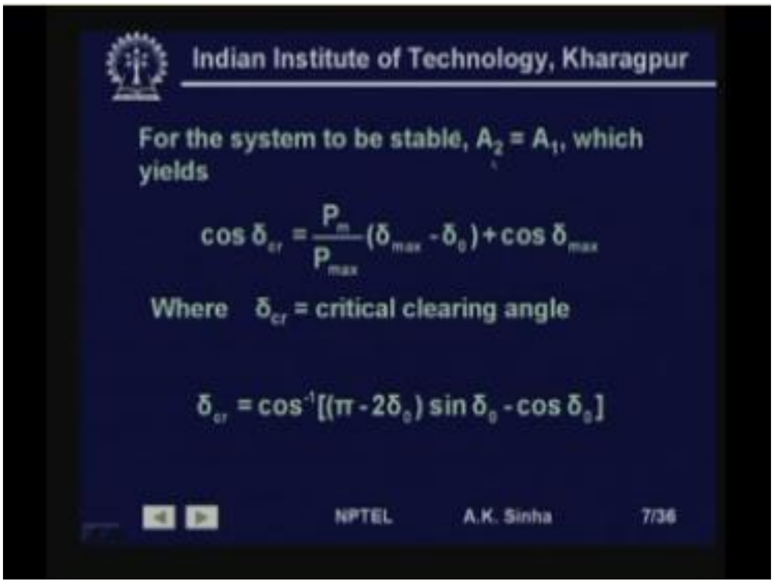

For the system to be stable A 2 must be equal to A 1. That is the decelerating area must be equal to accelerating area. And from these two relationships, that is, using the relationship A 1 is equal to, that is, this is equal to this, we can simplify. And finally, get cos delta c r is equal to P m by P max delta max minus delta 0 plus cos delta max. Now, the value of delta max is known, because delta 0 is known and P m is known and P max is also known from the system. Therefore, we can calculate delta c r.

So, delta c r as I said earlier, we call this as critical clearing angle. Because, if we clear the fault at with when the delta angle has not reached delta c r. Then, the system will be stable, if we clear the fault after the delta c r angle. That is, when the rotor has exceeded delta c r, then the system becomes unstable. And this is what, we have written, once u has cos delta c r, you can find out delta c r. So, this is how, we find out the critical clearing angle.

Now, in this case, since P m remains constant, during the time of the fault, the accelerating power is remaining constant. Therefore, for this kind of a situation, we can also find out the critical clearing time. So, because we need to find out critical clearing time, that is, what is the time in which we must clear this fault?

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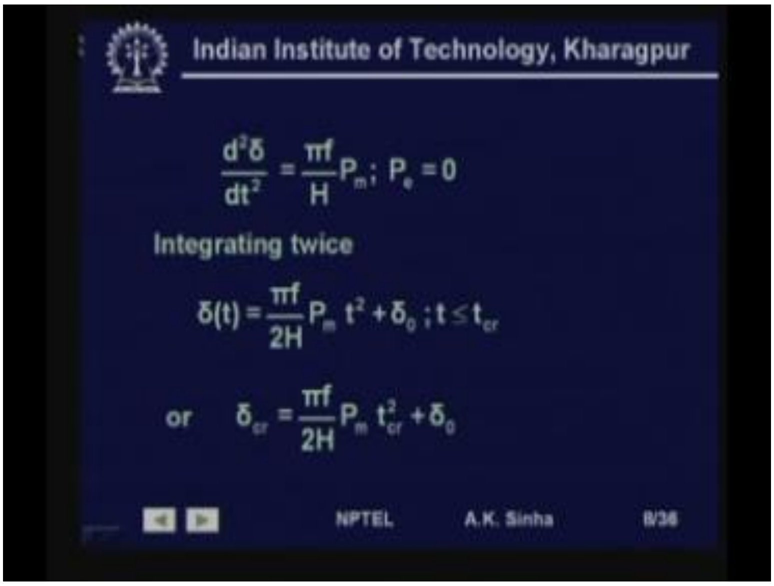

So, let us, we have the swing equation, d 2 delta by d t 2 is equal to pi f by H into P m minus P e, P e is equal to 0, for the fault condition. So, we are writing this equation for the faulted condition. That is, when the fault is on. So, this is the condition, we integrate it twice, then we will get delta, which is a function of time is equal to pi f by 2 H, P m t square plus delta 0. This is valid for t up to that is clearing time, up to the clearing time; this is going to be valid.

So, I should not write this as critical clearing time, but up to the clearing time. So, delta is given by this and therefore, since delta c r will be equal to pi f by 2 H, P m, t c r square plus delta 0. So, that is, what we have done is, here we have substituted for t, t c r, that is the clearing time. So, at the clearing time, the critical and clearing angle is given by this relationship. So, delta c r is equal to pi f by 2 H, P m in to t c r square plus delta 0. And since, we know delta c r, therefore we can calculate the critical clearing time, from this relationship.

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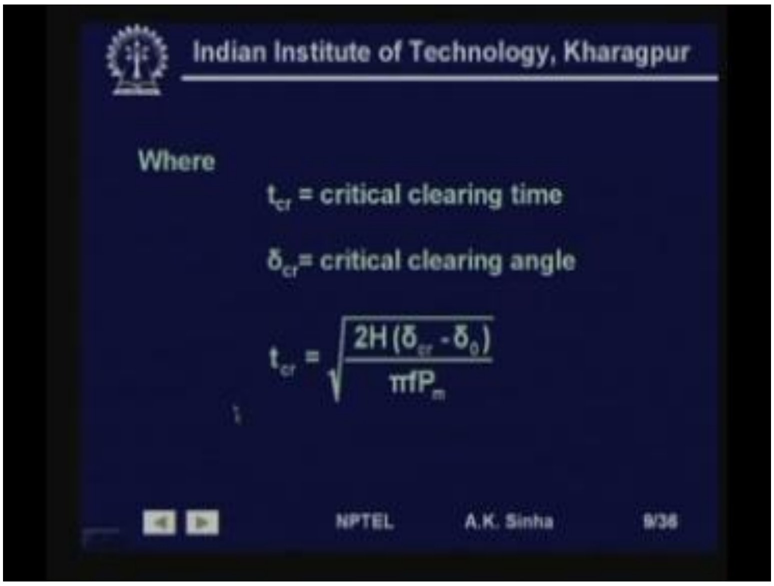

So, let us take an example to work this out. So, from this relationship, we can calculate t c r, which we have written here like this. T c r is equal to square root of 2 H into delta c r minus delta 0 by pi f into P m. That is, simply finding out this, we can take on this side. So, delta c r minus delta 0 and this whole term, which is a constant, can be taken on this side. So, this is 2 H by pi f into P m and this is equal t c r square. So, taking the square root, we will get t c r is equal to square root of 2 H delta c r minus delta 0 by pi f into P m.

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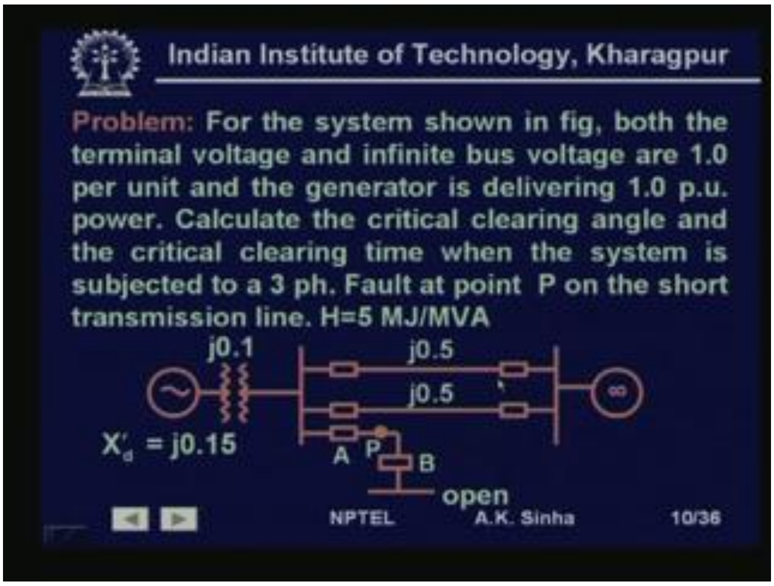

Now, let us taken an example, to see how this works. So, for the system shown, both the terminal voltage an infinite bus voltage is 1 per unit. For this system, the terminal voltage of the generator here and the infinite bus voltage are 1 per unit. And the generator is delivering 1 per unit power. So, 1 per unit power is flowing from this generator up to the infinite bus. Calculate the critical clearing angle and the critical clearing time, when the system is subjected to a three phase fault, at point P, on the short transmission line.

