Power System Analysis
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Lecture - 34
Power System Stability – II

Welcome to lesson 34 in Power System Analysis. In this lesson, we will continue with our discussion on Power System Stability.

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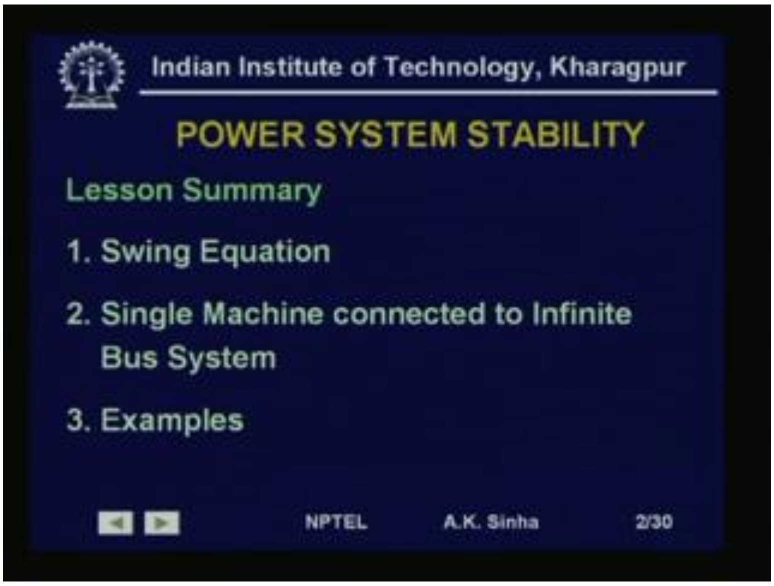

We will start with the Swing equation, which we had developed in the last lesson. And we will see, how we can include the damping term, in the swing equation. Then, we will discuss about the most important stability analysis model, which is the single machine connected to infinite Bus system. We will see, how we can convert a two machine system or even a multi machine system, in to this kind of a model, for stability analysis. And then we will take up a few examples to show how we can apply these concepts to power systems.

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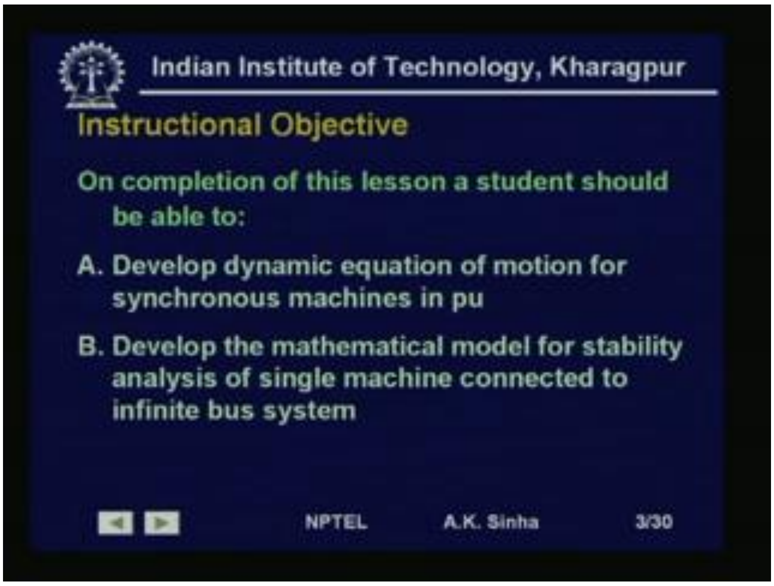

Well, on completion of this lesson. You should be develop the dynamic equation of motion for synchronous machines, which you have already done in the last lesson. But, what we will do is, we will extend this to include the damping terms. Next, you will be able to develop the mathematical model for stability analysis, of single machine connected to infinite bus system.

That is what we will do more in detail, in this lesson. And we will also see, how we can convert the different kinds of systems into this kind of a model. So, let us start with the swing equation. That we had developed in the previous lesson. We will go about it a little quickly.

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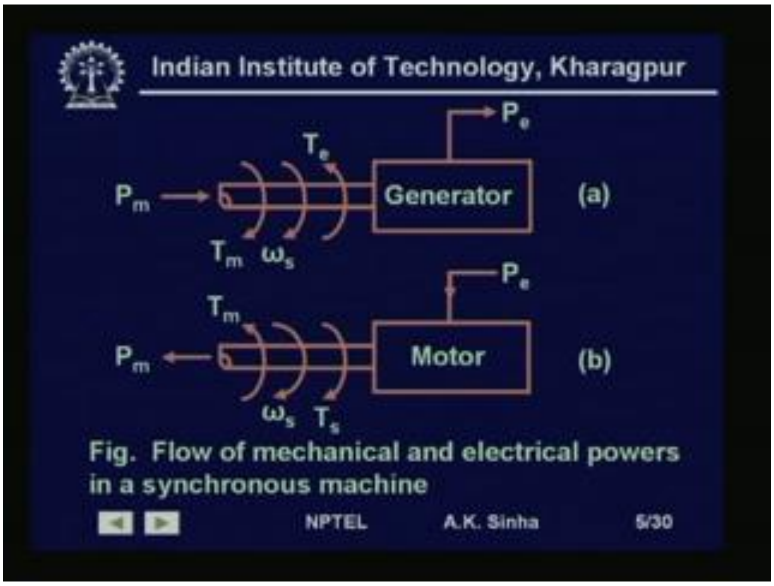

Well, we had said that if we have a system, where we have a generator. And which is supplying electrical power output to some loads. This generator to supply this electrical power output is run, by means of a prime mover. This prime mover power, that is the turbine power is the mechanical input P m. And this mechanical input is driving this rotor of the turbine, and the generator. And with this causes a torque on the turbine shaft, or the rotor shaft of the generator turbine.

This torque, we call as T m. And since, there is an electrical power output, from the generator, this electrical power is going to also exert a torque on the rotor of the synchronous machine. This torque will be opposing the mechanical input torque. So, this we have shown as T in the opposite direction. When these two torques are equal or balanced. Then, we get a constant speed, which is the synchronous speed of the machine. So, we get omega s, which will be a constant if T m and T are equal.

The same situation holds good, in case of a motoring machine. That is when we have a synchronous motor. Only thing is here, the electrical power will be an input. And the mechanical power will be an output. So, the same relationship will hold except that both the electrical and mechanical power. In this case, will have to be taken as negative of what we have taken in case of generator system. So, for generator system, we consider this P m and P e as positive, similarly this T m in this direction and T e in this direction.

We consider T e, in this direction, we consider them as positive, whereas in case of motor, they all will be having negative sign.

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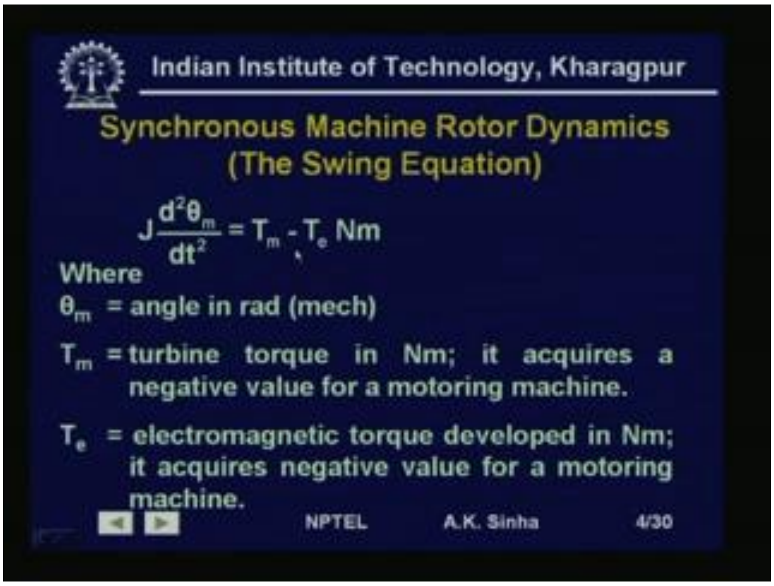

So, going back here, whenever there is a difference in these two torques. That is the torque T m and T, if T m is more than T e. Then, what is going to happen is this rotor shaft is going to accelerate. And that is, there is going to be an acceleration of this rotor shaft. And this acceleration will be given by this relationship, J d 2 theta m by d t 2, where d 2 theta m by d t 2 is basically the acceleration.

That we have theta m is the rotor position, with respect to a stationery reference frame. And T m of course, is the mechanical torque input, T e is the electrical torque, which is the output. So, T m minus T e all these are in N Newton meters. So, theta m as we said is the rotor position, which is in mechanical radiance. T m is the turbine torque in Newton meter, and where as we said in case of motoring machine, since this will be an output. So, this will have a negative value.

Similarly, T e is the electromagnetic torque, developed in Newton meter. This is the electrical, since electrical power is an output. So, the torque develop is the electromagnetic torque. And this we as a Newton meter, if we have the synchronous motor. Then, this will be negative value. So, this all we had already discussed in the last lesson.

