Power System Analysis
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Lecture - 36
Power System Stability – IV

Welcome to lesson 36, on Power System Analysis. In this lesson, we will continue our discussion on Power System Stability. In the last few lessons, we talked about rotor angle stability. In which we discussed, how we can analyze the stability of a system for small disturbances. That is we use small signal analysis, where we use the linearized model of the system.

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In today's lesson, we will talk about large signal stability. That is, we are going to talk about larges disturbances, when they take place; how the system responds to that. So, we will start with large signal stability, what we understand here. We will then go into the equal area criterion method for analyzing the stability of a single machine connected to infinite bus system, for large disturbance. And we will take some example to clarify this idea, how we use this equal area criterion.

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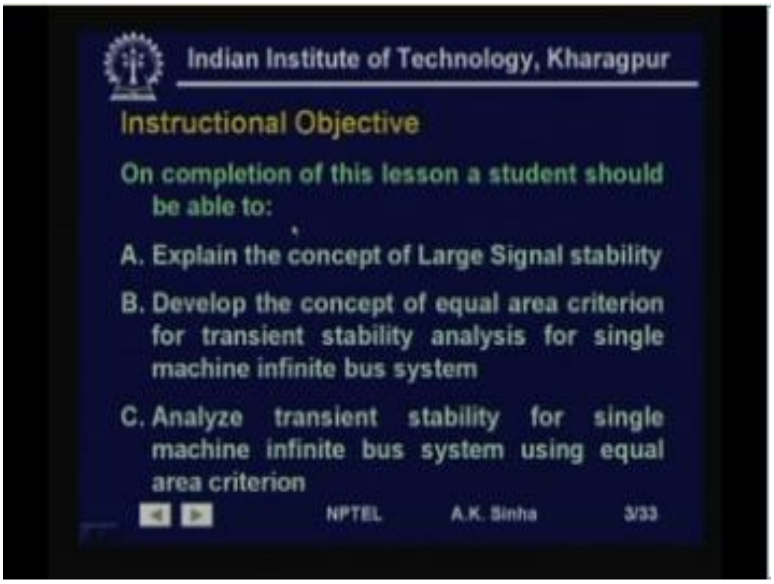

Well, once we complete this lesson, you should be able to explain the concept of large signal stability. After that, you should be able to develop the concept of equal area criterion for transient stability. In fact, the large signal stability that we talk about is popularly known in power system literature as transient stability. So, we will interchangeably call them, as large signal stability or transient stability.

So, develop the concept of equal area criteria for transient stability analysis, for single machine, infinite bus system. And you will be able to analyze transient stability for single machine infinite bus system using equal area criterion. That is, we will work out some problem on this and since, you would known how to solve this, you should be able to analyze for any other system as well.

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So, first, we will get into what we really mean by these large signals or large disturbances, which takes place in the power system. And how, power system responds to these large disturbances. As we said, this large signal rotor angle stability is popularly termed as transient stability. The large disturbances are mainly disturbances, such as faults or short circuit is in transmission lines or equipments outages of generators or heavily loaded lines also constitute large disturbances. Because, there is sudden large changing power flow in the system.

Now, what happens, when these take place is sudden large changes in power outputs of the generators takes place. Because, if a transmission line is tripped or a short circuit occurs, the generator outputs change considerably. And this change in generator electrical output causes an imbalance between the mechanical inputs to the turbine generator system. And the electrical output of the generator, causing acceleration or deceleration of the generator rotor.

These create, since this disturbance is large. This acceleration or deceleration caused by the large difference in the mechanical input and electrical output of the turbine generator system will cause large deviation in the rotor angles. And this large deviation of rotor angle and rotor oscillations about the operating point, takes place in such situation. And since, this excursion of rotor angle delta is large. We can no longer use the linearized analysis that we did for small signal case.

That is, when we talked about small disturbances. There, we had linearized the system equations about the operating point and carried out a linear system analysis to study the dynamics of the system. Here, this is not possible, because we have large deviations, which take place in the system. When large disturbances occur, which means, that the linearization of the electrical forward characteristics. That is P e is equal to P m, P max sin delta is no longer feasible.

And we have to work with the non-linear swing equation. That is, linear resolution swing equation in this case will not be feasible. Because, the delta angle changes are very large and we cannot linearize the equation anymore. So, linearized dynamic model cannot be used in this case. That means, we have to use the non-linear swing equation and this non-linear swing equation has to be solved for each machine.

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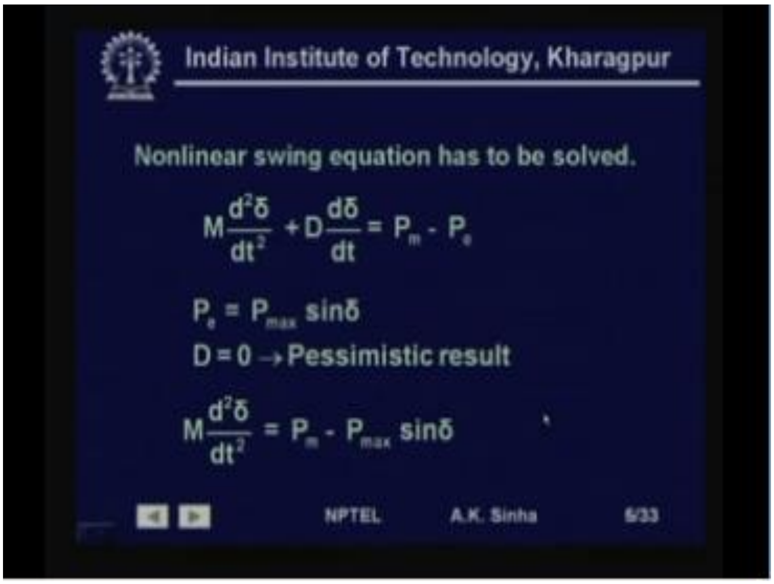

As we have already seen, the swing equation we write as M d 2, delta by d t 2 plus D, d delta by d t is equal to P m minus p e. Where P m is the mechanical power input to the turbine generator system. And P e is the electrical power output from the synchronous generator. M as we know is the angular momentum. All these quantities, as we have seen earlier we always write in per unit system. D is the damping coefficient. Now, we know that P e is equal to P max, sin delta. This is what introduces the nonlinearity in this swing equation. So, p is P e is equal to P max sin delta.

Now, most of the time, what we do is, when we are doing this analysis. Since, it is going to be a very complex analysis; we try to make certain assumptions to simplify the problem at hand. That is, what we do is, we assume this D, that is the damping coefficient to be very small and we neglect this. That is, we make D is equal to 0. Of course, if there is no damping in the system, what we are going to get as an output is a sustained oscillation or a runaway situation, as we will see later.