That is, at this point, that is again same as this point, which is this fault here is similar to a fault on this bus, because the voltage drop here is going to be extremely small. So, this voltage here, when the fault occurs is 0. So, the voltage at the bus will also be 0. Fault point P on the short transmission line, H is equal to 5 Mega Joules per MVA. The reactance for the transmission line is shown here. The transient direct axis transient reactance for the synchronous machine is given as 0.15 and for the transformer, it is 0.1 per unit. All the values are in the same base.

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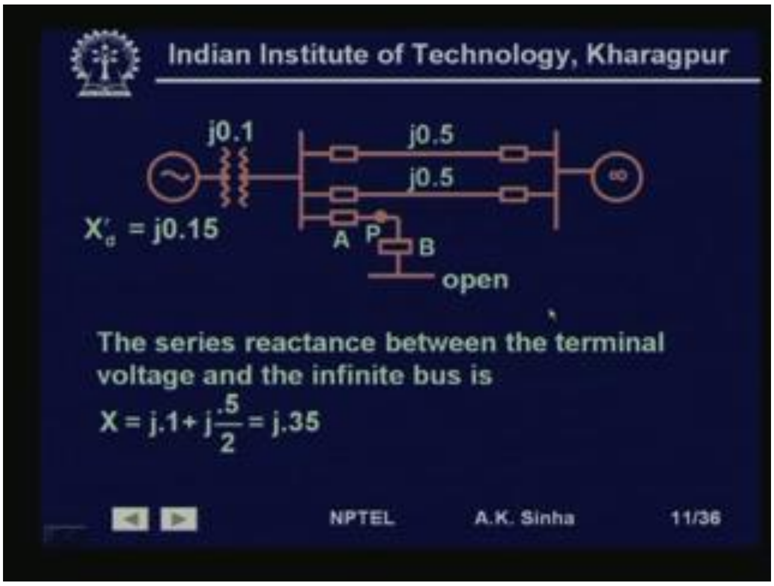

Now, what we have to do is, when we need to find out the power transfer relationship between this, machines to the infinite bus. We need to find out the internal voltage of the synchronous machine. So, first thing we need to do is, find out the internal voltage of the synchronous machine. Because, what we have been given is the machine terminal voltage is 1 per unit. Now, from the terminal of the machine up to the infinite bus, what is reactance of the systems? The series reactance between the terminal voltage and the infinite bus is coming from this transformer and these two lines in parallel. So, it is j 0.1 plus j 0.5 by 2. So, this comes out to be j 0.35 per unit.

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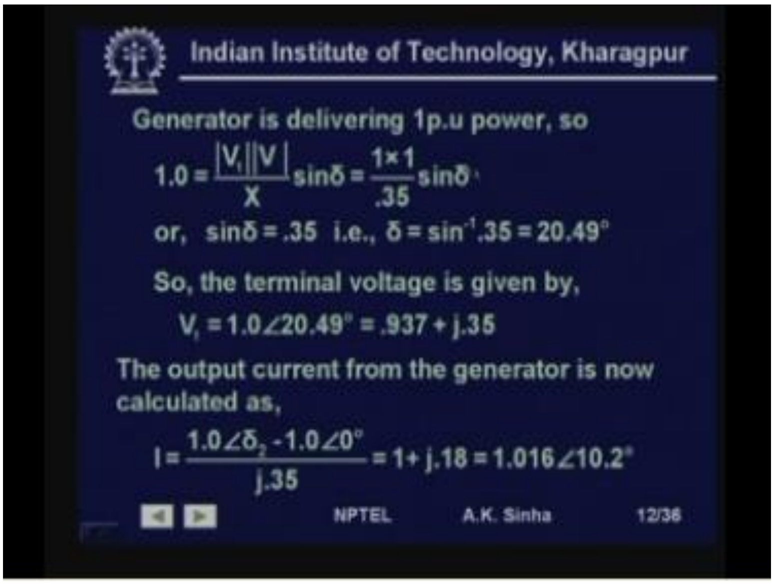

Now, generator is delivering 1 per unit power. So, we know that P e is equal to V t in to V by X in to sin delta, where V t is the terminal voltage of the machine. V is the infinite bus voltage. X is the transfer reactance. That is reactance between the terminals of the generator, up to the infinite bus. So, V t V by X in to sin delta, V t is 1, V is also 1 and the transfer reactance is 0.35 as we have calculated just right now.

So, 1 into 1 by 0.35 sin delta, which gives that, sin delta, is 0.35 or delta is equal to sin inverse 0.35 or 20.49 degrees. That is the initial operating point; we have the terminal voltage with an angle of magnitude of 1 and angle 20.49. So, terminal voltage, we have got like this. Now, we can calculate, once we know this voltage and the voltage of the infinite bus, we can calculate the current, which will be simply V t minus V, divided by the reactance of the system.

So, V t, which is, we are saying this is angle delta, we have written here delta 2. So, 1, 0 angle delta 2 minus 1.0 angle, 0 degree, this is V. We have chosen as we said earlier, we always chose the reference angle, which is the angle for the infinite bus. So, voltage at infinite bus is having the reference angle 0. So, 1 angle delta 2 minus 1 angle 0, divide by j 0.35. This comes out to be 1.016, with an angle 10.2, degrees. So, now, we know the current and we know the impedance of the system.

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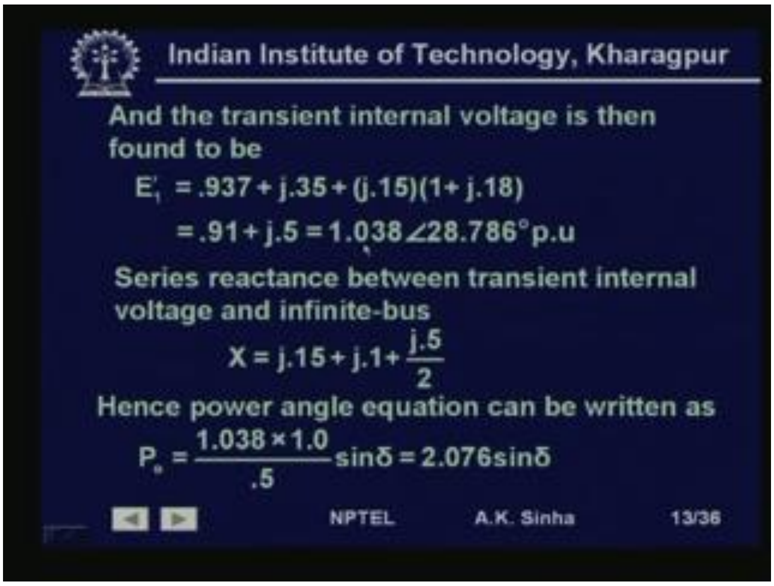

The transient internal voltage, that is, we are trying to find out the voltage, behind the reactance is equal to then, this is the voltage. This is the terminal voltage, so terminal voltage plus the drop in the transient reactance of the machine. So, terminal voltage plus the drop the reactance is j 0.15 and the current is 1 plus j 0.18. That is, what we had seen here, 1 plus j 0.18.