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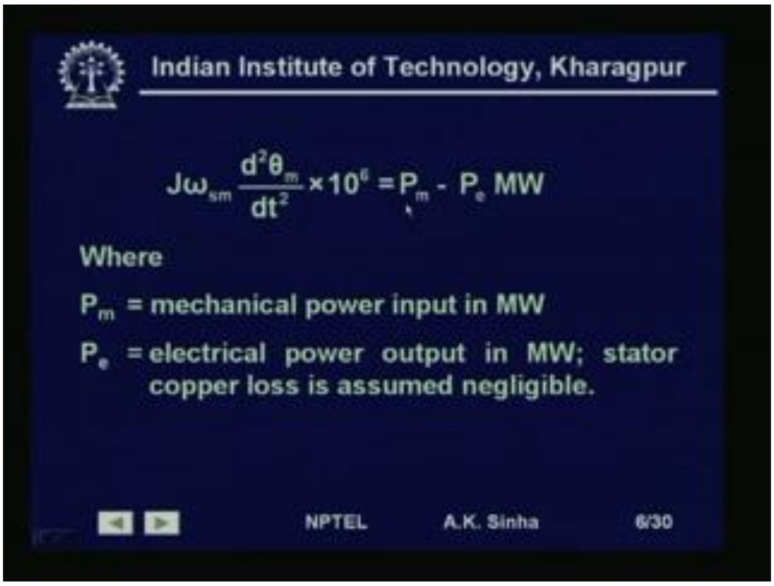

Now, instead of working with torque, we normally work with power. So, what we do is we multiply this relationship with the mechanical speed, in radiance per meter. Then, we get J omega s m, where this is the mechanical speed of the rotor s is denoting synchronous speed. So, since rotor will be rotating very near to the synchronous speed. That is at the time, when we are talking of this acceleration taking place.

So, we write J omega s m into d 2 theta m by d t 2 into 10 to the power minus 6, we have taken, because we want to write this expression, in terms of mega watts into. So, into 10 to the power 6 is equal to P m minus P into mega watt, I think this should be 10 to the power minus 6. So, this should be 10 to the power minus 6. Because, we want to write this in mega watt, if we do not multiply this, then this will be in terms of watts.

So, J omega s m d 2 theta m by d t 2 in to 10 to the power minus 6 is equal to P m minus P e in mega watts; where P m is the mechanical power input. And P e is the electrical power output, all these are in mega watt. What we have done in this case, that we have assumed. That there is no losses, taking place in the electrical machine, that is P m minus P e, for a synchronously rotating rotor will be 0. There is no loss, which is taking place. One can always take care of losses, if any one wants by modifying P m or P e to take care of that.

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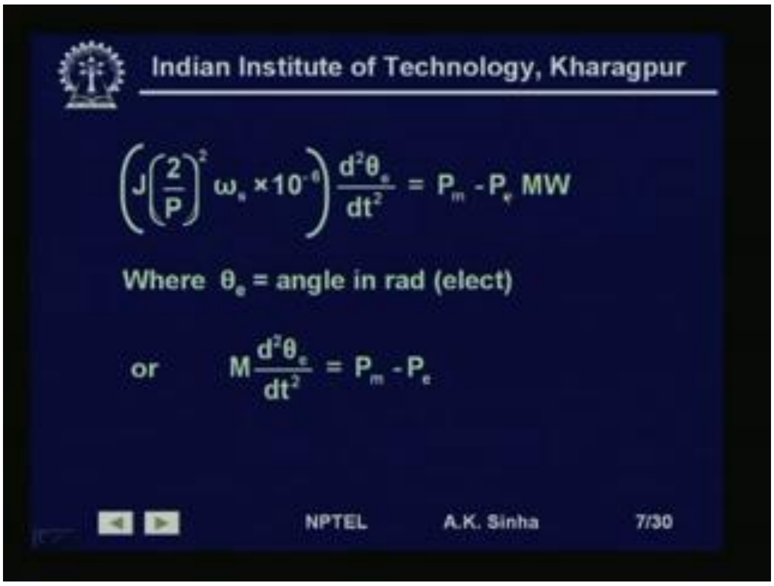

Now, instead of working with the mechanical speed. We normally work with the electrical speed. And we had already seen the relationship, between the electrical speed and the mechanical speed or the electrical angles and the mechanical angle. This relationship is the electrical quantities are equal to P by 2 times the mechanical quantities. In terms of speed or in terms of angle

So, what we will get is since we have omega s m, which we want to write as omega s. Then, omega s m will be equal to 2 by P into omega s. Because, omega s is equal to P by 2 omega s m, so we will get 2 by P here. Similarly, here we have the theta m, in the earlier expression. That is this was theta m. And we want to write it in terms of electrical angle, so our electrical position of the rotor. So, we will get again 2 by P into theta e here.

So, this 2 by P is again taken here, so it becomes 2 by P square. So, J into 2 by P square omega s into 10 to the power minus 6 d 2 theta e d t 2 is equal to P m minus P e mega watt, where theta is the angle in electrical radiance. Now, this term in the bracket as we shown earlier. Is what we call the angular momentum of the rotating mass, which is the rotor of the turbine and the generator system.

So, this we write as M, so we get M d 2 theta e by d t 2 is equal to P m minus P e. Now, what we normally do is, because this theta e will keep changing with time. That is with every rotation theta e goes, that is every 1 cycle of electrical rotation. This theta e changes by 360 degrees electrical. And therefore, it will keep on changing with time. So, instead of working with this theta e, which is with reference to a stationery reference frame. We work normally with a rotating reference frame, which is rotating at synchronous speed, so with respect to a synchronously rotating reference frame. This value of theta, will be much smaller. And it will not keep changing, unless there is a change in the speed of the rotor from the synchronous speed.

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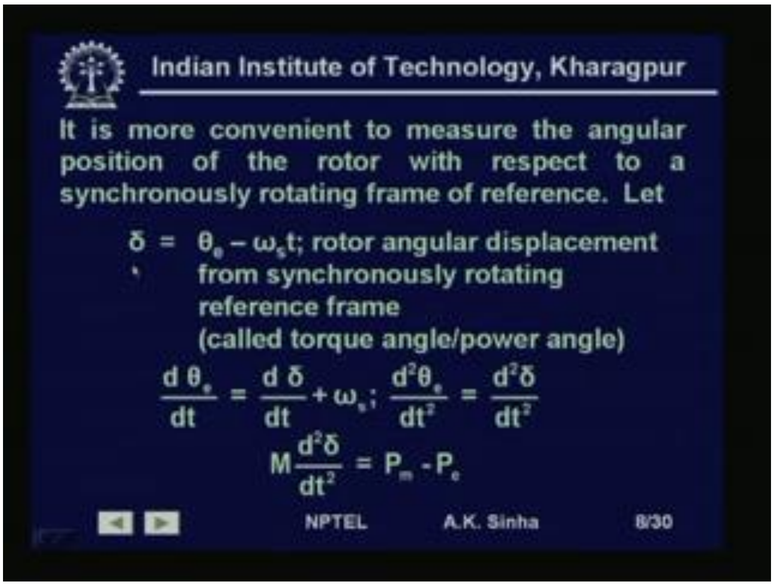

So, as I said, it is more convenient to measure the angle or the position, angular position of the rotor with respect to a synchronously rotating reference frame. So, if we do that, that is we write that angular position as delta. Then, delta will be equal to theta e minus omega s into t. At synchronous speed of course, theta e will be equal to omega s into t plus the angle, which is going to be it is reference.

So, we have delta is equal to theta e, because this theta e every cycle is equal to omega s for into time for 1 cycle. So, we subtract this, then we get the angular position with respect to the rotating reference frame. So, delta is equal to theta e minus omega s into t. Rotor angular displacement from synchronously rotating reference frame. So, delta is this rotor angle, displacement from synchronously rotating reference frame. This delta is generally called as the torque angle, or the power angle.

Now, in this relationship, we want to replace this theta e with delta. So, we need to find out the value of d 2 theta e by d t 2, in terms of delta. So, what we do is, we take this omega s t on the other side, and differentiate it once. Then, we will get d theta e by d t. So, d theta e by d t will be equal to d delta by d t. Plus this omega s into t was on this side. So, this was plus omega s t and once, we when we differentiate it with respect to t, we will get omega s here. So, d delta by d t plus omega s.

Differentiating it again, we will get d 2 theta e by d t 2 is equal to d 2 delta by d t 2. Here, omega s is a constant, so this differentiation will be 0. So, we get d 2 theta e by d t 2 is equal to d 2 delta by d t 2. And therefore this relationship, which we had, in terms of the electrical angle with respect to our stationery reference frame. Can be now, written in terms of rotating reference frame. Or in terms of power angle as M d 2 delta by d t 2 is equal to P m minus P e.