Now, if it is a sustained oscillation, which means, that in actual system. Because, of the presence of damping, the oscillations will slowly die out and the system will come back to a new steady state or a new synchronous operating point. So, assuming this D to be 0 we will get M d 2, delta by d t 2 is equal to P m minus P max sin delta P e. We are writing as P max sin delta.

So, D is equal to 0, when we write, we are going to get sustained oscillation, which will say that the system is going to have oscillations all the time. Of course, that will not be the case, because in actual system there will be D. So, taking D is equal to 0, basically leads to a pessimistic result.

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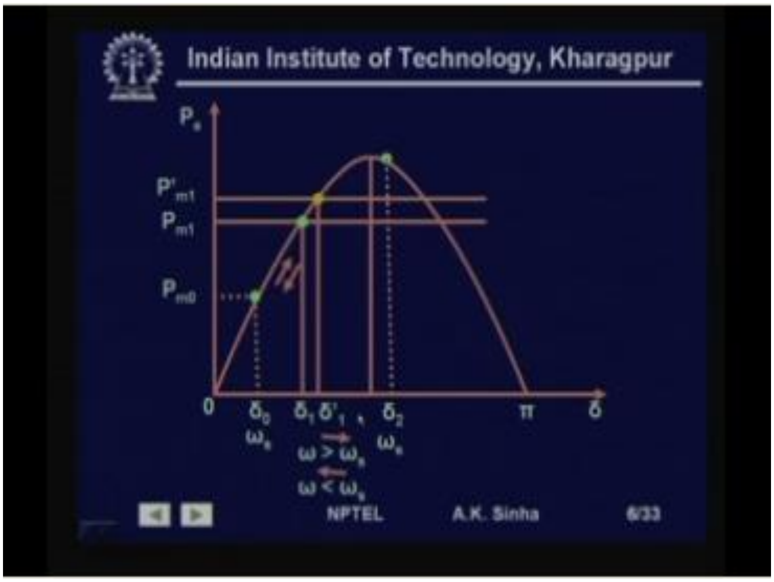

Now, let us see, how this whole dynamic process takes place. Let us say initially, we have the system, which is operating with the mechanical input P m 0 and electrical output as P e, which is equal to P m 0. And if P is equal to P m 0, then the power angle of the generator will be equal to delta 0. So, this is the initial operating point, the steady state point, which is there.

And system is working at synchronous speed, at this point. That is, mechanical input P m 0 and the electrical output P e, which is equal to P m 0 and the power angle of the machine as delta 0. This P of the system is synchronous speed, because P m and P e are equal. So, it is a stable operating point, where the system is working or this is the steady state operating point, where system is working.

Now, suppose, we suddenly increase the mechanical input to the turbine generator system from P m 0 to P m 1, now what is going to happen? Now, we have P m 1 minus P m 0 amount of mechanical input, which is, now excess? Because, the electrical output at this instant T is equal to 0 plus is still equal to P e, which is P m equal to P m 0. And suddenly, we have changed the mechanical input to this value.

So, this is the amount of accelerating power, which will be available with us, because this is the excess mechanical power available to the system. Because, of this excess mechanical input to the system. There is going to be acceleration, because now we have excess input and less output. Because of this what is going to happen is the rotor of the turbine generator system is going to accelerate.

If it accelerates, what happens is, it will start moving on this line, where we have the P delta characteristics. That is electrical output characteristics of the machine. So, as it accelerates delta angle increases, what is going to happen is the electrical output will also increase. That is P e will increase. Now, this will keep on happening. That is delta will keep on increasing, because still we have more mechanical input to the system and less electrical output. Now, this will keep on going, till we reach this point.

Now, when we reach this point, then the power angle or the rotor angle of the machine is equal to delta 1. And the electrical output of the machine is going to be equal to P m 1, at this point. So, at delta 1, the electrical output will be same as this mechanical input to the system. So, now, there is no accelerating power available. That is both electrical and mechanical power are equal to each other and so we would expect the system to start operating in this point.

However, this does not happen, because while moving from this point up to this point, the system has gained some speed. That is, at this point when the angle has come from delta 0 to delta 1. The speed of the system will not be synchronous speed, but will be a little higher than the synchronous speed. Now, because the speed is a little higher than the synchronous speed. In this case, what is going to happen is that delta angle will keep on increasing.

So, though the mechanical power input and electrical outputs at this point are equal. Since, the speed of the rotor is somewhat higher than the synchronous speed the delta angle will keep on increasing, because we are measuring delta with respect to the synchronously rotating reference frame. Since, the rotor is rotating at a speed higher than the synchronously rotating reference frame. So, delta will keep on increasing.

Now, this as soon as this happens, what happens the electrical output will now be more than the mechanical input P m 1. So, as soon as we go beyond this point. Now, the electrical output is more than the mechanical input. So, what is going to happen to the rotating system output is more, input is less. So, it is going to cause a deceleration of the rotor.

And so, what is going to happen is as we keep moving on this more and more deceleration will take place. And because of which, the speed will be now returning back to the synchronous speed. So, from a super synchronous speed, that is the speed higher than the synchronous speed, when as we move on this, the speed will start reducing down. Though, as we move on this, P e will also keep on increasing and because of this, there is going to be deceleration or decelerating power will be available, which will try to reduce the speed of the system.

And so, we are going to have lesser and lesser speed coming into picture, but all these time still the speed will be super synchronous. Now, suppose at this point, when the rotor has reached this point, then the speed becomes equal to the synchronous speed. Now, what is going to happen? That is all through this period the speed was gaining, because we had accelerating power available as soon as we cross this, the speed was more than synchronous speed.

But, now we have decelerating power. So, speed will start reducing and when, we reach this point. Now, the speed has come back to the synchronous speed. So, what is going to happen now? At this point the speed is equal to the synchronous speed, but the electrical output is much more than the mechanical input, which is P m 1. So, we have decelerating power available. That is more output and less input.

So, what is going to happen is, now the speed will further try to decrease. Because, the speed will try to decrease, which means delta angle will start reducing. Because, we are measuring this delta with reference to the synchronously rotating reference frame. Now, the speed is becoming less than the synchronous speed. So, delta with respect to the synchronously rotating reference frame is going to keep on reducing.

So, we will get delta reduced, as once we have reached the synchronous speed at this point the rotator will start decelerating and delta will keep on reducing. Again, when we come to this point, what we will have or from this point up to this point, all through we will still be having decelerating power. Because, electrical output is more than the mechanical input, so speed will keep on reducing.

