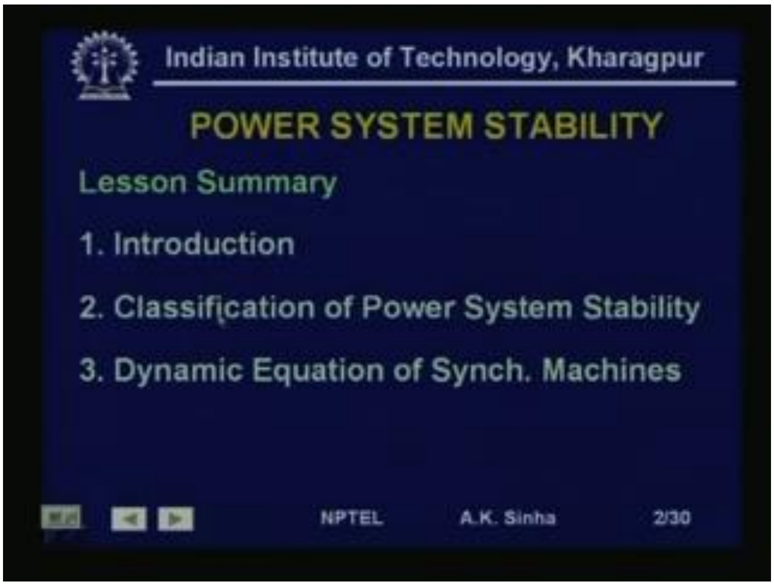
Power System Analysis
Prof. A.K. Sinha
Department of Electrical Engineering
Indian Institute of Technology, Kharagpur

Lecture - 33
Power System Stability-I

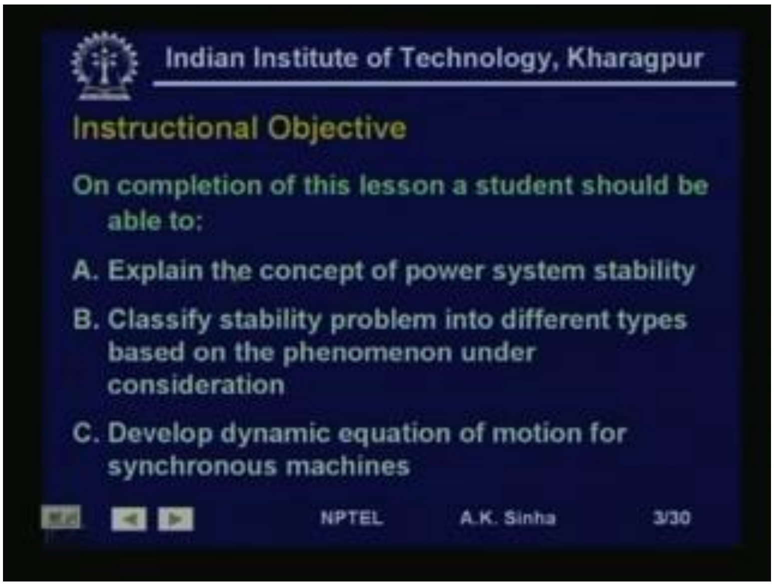
Lesson 33 on Power System Analysis. In this lesson, we will discuss the Stability of Power System. When it is subjected to disturbances, we will start with an introduction to the problem of stability.

(Refer Slide Time: 00:51)



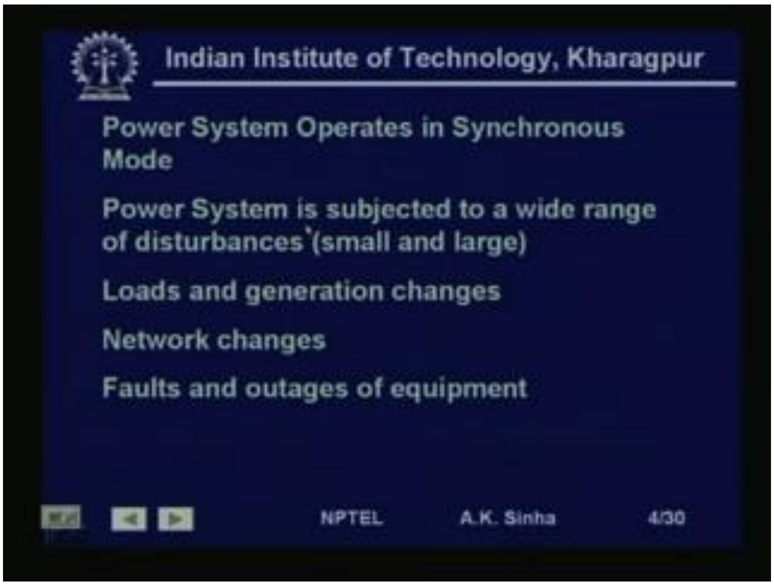
Then we will go into the classification of different types of stability, in case of power systems. And finally, we will try to develop the dynamic equation for synchronous machines in the power system.

(Refer Slide Time: 01:09)



On the completion of this lesson, you should be able to explain the concepts of power system stability. You should be able to classify stability problem, into different types based on the phenomenon under consideration. And you should be able to develop, dynamic equation of motion for the synchronous machines.

(Refer Slide Time: 01:35)



Well, as we all know, power system in it is normal operating state, works as a synchronous system. That is all the machines in the power system are synchronized with each other, that is they run at a common frequency, as we have talked about in our earlier lessons, when we dealt with short circuits. We had seen that short circuits or faults, create a condition which is very different from the normal operating condition.