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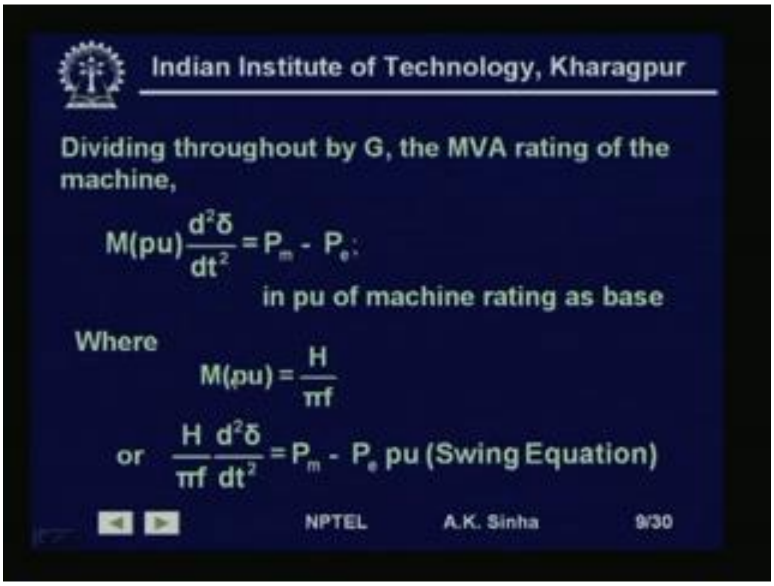

And many times since we have said from the very beginning, that we work mostly with the per unit system. Therefore, dividing this expression by the rating MVA rating of the machine, we will get the per equation in per unit. So, we will get M per unit into d 2 delta by d t 2 is equal to P m minus P e, where the power P m and P e are also in per unit instead of in mega watts.

Here, M per unit as we had seen earlier, we had written that kinetic energy is equal to G into H, where G is the rating of the machine. And H is the inertia constant. Or we have defined the inertia constant H, as the kinetic energy of the rotor at synchronous speed divided by the rating of the machine in MVA. Whereas, kinetic energy we have taken in terms of mega joules.

So, using that relationship, we had shown in the last lesson. That M is equal to H by pi f. So, replacing this M by this value H by pi f, we can write this expression as H by pi f d 2 delta by d t 2 is equal to P m minus P e in per unit. This equation, we call as the swing equation of the synchronous machine. Now, in this whole expression, we have not considered the damping torque, at all. The mechanical torque input and the electrical output torque, were considered, whereas we did not consider the damping torque. But, whenever there is going to be a relative motion, between the synchronously rotating magnetic field, in the air gap. And the rotor positions, or the rotor speed. Then, there is going to be a voltage developed in the rotor bars. And since, these damper bars placed on the rotor. And since, these damper bars are short circuited, there is going to be a current which will flow in these damper bars.

This interaction of this current with the field, will produce a torque, which is going to oppose the motion. So, whenever there is a relative motion, this motion is going to be opposed by the damping torque. So, damping torque will always be opposing the relative motion.

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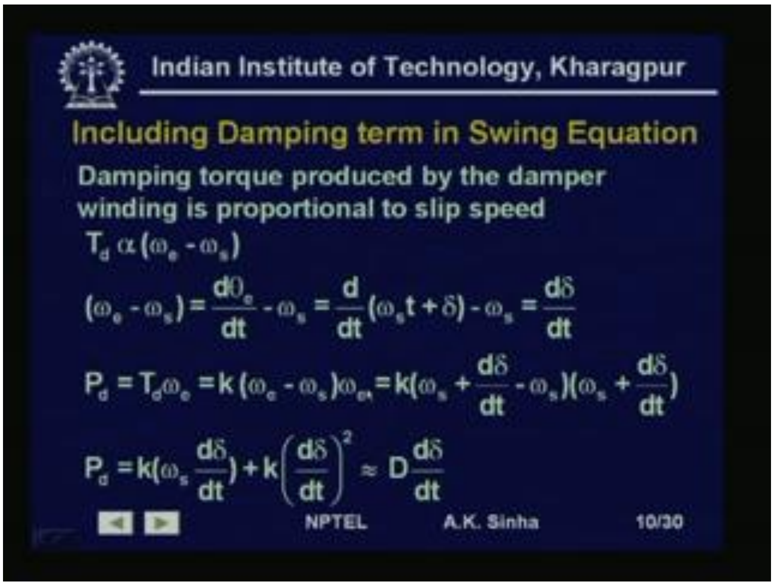

So, damping torque produced by the damper winding. As we said is proportional to the slip speed. Because, now this damping torque is produced only, when there is a relative motion. Whenever, there is a relative motion, depending on this relative motion only the voltage gets produced. So, the voltage is going to be proportional to this relative motion and since, the bars will have a constant resistance.

So, the current will also be proportional to this relative motion. And thereby the torque produce is also going to be proportional to this relative motion, which we call as the slip speed. So, the damping torque is proportional to the slip speed, the speed difference from the synchronous speed, and the actual speed of the rotor. Therefore, we are writing damping torque T d, as proportional to omega e minus omega s; where omega e is the actual rotor speed in electrical radiance per second.

Omega s is the synchronous speed of the rotor, in electrical radiance per second. So, T d is proportional to omega e minus omega s. And we have omega e minus omega s, can be written as omega e is nothing but d theta e by d t. That is with respect to the stationery reference frame. The angular position, the derivative of the angular position will give us the speed of the rotor, in terms of electrical radiance per second. So, we are writing this as omega e is d theta e by d t and omega s of course, is the synchronous speed.

So, this we can write now theta e. We can write as omega s t plus delta. As we had seen earlier, when we define the synchronously rotating reference frame, ((Refer Time: 19:20)) theta e is equal to omega s t plus delta from here. So, we are simply substituting this value here. So, d by d t omega s t plus delta minus omega s. So, this comes out to be d delta by d t s, the differential of derivative of d omega s t with respect to time t is equal to omega s, which will cancel with this omega s. So, we have got d delta by d t.

Now, the damping power P d will be equal to T d into omega e. And T d, we are saying is proportional to omega e minus omega s. So, we can write this as k times omega e minus omega s into omega e this omega e is given here. So, P d is equal to K omega e minus omega s into omega e. This is equal to k times omega s plus d delta by d t minus omega s. That is from here, we are writing this.

So, actually we can write this omega e as that is. So, omega e is omega s plus d delta by d t minus omega s into this omega e. Again we can write this as omega s plus d delta by d t. So, we have got this relationship now P d is equal to k omega s plus d delta by d t minus omega s into omega s plus d delta by d t. This omega s omega s will cancel out.

So, we have got this relationship now, as k into omega s into d delta by d t. That is d delta by d t. We multiply with this omega s.

So, we get omega s d delta by d t, this d delta by d t we multiply with this d delta by d t. So, we get d delta by d t whole square. So, we get this as k omega s d delta by d t plus k d delta by d t whole square. Now, d delta by d t will be generally small, the change in the rotor speed from synchronous speed is not very large. Therefore, this term d delta by d t whole square is going to be very small, and we can neglect it.

So, we can write this as k omega s d delta by d t, approximately equal to that. And this we generally write as d into d delta by d t, where d is called the damping coefficient of the synchronous machine. So, with this P d is equal to d delta by d t, we can now put this damping power in the swing equation of the synchronous machine.

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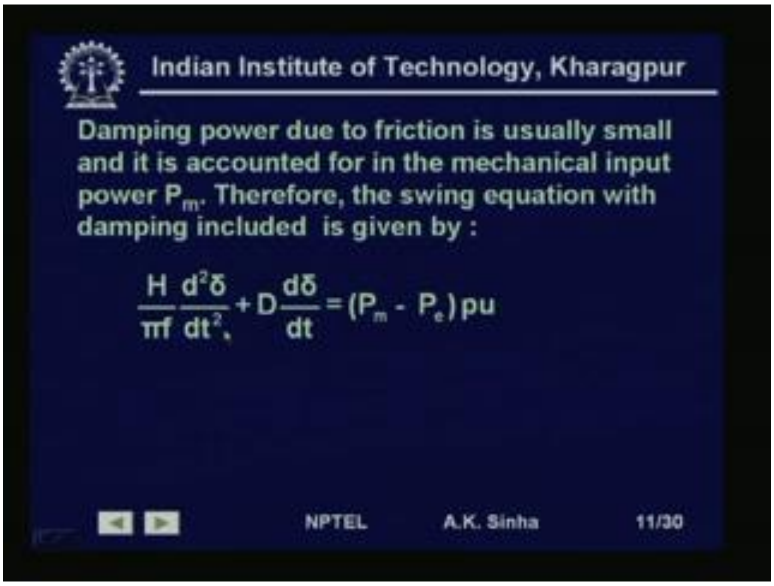

In that case, we will get this as H by pi f d 2 delta by d t 2 plus d delta by d t is equal to P m minus P e. That is the expression that we will get. In this case, we have not included one more damping term, which comes because of the friction. That is there is a friction power, or damping power due to friction, is also there. Normally this power is much smaller. And this is more or less constant. And this is normally taken care of by subtracting it, from the mechanical input itself.

So, when we talk about the mechanical input. We are taking about the net mechanical, input which has already taken care of the frictional losses. Or the frictional power loss, which takes place in the machine. So, this is already taken care of and therefore, with the damping the swing equation, now we have is like this. Now, we will take one small example to show, how we can write the swing equation for a synchronous machine.

