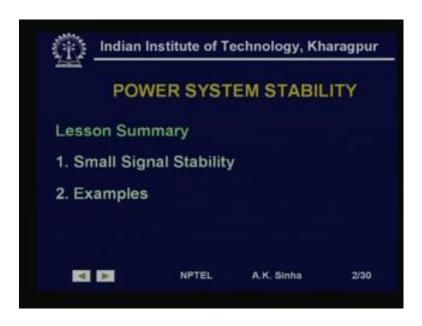
Power System Analysis Prof. A. K. Sinha Department of Electrical Engineering Indian Institute of Technology, Kharagpur

Lecture - 35 Power System Stability – III

Welcome to lesson 35, in Power System Analysis. In this lesson, we will continue our discussion on Power System Stability. Till now we have talked about the basic problem of power system stability. Then, we tried to classify the stability problem into different types and we started with the most important one.

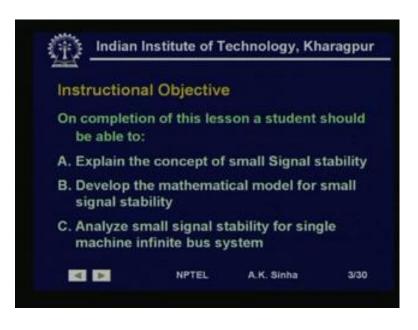
That is the rotor angle stability problem and we developed the dynamic equation for rotor angle, dynamics of a synchronous machine. Then, we went into the single machine infinite bus problem. And we developed this, swing equation for this kind of a system, including the damping term in this system. In this lesson, we will continue the discussion. We will start with the small signal stability. That is, when the disturbance to the system is much smaller.

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Then, we have seen that, power system swing equation. That governs the dynamics of the rotor angle as a non-linear equates differential equation. But, when we are talking about the small signal disturbances, then we can think of linearizing the system and use this linear system analysis for this. So, we will start this lesson with the problem of small signal stability and have to analyze small signal stability, for a single machine infinite bus system. We will take up some examples to illustrate, how we do this.

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Well, on the completion of this lesson, you should be able to explain the concept of small signal stability. Develop the mathematical model for small signal stability and analyze small signal stability for single machine infinite bus system.

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Small Signal Stability

$$M \frac{d^2 \delta}{dt^2} + D \frac{d \delta}{dt} = P_m - P_o pu$$

 $M = \frac{H}{\pi f}$ in pu system
 $P_a = \frac{|E||V|}{X_d} \sin \delta = P_{max} \sin \delta$
For small disturbance the swing eqn.
can be linearized about the initial
operating point $\rightarrow \delta = \delta_0 + \Delta \delta$
MPTEL A.K. Sinha 4/30

Well, as we said, we will start this lesson with a small signal stability problem. That is, what we are considering here is the system is running under normal operating condition

with all the variables being stable. That is having constant values. Now, there is a disturbance, which takes place. This disturbance is not of very large magnitude, but a small magnitude. But, this is going to perturb the system variables and these perturbations, because the disturbance is small are going to be perturbations.

Therefore, we know that the dynamic equation of power system is non-linear. That is, if we see the swing equation, we have the swing equation as M d 2, delta by d t 2 plus d, d delta by d t is equal to P m minus P e. And P e term, as we had seen is a term, which is a non-linear term. It is the function of delta and in fact, for cylindrical rotor machine, the value of this P e is equal to P max sin delta.

So, this equation is a non-linear equation, but at the operating point, normal operation, where it is taking place, before the disturbance. If the disturbance is small, we can linearize this dynamic equation, about that point and study the perturbation, as a linear system analysis. This is what we do, when we try to study the small signal stability for power system.

So, let us start with this swing equation, that we have M, we write as H by pi f, where H is the inertia constant in per unit. And we have P e is equal to E V by X d sin delta, which we call as P max sin delta. Now, here we have seen this P e being non-linear. So, if we are talking for small signal stability, we will have to linearize this dynamic equation, which is a non-linear equation, about the operating point.

So, for small disturbances, the swing equation can be linearized about the initial operating point. That is at delta is equal to delta 0 and we take a small perturbation of delta, delta. So, what we are saying is, we are linearizing this expression for the dynamics of the synchronous machine, about the operating point delta 0 and we are considering small perturbations. That is the change in delta is small delta, delta is small and therefore, we will use the linear relationships. So, let us see, how we linearize this swing equation for the synchronous machine.

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Indian Institute of Technology, Kharagpur $M\frac{d^2\delta}{dt^2} + D\frac{d\delta}{dt} = P_m - P_s(\delta)$ $M\frac{d^{2}(\delta_{0}+\Delta\delta)}{dt^{2}}+D\frac{d(\delta_{0}+\Delta\delta)}{dt}=P_{m}-P_{max}Sin(\delta_{0}+\Delta\delta)$ $P_{max}Sin(\delta_0 + \Delta \delta) = P_{max}(Sin\delta_0Cos\Delta \delta + Cos\delta_0Sin\Delta \delta)$ $Sin \Delta \delta = \Delta \delta$; $Cos \Delta \delta = 1$ $M\frac{d^{2}\delta_{0}}{dt^{2}} + M\frac{d^{2}\Delta\delta}{dt^{2}} + D\frac{d\delta_{0}}{dt} + D\frac{d\Delta\delta}{dt}$ = $P_m - (P_{max} \sin \delta_0 + P_{max} \cos \delta_0 \Delta \delta)$ NPTEL A.K. Sinha * 5/30

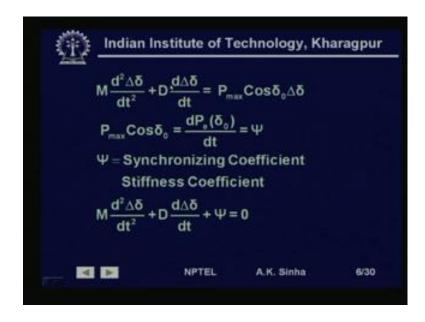
So, now again writing the same expression M d 2 delta by d t 2 plus d delta by d t is equal to P m minus P e, which is a function of delta. Now, we say that, delta is being perturb by a small amount from delta 0. So, we can write this delta as delta 0 plus delta, delta, so replacing this delta by delta 0 plus delta, delta. In this expression, we get M d 2 delta 0 plus delta, delta by d t 2 plus D, d delta 0 plus delta, delta by d t is equal to P m minus P max sin delta 0 plus delta, delta. That is, we have replaced all delta by delta 0 plus delta, delta.