So, speed from this point up to this point, as delta is reducing is going to be less than the synchronous speed. That is, it will be in the sub synchronous speed. When it reaches this point, again electrical output is same as the mechanical input, but what is the speed now? The speed is, now sub synchronous speed. That is less than the synchronous speed. So, even though mechanical input and electrical output are equal, since the speed of the rotating system is less than the synchronous speed and we are measuring delta with respect to synchronously rotating reference frame. So, delta will keep on reducing.

So, it will keep on reducing further, as it goes below this point, what happens electrical output now, becomes less than the mechanical input. So, we will get accelerating power available. That is from sub synchronous speed at this point, we will start gaining more and more speed. And again, we will find that, the speed will keep on increasing and may be somewhere around this point, we will again the speed will reach the synchronous speed.

And at this point, when it reaches the synchronous speed, what is the position? The position is the electrical output is much less than the mechanical input. So, it will again start accelerating. So, acceleration will take place. So, what is going to happen at this point, once we reach that is speed is synchronous speed. But, now, again since acceleration is taking place, the speed is going to increase to more than the synchronous speed. So, again, delta will increase and so we will find delta will keep on increasing.

Once, it reaches this point, again what we will find as that the speed of the rotor has increased beyond the synchronous speed. Though, P m and P e will be equal, still the speed beings more than synchronous speed, delta will keep on increasing and it will again come to this point. And so what we are going to get is a sustained oscillation between these two points.

In actual system, we know, since there is going to be a damping. So, these oscillations will be damped out and after sometime, the system will start operating at this point. So, this is, what we see is the kind of dynamics, which is going to take place. When, we have certain changes in the mechanical input or electrical output from a turbine generated system, which is connected to an infinite bus bar.

So, this is, what is going to happen? That is, we will get a sustained oscillation between these two points. If there is no damping, once there is damping these oscillations will slowly die out. And we will have the stable operating point, at or a steady operating point at this place, where the speed will be equal to synchronous speed. And the new power angle will be delta y and the mechanical input is P m 1 and electrical output is P e, which is equal to P m y. So, this is the scenario that we have.

Now, suppose, instead of raising, this mechanical input to P m 1, we raise it to P m 1 dash or P dash, m 1, if we do that, then what happens. Now, suppose we have done this, then now we have much more accelerating power available. So, the acceleration is going to be much faster and we will be going from this point moving up, as we keep moving up. The speed will be picking up, delta angle is increasing, the speed is increasing the accelerating power is reducing all the time.

When, we reach this point, what happens is, we have electrical output, which is equal to the mechanical input. But, as we have seen the speed at this point will be super synchronous. That is a little more than the synchronous speed and so delta angle will keep on increasing. Now, this delta angle will keep on increasing, may be if it crosses this point, then what happens? That is as soon as these go beyond this point, what we are having is electrical output more than the mechanical input.

So, there is deceleration and speed will try to keep coming down. But, still it since it is super synchronous speed, the delta angle will keep on increasing. Now, suppose, we have not reached the synchronous speed, till this point, that is we have crossed this point, then what happens? Now, we have mechanical input, which is equal to this much electrical output, which is less than this.

Then, since the speed is still not synchronous speed, it is still more than synchronous speed. And now, we have accelerating power available. That is mechanical input is more than the electrical output, once we cross this point. Then, what is happening is the speed is higher than the synchronous speed, delta has is increasing and we have accelerating power available. So, delta will increase, even at a faster rate, because now, there is acceleration, which is taking place for the rotating mass of the system.

So, it is going to keep following this curve. That this delta will keep on increasing as it keeps on increasing the electrical output keeps on reducing. That means, more and more accelerating power is available, which means rotor will keep picking more and more speed and this will be a runaway situation. That is the generator rotor will rotate at much faster speed and it will lose synchronism with the system.

So, this is, what can happen? So, here, what we see is, for a particular amount of change in the mechanical input the system is stable. Whereas, if for a definite amount of mechanical change, the system may become unstable and that is why, we say that, the stability of the system depends, not only on the operating point. But, also on the magnitude of the disturbance that takes place. The same situation we could have seen. if we would have started at this point.

Then, an equal amount of change in mechanical input would have made this system unstable. Because, we would have got accelerating power available all the time and this would have crossed this point very easily. So, again, the stability or the rotor angle stability of the system for large disturbances depends on both the size of the disturbance as well as the operating point, where we are operating.

And this is one of the reasons, why when we worked this system, we always work at a rotor angle, which is much less than the maximum power. That can be transmitted through the system. That is, we work at an operating point, which is much below the steady state stability limit of the system. Otherwise, a certain disturbance may make the system transiently unstable. That is, it may be unstable for transient condition. That sudden changes, may bring make the system unstable.

So, this is the physical idea of what happens, when there is a sudden disturbance, which takes place, which brings out a difference between the mechanical inputs. And the electrical output power of a synchronous machine.

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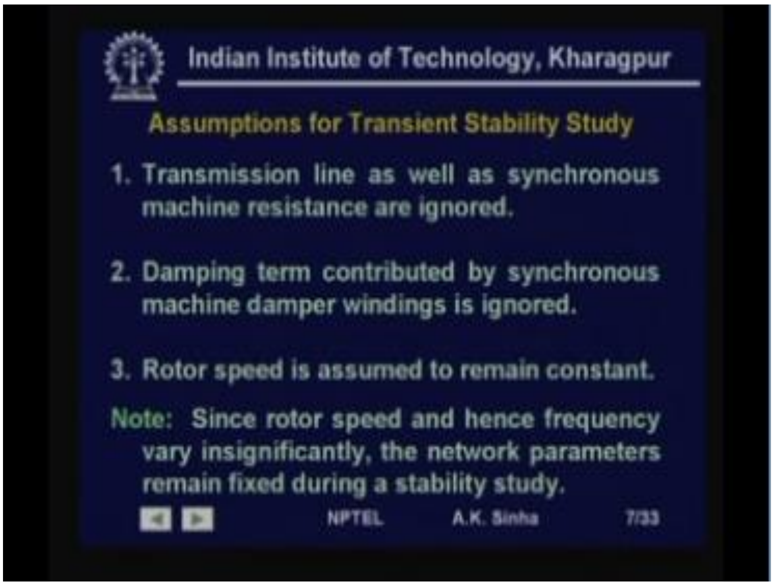

Now, as we have seen, from these characteristics that we have to work with a non-linear system analysis, which is a complex analysis? And therefore, we try to make certain assumptions for the transient stability study of the system. Now, some of these assumptions are valid or they are very much appropriate, whereas some are not so appropriate. But, we make them to make our analysis much easier and also, to gain an insight into the system behavior.