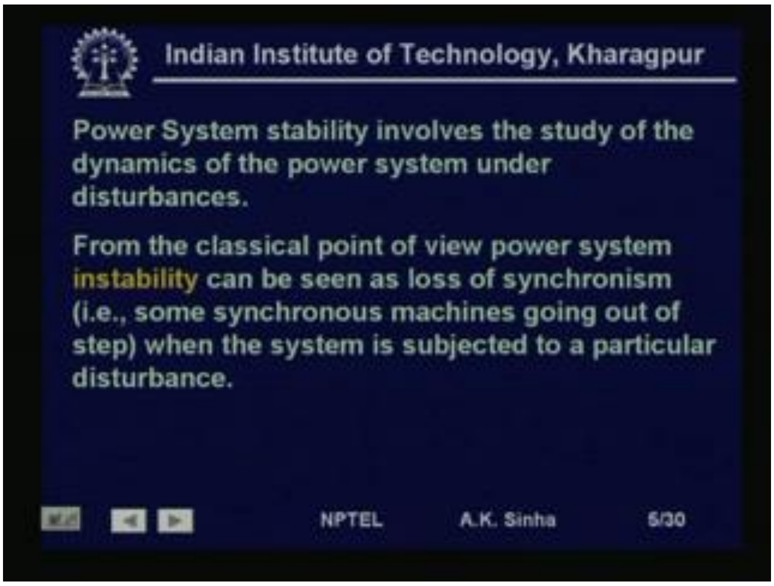
In terms, that a short circuit will make very heavy currents flow in the circuit, as well as the voltages at different parts of the system, will go to very low values. These are abnormal operating conditions. So, a fault does create a disturbance, in the power system operation. And due to these disturbances will the power system remain in synchronism, or not is a the kind of study. That we would be taking up in power system stability.

We know that power system is subjected to a wide range of disturbances. Some of these disturbances are small, some of these disturbances are large. I have already talked about some large disturbance, like short circuits, whereas small disturbances are the normal variation in load and the generation in the system. That is the loads and generation in the system keep changing all the time.

We do also have occasionally network changes, when we take out some line for maintenance. Or some line is stripped, because of a fault or some other reason. Then, as we said there are faults and outages of equipment, which take place. And all these create disturbances. The first one is a case or an example of small disturbance, whereas the second and third one are the examples of large disturbances.

And we do have these disturbances, on the power system. And what we are trying to study in this lesson, is how these disturbances influence the operation of the power system. Since, the power system is a dynamic system, because it consists of synchronous machines, which have rotating mass. And they are rotating at a particular speed, which we call the synchronous speed. All the machines in the system are synchronized. That is all of them rotate at synchronous speed.

(Refer Slide Time: 05:07)



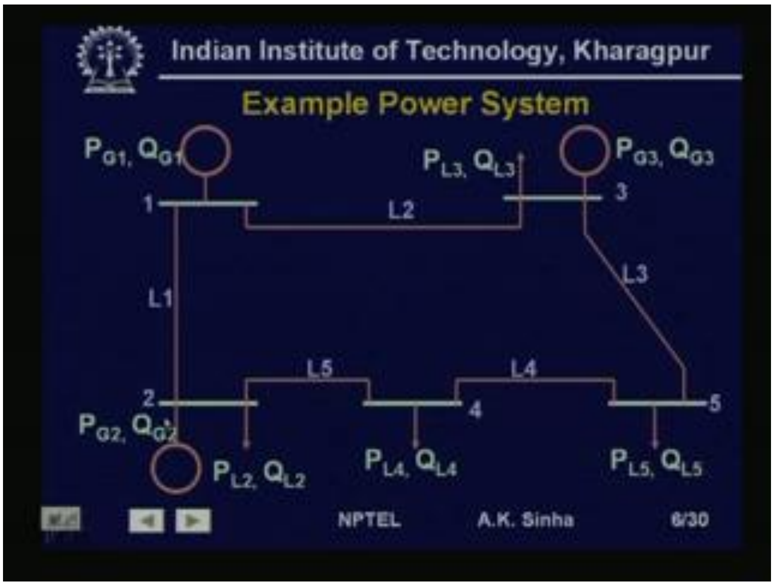
So, this is a dynamic system. And whenever there is any disturbance which occurs, the operation of the dynamic system changes. That is the system undergoes a dynamics and starts working at some other operating point. This change from one operating point to the another operating point is depending on the type of disturbance, that we have. Sometimes, when these disturbances are very large. We may not get a stable operating point or a synchronously operating point for the power system.

So, power system stability, we can say involves the study of the dynamics of the power system, under the influence of disturbance. So, we are trying to study the dynamics of the power system, under the influence of disturbances. This is what we mean by power system stability study. Now, from the classical point of view, power system instability can be seen as loss of synchronism.

That is we can see the power system operation, as a synchronous operation. And in case synchronism is lost by some of the machines. Then, we say that the power system has become unstable, or the system has shown an unstable behavior. So, we say that, the power system instability can been seen as loss of synchronism. That is some synchronous machines going out of step, when the system is subjected to a particular disturbance.

So, when the system is subjected to a particular disturbance. If all the machines, they still remain in synchronism after the disturbance. Then we say the system is stable, but if we find some of the machines have become unstable in the sense. That they have lost synchronism with the system, then we call that the system has become unstable.

(Refer Slide Time: 07:35)