Now, we can write P max sin delta 0 plus delta, delta as, that is we expand this term, then we get P max sin delta 0 cos delta, delta plus cos delta 0 sin delta, delta. Now, since delta, delta is small. So, sin delta, delta will be very much equal to delta, delta in radiance and cos delta, delta will be very nearly equal to 1. So, substituting these values, we will get M d 2 delta 0 by d t 2, plus M d 2 delta, delta by d t 2, plus D, d delta 0 by d t plus D, d delta by d t is equal to P m minus P max sin delta 0 plus P max cos delta 0 into delta, delta.

That is, we have substituted for this as delta, delta and cos, delta, delta is equal to 1, so substituting that, we will get this. Now, here M d 2, delta 0 by d t 2, plus d delta 0 by d t is equal to P m minus P max sin delta 0. Because, initially the system is operating in a steady state, before this disturbance took place. So, the relationship at delta 0 was

holding good, like this. That is M d 2 delta 0 by d t 2 plus d delta 0 by d t is equal to P m minus P max sin delta 0.

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So, removing this term, away, we will get this as M d 2 d 2 delta, delta by d t 2 plus D, d delta, delta by d t is equal to P max cos delta 0 into delta, delta. So, this is the expression or the linearized expression for the swing equation. That is the dynamics linearized equation for the dynamics of the synchronous machine or the rotor dynamics of the system. So, we can we normally write this P max cos delta 0. Actually, what is this P max cos delta 0.

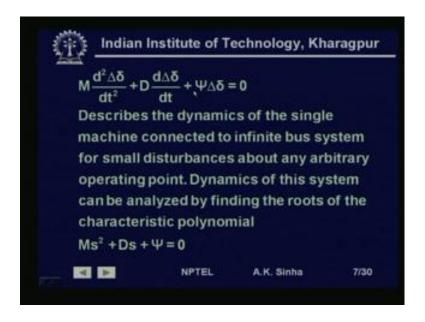
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Indian Institute of Technology, Kharagpur Small Signal Stability $M\frac{d^{2}\delta}{dt^{2}} + D\frac{d\delta}{dt} = P_{m} - P_{e} pu$ $M = \frac{H}{\pi f}$ in pu system $P_{a} = \frac{|E||V|}{X_{a}} \sin \delta = P_{max} \sin \delta$ For small disturbance the swing eqn. can be linearized about the initial operating point $\rightarrow \delta = \delta_{A} + \Delta \delta$ A.K. Sinha 4/30

If we see this expression, P m, P e is equal to P max sin delta 0. And if we differentiate this, then we will get this as P max cos delta. So, basically this expression, that we have got, that is the P by delta will be equal to P max cos delta. This is basically, giving us the slope of the power angle characteristics of the machine about the operating point delta 0 and we call this, as the psi. That is d p e at delta 0 by d t. This is the slope of this power angle curve of the synchronous machine at delta 0.

We are denoting it by psi and we call this as the synchronizing coefficient of the synchronous machine or it is also called the stiffness coefficient of the machine. It is basically, the slope of the power angle characteristics at the operating point delta 0. Therefore, now we are writing this expression as M d 2, delta, delta by d t 2 plus D, d delta by d t plus psi is equal to 0. That is, we will take this term, that is, what we are saying is, we are writing the characteristic equation for this relationship. So, we will write this as M d 2, delta, sorry, d 2, delta, delta by d t 2 plus D, d delta, delta delta by d t plus psi is equal to 0.

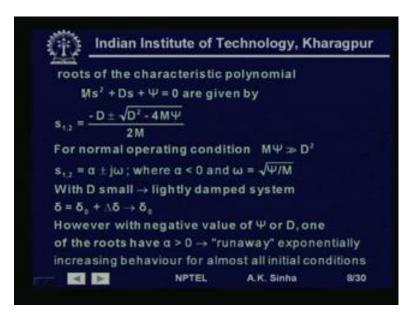
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Now, this expression, here we had missed this term delta, delta. So, finally, what we get is, here we had missed this expression delta, delta. So, finally, we are getting is, basically this M d 2, delta, delta by d t 2 plus D, d delta, delta by d t plus psi delta, delta is equal to 0, describes the dynamics of the single machine, connected to infinite bus system, for star small disturbances, about any arbitrary operating point. That is the operating point, wherever we want that delta 0, about that delta 0.

This is describing the dynamics of a single machine connected to infinite bus system. That is the dynamics of the single machine, about the arbitrary or any starting operating point. Dynamics of this system can be analyzed by finding the roots of the characteristic polynomial. That is, this is the second order polynomial that we have, writing d 2, that is second derivative as s; that is taking the lap loss transform. We will get M s square delta, delta plus D s delta, delta plus psi delta, delta is equal to 0. So, writing the characteristic polynomial we will get M s square plus Ds plus psi is equal to 0. And we can find out the roots of this polynomial, which will tell us about the dynamics of this system. This is how; we analyze the dynamics of a linear system.