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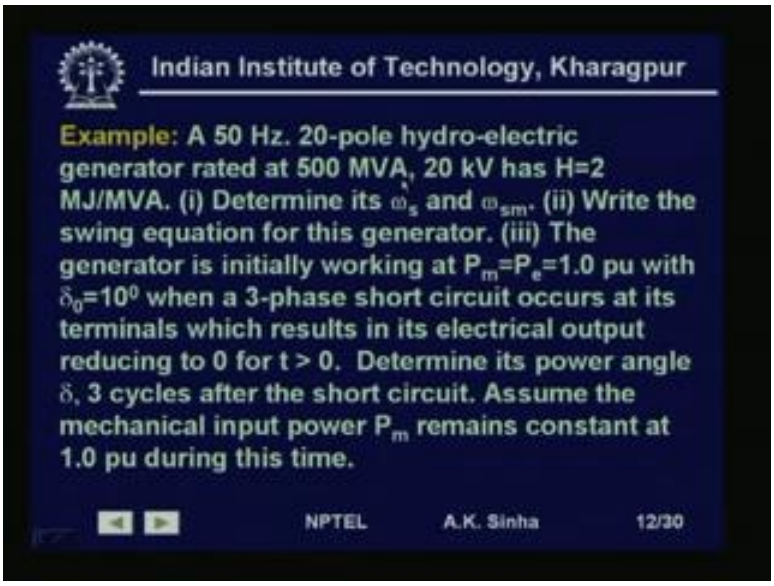

So, let us take this example, we have a 50 Hertz, 20 pole hydroelectric generator, rated at 500 MVA, 20kV. And it has H is equal to 2 mega joules per MVA. Now, here we are seeing synchronous machine with 20 poles. Now, normally with hydroelectric generators, the speed is much lower compared to that of a thermal generator or turbo generator, where the steam turbine work at high efficiency at high speeds.

And normally, the speed for these generators will be of the order of 3000 rpm. So, we need only 2 poles for these machines. Whereas, for hydroelectric generators the speed depends to some extent, on the type of the turbine we have. As well on the head available, that is the height of the water, which is available. And these speeds are generally much lower. And just so in order to generate power at 50 Hertz, we need much larger number of poles.

That is when we have 20 poles on the system, the speed is going to be much. That is at much lower speed, we will be able to generate power at 50 Hertz. So, determine it is omega s, that is what is the synchronous speed of this machine? And what is the synchronous speed in mechanical radiance per second, as well as in electrical radiance per second. Second part is write the swing equation for this generator. So, we need to write down the swing equation. The third part is the generator is initially working at P m is equal to P e is equal to 1 per unit, with delta is equal to 10 degrees.

That is initially the delta angle or the power angle is 10 degrees. When a three phase short circuit occurs, at it is terminals, which results in it is electrical output reducing to 0 for t greater than 0. That is a t is equal to 0 plus, we are assuming of three phase short circuit occurs. At the terminals of the machine since a three phase short circuit has occurred on the terminals of the machine, the voltage goes to 0. And thereby, there is no electrical power output from the generator.

So, at t is equal to 0, the electrical output with at 0 plus, electrical output becomes 0 for all t greater than 0. Determine it is power angle delta 3 cycles after the short circuit. That is after the short circuit, after 3 cycles what is going to be the angle delta? Actually, what happens, we have the mechanical input to the machine, before the fault was equal to 1 per unit which was same as the electrical output. So, P m minus P e was in that case 0.

And the speed of the machine was synchronous speed. And the delta angle was constant, whereas when at t is equal to 0 the fault occurs. The P m remains same, because the mechanical input to the generator changes much slowly. Whereas, the electrical output from the machine at t is equal to 0 plus has reduced to 0. So, P m minus P e, now is equal to 1 per unit. And this is going to cause an acceleration of the rotor, of the turbine generator system.

And because of this acceleration of the rotor, that delta angle or the power angle of the machine is going to change. And we are asked to find out this power angle after 3 cycles. So, this says determine it is power angle delta 3 cycles after the short circuit, assume the mechanical input power P m remains constant at 1 per unit during this time. This is what we said, normally the power input mechanical power input it does not change, in this short period. And therefore, we are assuming it to remain constant at 1 per unit.

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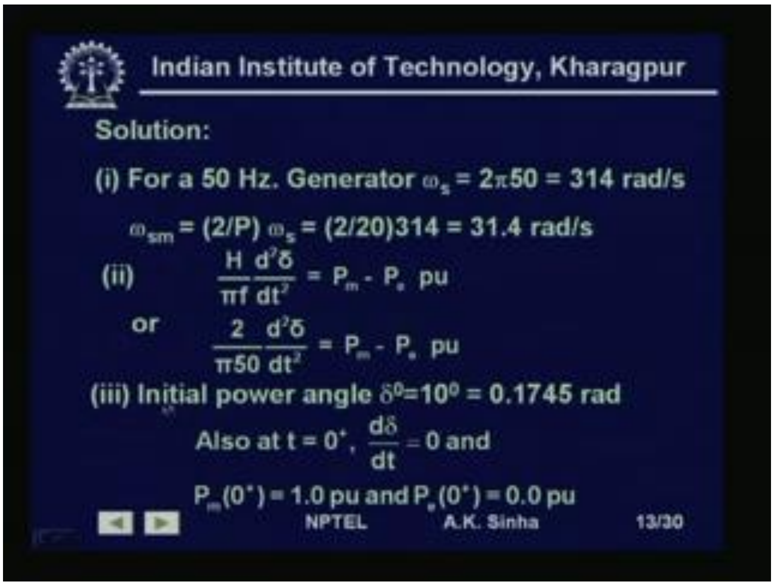

Now, for the first part, we know that the frequency is 50 Hertz. So, the speed of the synchronous speed of the generator omega s, will be equal to twice pi f, which is equal to 314 radiance per second. So, twice pi 2 pi into f, f is 50 Hertz. So, this comes out to be 314 radiance per second. Now, omega s m the mechanical speed is given by 2 by P into omega s. Now, in this case we have p is equal to 20.

So, we have 2 by 20 into omega s 314 radiance per second. So, this speed comes out to be 13.4 radiance per second. So, we see that, the speed of this generator will be in terms of rpm will come out to be 300 rpm, whereas in case of for a 2 pole generator, if we were running, then the speed would have been 300 rpm. So, for a 20 pole generator, the speed is 31.4 radiance per second. And we can convert it in rpm, then we will find that this comes out to be around 300 rpm.

Second part is we need to write down the swing equation for this machine. Swing equation as we have seen earlier. We write down as H by pi f into d 2 delta by d t 2is equal to P m minus P e in per unit. Now, here the H of the machine has been given as 2 and f is 50. So, we simply write 2 by pi 50 d 2 delta by d t 2is equal to P m minus P e, this is the s, we swing equation for this machine.

So, we can write down the swing equation for any machine, knowing it is inertia constant and the frequency. And if we know these powers, then this is P m is equal to 1 and P e, whatever is the output. If there is difference, we are going to get an acceleration. If these two are equal, the acceleration will be 0, the speed will remain constant. Now, third part is, we have been told that initial power angle delta 0 is 10 degrees, which is equal to 0.1745 radiance.

Also at start t is equal to0plus d delta by d t is equal to 0. That is just after the fault has occurred the speed cannot change much. So, the speed at t is equal to 0 plus is 0, only that is d delta by d t is 0 to change in speed from the synchronous speed is going to be 0. So, d delta by d t is equal to 0 . Now, P m 0plus is equal to 1, that is we have assumed, that the mechanical power input has not changed. After the fault for the time under consideration. And P e 0 plus is equal to 0.0, since the fault has occurred. So, after the fault we have, immediately after the fault we have this power output as 0.

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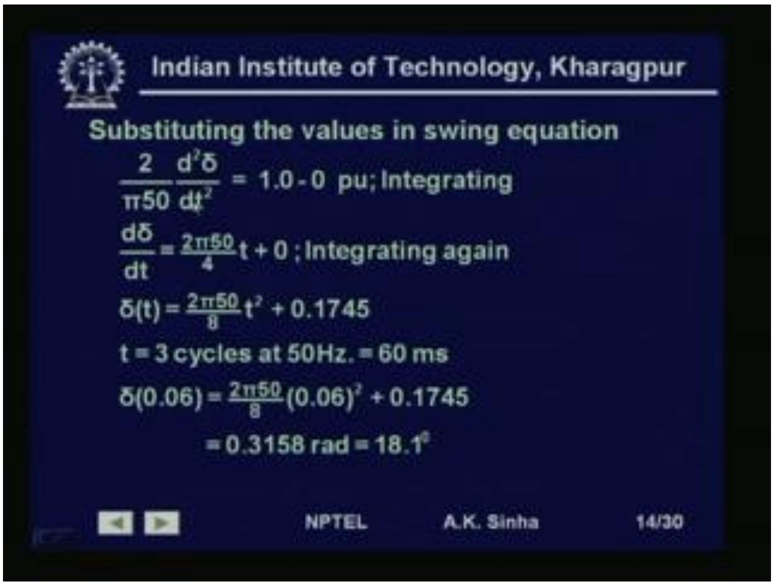

Therefore, now substituting these values in the swing equation, we will get 2 by pi 50 into d 2 delta by d t 2 is equal to P m is 1and P e is 0. Now, this is the swing equation which is valid for this operating condition. Now, integrating this once, we will get d delta by d t is equal to twice pi into 50 by 4 into t plus 0. That is we are writing instead of pi into 50, we are multiplying it by 2 on numerator and denominator. So, twice pi into 50 and this becomes 2 into 2 4.