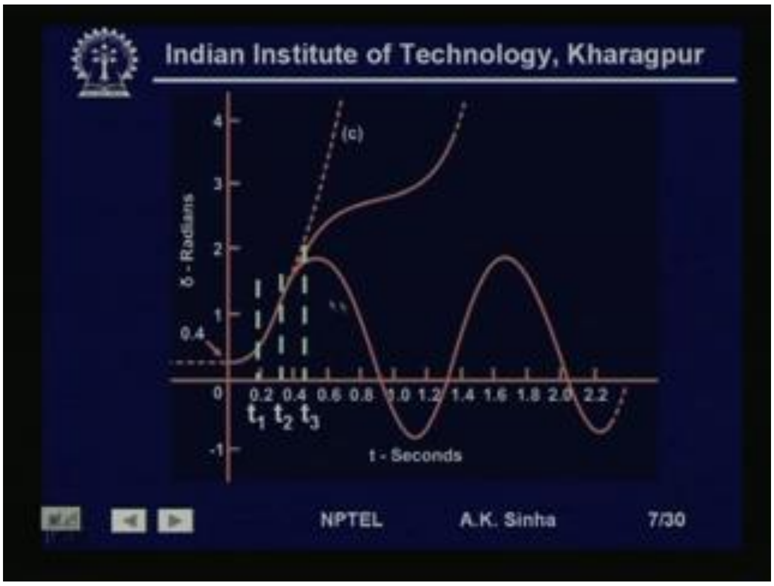
Now, this we can try and understand from a very simple example. Here I have taken a simple example, which is a 5 bus example, which is connected by 5 different lines. In this system, we have three generators, here at bus 1, bus 2 and bus 3. We have loads at bus 2, bus 3, bus 4 and bus 5. Now, let us assume that a fault has occurred at a point on line L 2. And this point let us say is very near to bus 1, then in that case when a three phase short circuit occurs at this point. Let us say, then what happens is you are going the voltage of this point, which is very close to this point. So, it is effectively the voltage of this point will go to 0. So, when the voltage of this bus goes to 0, what is going to happen is. The power transfer, or the power supplied from this generator, well electrical power supplied from this generator will also go to 0, because the voltage here is 0. So, E v by x in to sine delta is the electrical power output and since, V is 0. So, this value is going to be 0. So, there is no electrical power output from this generator. Whereas, the mechanical input of the generator remains same. That is whatever was the mechanical input of the generator, which was equal to the electrical output from the generator before the fault. So, that remains same.

So, what is going to happen. We have large amount of mechanical input, to the turbine generator system. Whereas, the electrical output from the generator is has become 0. So, the rotating mass of the synchronous machine connected to this bus. is going to accelerate. Because, output is 0, input is very large. Now, this acceleration means. the phase angle or the rotor angle of this machine will keep on increasing. It will keep on accelerating.

And because of this, this angle, rotor angle of machine will keep on increasing. Now, suppose we clear the fault by opening this line. Then, what will happen, again the short circuit has now being removed. So, again there is going to be power flow from this generator. Now, this power flow from the generator, electrical power output from the generator at that instant, is going to depend on what is the voltage phase angle.

Or the rotor angle of the machine, at that particular instant. Because, we know the electrical power output from this machine will be approximately equal to E V by x into sine delta. So, depending on what is the angle delta, at that time the electrical output will depend.

(Refer Slide Time: 11:19)



Now, this kind of a situation, we can show very easily from this graph, where we say that at time t is equal to 0, the fault has occurred. Now, because of this fault has occurred, the angle delta will keep on increasing like this. As shown in this graph, because there is accelerating power available. Now, when this delta keeps on increasing, if suppose we clear the fault very quickly.

Let say n less than 0.2 seconds. Then, what is going to happen is that, electrical output now has increased. Now, the electrical output will be somewhere around this point. And because, the mechanical input is only this much, it will be constant. Because, this is a very small time duration. So, there is going to be deceleration, which will take place. And because of this reason, the delta angle will not just keep on increasing. It will increase for some time and then it will go down.

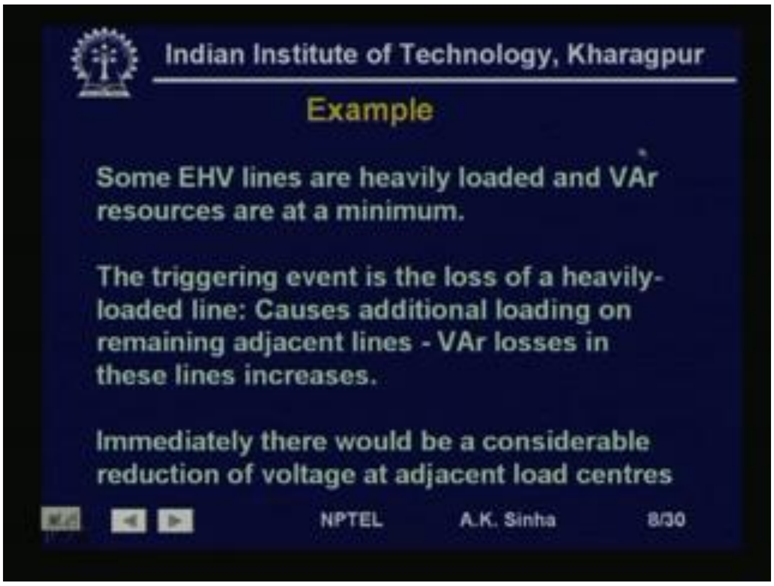
So, we will have a dynamics like this, which will take place. And finally, the system will settle down to a new operating point. But, suppose the fault is cleared at this time t 2, which is say around 0.35 seconds. Then, in that case the delta angle would have come up to this point. And the acceleration would still be there. And because of which we will find that the delta angle will keep increasing like this.

And we will see that, the synchronous machine is keeps on accelerating. And because of this acceleration, the machine has lost synchronism, with the rest of the system. So, the machine would lose synchronism like this. Suppose, we clear the fault after much longer time say at around 0.45 seconds. Then, what we will find is, this delta angle would have reached up to this point at this time. And the acceleration will be much more.

And what happens is, the system will, that is this machine will become unstable with it is delta angle. We will keep on increasing at a much faster rate. So, what we have is for the same fault, which has occurred, if we clear the fault at different times. The machine can be stable or it can become unstable. Now, in this example, what we are saying is that, what is happening is this delta angle of the machine.

That is the power angle or the rotor angle of the machine with respect to the synchronously rotating reference frame, keeps on increasing. Or it goes through an oscillation depending on when we clear the fault. So, we are trying to study the rotor angle dynamics of the machine. And this rotor angle dynamics is occurring, because of a mismatch between the electrical power output. And the mechanical power input to the system. So, it is a real power mismatch, which is taking place. Because of which the machine is going through a dynamics and the rotor angle keeps changing. So, this is one phenomena of stability that we can see.