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So, looking at this, the roots of the characteristics polynomial M s square plus D s plus psi is equal to 0 are given by there will be this is a second order equation. So, there are going to be two roots and we will get the roots as S 1 and S 2. This will be equal to minus D plus minus square over square root of D square minus four M psi divided by 2 M. This is a quadratic equation. So, we know how to find out the roots. So, the two roots are given by this relationship.

For normal operating condition, normally what we have is, M psi is much larger than D square. The damping terms are normally much smaller. And therefore, we will get this term as S 12. That is the two roots as alpha plus minus j omega, where alpha is generally less than 0. That is alpha is negative and omega, which gives the frequency of oscillation is given by root, over psi by M.

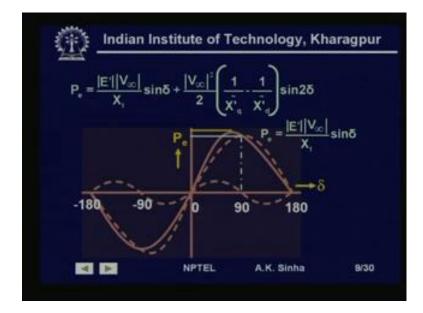
So, with D small, that is for lightly damp system delta is equal to delta plus delta, delta, which will be finally, leading to the same delta 0. That is, what we are trying to say is, if the system is operating initially at delta 0. We somehow, give a perturbation to the system. That is, we make a delta, delta change in the rotor angle of the system. Suddenly, then, what happens is, the system will go through a dynamics.

And will finally, settle down, again to delta 0. That is the dynamics will slowly be damped down and it will go back to the same stable operating condition with the rotor angle as delta 0. However, if either psi or d, anyone of them is negative, then what we

get is that, the value of alpha becomes positive. That the meaning of this is, that the oscillations will build up and the system will go into a runaway situation. That is this machine delta angle will keep on increasing and it will be a runaway situation. This means, that the generator will lose synchronism from the system.

So, this is, what we normally get and therefore, basically, when we are trying to analyze the small signal stability for the system. What we are looking at is, whether due to the small perturbations about it is operating point. The system is going to come back to the same operating point or it is going to run away. That is, whether the oscillations get damped out or these oscillations will build up and the system will get desynchronized from the rest. That is the machine will get desynchronized from the rest of the system.

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Now, let us see this same situation. Here, we have this curve, which is shown here is the curve, which is showing this P e is equal to E dash V infinity by X t sin delta. This is the term; that we get for a cylindrical rotor machine. That is the power angle characteristics for the cylindrical rotor machine in the generating mode this like. And in the motoring mode, it will be like this. This should go beyond this.

This curve is not very proper, for generating mode, it is okay. This curve is like this and for a salient pole machine; the electrical power output is given by this relationship, where, we find this is the term, which is same for the cylindrical rotor machine. But, this is a term, which is added, which is having a sin 2 delta terms involved, which means it is a second harmonic term and this is shown here.

So, this term that is the second term in this is shown here. And finally, for the salient pole machine, the power angle characteristics will be the sum of these two terms. So, it is given by this line here. Now, what we find is, that the salient pole machine has a little more maximum power available than the cylindrical rotor machine. This happens, because of the reluctance term or the second harmonic term.

Here, which we normally call as the reluctance term, because it is the reluctance power, this term is coming mainly, because of the saliency of the synchronous machine poles. That is, what we are saying is, that the reluctance in the direct axis and the quadrature axis are not same. And because of that, only this term is coming and that is why we call this as the reluctance term. And in fact, there are machines, which are built based on this reluctance power or reluctance torque, which are called reluctance motors, themselves.

Anyway, so we have these two characteristics and what we see from these characteristics. At any operating point, let us take this point on either of the cylindrical or rotor machine or on salient pole machine. Let us, start with the cylindrical rotor machine, suppose we are operating at this point. Now, at this point, we have angle delta 0 and what we find is, that the slope here, which is the synchronizing coefficient is positive here.

Now, suppose, we slowly keep on increasing the power output of the machine, that is we very slowly increase the mechanical input to the machine. What is going to happen? Because of that, the delta angle will keep on increasing. So, it will start from here say initially it is starting here. So, we will keep on increasing very gradually. So, machine operating point will keep on moving. That is delta angle will keep on increasing.

Since, we are increasing the mechanical input and electrical power output is also increasing. Because, of the increase in delta angle, therefore these are all the time getting matched. And the machine is running in synchronous condition. That is the machine speed remains synchronized. We keep on doing this, till we reach this point. Now, suppose, if we increase the mechanical input by a very small amount at this point. What will happen, delta angle will again increase from here. That is, at what is going to happen, because of this increase of delta angle.

Now, delta angle has crossed 90 degrees and what is happening? The electrical output of the machine is going to reduce the mechanical input has been increased. Whereas, the electrical output has reduced, which means, there is going to be a difference between these two powers. So, there is going to be some acceleration, which will take place. And because of this acceleration what happens, delta angle will keep increasing.

So, delta angle increases, then further the power output of the machine electrical power output of the machine will decrease. And this will continue like this and the system will lose synchronism, because the speed will keep on increasing. Even, if we have not increased the mechanical power, beyond this value. So, this point is, what we can get is the maximum power output from the machine. We cannot get any more power output from the machine.

Even, if we work the machine very gradually. So, we call this point as the maximum power output or P max and we also call this point as the steady state stability limit. That is the machine will lose it is stability, if we try to operate this, beyond this point. So, if we want to work the machine, beyond this maximum power, the machine loses stability and we have worked this very gradually.

So, we have assumed that, there is no dynamics or transient taking place and the system is all the time working in steady state. And that is why; we call this as steady state stability limit. Same thing, can be seen for the salient pole machine, here also we will get a P max value. And this is the value, which we call the steady state stability limit. And what we find that, as we keep moving on these power angle characteristics.