And when we take this on this side, then we have d 2 delta by d t 2 is equal to 2 pi 50 by 4 into1. Now, integrating this, we will get d delta by d t is equal to 2 pi 50 by 4 into t plus 0. Because, we will get plus d delta by d t at 0 plus we have seen is 0. So, the constant of integration here is 0. Now, integrating again, we will get d delta with respect to time. This is the function of time is equal to twice pi into 50 by 8. We are integrating this t, so we will get t square by 2.

So, just substituting that, we will get twice pi 50 by 8 t square plus, the constant of integration. And we have seen delta 0 is equal to 0.1745 radiance. So, we are substituting that value here, and we get d delta at any time t from this expression. Now, we have said t is, we want to find out the delta after 3 cycles. So, t is equal to 3 cycles and at 50 Hertz this comes out to be 60 millisecond

Therefore that is 0.06 second. So, substituting for t delta at 0.06 second is equal to twice pi into 50 by 8 into t square, that is 0.06 square plus delta 0, which is equal to 0.1745. And when we do this we get this value as 0.3158 radiance, which is equal to 18.1degrees. That is in just 60 milliseconds, we have seen the rotor angle has changed from 10 degrees to 18.1 degrees. So, this is how we can try and find out the rotor angle, by solving the swing equation for the given machine.

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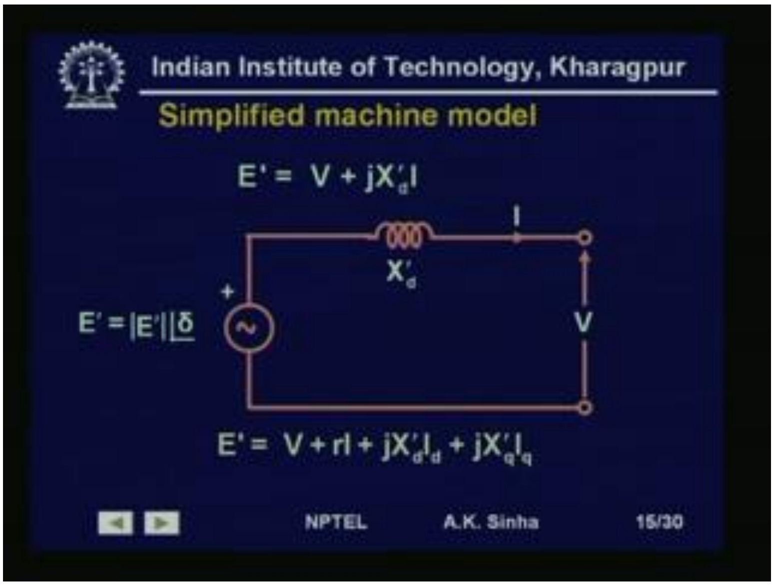

Next, we will try to see how we apply this to for a stability analysis of different systems. Now, here we will start with first, using the synchronous machine electrical model. Now, here, we have already seen earlier, that simplified model for the electrical machine can be seen as a voltage source. In series with a reactance, which is direct axis reactance, when we are talking about the cylindrical rotor machine. Then, we have the internal voltage E dash is equal to V plus j X d dash into I.

Normally, when we are working for stability analysis. Since, we are talking in terms of transient small periods, in which we are talking about the dynamics of the rotor of the machine. So, we are using the transient reactance of the machine. So, this is what we will get, in case of a cylindrical rotor machine. But, if we are using a salient pole machine model, then the internal voltage E dash will be given by V plus r into I. I have used the resistance also, normally this resistance is negligible, and one can neglect this term r into I.

Otherwise, if we want to use it, then this can be used. So, V plus r I plus j time X d dash into I d plus j times X q dash into I q. This is the expression, which will give the internal voltage of the machine. And that is the internal voltage E dash, will have a magnitude E dash. And an angle delta and this internal voltage, with it is reactance as shown for this salient pole machine. Or for the cylindrical rotor machine will give us the machine model.

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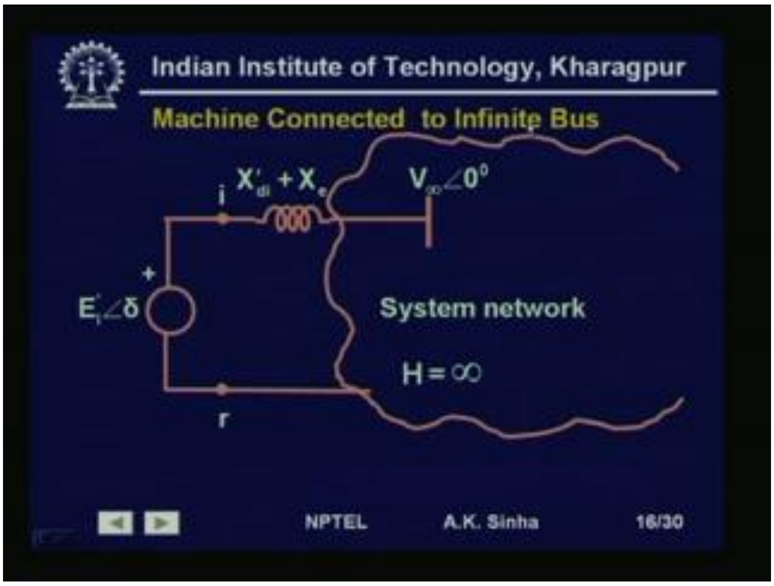

Now, we will go into the most commonly used model, for transient stability analysis, as we had said earlier, when we talked about the definition of the stability. That many times, we are interested in studying the stability of a single machine, or a group of machines. Not all the machines in the system or the complete system as such. So, this is one model, which is very, very used in power system stability analysis, which we call a single machine connected to infinite bus system.

Actually what we can see this as, if we have connected any machine to a very large system. Then, this system being very large, this will have very large inertia. And any change in power in this is hardly going to affect the speed, or the frequency of this system. Also since, this system is very large and it has a very large number of generators. the reactive power reserve will also be large enough.

So, any changes in reactive power flow from this. is not going to change the voltage magnitude of this bus. That is in terms of active power. As well as reactive power, the changes caused by this machine are small to create. any changes in the behavior of this machine. And that is behavior of this system. And that is why we call this system as an infinite bus.

That is we say, this machine is connected to an infinite bus, that voltage of which we write normally as V infinity. And since, the rotor angle for this system, is not going to change with the power flow from this machine. We normally use this voltage angle, or the power angle of this bus, where this machine is connected to this system, as the reference voltage V's, which we put as 0 degrees. So, the voltage at this bus V infinity is not going to change.

It remains constant or invariable with any change, which takes place with this machine. As well as it is angle does not change. And therefore, we can think of this infinite bus, as a ideal voltage source for complex voltage. That is it is a ideal voltage source, whose voltage magnitude, as well as angle do not change at all. So, in terms of circuit analysis, we had talked about ideal voltage source, where the voltage does not change. Whatever may be the power, which we draw from it.

Similarly, the voltage of this bus is not going to change. That is the complex voltage of this bus is not going to change, whatever may be the power drawn or supplied from this bus. So, we can think of this system, which we call as the infinite bus, as an ideal voltage source for complex voltage. So, if we look at that, then we can see here this machine is connected to this infinite bus or this large system, by means of an external reactance.

This reactance will normally be the reactance of the transformer. And the transmission line through which this generator gets connected to the system, which we write as X e. And X dash d is the reactance of this machine. That is direct axis transient reactance of this machine. Here, I had shown this is the i th machine. Similarly, we can think of all different machines. And when we are trying to study, the stability of any single machine this can be any machine one machine in the system.

So, this is how this model is prepared. And the electrical model of this can be given as the machine internal voltage, E dash angle delta. X d dash is the direct axis reactance of the machine. And X e is the external reactance, which is the reactance of the transformer or the transmission line. This is connected to a infinite bus, whose voltage is V infinity. And angle is 0 degree. And this machine is supplying a power to the system, or the infinite bus and this power is P e. So, this is the system, that we call as the single machine connected to infinite bus system, and as we had seen, in case of the synchronous machine analysis.