So, the first assumption that we make is transmission line, as well as synchronous machine resistances are ignored. This is very much valid in one sense that, the resistance of transmission line is much smaller compared to the reactance of the line same is the case with the synchronous machine windings. That is the resistance is much smaller compared to the transient reactance of the machine. So, they are generally ignored.

The other aspect of this is, that this is going to lead to a conservative result in the sense that if the resistance is included. We know that, they are going to have damping in the system. And therefore, this is going to reduce the oscillations as the time crosses or the oscillation will die out, because of the damping provided by the resistance. After all resistance, what they do? There is a power loss, which takes place, because of the resistance, high square or losses and this is acting as a damper on the oscillation.

Damping term contributed by synchronous machine damper winding is ignored. Same thing, as the previous one this damping term is generally much smaller. And also this, if we ignore this, this leads to a pessimistic or conservative result, where we are going to get sustained oscillation, instead of a damp down oscillations. So, if we get sustained oscillations, we can always say that the system, actual system is going to be stable.

Because, there is going to be damping, because of the damper winding as well as damping, because of the resistance in the transmission system as well as synchronous machine windings. So, there are power losses, which will take place, because of the resistance in the system.

Third assumption we make is, rotor speed is assumed to remain constant. Well, the rotor speed actually does not remain constant. But, the change in rotor speed from the synchronous speed is very, very small. That is, when we are talking about the change in speed, it will be say from 3000 r p m to may be 3005 r p m or so. That is, we are saying that the change in speed is negligible or very small and it does not cause much error, if we assume that the speed remains constant.

Though, in actual practice, we know the speed will vary it will be a little more or little less depending on the acceleration or deceleration of the rotor. But, for the time period, over which we study the stability of the system for large disturbances, this change in speed is very, very small and can be ignored. Normally, the time period of study for this transient stability study is between 3 to 10 seconds. So, within this period the change is not significant and we ignore this change in speed. So, we say rotor speed is remaining constant.

Another advantage of keeping this or making this assumption is, since the rotor speed remains constant the frequency is constant. And therefore, the network parameters do not change. Since, we know that reactance is proportion to the frequency of the system, it is X is j omega l and y, which is the capacitance is equal to j omega c. So, the admittance part is j omega c and the reactance part is j omega l. All these are dependent on frequency, because omega is twice by f. Since, the speed is remaining constant, F is constant. So, the parameters will also remain constant.

If we assume the variation in speed, then we will have these parameters also which will be varying. Though, the variation, because of the speed change, which is negligible this also negligible. So, we say that, the network parameters remain fixed during the stability study.

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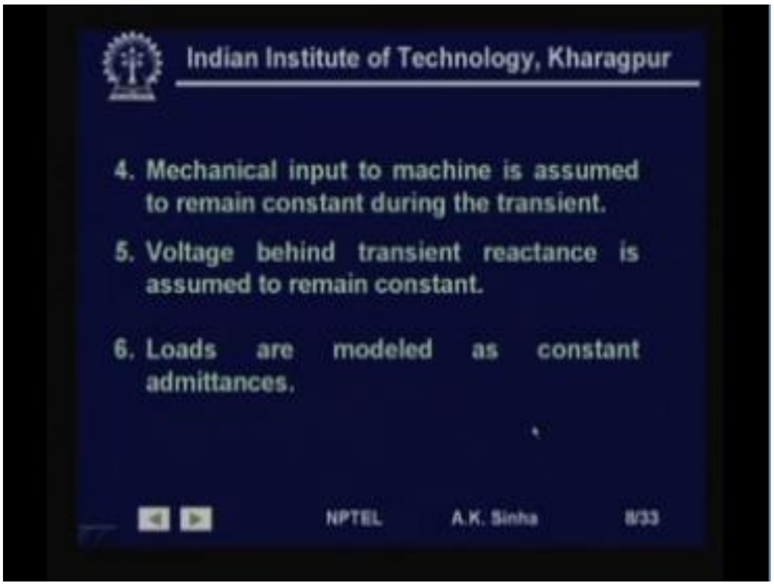

The fourth assumption, that we are making is mechanical input to the machine is assumed to remain constant, during the transient. That is the short period of study that we are doing. In that period, we assume that the speed governor system of the generator is not active enough to increase the mechanical input or decrease the mechanical input as required by the speed of the system.

That is, if the speed goes up, the speed governor will try to reduce the mechanical input. If the speed goes down, then it will try to increase the mechanical input by opening the valve a little more. So, what we are assuming is that the speed governing system is slow and it is effect we are ignoring. Voltage behind transient reactance is assumed to remain constant.

Now, this assumption again says that, that automatic voltage regulator action, we are ignoring. That is automatic voltage regulator of the generator; we will try to keep the terminal voltage constant. And thereby, increasing the excitation and so that, voltage behind the reactance. However, we are assuming for this time frame, that we are considering the effect of the automatic voltage regulator is neglected. Though, in practice, this is not true, because automatic voltage regulators are generally very fast acting.

And that, this also leads to a pessimistic result, because whenever a fault occurs, the voltage, terminal voltages are going to go down, which means that, AVR will try to boost the voltage behind the reactance. By increasing the excitation of the excitation to the synchronous machine and which means, the value of e will increase. And therefore, E v by X is going to increase and this is going to change the power angle characteristic of the machine.

Because, now the maximum power that can be transmitted is going to become more than what was valid with the previous value of e. However, this will make the analysis much more complex. Therefore, we neglect this for studying when we are doing the study for small systems. Though, nowadays, when we have computer programs for studying the transient stability, the mechanical input changes are also taken care. That is, we model the speed governors.

Similarly, we model the automatic voltage regulators, along with it is saturation and then it is all these things are modeled nowadays. When, we have where we have the model transient stability programs running. However, to understand the phenomena, we are making these assumptions. That is speed governor action is ignored, automatic voltage regulator action is ignored. That is all control us on the system we are ignoring. Loads are modeled as constant admittances.

This again is true to a great extent; however, with induction machine, this may not be true all the time. Because, if the voltages go down to very low value, we have the machine behavior, which is quite different and we have what we call the machine get stalled or things like that. So, those things are can also be modeled in the model programs.

But, for our purpose of analysis or trying to understand, how we do this analysis. We are assuming that the loads, they are constant impedance loads. That is the impedance value; we calculate at the voltage level at the pre fault condition or before the changes occur at T 0 minus. Whatever is the voltage at that time, we calculate the admittance knowing the power and we use that admittance all the time assuming it to remain constant.