(Refer Slide Time: 15:11)



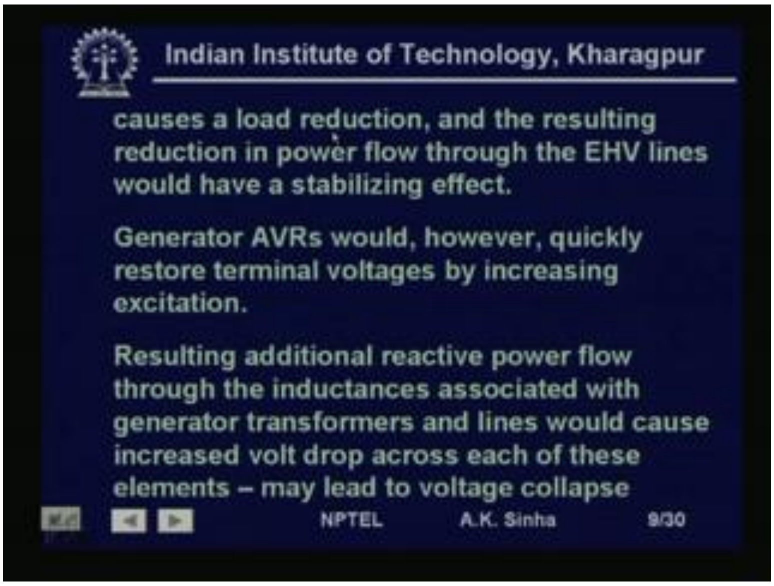
Similarly, suppose we take another example. The system is working under heavy load. So, some of the extra high voltage lines, are heavy loaded. And the reactive power resources are at minimum. That means, to just to maintain the voltage. all the generator excitation is to their limits. As well as whatever shunt capacitance or shunt compensation we had in the system, are also stretched to almost their limits.

So, we have the under a heavily loaded condition EHV lines are heavily loaded. And the VAr resource are at minimum. Now, suppose one of the heavily loaded line, because this line was somewhat overloaded. The relays it is sense the overloading, and trip the circuit breaker. Because, we cannot tolerate overload on lines large overload on lines for long period of time.

So, the protection system, in order to prevent physical damage to the transmission system, will trip the line. So, the triggering event is the loss of heavily loaded line. Now, what will happen because of loss of this heavily loaded line. The other lines, since the load and the generation in the system has not changed. The power will flow now through the other lines, the power which was flowing through this line, which now has been tripped, will now flow through other lines.

And thereby causing additional loading on the remaining adjacent lines. Now, when this additional loading takes place on the other lines, the Var losses in these lines will also increase, because lines have large reactance and very little resistance. As well as, when the lines are heavily loaded, what we find as the receiving end voltages drop down. This is what we had seen when we studied the transmission line. And power flow on transmission lines. So, VAr losses in these lines will increase. Now, what is going to happen? Because of this increase in VAr losses, the voltages at the load ends are going to drop.

(Refer Slide Time: 18:11)



Now, because there would be a considerable reduction of voltage, at the adjacent load centers. That is at the load ends, the voltages are going to drop. So, what is going to happen. This will cause a reduction in the load. Because, we know if we have a constant impedance load, then the load will decrease. Because, it is V square by Z. which is the power which is going to the load. So, since V drops.

So, the load also reduces. Even for induction machine loads, when the voltage drops the load decreases to some extent. And because of this voltage drop, we have a load reduction. This reduction will result in a reduction in power flow, through the EHV lines. That is what we would like to have, because these lines are already heavily loaded. Now, because the load has reduced, because of reduction in voltage. So, the power flow on these lines will reduced, and this will have stabilizing effect.

But, this is not going to remain there for a long time, because generator voltage regulator systems, will try to restore the terminal voltages quickly, by increasing the excitation. And the other VAr resources, control VAr resources will also the controllers on them will act, to increase the VAr resources to maintain the voltage. And again if we are now trying to maintain the voltage. That is increase the voltage at the load ends. What will happen, the loads will now again increase.

So, this resulting additional reactive power flow through the inductances. So, this will result in additional reactive power flow, through the inductances associated with the generator transformers, and the lines. So, this is what is going to happen. Because, now we are trying to increase the voltages have increase, the loads will increase. And that means, more power flow will take place on the lines.

And this will result in more VAr losses on the line, because line has large inductance, and the current flows through that. So, you are going to have more VAr losses on the lines. And this will cause increased voltage drop on each of these lines. This kind of a situation can result in a complete voltage collapse Because, the voltages will drop to a very low value. And since, there is no more VAr resources available.

And this voltage will keep on dropping. And we will get to a phenomena, what we call as the voltage collapse. So, here what we are seeing is some of the load buses, will lose voltage completely. That means, they will be tripped out or the voltages will become 0. And because of the heavy current flowing, the lines will also be tripped. So, loads will be tripped in this case. And this again is a dynamic phenomena, which is taking place, because of the VAr imbalance. That is the imbalance in the reactive power.

And this kind of a problem of voltage collapse has occurred on a number of occasions, in power system. And has created blackouts in many portions of the system. So, here we are again seeing that, the system is becoming unstable. In the sense that, the normal operation of the system is not possible the loads are lost, in such a situation. And so this is again a problem of power system stability. Here, what we are looking at is basically the mismatch in the reactive powers.

And the major phenomena under consideration is the voltage. How the voltage at different buses are behaving, whether it can regain. And come back to a normal position, or this voltage keeps on going down. And the system or the loads, at those buses are tripped. ((Refer Time: 23:03)) We can take another example. Again, let us take the same power system, the 5 bus power system with these three generators.

Now, let us say that we have a normal operation of the system, which is taking place with line L 1 out of the system. That is line L 1 is tripped out, because of maintenance or because of a fault which had occurred earlier. And this line has been now out of service. And still what we have is due to these three generators, are able to supply load through the other lines. And this system is working. Now, suppose this line L 4 is tripped, because of a fault on this line, under the situation, then what happens.

We have now a system which is separated into two parts. That is bus 1, 3 and 5 remain in one part. Bus 2 and 4 remain in another part. Now, what we can see is that, the system has got now islanded. That is there are two islands, instead of one contagious system, that we have. So, now what we are seeing is, in this case we will find that, this part of the system. That is which consists of generator two at bus 2 and loads at bus 2 and bus 4.