The synchronizing coefficient; that is the slope is positive up to the P max value. That is up to the steady state stability limit. At steady state stability limit, the value of the synchronizing coefficient is 0. That is the slope at this point, slope of P del P by delta is 0 at this point. That is the stability limit point and beyond this point the slope becomes negative. That is, what we said that, if either of these that is, if we get either psi value, that is this value psi becomes negative.

Then, we get alpha as positive, which means the system is going to become unstable. That is the roots come in the right half of the S plane, which shows instability, because the dynamics. The oscillation in this case are going to build up and this is, what we have seen here, that if the psi becomes negative. That is, we start try to operate in this region. Then, we use synchronism, because any small change in P m, where we would like to raise the output.

In fact, when we changed P m by a small amount increase it, then delta angle increases. But, P e reduces, which means the difference increases and the rotor will keep on accelerating. So, therefore, the speed will keep on increasing and it will lose synchronism. So, this is, how we understand the steady state stability of the system. That is with very gradual changes, if we work.

Then, we call this kind of a stability; that is the maximum allowable power, from a machine to the system is, what we call as the steady state stability limit. That is beyond that point the stability of the system; even under steady state operating condition is lost. Anyway, we are talking mostly about small signal analysis, where we are saying that, the effect, what is the effect of damping and what is the effect of synchronizing coefficient. We have seen synchronizing coefficient or the stiffness coefficient, if it in goes becomes negative, the system loses stability.

Anyway, also we have one thing, which we can see from these characteristics is, the slope for the salient pole machine is larger than the slope for the cylindrical rotor machine in the normal operating region. That is in the stable operating region. That is what we see is, that the salient pole machines are more, stiff and the change in their delta angle. For a given increasing power output is much less compare to that of the cylindrical rotor machine.

Another thing, that we see normally is, that the maximum power point for a salient pole machine is reached, below 90 degrees. Whereas, for a cylindrical rotor machine is the maximum power is reached at 90 degrees. That is, when sin delta becomes equal to 1. So, this is, about the steady state stability part, where that is, where we are talking about the dynamics, when we move a very, very small rate, from one steady point to another steady point, very gradual changes taking place, where we are considering, no dynamics building up.

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Let us go back to the small disturbances. So, writing the linearized swing equation M d 2 delta by d t 2 plus D, d delta by d t plus psi delta, delta. We are replacing M by H by pi f as we had seen earlier, that is writing instead of M. That is angular momentum, we are writing in terms of inertia, constant. Now, this expression can be written in a standard form like this.

That is d 2 delta, delta d t 2 plus twice zeta omega n, d delta, delta by d t plus omega n square delta, delta is equal to 0. This is a standard form of second order differential equation for analyzing the dynamics of linear system. Now, if we write this expression in this form, then we will get omega n is equal to square root of pi f 0 by H into psi and zeta will be equal to d by 2, square root pi f 0 by H psi. Where, f 0 is the synchronous frequency of the system and D of course, is the damping coefficient.

Now, damped frequency of oscillation for this particular system is given by omega d, which is equal to omega n square root of 1 minus zeta square. I am not going into the details of, how to get all these values. One can go through any standard book on control system to understand that, what we are saying is, only we are putting the results here. So, the damped frequency of oscillation omega d is given by omega n into root over 1 minus zeta square.

And for small perturbations delta, delta 0 about the initial operating point delta 0. The dynamic response of the system, that is if we give a very small perturbation equal to

delta, delta 0 at the starting point delta 0. That is suddenly, we change that delta angle at time t is equal to 0 from delta 0 to delta 0 plus delta, delta 0. Then, how this change in angle is going to take place is, what we are writing here, that is the solution for the dynamic equation linear dynamic equation, that we had can be written in this form.

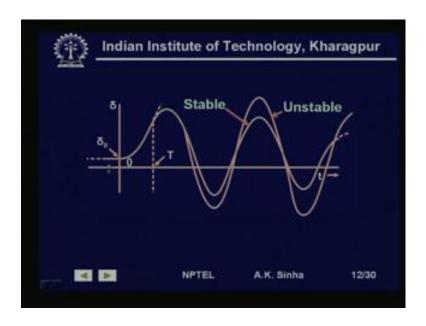
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Indian Institute of Technology, Kharagpur For a small purturbation Δδ_n about the initial operating point δ_n the dynamic response of the system is $\Delta \delta = \frac{\Delta \delta_0}{c} e^{-\zeta \omega_0 t} \sin (\omega_0 t + \theta)$ $\Delta \omega = -\frac{\omega_n \Delta \delta_0}{\sqrt{1 - \varsigma^2}} e^{-\varsigma \omega_n t} \sin \left(\omega_d t \right)$ $\delta = \delta_{\alpha} + \Delta \delta$; $\omega = \omega_{\alpha} + \Delta \omega$ time constant(τ) = A.K. Sinha 11/30 NPTEL

That is delta, delta at any time t. The change in the angle from delta 0 is equal to delta, delta 0 divided by root over 1 minus zeta square e to the power minus zeta omega n t into sin omega d into t plus theta. Now, this is giving us the change in delta from delta 0. Similarly, the change in frequency, that is delta omega from omega 0 is given by minus omega n into delta, delta 0 divided by root over 1 minus zeta square e to the power minus zeta omega n t into sin omega d t.

And the delta at any time t, for t greater than 0 is given by delta 0, plus delta, delta, where delta, delta is given by this relationship and omega, that is frequency at any time t at t greater than 0 is given by omega 0 plus delta omega. And the time constant for this oscillation is given by 1 by zeta omega n, which is equal to twice H by pi f D. So, these are the results, which we which I have put here. That is, how this dynamics are seen, when we give this small perturbation to the system.