So, using this, we will get the voltage, behind the transient reactance as 1.038, angle 28.786 per unit. That is delta 0; we have got as 28.786 degrees. Now, the series reactance between the transient internal voltages, an infinite bus, that is between E dash an infinite bus is equal to how much? This is 0.15, which is the transient reactance of the machine plus 0.1, which is the transformer reactance plus 0.5 parallel two lines, so 0.5 by 2.

So, this is, what we got here, total transfer reactance is 0.15 plus j 0.1 plus point j 0.5 by 2 this comes out to be 0.5. So, the power angle equation for the machine now can be written as, E v by x into sin delta, this is equal to 2.076 sin delta.

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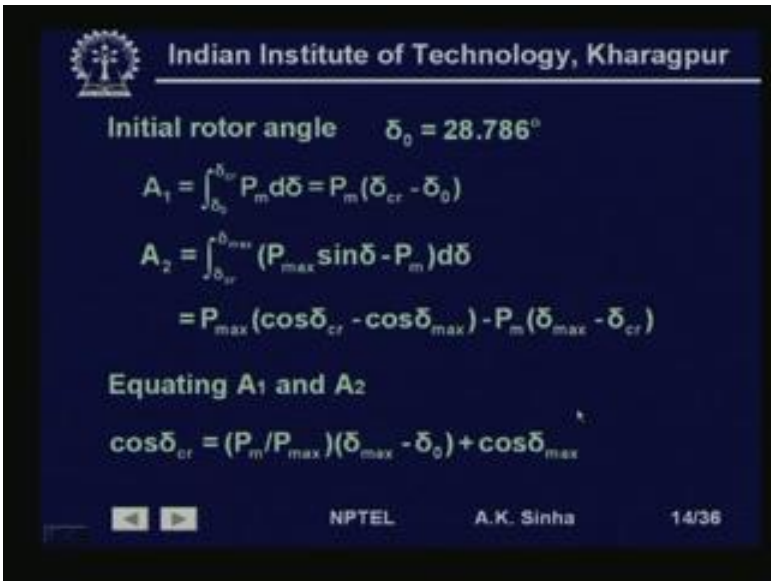

Now, the initial rotor angle, as we have seen is the angle of E dash, which is coming out 28.760. This we can find out from this relationship also. That is P e initially is equal to 1 per unit. So, putting this 1 per unit is equal to 2.076 sin delta from there also we will get the same value. That is delta 0, initial operating point power angle is 28.786 degrees.

Now, we can calculate the accelerating area, which will be from delta 0 to delta c r, P m, d delta. Because, P e in this case is equal to 0, when the fault has occurred at this point. Then, the power transfer from this will be 0 because the voltage at this bus will be 0 and all the circuit only has reactance. So, the short circuit current will be purely reactive, no active power will be flowing in this case.

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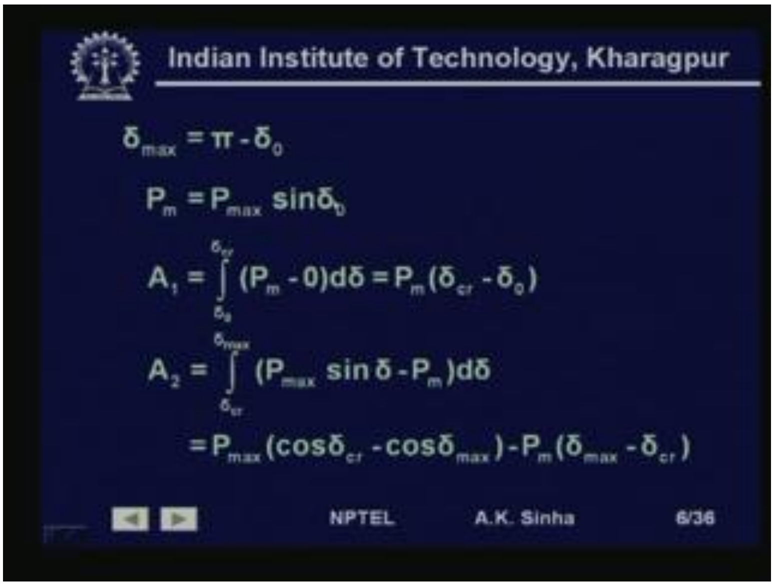

So, we have accelerating area given by P m delta c r minus delta 0, as we have seen in the previous case. That is, here we had seen this is Pm delta c r minus delta 0. So, same thing, we are writing in this case also. So, this comes out to be P m delta c r minus delta 0. A 2 will be from delta c r to delta max, P max sin delta, that is P e minus P m d delta. And this is equal to P max cos delta c r minus cos delta max minus P m delta max minus delta c r, where these are in radius. Equating A 1 and A 2, we will get delta c r is equal to P m by P max into delta max minus delta 0 plus cos delta max. So, this is the relationship that we get for calculating the critical clearing angle.

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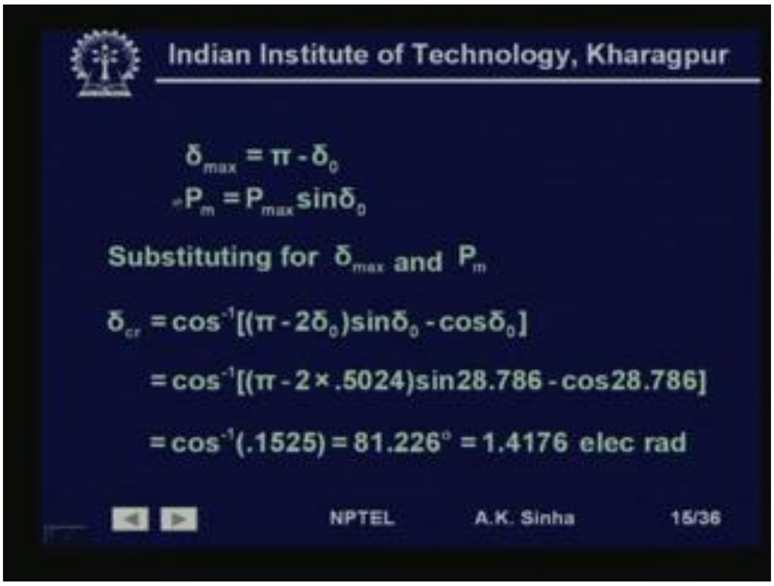

So, now delta max as we have seen is equal to pi minus delta 0 and P m is equal to P max sin delta 0. So, substituting for delta max and P m, we will get delta c r is equal cos inverse pi minus 2 delta 0 sin delta 0 minus cos delta 0. Substituting the value of delta 0 in this relationship, we will get this as cos inverse pi minus 2 into 0.5024 sin 28.786. Here, as we said whenever we are using delta 0, we are writing the value not in degrees, but in radius.

So, 2 into delta 0, in radiance, that is 28.786 in radiance. That is 28.786 into pi by 180. So, this is equal to this sin, this much minus cos, this much. This, when we solve, we get this as 1.4176 electrical radiance, which we can convert into angles, if we want.