It may not be feasible for this system to remain working in synchronism. Because the load in this case at bus 2 and bus 4, may be much larger than the load, the generation which can be supplied from this generator at bus 2. In such a situation, what is going to happen is, load real load being much larger than the generation, that is feasible. So, the frequency will keep falling. And this system will go into a blackout after sometime.

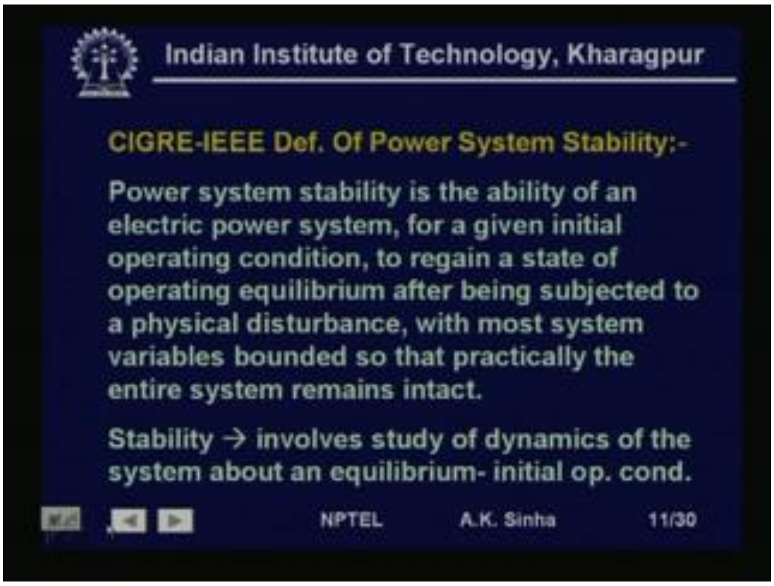
Whereas, it may be feasible for this part of the system, that is with bus 1, 3 and 5 to remain working in synchronism. Of course, these generators will initially will be generating more power, than what the load at bus 3 and 4 are together. And so there is going to be an acceleration, and the frequency of these generators will increase. Because, the generated power is more than the loads. So, frequency is going to increase.

But, the speed governors at these generators will reduce the mechanical input to these generators. And thereby bringing the frequency down to back to the synchronous speed. And this system, may work in synchronism that is under normal condition, whereas this part of the system may collapse completely. So, such a situation can also occur this again is a dynamic phenomena, which will take place over a period of time. Of course, the time period will be a few seconds only.

So, again study of such kind of a situation is the study of power system stability. But, this phenomena is somewhat different from the other phenomenas, that we discussed. Here again the imbalance, that we get because of the disturbances real power imbalance. But, the phenomena which is of concern here is, whether the frequency can be maintained, at the synchronous frequency or not.

And therefore, we see this as a different phenomena as compared to the first one, where we were looking at again the real power imbalance. But, the delta angle or the machine rotor angle dynamics here, we are looking at the dynamics of the frequency of the system. So, from these examples, we can understand now that we have power system stability problem, can be manifested into different types of problems.

(Refer Slide Time: 28:02)



So, therefore, the classical definition of stability, where we talked only about the rotor angle. That is synchronism being maintained or not, is not really giving us the complete definition for the stability. And because of this reason, the CIGRE and IEEE committee formed a task force, to define the power system stability. According to this task force definition, they tried to take care of the all kinds of stability problems in power system.

In the definition, the definition says power system stability is the ability of an electric power system. For a given initial operating condition, this is very important. That is we are talking about stability, which is for a given initial operating condition. That is initial operating condition is an important aspect in stability. For different operating conditions, the same disturbance may make the system stable or unstable.

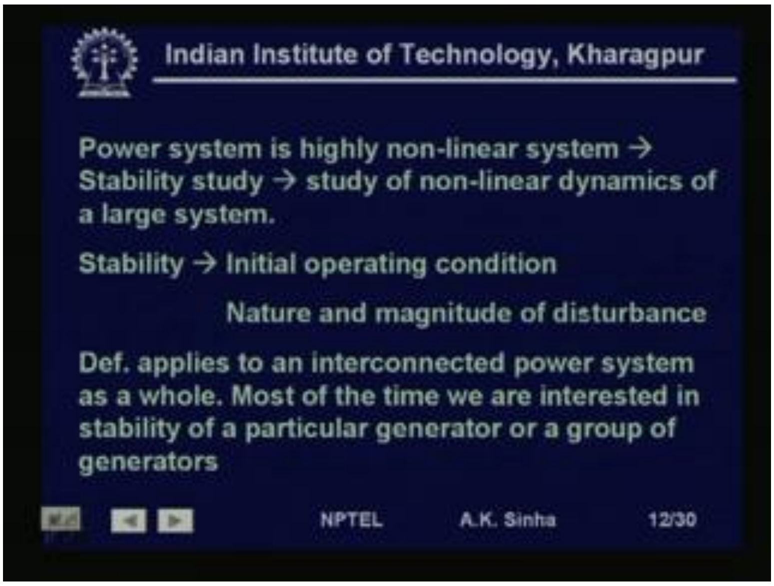
So, it says power system stability is the ability of an electric power system. For a given initial operating condition to regain a state of operating equilibrium, that is to regain a state of operating equilibrium which mean, where the real and reactive power balance is maintained. That is the system is working in synchronism. And the voltages at various points in the system are within the normal operating limits.

So, to regain a state of operating equilibrium, after being subjected to a physical disturbance. So, all this we are saying after the system has been subjected to a disturbance. Whether, there is going to be, that is system is going to regain a state of operating equilibrium, or not with most system variables bounded. That is the frequency the voltage, all these are within the limits. None of these are exceeding or keep on increasing all the time.