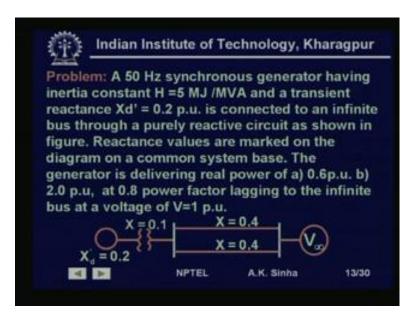
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I am showing this on this diagram. That is, we started at this delta 0 at t equal to 0. We have given a perturbation of delta, delta 0 and then the system will go through a dynamics like this. If we have the damping coefficient d, which is positive? That is, what we are seeing is the system goes through an oscillation. But, these oscillations are slowly dying out and finally, the system will reach a time at after sometime the angle delta 0 again.

But, if the value of d the synchronizing, the damping coefficient is negative, then we see, we go through this curve. We go through this second curve, where we find that these oscillations are gradually building up. And the system will finally, lose synchronism, because delta angle will keep on going higher and higher and after sometime, it will be a runaway situation.

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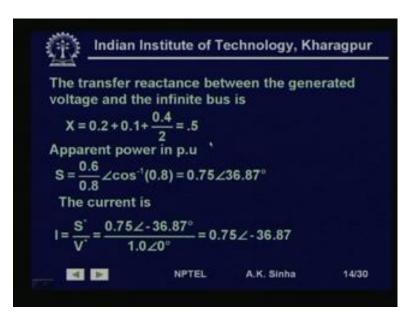


Now, let us try to see, this through some example. So, we take a very simple example. We have a 50 hertz synchronous generator, having inertia constant H is equal to 5 mega joules per MVA and a transient reactance X d dash is equal to 0.2 per unit. It is connected to an infinite bus, through a purely reactive circuit as shown in figure. That is, it is connected through a transformer and a double circuit line to the infinite bus system.

The reactance values are marked on the diagram. That is transformer has a reactance of 0.1 per unit. The line each circuit has a reactance of 0.4 per unit. The generator is delivering real power of a 0.6 per unit and b at 2 per unit at 0.8 power factor lagging. That is, what we are trying to do is, to try to analyze the dynamics of the system for two conditions.

One, when the system initial operating point is 0.6 per unit. That is the electrical output from the generator in steady state is 0.6 per unit and the second condition. When, we have the initial operating point, where the generator is delivering 2 per unit power. So, and for both this, we have assumed that the power factor is 0.8 lagging. So, this is and the infinite bus voltages, infinity, V infinity is 1 per unit. The damping coefficient in both these cases is assumed to be 0.15 per unit.

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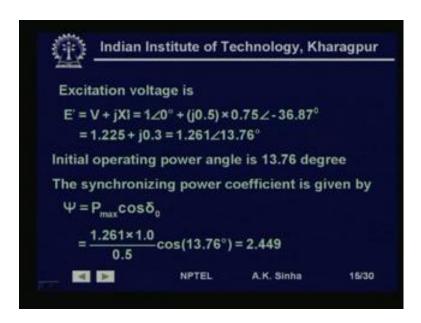
So, first what we will do is, the transfer reactance between the generated voltage and the infinite bus. That is the voltage behind the reactance for this machine, as we have seen earlier. The simplified model had a voltage behind the transient reactance and so for this voltage source, what is the transfer reactance up to the infinite bus. That is 0.2 plus 0.1 plus 0.4, two of these 0.4 in parallel. So, this is the reactance 0.2 plus 0.1 plus 0.4 by 2, that is 0.5 per unit.

So, the reactance is 0.5 per unit and we have the apparent power in per unit as S is equal to 0.6 by 0 0.8. In fact, this since we are writing reactance I have not written j here, otherwise you can write this j. Because, the angle of this is always 90 degrees, it is a inductance. So, S is equal to 0.6 by 0.8, 0.6 is the power per unit and 0.8 is the power factor. So, 0.6 by 0.8 into cos inverse 0.8, the angle will be that.

So, the apparent power in per unit can be written as 0.75, which an angle of 36.87 and the current for this, I will be equal to S conjugate by V conjugate. Because, we have S is equal to V I conjugate. So, S conjugate is V conjugate, I. Therefore, I is equal to 0.75 with an angle of minus 36.87, because S with an angle 36.87. So, when we are taking conjugate this becomes minus divided by V conjugate. This is 1 angle 0. So, 1 minus 0 is same as 0. So, the current is 0.75 angles, minus 36.87.

So, this is the value of current in per unit, that we get and from this current. We can now find out the voltage behind the transient reactance for the machine. Because, the voltage at the infinite bus is given as 1, angle 0.

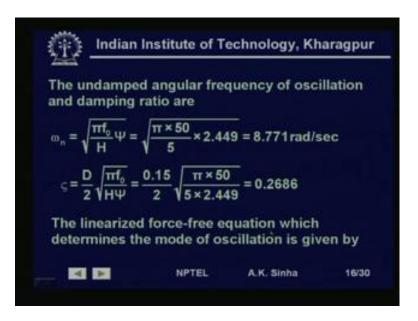
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So, E dash, the voltage behind the transient reactance is equal to V plus j X into I, where X is the transfer reactance between the voltage behind transient reactance to the infinite bus. So, E dash is equal to V plus j X I, this is equal to 1 angle 0 plus j 0 0.5 into I is 0 0.75 into with an angle of minus 36.87, this. When, we solve this, this comes out to be 1.225 plus j 0.3. That is equal to 1.261 with an angle 13.76 degrees, that is delta 0 is 13.760.

So, initial operating power angle is 13.76 degrees and the synchronizing power coefficient at this operating point psi is equal to P max cos delta 0. So, this will be equal to 1.261into 1 this is E. This is V divided by X into cos of angle delta 0. So, this comes out to be 2.449. So, synchronizing coefficient at this operating point is equal to 2.449.

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The undamped angular frequency or natural frequency of oscillation and the damping ratio, that is omega n and zeta, we can calculate as we had seen from these expressions earlier.