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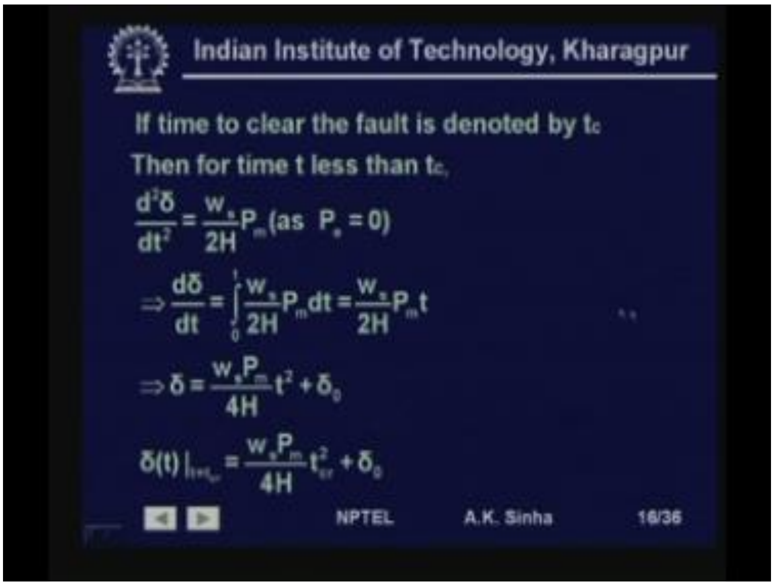

Now, if the time to clear the fault is denoted by t c, then for the time t less than t c, we have this relationship that is less than equal to t c. That is, till the fault is on. We have the relationship d 2 delta by d t 2 is equal to omega s. That is twice by f by 2 H, P m as P e is equal 0. Now, from this we will integrating once, we will get d delta by d t 0 to t omega s is not w s, omega s by 2 H, P m, d t. This is equal to omega s by 2 H, P m plus t plus d delta by d t at t 0, which is the initial operating point.

And since, the change in speed does not take place instantaneously. So, d delta by d t at t is equal 0 plus is 0. Therefore, we have here no term for integrating in constant of integration. That is constant of integration in this case is 0. Again, integrating again, we will get omega s, P m by 4 H, t square plus delta 0, the same relationship that we had shown earlier.

And in this, delta t at t is equal to the clearing time is equal to omega s, P m by 4 H, t square, that t at critical square plus delta 0 at t c r square plus delta 0.

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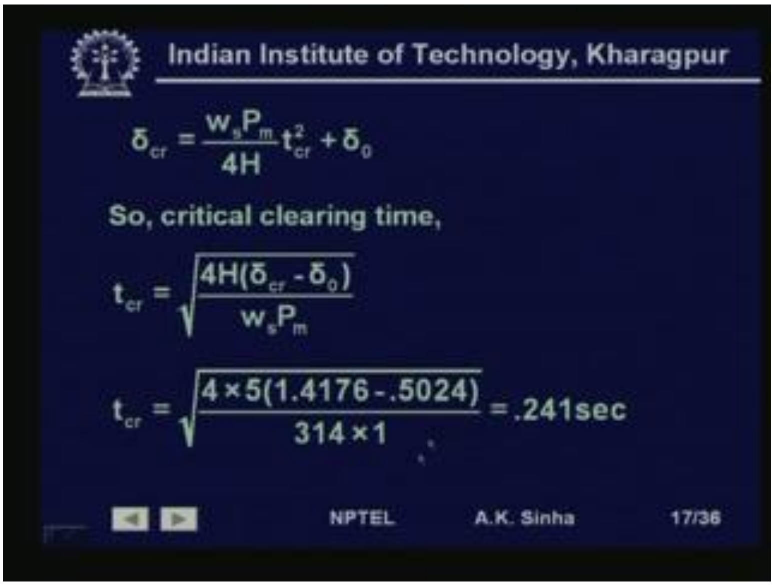

From this we can get delta c r is this much and since, we know the value of delta c r. So, we can calculate t c r in that term. So, this comes out to be 4 H delta c r minus delta 0 by omega s into P m. Substituting the value t c r is equal to 4 in to 5 is H is 5 Mega Joules per MVA, delta c r is 1.4176, as we have calculated earlier, delta 0 is 0.5024, radiance as we have calculated earlier, divide by omega s, which is twice by f.

So, this f is 50. So, 314 into P m, P m value is 1. That is, initially the machine is supplying 1 per unit power. So, we get the critical clearing time as 0.241 seconds. So, this is, how we can calculate the critical clearing angle, as well as critical clearing time for a system. When, the fault occurs, the electrical power output becomes 0, during the faulted conditions.

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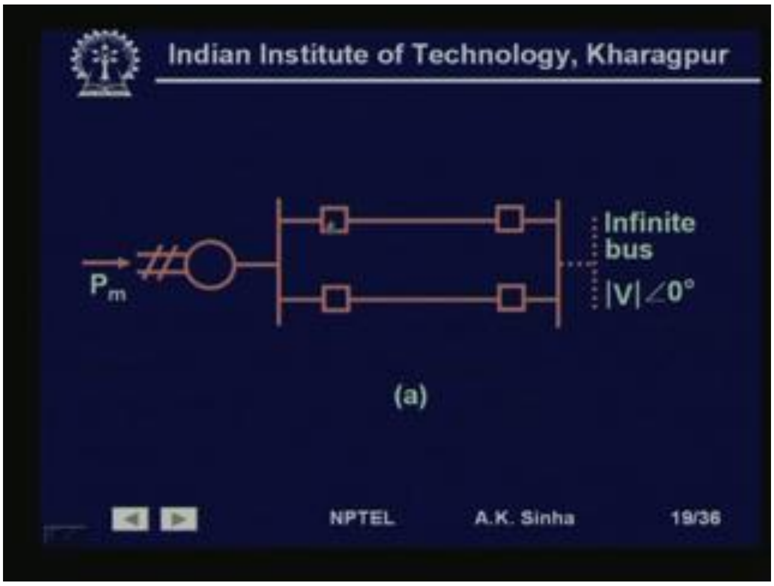

Now, let us take another condition. We have a system, where we have synchronous machine providing power to the infinite bus by means of parallel transmission lines. Now, if we lose, one of the transmission lines what is going to happen? So, let us see this condition.

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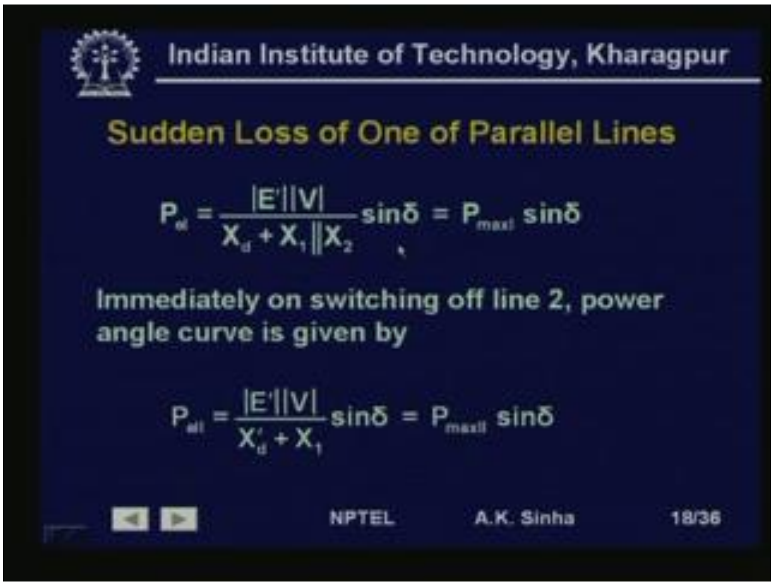

So, sudden loss of one of the parallel lines. Now, the electrical power output P e is equal E dash V by X d plus X 1 and x two in parallel. That is the two transmission lines. The reactance’s we have as X 1 and X 2. So, normally these will be same. So, we have this X

1 and X 2 in parallel. So, E dash V by X v plus X 1 and X 2 in parallel into sin delta, this is we call as P maximum sin delta.