So, that practically the entire system remains intact, which again says that most of the part. Or the major part of the system still remains intact works in synchronism, with voltages within normal limits. That is a normal operating condition for the system is maintained, even after the disturbance. Then, we call the system is stable. The stability involves study of dynamics of a system about an equilibrium.

That is what we are saying this definition says that. We are trying to say the initial operating condition is an equilibrium operating condition; where real and reactive power balances. For load and generation is maintained. So, it is an equilibrium operating condition. And the dynamics of the system about this equilibrium operating condition, is the study of power system stability. So, involves the study of dynamics of the system about an equilibrium initial operating condition.

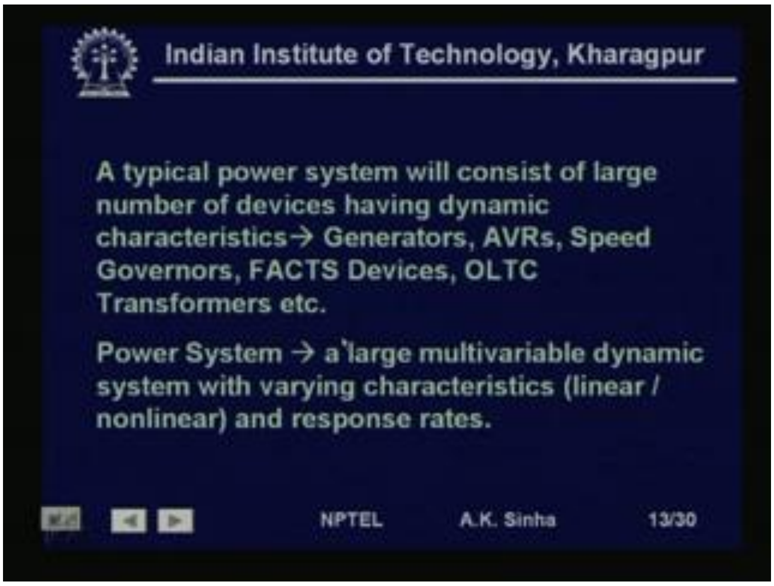
(Refer Slide Time: 31:30)



Now, one of the problems in stability study is. That power system mathematical model, that will get or is a non-linear model. That is power system is a non-linear dynamic system. So, power system is a highly non-linear dynamic system. And therefore, stability study means, study of non-linear dynamics of a large system. Power system is also a large complex system. And it is a non-linear system, therefore the power system stability study involves study of non-linear dynamics of large system.

Stability, can be defined only in terms of initial operating condition. And nature and magnitude of disturbance. What this really means, because the system is non-linear, for the same operating point. If the magnitude and nature of disturbance are different, the system for one case may be stable, whereas, for the other case it may become unstable. And similarly, for the same kind of disturbance at two different operating points, the system may be stable for one, and may become unstable for other. Therefore, whenever we talk about the stability of a power system. We must also indicate the initial operating point, as well as the nature of disturbance. Now, this definition that we have just seen, applies to an interconnected power system. But, most of the time, we may be interested only in the dynamics of one machine or a group of machines, with respect to the rest of the system. That is what most of the time, we would like to study the dynamics of one machine or may be a group of machines with respect to the entire system.

(Refer Slide Time: 33:48)



Now, as we already know, a typical power system will consists of large number of devices having dynamic characteristics. That is we have large number of generators. All these generators will have automatic voltage regulators, AVRs. We have these will also have speed governing systems. And we also have systems like FACTS devices, which are used for controlling real and reactive power flow on the transmission line. So, FACTS devices are basically what we call, flexible AC transmission system.

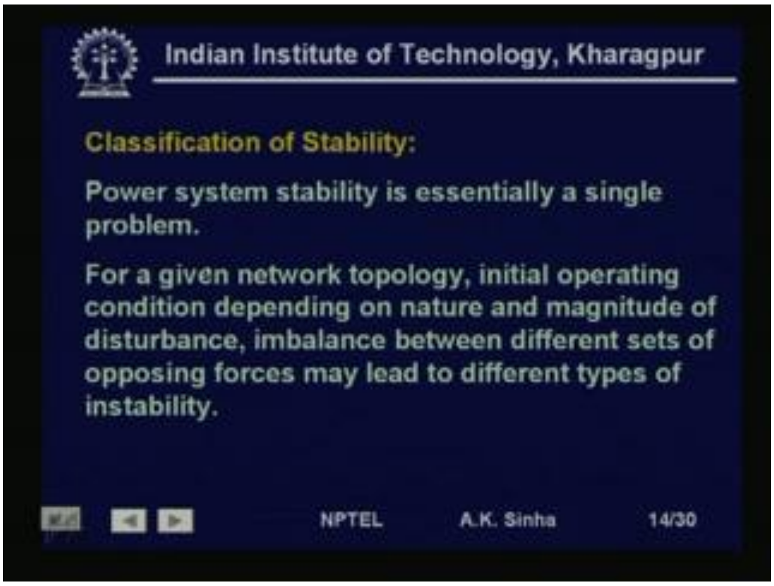
So, these are power electronic devices, which can control real and reactive power flow on the transmission line. We also have on load tap changing transformers, etcetera. So, we have large number of controlled devices, in the system. And the system consists of a large network of transmission lines, as well. So, we can say that, the power system consists of a, or can be considered as a large multivariable dynamic system, with varying characteristic.

That is some of these controllers will have, or the controllers will have linear characteristics. Some of them will have non-linear characteristics. And therefore, and also some of them will have very fast response rate. Whereas, the others response rates will be very slow, like we have response rates for FACTS devices, which will be in milliseconds or even faster. Whereas, on load tap changing devices have response rates which are in terms of a minute or so.

So, we have all kinds of devices. Devices which are linear devices, which are non-linear devices, which are fast response time devices, which have slow response time. And therefore, to study the dynamics of such a complex system is very difficult. And thereby, what we do is, we try make certain assumptions. And we try to study the stability, under these assumptions or under different types of phenomenas. We make different kinds of assumptions.