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Omega n is given by this and zeta is given by this expression. So, simply substituting the values here omega n is equal to square root of pi f 0 by H in to psi. So, pi into 50 by 5 into 2.449, this is psi value that we have calculated. So, it comes out to be 8.771,

radiance per second and zeta is equal to D by 2 square roots by pi f 0 by H psi. This is equal to 0 0.15 by 2.

This is the value of D, that we have said, we will we are using square root of pi into 50 divided by 5 H value into psi 2.449. So, this comes out to be 2.2686. So, the damping coefficient is 0.2686.

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Indian Institute of Technology, Kharagpur $\frac{d^2\Delta\delta}{dt^2} + 2\varsigma\omega_n\frac{d\Delta\delta}{dt} + \omega_n^2\Delta\delta = 0$ or, $\frac{d^2\Delta\delta}{dt^2}$ + 4.71 $\frac{d\Delta\delta}{dt}$ + 76.93 $\Delta\delta$ = 0 The damped angular frequency is $\omega_{d} = \omega_{c} \sqrt{1-c^{2}} = 8.448 \text{ rad/sec}$ Frequency of damped oscillation $f_d = \frac{\omega_d}{2\pi r} = 1.345$ Hz A.K. Sinha 17/30 4 4 NPTEL

Now, the linearized force free equation, which determines the mode of oscillation, as we had seen earlier, is given by this relationship, d 2 delta, delta by d t 2 plus twice zeta omega n d delta, delta by d t plus omega n square delta, delta This is equal to 0. So, this is the expression that we have. Now, substituting the values of zeta omega n in this expression, we will get the expression for the machine connected to infinite bus at the initial operating point. That is delta 0 is equal to 17.36 degrees.

Then, we get this expression as d 2, delta, delta by d t 2 plus 4.71 twice zeta omega n value into d delta, delta by d t plus omega n square which is 76.93. That is omega n we had seen here and zeta we have seen here. So, substituting these values we will get 76.93, delta, delta. This is equal to 0. The damped angular frequency omega d, as we have seen is given by omega n square root of 1 minus zeta square.

This comes out to be 8.448, radiance per second and the frequency of the damped oscillation f d is equal to omega d by twice pi, because omega is twice pi f. So, this

comes out to be 1.345 hertz. That is the oscillating; the frequency of oscillation for this system is 1, 345 hertz. This is over and above the 50 hertz synchronous frequency. That we have for the system, that is, if we look at this diagram.

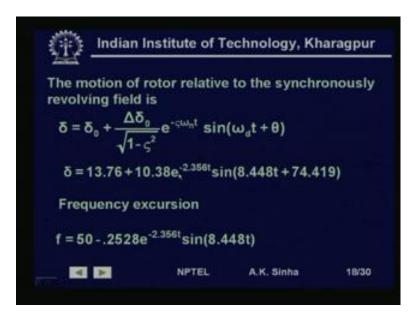
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Then, we are saying the frequency of oscillation for this system is 1.345 hertz. This is a normal thing that we see, that is the damped frequency of oscillation for the dynamics of the system is normally of the order of 1 hertz or so. So, in this sense, that is why, when we are trying to analyze this system, we normally take a time period of a few seconds. Because, the frequency of oscillation is 1 hertz, so you get one cycle in one second or so.

So, when you are doing it for 3, 4 seconds, you have got the dynamics for 3, 4 cycles. Also, the other thing is, we can say that the time period of for oscillation is given by 1 by zeta omega n, that is sorry. We had seen here time constant is 1 by zeta omega n and normally around 4 times constant. You will find the system would have settled down to less than 1 percent, the magnitude of oscillation.

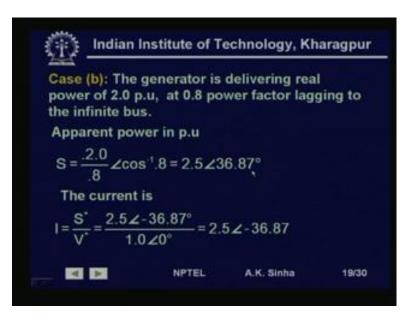
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So, the motion of rotor relative to the synchronously rotating field is delta is equal to delta 0 plus delta, delta. This is the expression that we had seen earlier and we see delta is equal to 13.76 this is the starting point plus 10.38 e to the power minus 356 t. Now, this is showing as time increases this term is going to reduce. And that shows that, the damping is going to reduce the oscillation magnitude. And after some time, that is normally around 4 time constants, it is going to settle to settle back to delta 0.

So, this term, as t keeps on increasing, will keep on reducing and will reduce to 0 as t times to infinity, which shows that the system is going to be stable. When, as t tends to infinity and the frequency of excursion, that is given by this relationship 50 minus 2.2528 e to the power. Again, which we are finding as t increases, this frequency will also settle down to 50 hertz.

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Now, when we have the initial operating point, where the power being delivered by the machine is 2 per unit what happens is, we again calculate the apparent power with it is angle and the current; we get an S 2.5 angle 36.87. In this case, we have assumed that, the exciter of the machine is able to make the voltage of the machine go up without any limits.

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-	Name In		
Excitation vo	oltage is		
E' = V + jXI =	=1∠0° + (j.5)	×2.5∠-36.8	7
= 1.75 + j	1.0 = 2.015∠2	9.74°	
	ng power angl		ree
The synchron	izing power c	oefficient is g	iven by
$\Psi = P_{max}$	cosδ₀		
= 2.015×1.	0cos29.74° =	3.5	

And therefore, what we find is that the voltage behind the transient reactance, in this case is going to a very high value, because a very large amount of current is flowing. And because of this, the change in delta angle is not really much, but if we would have kept the excitation voltage same as that of 0.6. Then, we would have found that the delta 0 is a very large value.