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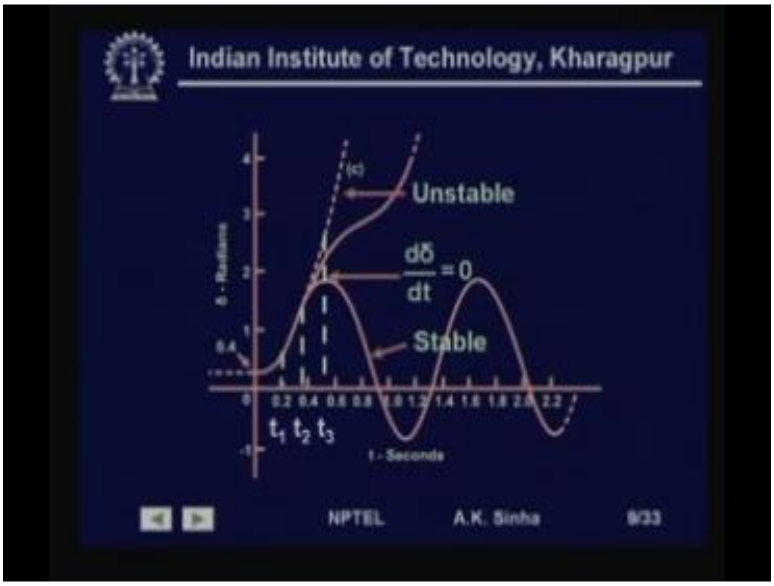

So, as we have seen that, when we make these assumptions, we have seen that. If we are working with ((Refer Time: 36:56)) the changing from P m o to P m 1. What we will find is, we are going to get an oscillation or a delta angle change, which will look like this. Here, I have model small damping, that is, if you see the oscillations are diagonal a little.

So, this is, what will happen, we will have delta angle oscillations, as we had seen earlier. So, this is delta versus time clock will look like this. If we increase the mechanical input to that P dash m 1 level, then what we will get is an unstable system. That is, it will have up to this region, where we will have deceleration and then, again it will accelerate very fast.

So, again, what we are finding is, now the delta angle keeps on increasing and we say that the system has become unstable. If we would have increased this P m value; that is change in P m we would have taken much larger than the response of the system would have been like this. That is, there is going to be a very fast acceleration as the system is going to become unstable. This we can also as we had see earlier, we can see with the clearing time aspects also.

Now, here, we can see a something very peculiar, which happens. If we see these three responses, this response is the response for a stable system and these two responses are response for unstable system. Now, when we see this response for a stable system, what we find is that the delta angle keeps increasing. And after sometime, it starts falling down. That is, there is a point on the P delta, sorry, delta versus time characteristics, where d delta by d t is equal to 0.

That is a point at which the change in delta angle with respect to time is 0. That is all these time, it is increasing, then it keeps on reducing this change keeps on reducing it becomes 0. And then, it becomes negative. That is delta angle keeps on reducing, this is what we had seen that we will get sustained oscillation in that situation. Whereas, for unstable system, we will not get any point, where we will get this d delta by d t is equal to 0.

And we try to take advantage of these particular phenomena. That is this particular aspect of the delta versus time characteristics. That for a stable system, we will get a d delta by d t is equal to 0 at some time t. This is what happens and we will take advantage of this in trying to build up, what we call the equal area criteria for transient stability analysis.

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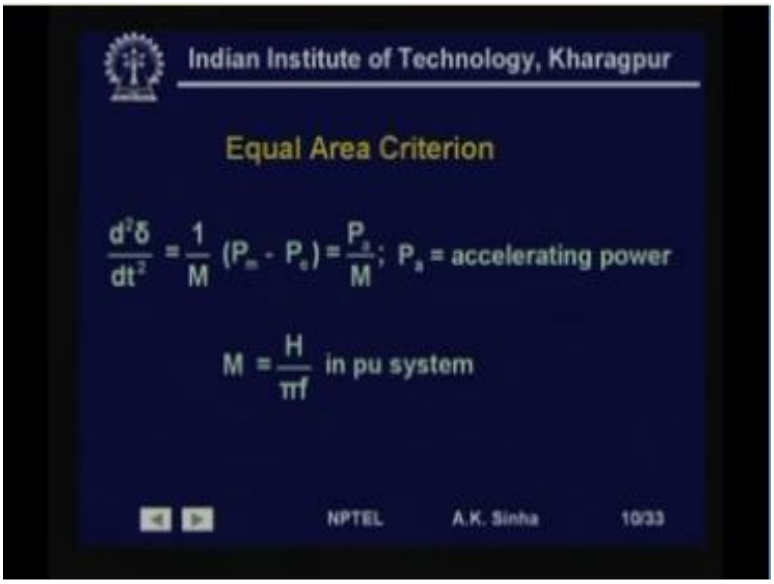

So, let us see what how we do that. We have the swing equation which we write as, M d 2, delta by d t 2 is equal to P m minus P e. I have taken this M from the left hand side to the right hand side. So, we write d 2, delta by d t 2 is equal to 1 by M P m minus P e, P m minus p e is the accelerating power. That is if the difference is positive, it is accelerating power, if the difference is negative, it is decelerating power.

So, we write this P m minus P e is accelerating power. So, we will write this as equal to Pa by M, where P a, we call as the accelerating power. M as we have already seen is the angular momentum in per unit, which is also called as a ratio constant. M is equal to H by pi f in per unit system. That is what we had seen earlier. So, I am using the same relation here.

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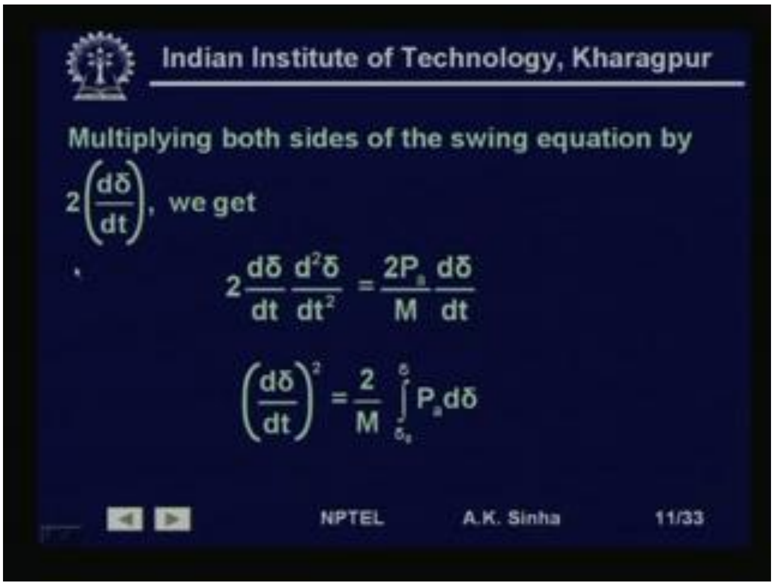