That is what we do is, as we had seen earlier, that we had seen three different kinds of stability problem. That is one case, we had rotor angle dynamics, in another case we had voltage dynamics. Whereas, the third case, we saw the dynamics or the frequency. So, depending on what phenomena, we are more interested in studying. We can classify the power system stability into different types.

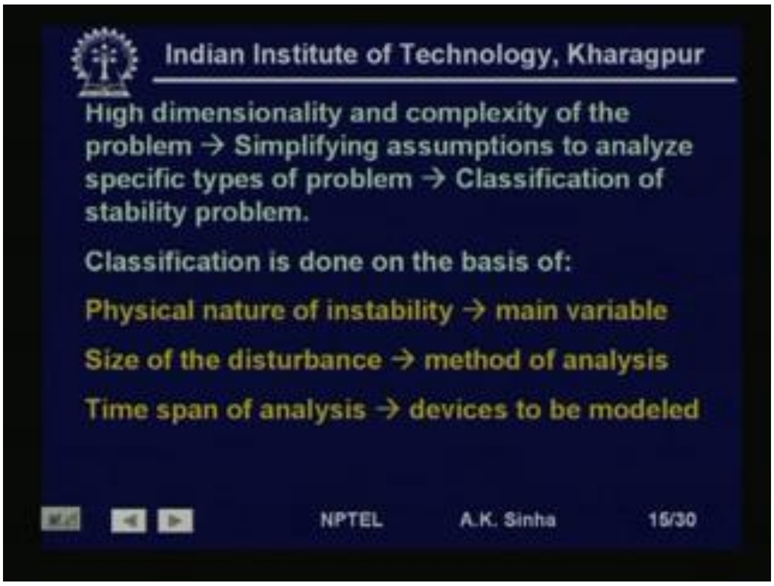
(Refer Slide Time: 36:54)



So, all though power system stability is essentially a single problem. For a given network topology, initial operating condition. And depending on the nature and magnitude of disturbance. Imbalance between different sets of opposing forces, that is the loads and generation, the mechanical input and the electrical output from the machines, so,whether these get balanced or not.

So, imbalance between different sets of opposing forces, may lead to different types of instability. That is what we had seen real power imbalance. That is mechanical input and the electrical output, will result in rotor angle in stability or stability. That is rotor angle dynamics, whereas the reactive power imbalance can lead to voltage dynamics. And incase of the electrical power generation imbalance, in electrical power generation and load can lead to a frequency dynamics.

(Refer Slide Time: 38:04)



And therefore, we can classify the power system stability problem, into different kinds of stability problems. Classification has the advantage, that we can make simplifying assumptions. For these different kind of problems, since we are concerned with one particular kind of problem. We may neglect the controller actions, which are not going to govern this particular parameter at all.

So, high dimensionality and complexity of the problem is can be solved to some extent. By simplifying assumptions to analyze, specific type of problems. And that is what, why we need classification of the stability problem. Generally, the classification is done on the basis of following criteria. One is physical nature of the instability. That is what is the main variable, in which we are interested just like we saw the rotor angle, or the voltage or the frequency.

Size of the disturbance, whether it is a small disturbance, or a large disturbance. This is going to govern the method of analysis. That is if the disturbance is small, we can use a linearized model of the system, about the operating point. But, if the disturbance is large, then we have to use the non-linear model of the system. Time span of analysis, this governs the devices to be model. If we are interested in the fast dynamics. That is going to take place for rotor angle or frequency or whatever it is.

We need not model those devices, whose time responses are much larger. That is if the time of study, that we are going to have is just a few seconds. Then, why model an OLTC in this kind of a study. Because by the time the study is over OLTC would not have even responding. So, the devices to be modeled will depend on the time span of the analysis. So, on these basis, we classify the power system stability.

(Refer Slide Time: 40:35)

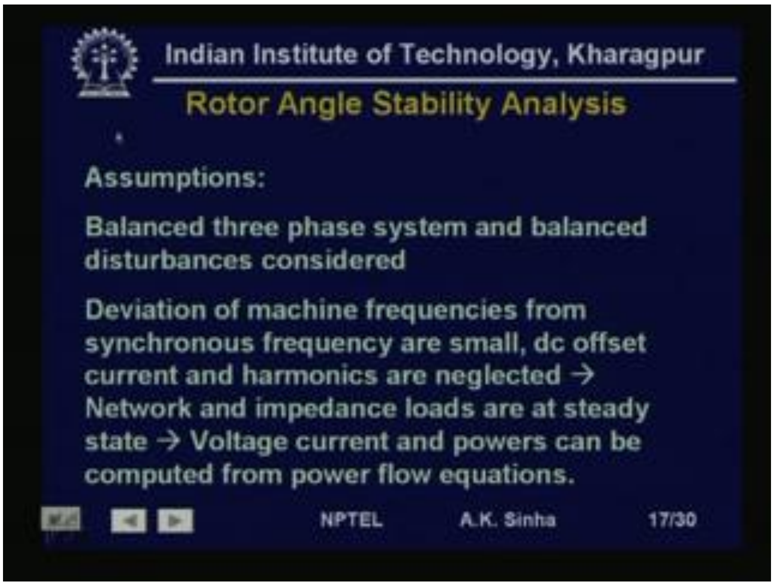
The classification is shown here, power system stability, can be classified as rotor angle based on the phenomena under study. Rotor angle stability, frequency stability, voltage stability. All these can have for small signal stability. That is small disturbances. And for large signal stability or large disturbances, which we normally call as transient stability problem. And these can be done normally for short term normally, these studies are done for 2 to 10 seconds time period.

Whereas, frequency stability is generally for large disturbances only. For small disturbances, the frequency variations are very small. And this does not lead to any frequency instability. So, it is generally done for very large disturbances. Only where you are going to have large variation or large mismatch in real, that is in the electrical power generation and the electrical load on the system.

So, frequency stability can also be studied on short term or a long term basis. That is you want to study with only a few seconds. If you want to take into account the action of other controllers, like boilers response and other things. Then, you do a study over a very long period of time may be 20 minutes 30 minutes, which will take care of these responses. So, you do that, so short term long term.