Immediately on switching off the line 2, power angle curve, that is, if we switch off, one of the line, suddenly, then what happens? The reactance now, in this case, when one of the lines goes off, say line 2, goes off, this is X 1 and this is X 2. Now, the line 2, goes off that is this line is open. So, the transfer reactance now will be the X d reactance for the synchronous machine plus X 1 the reactance of the transmission line. So, the transfer reactance from the synchronous machine up to the infinite bus is now X d dash plus X t.

So, this is, what we have seen here. So, in the electrical power output, under when the one line is switched off, we are writing is as P e 2 is equal to E dash V divide by X d dash plus X 1 in to sin delta, which we write as P max 2 sin delta. So, this, when both the lines are working, we have P max 1 sin delta, as providing us the electrical power output characteristics.

Or, the electrical power angle characteristics, for the machine with two lines, both the lines on. And this is providing us, the electrical power angle characteristics, when one of the lines is switched off. That is when line two is switched off.

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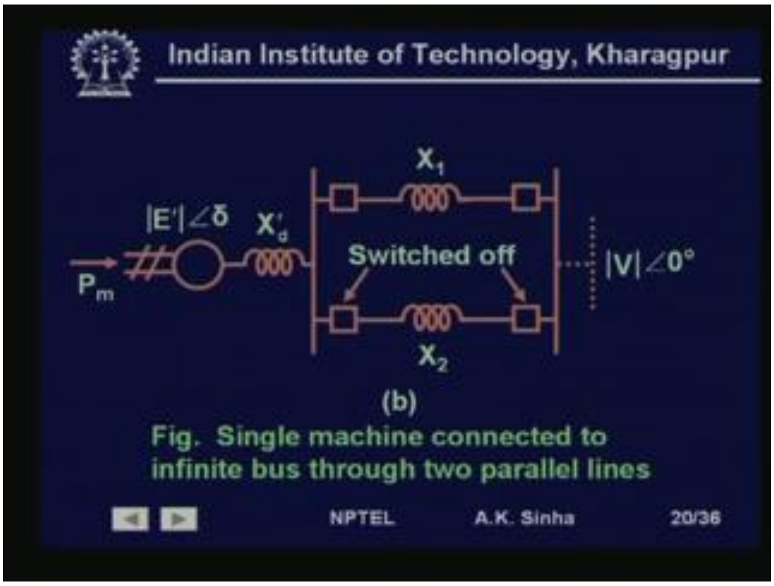

So, if we see this condition; that is, this line is switched off. Then, now what we are finding from here is, we have two different power angle characteristics. For two different operating conditions, one when both lines are on and another when one line is off.

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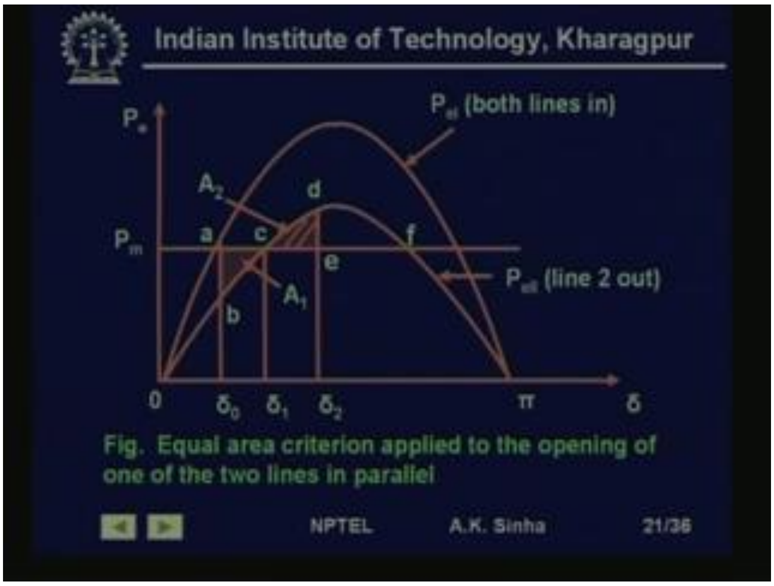

So, for a case, where we have a synchronous machine, supplying power to infinite bus bar, through two parallel transmission lines, we have the electrical power characteristics.

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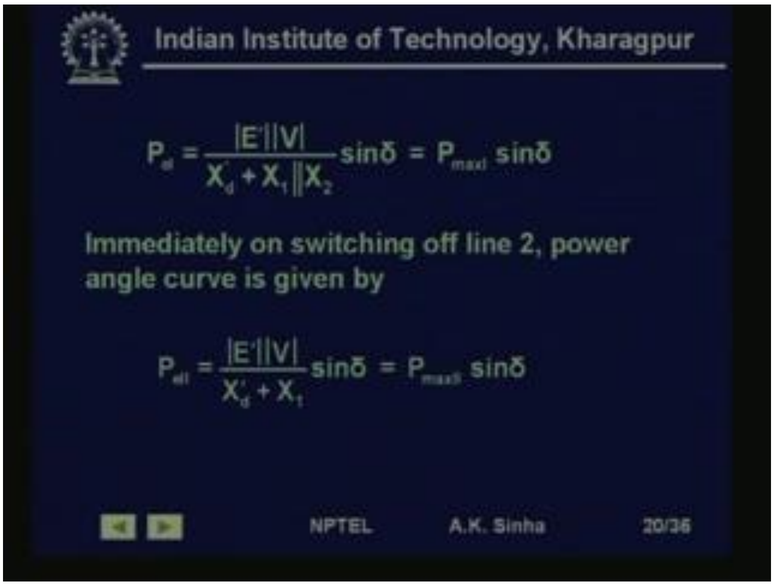

Under, when both the lines are operating given by this relationship P 1 is equal to P max 1 sin delta. Where, P max 1 is equal to E dash V divided by X d dash plus X 1 and X 2 in parallel, as we can see from this figure.

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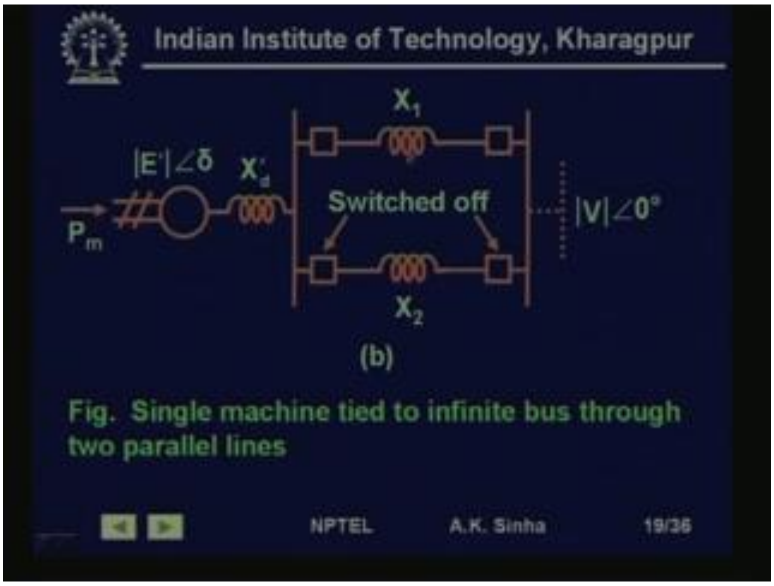

Where, we have X 1 and X 2 the two transmission line reactances. V is the infinite bus bar voltage and it is angle is taken as reference. X d dash is the transient direct axis, transient reactance of the synchronous machine. E dash is the voltage, behind the transient reactance for the synchronous machine. And delta s is the power angle, that is the angle by which, the voltage behind transient reactance leads the infinite bus voltage. And P m is the mechanical power input to the turbine generator system.