Now, if we look at this equation, that is d t delta by d t 2 is equal to P a by M. We multiply this by 2 into d delta by d t. That is multiply both sides by 2 into d delta by d t. Then, we will get 2 into d delta by d t multiplied by d 2 delta by d t 2 is equal to 2 p a by M into d delta by d t. That is both sides we have multiplied by 2 d delta by d t. Now, this term, if we integrate this with respect to P. We will get, this as d delta by d t square is equal to 2 by M, integral here will be from delta 0 to sum delta Pa d delta. So, integrating this, we are going to get d delta by d t square is equal to 2 by M, integral from the initial operating point delta 0 to the final operating point delta, so 2 by M, integral P a, d delta.

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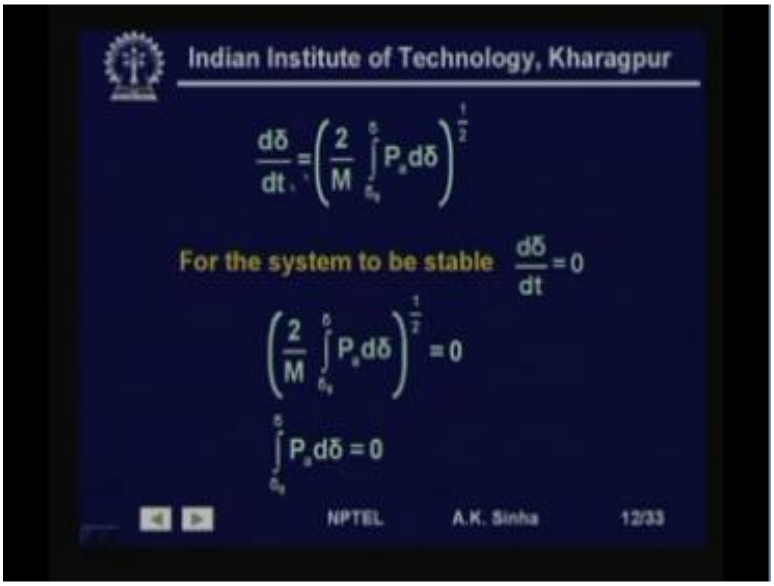

Now, from this, we can write d delta by d t is equal to 2 by M integral P a d delta from delta 0 to delta. That is limits to the power half that is square root of this. Since, this was square, so we remove this square, so we put a square root here. So, we have got this d delta by delta is equal to 2 by M integral this square root. Now, we have seen, that for the system to be stable, we must have d delta by d t is equal to 0 at some point.

So, using this criteria, we can write that 2 by M integral from delta 0 to delta P a d delta half is equal to 0, that is square root of this must be equal to 0. This is d delta by d t and this must be equal to 0 at some angle delta. So, we have this relationship, this is equal to 0. Now, since M is a constant, this is a square root, this 2 is a constant. So, we can write basically that integral from delta 0 to delta of P a d delta is equal to 0.

That is, what we are saying, that this relationship integral P a d delta is equal to 0, what does it really mean? It simply means that area under the accelerating power curve for from delta 0 to delta must be equal to 0. That is from initial operating point delta 0 to the final operating point delta. The accelerating power the area below the accelerating power curve must be equal to 0.

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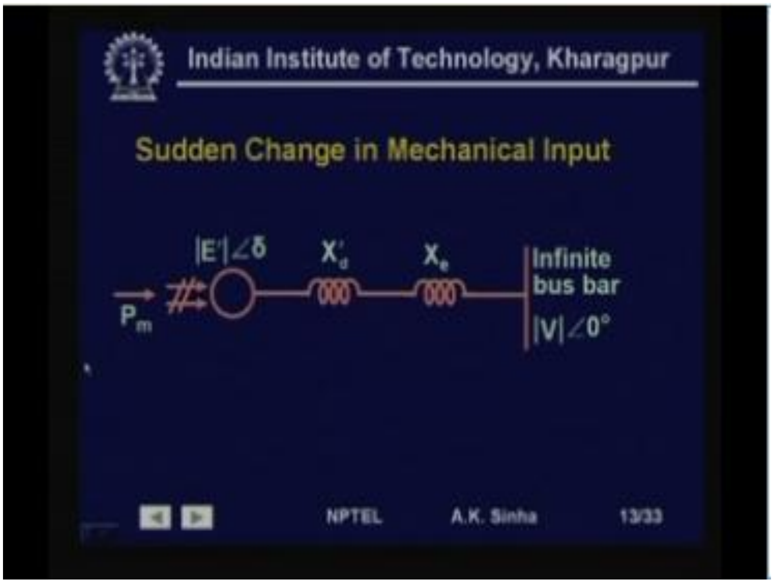

This let us see how we use for stability study. So, here we are talking about a sudden change in mechanical input the same thing which we discussed earlier. The machine internal voltage is E dash with an angle delta the X d dash is the machine transient reactance X e is the external reactance. And we have infinite bus bar, with a voltage V angle 0 degree; P m is the mechanical input to the system.

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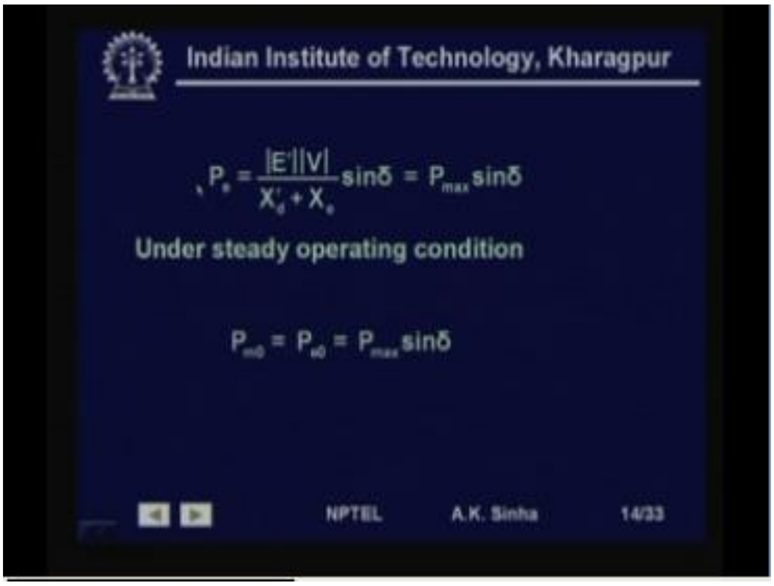

Now, P e the electrical output from this system will be given by E dash V by X d dash plus X e into sin delta. This is EV by X sin delta and total transfer reactance in this case is X d dash plus X e. So, this is, what we will get as the electrical output, which we call as P max sin delta. Because, this term is the maximum power that can be transmitted on this system. So, this is P max sin delta.