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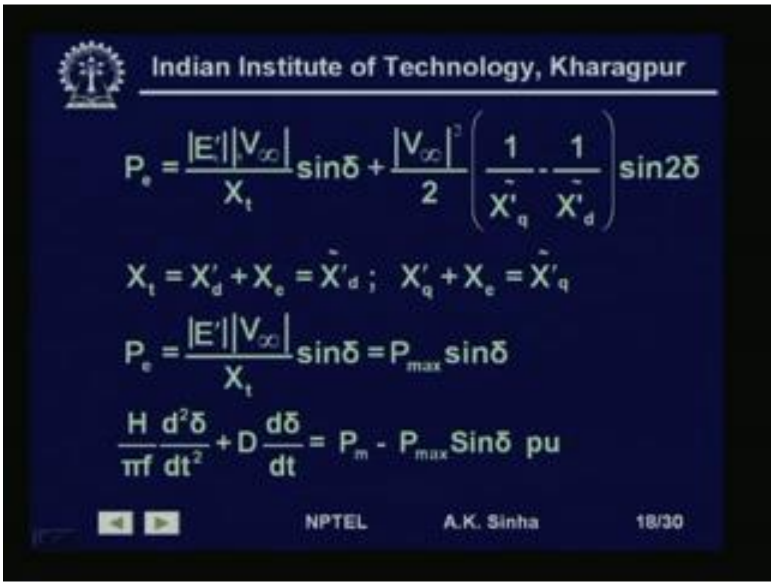

The power delivered by this machine to the infinite bus is given by this relationship P e is equal to E dash V infinity by X t, where X t is the transfer reactance. It is the total reactance between the internal voltage of the machine up to the infinite bus. So, X t sine delta plus V infinity square by 2 into 1 by X d X q dash minus 1 by X d dash. This hats are taken to take care of the total reactance, in the direct-axis and the quadrature-axis as such.

That is we define X t as X d dash plus X e and X d dash hat is equal to the same reactance. That is X t and X q dash hat is X q dash plus X e. That is what we have done is that, we have included the external reactance along with the direct, and the quadrature axis reactance of the machine. Normally, if we are considering a cylindrical rotor machine, which we do most of the time. Then of course, X d dash and X q dash will be equal.

And therefore, this second term vanishes, that is it goes to 0. This minus this is, since these two terms are same, so this will be 0. And we get the relationship, which is P is equal to E dash into V infinity by X t into sin delta, this is the relation which we had seen n number of times. We write this E dash V infinity by X t as P max. So, we get P is equal to P max sin delta. That is the electrical power output, from the machine to the infinite bus is dependent on the power angle delta of the machine.

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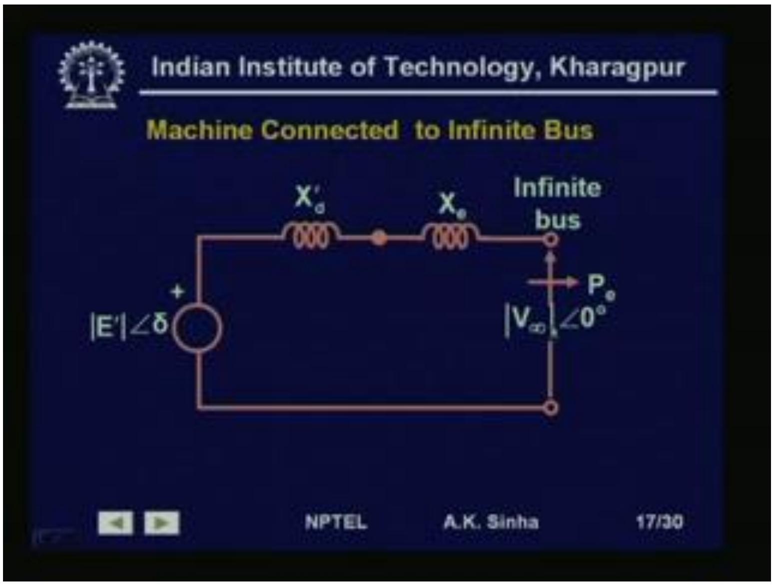

That is it depends on, by how much this angle delta is leading the voltage angle at the infinite bus. If it leads this, then the power is being transferred from the synchronous machine to the infinite bus. If it large, then the power will be transferred from the infinite bus, towards the machine. That it will start acting as a motor. ((Refer Time: 45:00)) So, now, since we know that the electrical power output is given by this relationship. So, we can replace this relationship into the swing equation of the machine.

So, we can write this swing equation for this machine. Now, as H by pi f d 2 delta by d t 2 plus d by d delta by d t is equal to P m minus P e. Instead of P e, we are now writing this as P max sine delta. This is a second order non-linear differential equation. Non-linearity cropping up, because of this trigonometric function here. And this is a second order differential equation. So, we need to solve this second order non-linear differential equation. For this simplified single machine infinite bus system, when we want to study the stability of the system.

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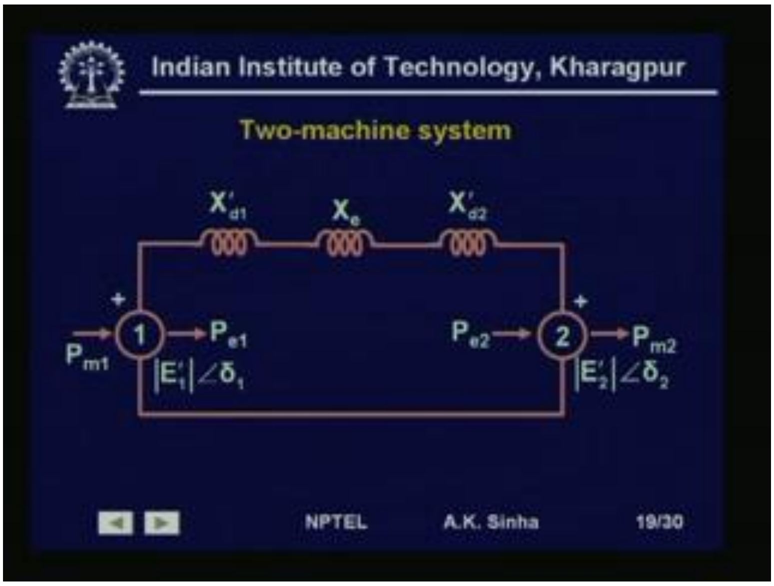

Now, let us see, how we can convert different systems into a single machine infinite bus kind of a model. Let us take the situation, where we have two machine, two synchronous machine connected to each other. We have X d 1 dash, as the direct axis transient reactance of machine 1 X d 2 dash. As the direct axis transient reactance of machine 2 and X e is the reactance by which these two machines are connected to each other. It is the reactance of that line which connects these two machines.

Now, certainly when we have connected these two synchronous machine. One will be acting as generator, another will be acting as a motor. So, machine1acting as a generator, we have mechanical power input and electrical power output. This machine internal voltage we are writing as E 1 dash with an angle delta 1. The machine 2 is acting as a motor. So, electrical power is input to this and mechanical power is output to this. So, and it is internal angle, we are writing as E 2 dash with an angle delta 2. Now, since we have assumed, there is no losses taking place in the system. So, we will have P e 1 is equal to P e 2. And similarly P m 1is equal to P m 2.

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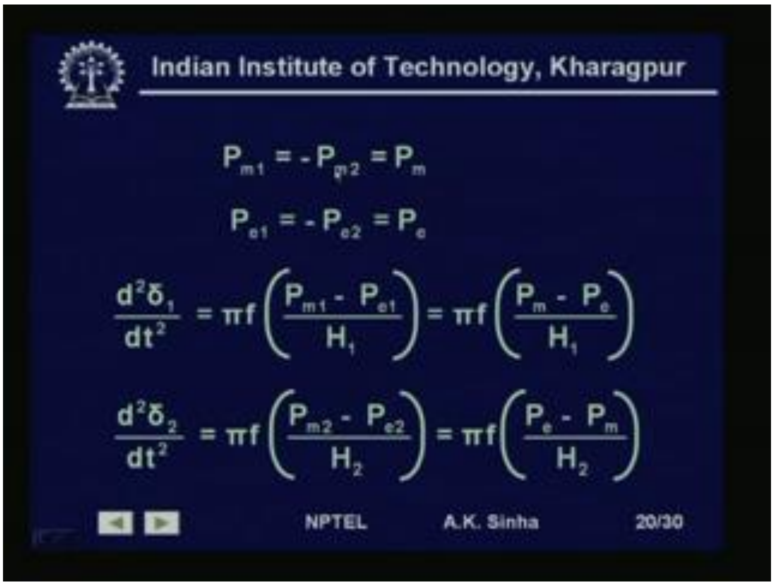

So, we are writing P m 1is equal to minus P m 2. Because, in the case of the motoring machine, machine 2, since it is motoring and the mechanical power is an output. So, we are writing this negative for that. So, P m 1 is equal to minus P m 2, which we write as P m. Similarly, P e 1 is equal to minus P e 2, because in case of machine 2, the electrical power is an input not an output. So, this negative sign is to take care of that.

So, P e 1minus P e 2 is equal to P e. And now substituting this expression for the swing equation, we will have d 2 delta 1 by d t 2 is equal to P i f by H 1 into P m 1 minus P e 1. So, we can write now this P m 1 minus P e 1, as P m minus P e from here P m 1, P e 1. So, P m 1 is P e P m and P e 1 is P e. So, we are writing pi f P m minus P e by H 1. Similarly, for the second machine, we will have d 2 delta 2 by d t 2 is equal to pi f by H 2 into P m 2 minus P e 2.

And since, we have P m 2 minus P m 2 as P m and minus P e 2 as P e. So, we this will become positive P e and this will become negative P m. So, we have got pi f by H 2 P e minus P m. Now, subtracting this two equations, we will get.