When, one of the line is suddenly switched off, which may occur, because of, we want to take this line out for maintenance or may be a mal operation one of the circuit breakers due to some reason. Then, in that case, the transfer reactance will change from X d dash and X 1, X 2 in parallel to X d dash and X 1 in series, because this is now switched off. So, in this case, the electrical power output characteristics will be given by P e 2, which will be P max 2 sin delta, where P max 2 is given by E dash V by X d dash and X 1 in series.

So, these two characteristics are on P delta plane can be shown here, where these characteristics is for the both lines on and the other characteristics P e 2 is with line 2, all out from service, only line one in service. Now, P m is the mechanical input to the system under steady state. The electrical output P e will be equal to P m, because we are considering the system to be a loss less system.

So, the operating point will be given by this point a and the angle delta will be equal to delta 0. This will be the operating point, where P m and the characteristics of the electrical power output is the same. Now, when the line two is suddenly switched off, then the transfer reactance will immediately change to X d dash plus X 1. That is, much larger value and the electrical power output characteristics will shift from this characteristic to the other characteristics.

Since, delta angle cannot change suddenly. So, the point will shift from a to this point b, where delta angle will be equal to delta 0. Now, we have a b amount of acceleration, accelerating power available, which will make the rotor turbine generator rotor system to accelerate. Since, we are measuring this angle delta with respect to a synchronously rotating reference frame. So, because of the acceleration of the rotor, delta angle will increase.

When, it reaches this point c, then we have electrical power output increase and up and it will become equal to P m. But, since the rotor has accelerated, all through this period, the speed of the rotor will be higher than the synchronous speed. So, still rotor angle will keep on increasing, but once it crosses point c, on this characteristic, the electrical power output now is more than the mechanical input. So, rotor will start decelerating and so speed will start decreasing from super synchronous speed.

And till say, it reaches this point t, where it becomes just equal to the synchronous speed. Now, this point d can be found out by equating these areas A 1 and A 2, where A 1 is a, b, c and A 2 is c, d, e. A 1 is the area, during which the rotor is accelerating, whereas c, d, e is the area during which rotor is decelerating. This is what we get from the equal area criterion.

So, once it reaches point d, what happens is we have so much of decelerating power available and the speed is synchronous speed. So, rotor will start, it is speed will start going, below synchronous speed as so delta angle will start decreasing. Again, it will come to c, where there will be neither acceleration nor deceleration. But, since the speed will be lower than the synchronous speed at this point delta will keep on decreasing.

Till it reaches point b, where it will again at a synchronous speed, because during this period, it will start accelerating and so, we will see, that the rotor will oscillate between these two points b and d, on these characteristics. However, since the system will have some damping available. These oscillations will slowly decrease and we will get finally, damped to. And will come steady state operation at this point c, which will be the new operating point and the angle, will be given by delta 1.

The maximum angle, which this rotor will attain, will be given by this point d, which is delta 2 and this we obtain, when A 1 is equal to A 2.

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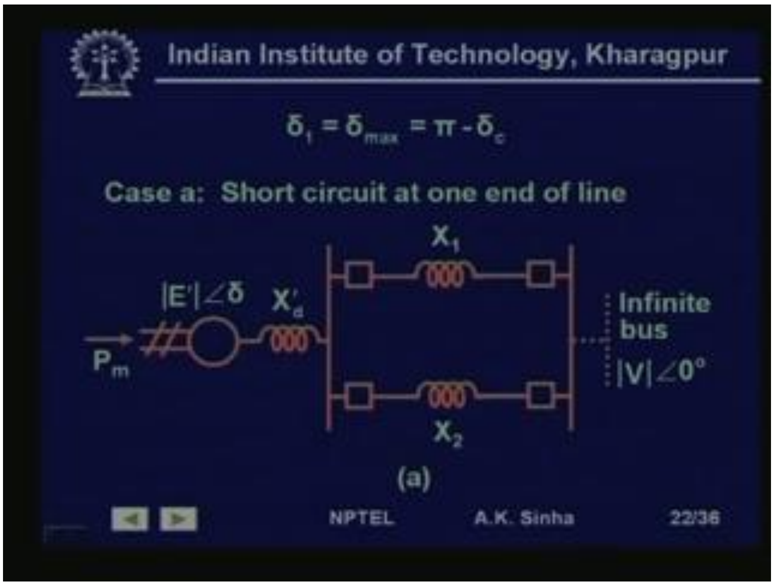

So, now, again if we want, that we want to find out the critical clearing angle for this case, then we can do that, because in this case, again critical clearing angle will come. When, we have the maximum angle, which is when the operation comes to this point d and this point, if you see, will be having an angle, which will be equal to pi minus delta 1. So, delta max in this case is pi minus delta c or delta 1, whatever we write. Here, we are writing in this as delta 1.

So, what we will say is, this is point delta 1, which is the operating point and the maximum, which we can reach will be this point. That is this point, which will be equal to pi minus delta 1. That is the point where P m cuts these characteristics, second characteristics, that is characteristics, when line 2 is out.

Now, let us take, another case, when a short circuit at one end of the line takes place. So, instead of now removing one of the lines, we have now a short circuit taking place at one end of the line, either this end or this end of the line. In either case, what happens is the fault is on either this bus. If the fault occurs here, the fault is same as it is on this bus; if the fault occurs here it is same as on this bus.

In both these cases, the voltage at this bus or this bus becomes 0. And therefore, there will be no power transfer taking place between the synchronous machine and the infinite bus under this condition. So, during fault condition, that is, when a three phase fault occurs at either end of the line, during the fault period. There will be no power transfer taking place between the synchronous machine and the infinite bus.

That is, we have taken the fault on this end of the line, this end of the line. So, what we have is, this end of the line is shorted, which is same as this end of the line is also shorted, it is grounded.

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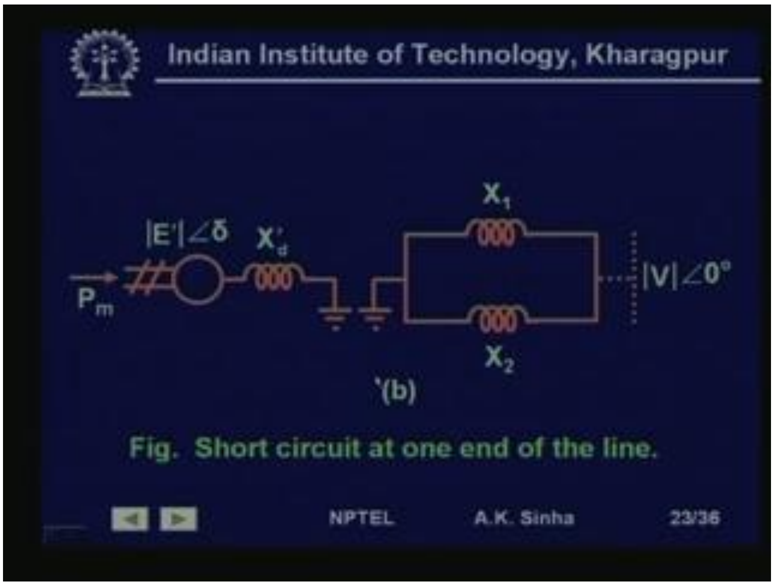

So, we have the power, no power transfer feasible, under this condition. Because, a three phase fault has occurred. So, this bus voltage has come down to 0.