Now, under steady operating condition; that is under the initial operating condition, when we had not changed the mechanical input the mechanical input and electrical output, were equal and the system was running as synchronous P. So, P m 0 is equal to P e 0 at the initial operating condition and this is equal to P max sin delta 0. In fact, we should write this as sin delta 0. So, P m 0 is equal to P e 0 is equal to P max sin delta 0 at the initial operating point.

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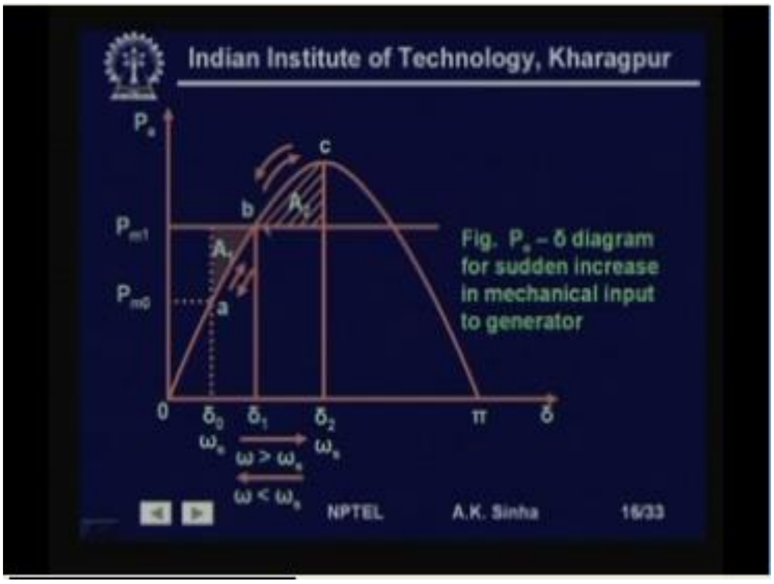

That is, what we have shown here, P m 0 is equal to P e 0 and the power angle is delta 0. Now, suddenly, we have increased the mechanical input to P m 1, as we have seen the rotor is going to accelerate, when it reaches this point the speed is still beyond synchronous speed. So, it will keep moving further, where when it reaches point c, the speed becomes equal to the synchronous speed. And since, P e at this point is greater than P m. So, delta angle, there is deceleration and delta angle will reduce.

So, delta will keep following this, when it reaches here, again P m and P e are equal. But, the speed now has become sub synchronous, because from synchronous speed, there is a deceleration which takes place and here the speed is less than synchronous speed. So, again, it will keep falling delta will keep on falling down. And when it reaches this point a, again we will have the speed reduced, as soon as it reaches goes beyond this, we have P e less than P m. So, there is acceleration.

And since, the speed is less, then synchronous speed at this point. That speed will keep on building and at this point again, the speed will become equal to the synchronous speed. But, again, since we have less electrical output than the mechanical input, delta will, because of the accelerating power available delta will keep on increasing the speed will keep building. And as we have seen, we are going to have a sustained oscillation between point a and c.

Now, if you look at the previous relationship here. We said that, for the system to be stable integral P a d delta must be equal to 0 and this is from delta 0 to delta. That is the final or the maximum excursion of delta angle. So, if we look at this, what we are finding is, that during all this time when the rotor moves from a to b. What we are seeing, we are getting accelerating power, which is the difference between P m 1 and the electrical output characteristics of the machine from a to b.

So, at this time, all this we are getting and what is P a d delta, integral is basically this area, which we are going to get, till at this has reached the point b. When, delta angle keeps on increasing here, what is the acceleration that we are getting? Since, we go beyond this; we have electrical power output, which is larger, than the mechanical input, so we have got deceleration. So, we have got deceleration, which is going to take place.

And what we are getting as, P a is the negative and the speed at this point will keep on reducing, till it reaches to synchronous speed at this point c. So, if we look the area between this P m and the electrical characteristics or the electrical power characteristics of the machine. Then, we get this area A 2, which is the decelerating area. So, this is the accelerating area, this is decelerating area.

And if accelerating area is equal to decelerating area, then we will get d delta by d t is equal to 0. That is we have reached the synchronous speed, at this point c. So, what we find is for the system to be stable the accelerating area, available must be equal to decelerating area. That is accelerating area A 1, should be equal to decelerating area A 2.

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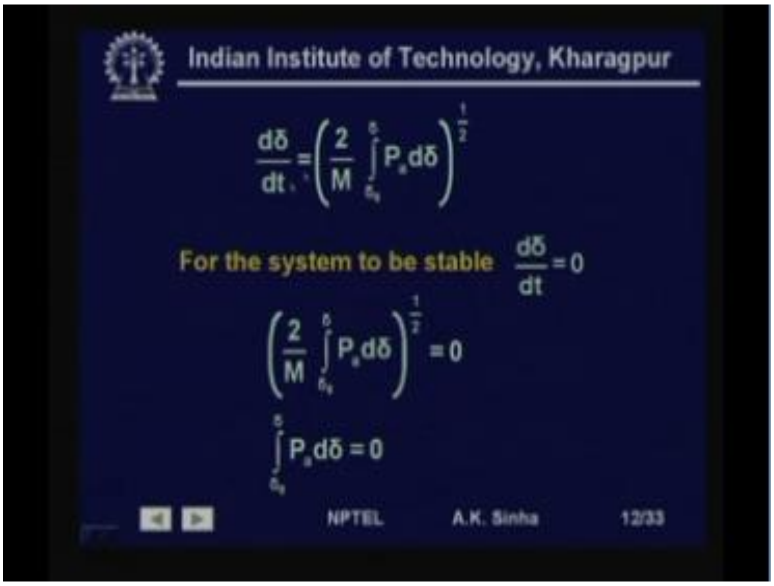

And if that happens, then what we have got is, basically this P a d delta integral is equal to 0, because A 1 plus A 2 is the total area, but A 2 is negative. So, A 1 minus A 2 is equal to 0 in this case. That is A 1 is equal to A 2, then we get this P a d delta integral is equal to 0. ((Refer Time: 51:40)). So, now, this is what we have seen that for the system to be stable. These two areas must be equal and that is why the name for this kind of analysis is equal area criterion.