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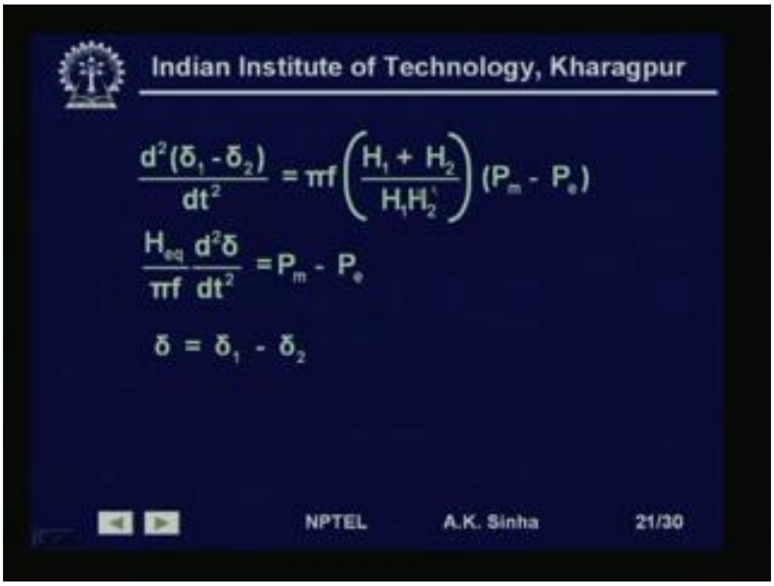

That is if we subtract these two equations, that is equation 2 from equation 1. We will get d 2 delta 1 minus delta 2 by d t 2 is equal to pi f H 1 plus H 2 divided by H 1 H 2 into P m minus P e. And this we can write as, if we take this term on this side as H equivalent by pi f d 2 delta d t 2 is equal to P m minus P e, in the same form as the equation we had. For a single machine connected to infinite bus.

Except that, in this case the H equivalent we are writing. And the angle delta we are writing. which is delta 1 minus delta 2. After all we have seen, it is we have used the reference angle as 0, which was the angle, voltage angle of the infinite bus. So, if we take delta 2 as the reference angle, here in this case. Then, we have delta 1 or the delta 1 minus delta 2 is basically the difference in the angle of the two machines.

So, this turns out to be an equation, which we are using for a single machine infinite bus system. So, the swing equation which describes, the motion of this two machine system. Can be written in this form, where we have H e q as the equivalent inertia. And delta which is delta 1 minus delta 2 is the angular difference of the internal voltages.

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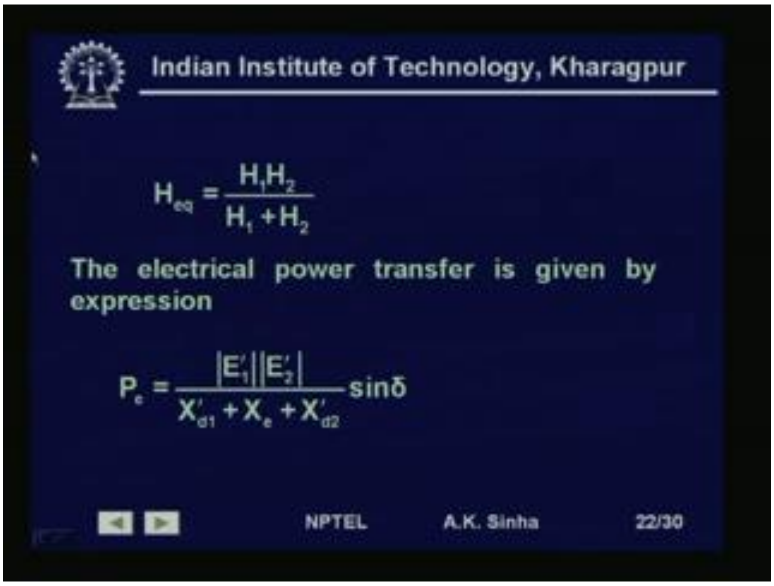

And H e q we have got as H 1 H 2 by H 1 plus 2. And the electrical power transfer in this case, will be given by we had E v by X t. So, here E is the machine 1 for internal voltage E 1dash. And V is the second machine internal voltage, that is E 2 dash. Divided by X t, which is X d 1 dash plus X e plus X d 2 dash into sin delta.

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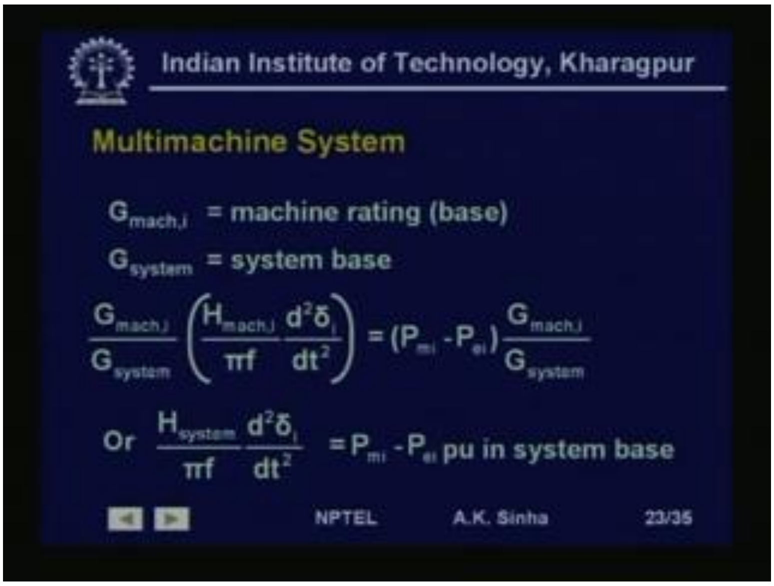

We have seen how we can convert a two machine system into a single machine system. However, this kind of a conversion to a single machine infinite bus system is not possible, for a large multi machine system. In this case, we need to write down the swing equation for each machine separately. And solve the swing equation for all the machines to look at the dynamics of the system.

Now, when we see this multi machine system. We have to understand one thing that, the machines have different ratings. And their inertia constants are normally provided in terms of the machine rating or machine base itself. So, for a machine i in the system, if the rating is given as G machine i, which is the base value. These values will be different for different machines in the system, because in a large system, we have large number of different ratings of machine.

So, we need to choose a single system MVA base for a multi machine system. So, let us say that we have chosen G system, as the system base for this multi machine system. Now, what we have to do is, we have to convert the inertia constants of all the machines like this. For machine H machine i has to be converted to the system base. And this we can do, by multiplying H machine i by G machine i divided by G system. Because, when we multiply by G machine i, we get the kinetic energy of the machine rotating part.

And we divide it by G system, then this becomes the inertia constant for that machine, in terms of the system base, because we define H as the kinetic energy of the machine, divided by the rating or the MVA base of the system. So, this is how we can convert it. So, we write the swing equation now in this form. G machine i by G system is equal to H machine i by pi f into d 2 delta I by d t 2, this is equal to P m i minus P e i.

Now, these ratings also initially would be given, in terms of machine ratings. So, here also we convert them to the system base. So, in terms of system base in per unit values. So, we will multiply it by G machine i, then it converts into MVA. And we divide it by system base. So, then this becomes converted to the system base in per unit. So, this is what will result, when we write this in on system base H system by pi f into d 2 delta I by d t 2 is equal to P m i minus P e i in per unit on the system base.

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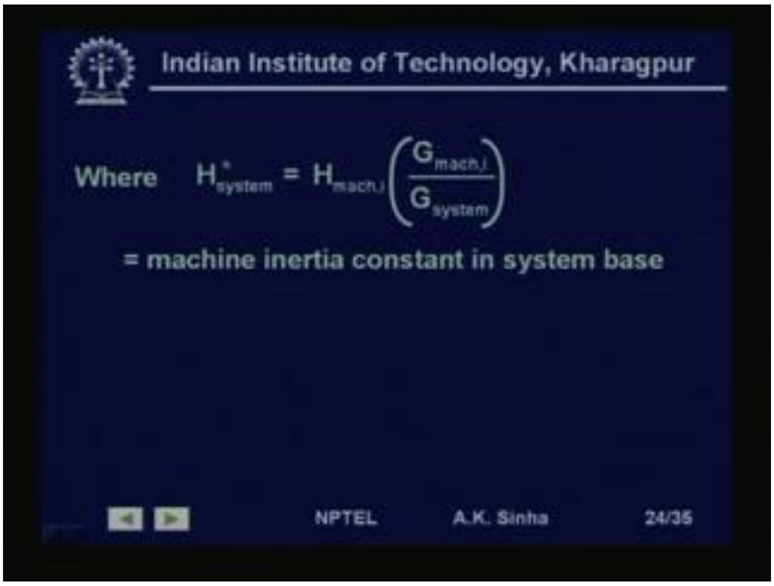

Where H system as we have seen is equal to H machine i into G machine i by G system. That is the inertia constant of machine i on a system based is given by this.