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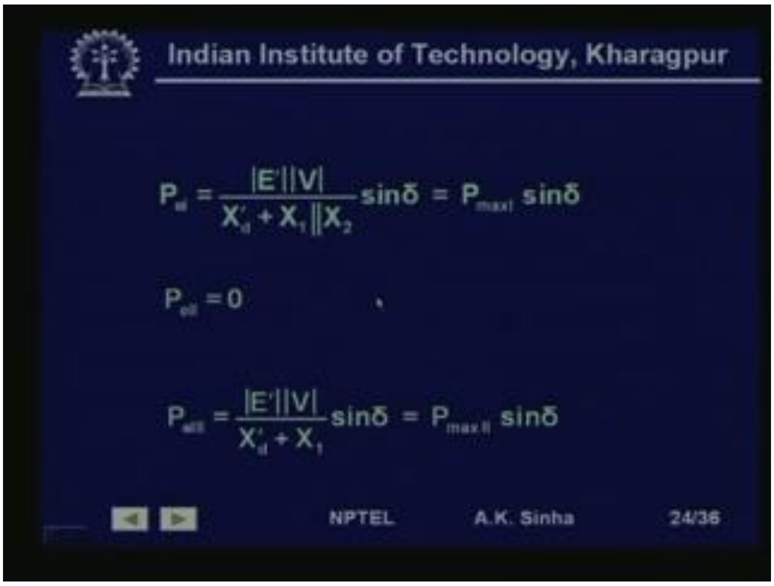

So, in this case, the power angle characteristics, before the fault is E dash V by X d dash with X 1 and X 2 in parallel into sin delta, that is P max 1 sin delta. And in the second case, when the fault is there, the power transfer electrical power output is 0. So, P e 2 is equal to 0. Now, if the fault is cleared by opening both the circuit breakers. So, the fault, once the fault on the line is cleared by opening both these circuit breakers. The system will recover, but now the transfer reactance will be X d dash plus X 1, only in this case.

So, we have the post fault condition, when the line, one of the lines is tripped. The faulted line is tripped, then we have P e 3 is equal to E dash V by X d dash plus X 1 sin delta. Because, now the transfer reactance is X d dash plus X 1, only one of the line is now tripped. So, this is equal to P max 3 by sin P max 3 sin delta in this case, this is written as P max 2, which it should be P max 3.

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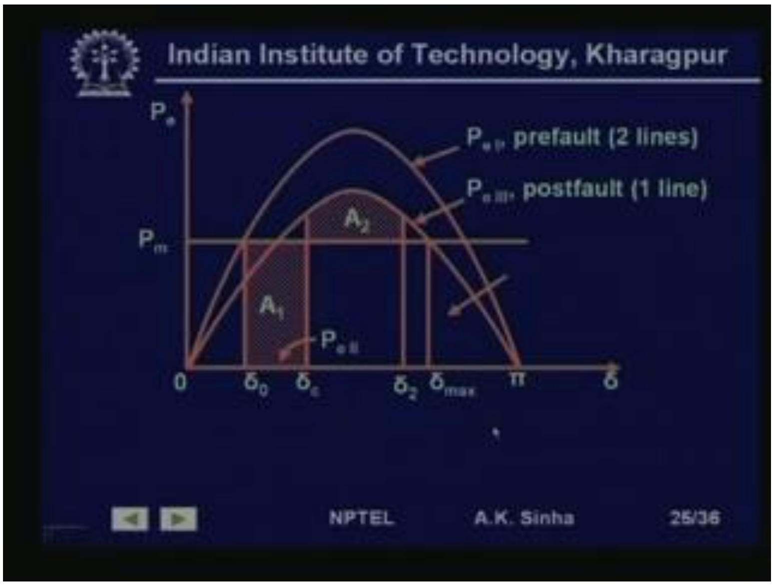

So, what we have is, system is initially operating at this point, where P m is equal to P e. The fault occurs, the electrical power output, goes down to 0. There is this much amount of accelerating power available and therefore, delta angle keeps on increasing. So, during fault, the delta angle profile will be like this. So, delta angle will keep moving along this line.

Now, suppose at this point, when delta angle has reached delta c, we clear the fault, what will happen, we have cleared the fault by opening line 2. So, what is going to happen? This electrical power output will jump now to this point and from this point, it will now be moving on these characteristics, which are the characteristics for the post fault condition. That is, when one of the line is now tripped.

So, this will keep moving on this and we will have the accelerating area given by this area A 1, whereas decelerating area will be given by the area, under this curve P e and this line P m. And as we have seen the maximum value of decelerating area can be obtained up to this point delta max, where delta max we will get equal to pi minus delta 1. That is, where this angle, we call as delta 1.

So, pi minus this angle. So, this is what, we will get as the maximum angle. And if we have A 1 is equal to A 2 at some, angle which is less than delta max, then system is going to be stable. If it is beyond delta max, then the system becomes unstable.

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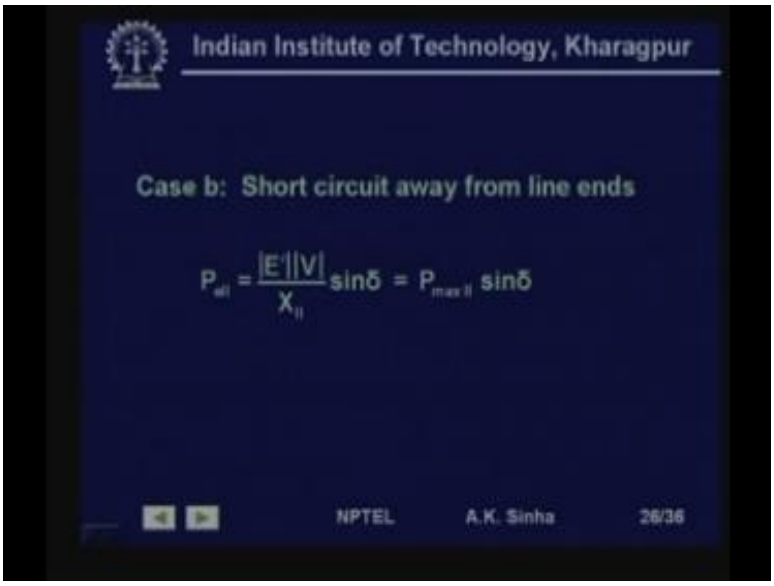

Now, short circuit, away from the line ends. That is, now we take a case, whether short circuit occurs, somewhere in between the two ends, that is in the middle of a line.

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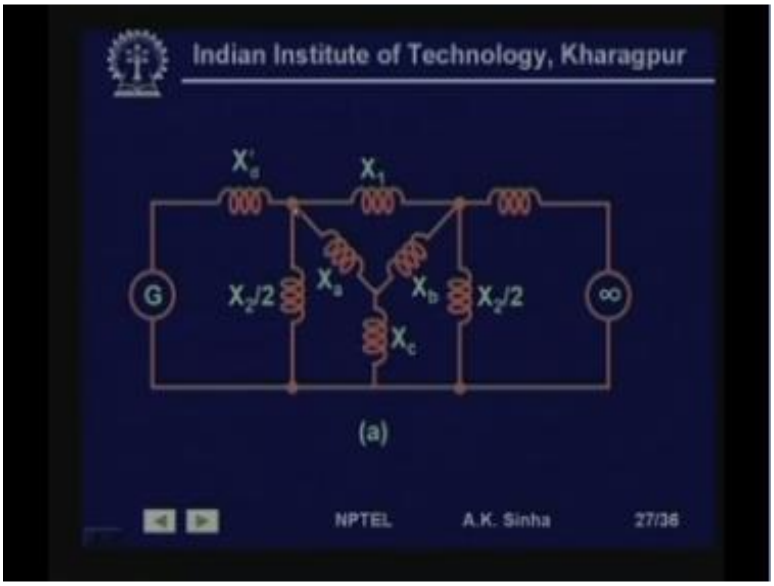

So, in that case what happens is, we have now, short circuit occurring in the middle of the line.