Here, we have assumed that, it is possible to keep the excitation voltage to go up to very high values. And therefore, we are finding voltage behind transient reactance as 2.015, with an angle 29.74 degrees. So, now, initial operation point delta 0 is 29.74 degrees. At this, we will get again the synchronizing coefficient psi, which comes out to be equal to 3.5. It has got a different value now.

But, if we would have used e dash as same value as earlier, that is at the value of 1.261, then we would have got a value of delta 0 in this case, which would have been very large sorry very small. It would have reduced considerably.

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Indian Institute of Technology, Kharagpur The undamped angular frequency of oscillation and damping ratio are $\omega_n = \sqrt{\frac{\pi f_0}{H} \Psi} = \sqrt{\frac{\pi \times 50}{5} \times 3.5} = 10.48 \text{ rad/sec}$ $\frac{D}{2}\sqrt{\frac{\pi f_0}{100}} = \frac{.15}{.2}\sqrt{\frac{\pi \times 50}{5 \times 3.5}} = .2246$ The linearized force-free equation which determines the mode of oscillation is given by NPTEL A.K. Sinha 21/30 4

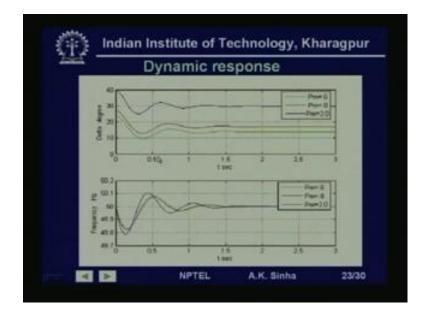
The undamped natural frequency, again we can find out for this case, comes out to be 10.48, radiance per second zeta comes out to be 0.2246.

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Indian Institute of Technology, Kharagpur $\frac{d^2\Delta\delta}{dt^2} + 2\varsigma w_n \frac{d\Delta\delta}{dt} + w_n^2\Delta\delta = 0$ or, $\frac{d^2 \Delta \delta}{dt^2} + 4.707 \frac{d \Delta \delta}{dt} + 109.83 \Delta \delta = 0$ The damped angular frequency is $\omega_{d} = \omega_{n} \sqrt{1-\zeta^{2}} = 10.212 \text{ rad/sec}$ Frequency of damped oscillation $f_d = \frac{\omega_d}{2\pi} = 1.6253 \text{Hz}$ A.K. Sinha 22/30 NPTEL

And we will get, again substituting in this expressing the values, again omega d, the damped natural angular frequency, comes out to be 10.212, radiance per second. And the frequency of damped oscillation is coming out to be omega D by 2 pi is equal to 1.6253 hertz.

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Now, we plot a variation in delta angle and frequency, when we give a small perturbation of delta angle to the system. Then we will get plots as shown here.

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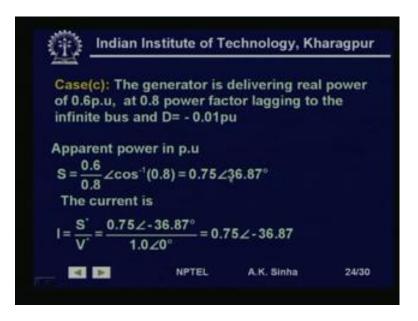


These can be obtained by looking at these relationship, delta, delta given by this relationship and delta omega is given by this relationship. So, we can plot delta with respect to time and for this particular system we have used delta, delta as 10 degrees. So, we have given a perturbation of 10 degrees and we have plotted delta and omega with respect to time. Then, we will get this, characteristics.

Now, here we have shown characteristics with P m is equal to 0.6 and for P m is equal to 2. The delta angle varies like this. Similarly, the frequency in the two cases, are shown here. We have also shown a third case with P m; that is the starting point being 0.8 per unit. So, initial output is 0.8 per unit, in that case also, we can see.

Now, we see that these plots are very similar, when the initial operating point is somewhat different. Then, there is a slight variation in frequency and delta angle. That is, in terms of their damped natural frequency, which we have already seen and we see that for positive damping, these oscillations. Finally, die out and after a few seconds, we get a steady state operating condition.

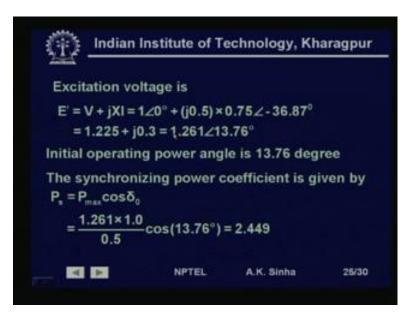
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If we take a case, where we have damping, which is negative. Now, this damping, negative damping does occur into the system, sometimes, when we have controllers in the system, which have very large gain. So, sometimes this does happen, that is the overall damping of the system becomes negative. And in such cases, how the system behaves, we can see from this example.

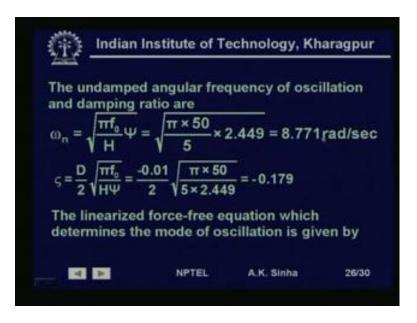
So, again, we are taking the example, where the generator is delivering real power of 0.6 per unit at 0.8 power factors lagging to the infinite bus and the damping D is minus 0.01 per unit. A very small negative damping, what happens because of this, again same thing apparent power we are getting this as 0.75? So, and with an angle 36.87, so the current is coming out to 0.75 with an angle minus 36.87.

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The excitation voltage, again the same thing 1.261 with an angle 13.76 degrees, that is the initial operating point is 13.76 degrees. Now, the synchronizing coefficient in this case is again 2.449, the same as earlier case.