This is a very simple analysis; this is a graphical way of trying to solve the swing equation. This is valid only for a single machine infinite bus case, because for multi machine case, we will not be getting these P delta characteristics, like this. So, we have reduced our system to a single machine connected to infinite bus, where the P delta characteristics for the machine can be shown like this. And with this, we can analyze the stability of a single machine, infinite bus system.

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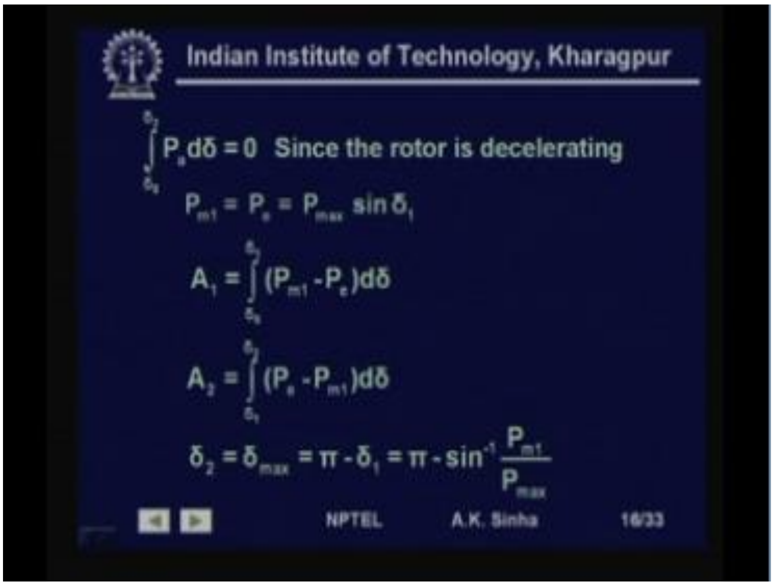

So, now, we have the integral from delta 0 to delta 2 of Pa d delta is equal to 0. Since the rotor accelerates in this part and decelerates in this part and reaches the synchronous speed at delta 2. So, we have P a d delta is equal to 0, that is P m 1 is equal to P e is equal to P max sin delta 1. This is delta 1. So, at this point b, we have delta 1 and P m 1 is equal to P e 1, which is P max sin delta 1.

Therefore, we write area A 1 what is the area A 1? Area A 1 is this area, that is between P m 1 and P delta characteristics of this machine. So, P m 1 minus P e, which is the function of delta from delta 0 to delta 1. So, we have got A 1 is equal to integral delta 0 to delta 1. P m 1 minus P e, d delta, this is the accelerating area. All this time P m 1 is greater than P e. So, this is the accelerating area.

Similarly, we have decelerating area which will take place, when the angle delta goes beyond delta 1. So, as it goes beyond this, P e is more than P m 1. So, there is deceleration, which takes place and this area is from delta 1 to delta 2, P e minus P m 1. Because, here, if you see P e is greater than P m 1, so we are writing this as decelerating area. So, decelerating area is P e minus P m 1.

Whereas, accelerating area is P m was greater than P e. So, P m 1 minus P e. That is why, the way we were writing it. So, A 2 is delta 1 to delta 2 integral P e minus P m 1 d delta. And we have, when we have these two equal we get the system to be stable.

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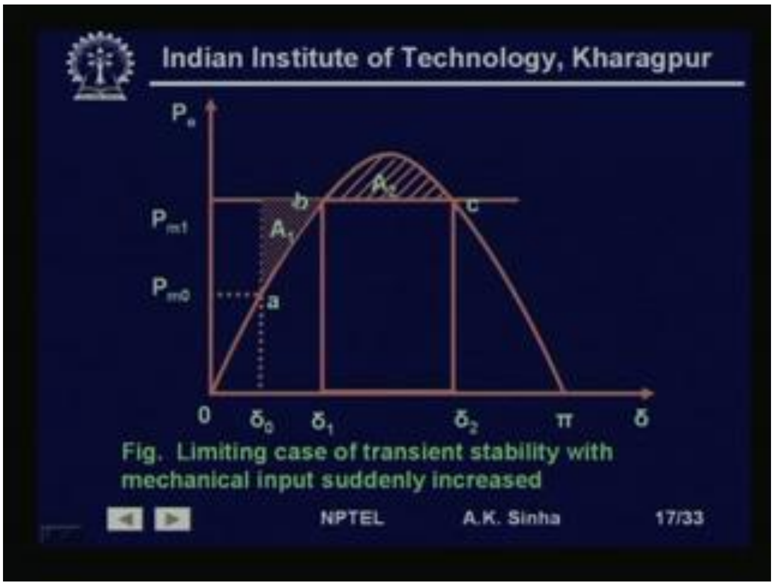

Now, suppose, we have a situation, where we increase this mechanical input from P m 0 to P m 1, such that. Now, this is the accelerating area, which is equal to the decelerating area, which will be coming up to this point. If this we would have increased this, even a little further, this point c would have come below and then, we would not have had any stability available. Because, we would have got accelerating power, still available and this area A 2 would have been less than the accelerating area available.

So, this is, what we call a limiting case. That is a case, where the system is just stable, that is area A 1 is equal to the maximum decelerating area, which is available and the maximum decelerating are available will be this A 2 up to this point delta 2. And therefore this is the maximum delta excursion. That is possible, because if delta excursion goes beyond this. We will start getting further acceleration of the system and delta will keep on increasing.

So, under such as a situation, when delta 2 is equal to delta max, that is the maximum delta which is possible for stability, which this delta max is equal to how much? This will be, this is delta 1 and this will be pi minus delta 1. So, delta 2 is equal to delta max delta 2 is equal to delta max, which is equal to pi minus delta 1, which is equal to pi minus sin inverse P m 1 by P max.

And this is, what tells us, that we have reached the maximum delta value. That is feasible for stability and this provides us the delta max value from this relationship. So, pi minus delta 1 is, what tells us the maximum value, which is pi minus sin inverse P m 1 by P max. So, this way, we can look at or find out the stability limit for the system. That is, how much power we can increase. So, that the limit is reached.

So, with this, we will be stopping today. And in the next lesson, we will continue and see how we can apply these equal area criteria for other kinds of disturbances. And we will take up some example to see how we apply this criterion for the single machine infinite bus system.

Thank you.