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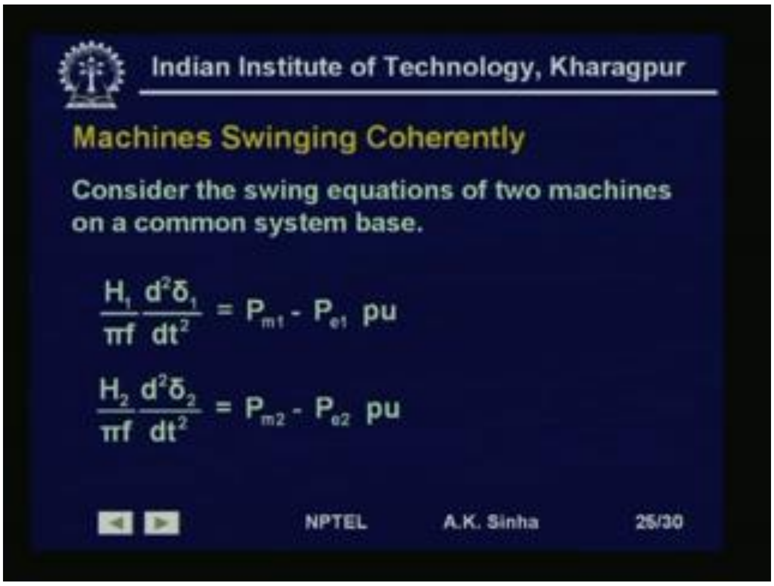

Now, when we have a number of machines connected to the system. We will find that some machines. are swinging together, whereas some other machines may not be, or may be having a different swing from each other. Specially, if we have number of machines at a particular generating station. This is going to happen, that is all those machines will be experiencing this same amount of disturbance. And therefore, they will be swinging together. These machines which swing together, we call them as coherent machines.

So, for coherent machines, we can write the swing equations and combine them. And make an equivalent single machine equation for them. So, consider the swing equation of two machines on a common system base. So, we have H 1 by pi f d 2 delta 1 by d t 2 is equal to P m 1 minus P e 1, this is for machine 1. Similarly on the same system base, we have H 2 by Pi f d 2 delta 2 by d t 2 is equal to P m 2 minus P e 2.

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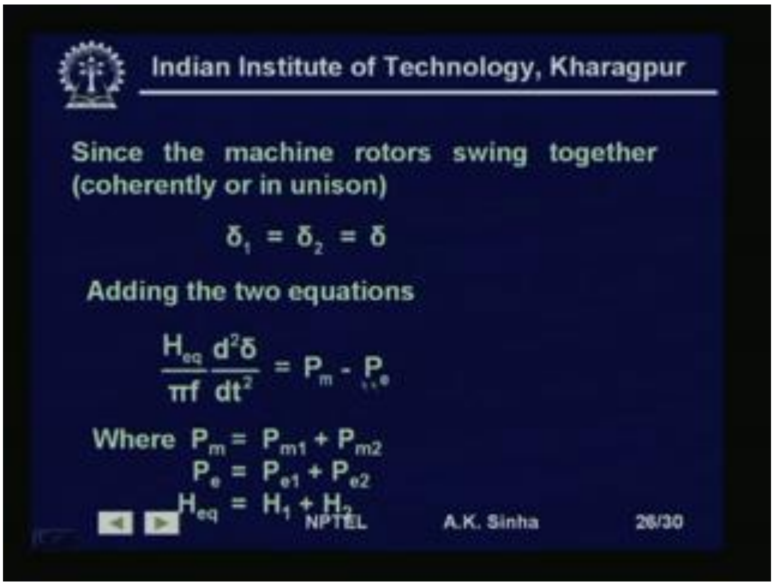

Now, if the machines are swinging together. Then delta 1 and delta 2 will be same. That is the relative motion of these 2 machines are same. So, delta 1 is equal to delta 2 is equal to delta. And therefore, adding these two equations, we can add them, if we add them, then we will get the equation in terms of H equivalent by pi f d 2 delta by d t 2 is equal to P m minus P e. Where P m is equal to P m 1 plus P m 2 P e is equal to P e 1 plus P e 2 and H equivalent is equal to H 1 plus H 2. So, this way what we have seen that the coherent machines. Their inertia can simply be added together. And we can make a single machine equivalent, for the coherent machines.

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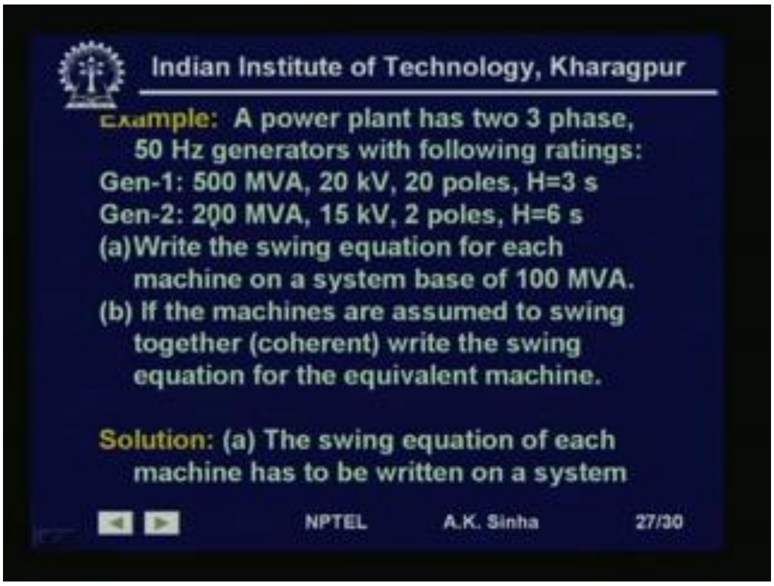

Now, let us take a very small example here. A power plant has two, three phase, 50 Hertz generators with following ratings. Generator 1, 500 MVA, 20 kv, 20 poles, H is equal to 3 seconds. Generator 2, 200 MVA, 15 kv, 2 poles H is equal to 6 seconds. Write the swing equation for machine on a system base of 100 MVA If the machines are assumed to be swinging together, that is they are coherent, write the swing equation for the equivalent machine. The swing equation for each machine, has to be written on a system base of 100 MVA.

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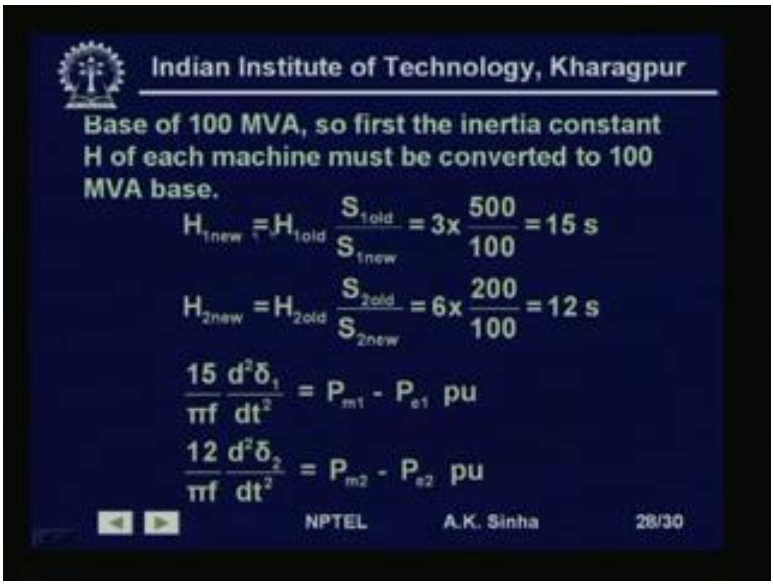

So, first we will convert the inertia of each machine to a 100 MVA base. So, H 1 new, which is the inertia of the machine 1 at 100 MVA base will be H 1old into S 1old divided by S 1 new, which is same as G old by G new. So, 3 into 500, which was the machine base for this 3 into 500. And this is the system base, so 3 into 500 by 100, which is 15 seconds. Similarly, for the second one, we will get this as 6 into machine rating 200. And this is the system based.

So, this comes out to be 12 seconds. And therefore, putting the value of H on the system base of 100 MVA, we get the swing equation as H. We have written here for machine 1, H for machine 2. As this so 15 by pi f d 2delta 1 by d t 2 is equal to P m 1 minus P e 1. And for second machine 12 by pi f d 2 delta 2 by d t 2 is equal to P m 2 minus P e 2. Now, if the machines are coherent, then we are saying delta t is equal to delta 1, t is equal to delta 2 t.

In that case, what we have seen is the mechanical powers will also add up. The electrical powers will add up and the inertia also adds up. So, we will add up the inertia, and we will write 27 by pi f, that is 15 plus 12. We have got this 15 and this 12, so inertia gets added on the same base both are there. So, 27 by pi f d 2 delta by d t 2is equal to P m t minus P e t, where P m is equal to P m 1 plus P m 2 and P e is equal to P e 1 plus P e 2. So, in this way, we can write the swing equation for coherent machines. So, with this we will stop today, in the next lesson. We are going to talk about small signal stability analysis, for these rotor angle derivations, so small signal stability analysis.