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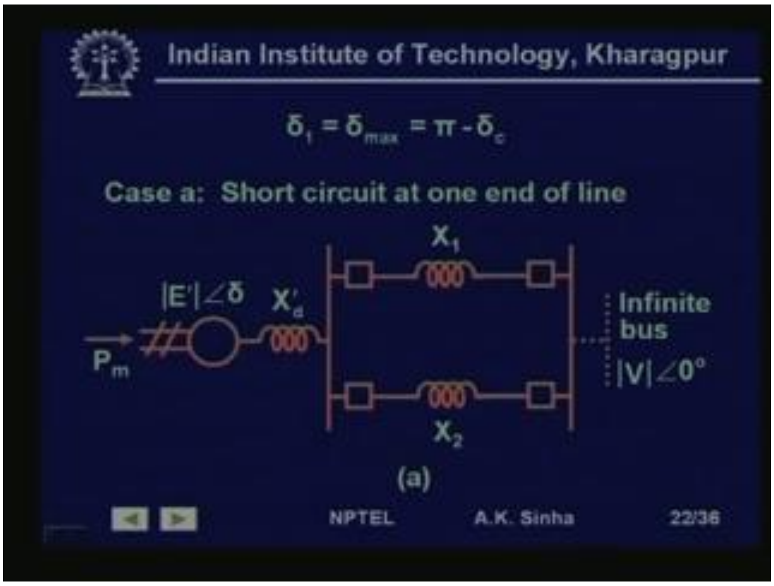

So, if we look at this diagram. Short circuit is occurring in the middle of the line. So, we have two parts of reactance's can be divided into two parts up to the fault point on this side and up to the fault point up to on this side. So, we can now have X 1, this is line 1 and for line 2. We have two parts X a and X b, we have taken the fault on the middle of the line. So, we have the reactance X 2 by 2 and X 2 by 2, on these two sides.

Now, that is, this is the reference or the ground potential. So, what we have is, when this fault three phase fault occurs on this point any point P here, which is in the middle of the line. Then, this part of the reactance is connected to the ground, as well as this part of the reactance is connected to the ground, because this point is now having 0 potential. So, we will see that, this is half of the reactance of this line to the X 2 by 2 and X 2 here.

So, what we have is X 2 by 2 X 1 and X 2 by 2 here falling at delta circuit and we can use star delta conversion. To convert it in to this part of the circuit, that is the star conversion for this delta is X a, X b and X c we can calculate these values of X a, X b and X c. Then, we have, we can further add this X a to X d dash and X b on this side for the reactance of the transformer, if we have.

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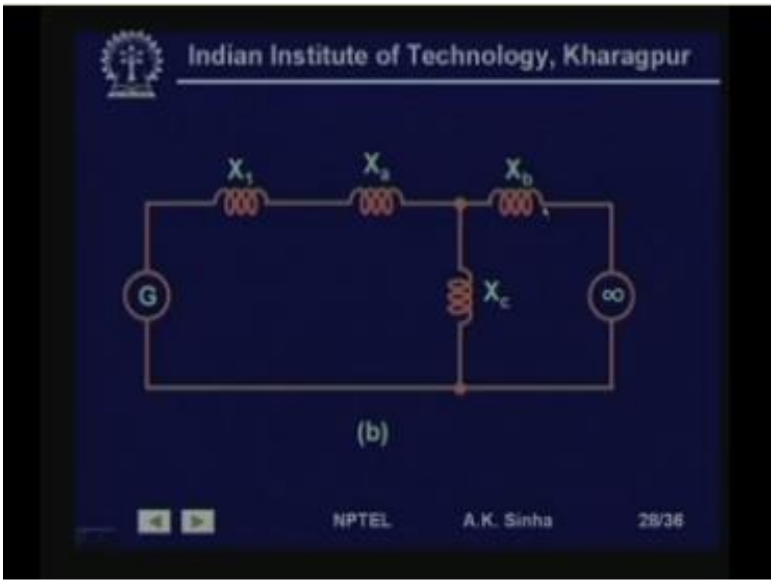

And we will have a star circuit, which we can further reduce to a delta circuit. So, we have the star circuit here X 1, X a, X b and any reactance on this side. That has been combined with that an X c.

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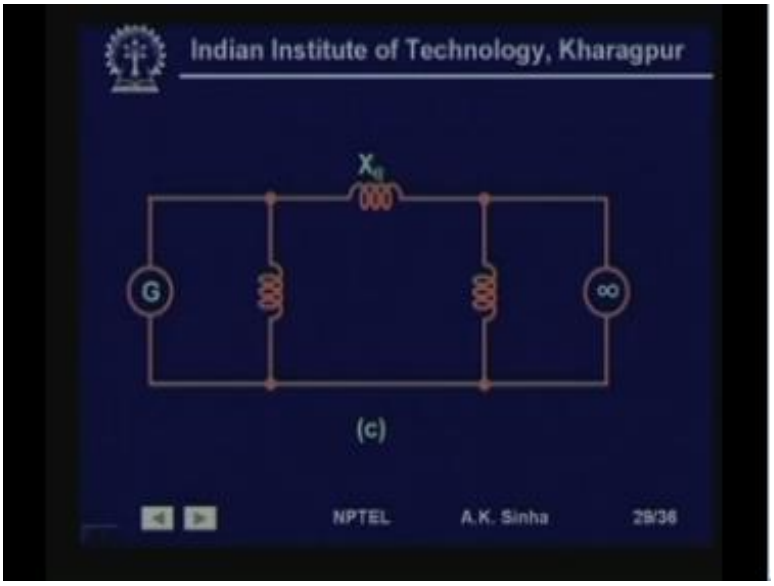

And this can again this star, can again be converted into a delta circuit or a pi circuit, where we will have, this reactance now, which is the reactance transfer reactance between two machine. Because, these two reactances will not be carrying any real power this voltage is same as the infinite bus voltage. This voltage is same as the generator internal voltage and this is the reactance between these two voltage points. So, this is the transfer reactance that we get.

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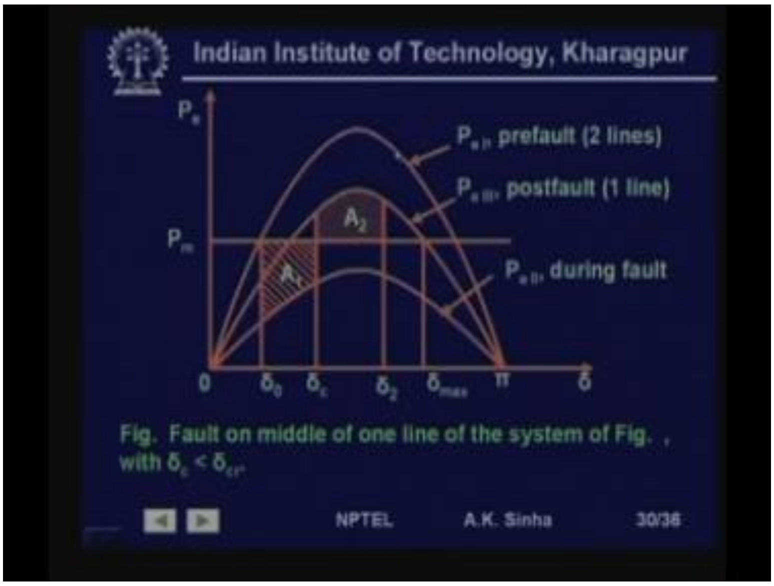

So, we need to calculate this X 2, in this case. So, what we find is, that we have now three curves that we get, first is the pre fault characteristics. Second is, during fault we have now, this transfer reactance X 2 coming into picture. So, we will have the P max in this case as E v by X 2 coming in to picture. And with this, we will have this characteristics coming here, which is the during fault characteristics. And the third one, once line is tripped by opening the circuit breaker will be this line.

Now, if the fault occurs, what happens is the operating point from this point drops down to this point. That is during fault, this is the characteristics. So, electrical output drops down to this and delta angle will keeps on increasing the electrical power output moves along this line. Now, at delta c, if we clear the fault, it will jump to this point, where one of the lines is cleared and it will move along with this. And then, we will talk about the numerical integration method for solving the swing equation.

Thank you.