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Now, here, we find that the omega n, the undamped natural frequency is again coming out to be same as 8.771, radiance per second. Whereas, the damping coefficient has now become negative, because D is negative, so it comes out to be minus 0.179.

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Indian Institute of Technology, Kharagpur $\frac{d^2\Delta\delta}{dt^2} + 2\varsigma\omega_n\frac{d\Delta\delta}{dt} + \omega_n^2\Delta\delta = 0$ or, $\frac{d^2\Delta\delta}{dt^2}$ - 0.9⁴2 $\frac{d\Delta\delta}{dt}$ + 76.93 $\Delta\delta$ = 0 The damped angular frequency is $\omega_d = \omega_n \sqrt{1-\varsigma^2} = 8.769 \text{ rad/sec}$ Frequency of damped oscillation $f_d = \frac{\omega_d}{2\pi r} = 1.396$ Hz NPTEL 4 1 A.K. Sinha 27/30

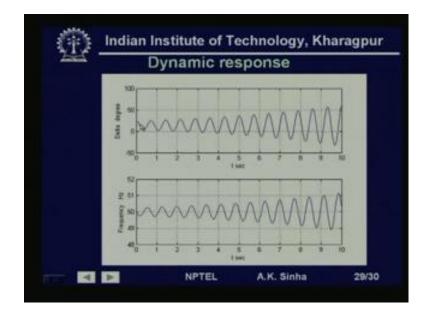
And because of this negative damping, the substituting the values in the expression for the dynamics of the system, we get d 2 delta, delta by d t 2 minus 0.942. This term has become negative, because zeta is negative, d delta; delta by d t plus 76.93 delta, delta is equal to 0. And the damped angular frequency for this again is 8.769, radiance per second, which again gives you a damped frequency of oscillation as 1.396 similar to, what we have got earlier.

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Indian Institute of Technology, Kharagpur The motion of rotor relative to the synchronously revolving field is $\delta = \delta_0 + \frac{\Delta \delta_0}{\sqrt{1 - c^2}} e^{-\zeta \omega_h t} \sin(\omega_d t + \theta)$ $\delta = 13.76 + 10.38e^{Q.471t} \sin(8.769t + 74.419)$ Frequency excursion f = 50 - .2528e^{0.471t}sin(8.769t) A.K. Sinha ¥ NPTEL 28/30

However, when we see this expression for delta angle, with time then we see this as delta is equal to 13.76 plus 10.38 e to the power 0.471 t. Now, this is positive. So, with as t keeps increasing, the second term will keep on increasing. Same thing happens for the frequency term as t increases the frequency will keep increasing, which shows that the system is going through a runaway condition. That is, it is frequency will keep on increasing with time.

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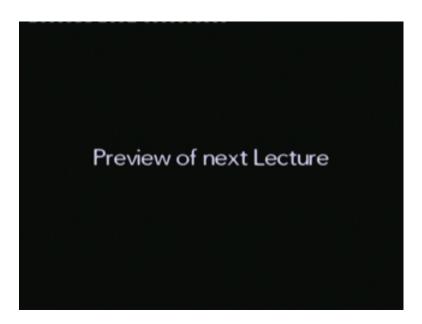


And if we do the plot, we see how the delta angle oscillations keep building up. Same thing happens, with the frequency. So, it will keep building up and the system will be in a runaway situation. So, here, we have seen, how even for small perturbations, the system can be stable or can become unstable. When either the damping term is 0 or the synchronizing coefficient, sorry, when the damping term is negative or the synchronizing coefficient is negative.

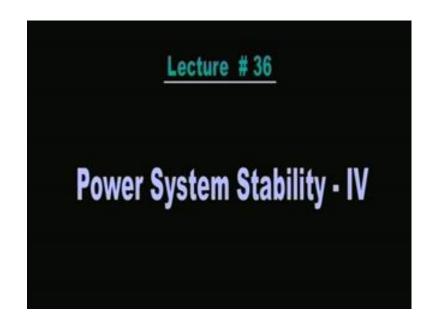
And we also saw that, there is a limit to the power transfer from the machine to the infinite bus or the system and this limit is what we call it a steady state stability limit. So, we cannot work beyond this power transfer value. So, with this, we will stop today. In the next lesson, we will talk about large signal rotor angle stability. That is, where we will talk about, what happens to the system dynamics, when large fall, large disturbances like short circuit is occur in the system. With this we end today.

And thank you very much.

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Welcome to lesson 36, on power system analysis. In this lesson, we will continue our discussion on Power System Stability. In the last few lessons, we talked about rotor angle stability in which we discussed. How, we can analyze the stability of a system for small disturbances. That is, we used small signal analysis where we used the linearized model of the system.

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In today's lesson, we will talk about large Signal Stability. That is we are going to talk about large disturbances, when they take place, how the system responds to that. So, we will start with large signal stability, when we understand here. We will then go into the equal area criterion method for analyzing the stability of a single machine connected to infinite bus system for large disturbance. And we will take some example to clarify this idea, how we use this equal area criterion.

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Instructiona	I Objective		
On completion be able to:	n of this les	son a sti	ident should
A. Explain the	e concept of	Large Si	gnal stability
	e concept o ent stability finite bus sy	analysi	
C. Analyze machine i area criteri	nfinite bus		
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Well, once we complete this lesson, you should be able to explain the concept of large signal stability. After that, you should be able develop the concept of equal are criterion for transient stability. In fact, the large signal stability that we talk about is popularly known in power system literature as transient stability. So, we will interchangeably call them as large signal stability or transient stability.

So, develop the concept of equal area criterion for transient stability analysis for a single machine, infinite bus system. And you would be able analyze transient stability for single machine infinite bus system, using equal area criterion. That is we will work out some problem on this. And since you would know how to solve this, you should be able to analyze for any other system as well.