Voltage stability again small signals stability, and large signal stability. That is for large disturbance and small disturbance. These can be done for short term and long term, depending on whether we want to take care of the devices. Such as on load tap changing transformers, and other devices, which are slow acting or not. If we are taking them into consideration, we are trying to study the stability over a longtime period. So, a long term stability. If we are not doing study only for a short period, then we need not model such devices.

(Refer Slide Time: 43:03)



So, now we will start with the first one, that is the rotor angle stability analysis. For this particular analysis, since we are interested only in rotor angle stability. That is the dynamics of the rotor angle, we are going to make certain assumptions. That is we assume that balance three phase system. And balance disturbances are considered, that is system is operating as a balanced three phase system, before the disturbance occurs.

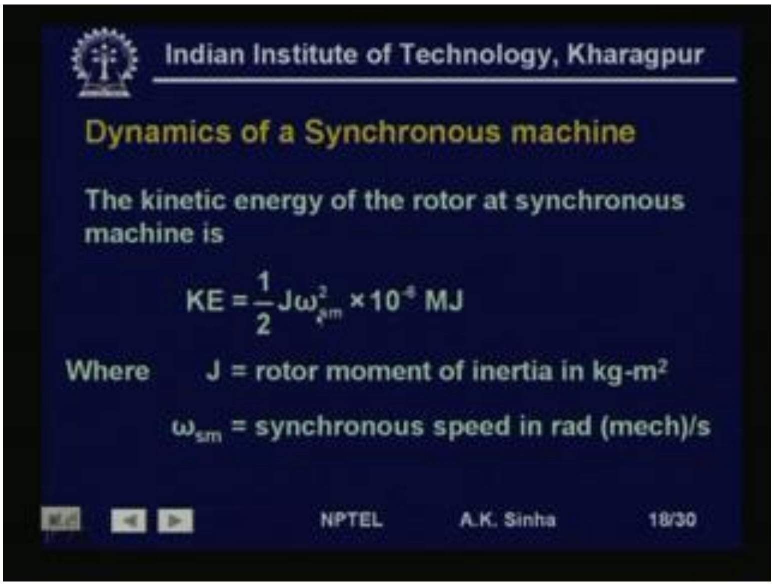
And the disturbance that we are going to consider is also a balance disturbance. That is it is a disturbance which means a three phase load change, or a three phase fault on the system. We are not considering unbalanced changes or unbalanced operation of the system, deviation of machine frequencies from synchronous frequency are small. Since, the time period of study is very small. And the system inertia is large. The deviation in frequency which takes place, that is speed change is very small in general.

And therefore, we neglect these changes to some extent. Dc offset current and harmonics are neglected. That is we had seen that, when we do a short circuit on the generator dc offset currents. As well as high frequency currents may be present and but, for the stability study we neglect these effects. Basically, also we say that network impedance loads are at steady state. That is network and impedance loads, are at steady state.

That is again because of the dynamics which is taking place, this dynamic is taking place mainly at the rotor of the generating machines. Or the synchronous motors, if they are present in the system. We are considering this dynamics is very slow. As compared to the dynamics, which occurs in the network. And we are assuming that the network dynamics, have died down much faster before it can affect the dynamics of this system.

So, the time periods for study is much larger here, compared to that of the network dynamics, which is of the order of in microsecond range. And this dynamics dies out very quickly. That is the transient that comes on the network dies out quickly. Voltage current and power can be computed. So, if we are taking network to be in steady state, and in loads as impedance loads. Then, voltage current and power can be computed from the power flow equations. That is the advantage of making this assumption is, we can use the well known power flow equations, with which we are very much conversed.

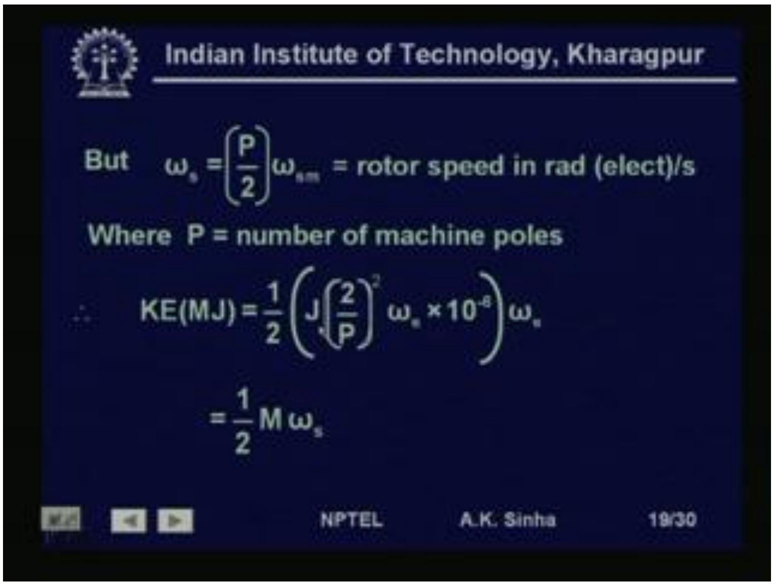
(Refer Slide Time: 46:12)



Now, let us start with the dynamics of the synchronous machine. The kinetic energy of the rotor at synchronous machine can be written as K E is equal to half J omega s m square. Now, we are writing this s m, because now each machine is rotating at synchronous speed. And the speed in mechanical radiance per second, we are writing as omega s m. That is synchronous speed in mechanical radiance.

So, kinetic energy is equal to half J omega s m square into 10 to the power minus 6, because we are using Mega Joules here. Otherwise, we will write this half J s m square omega s m squared joules. So, anyway we always work with mega watts and Mega Joules. So, we are using this 10 to the power minus 6 here. J is the rotor moment of inertia in Kg meter square, omega s m is the synchronous speed of the machine rotor in radiance per second. And this is mechanical radiance per second.

(Refer Slide Time: 47:29)



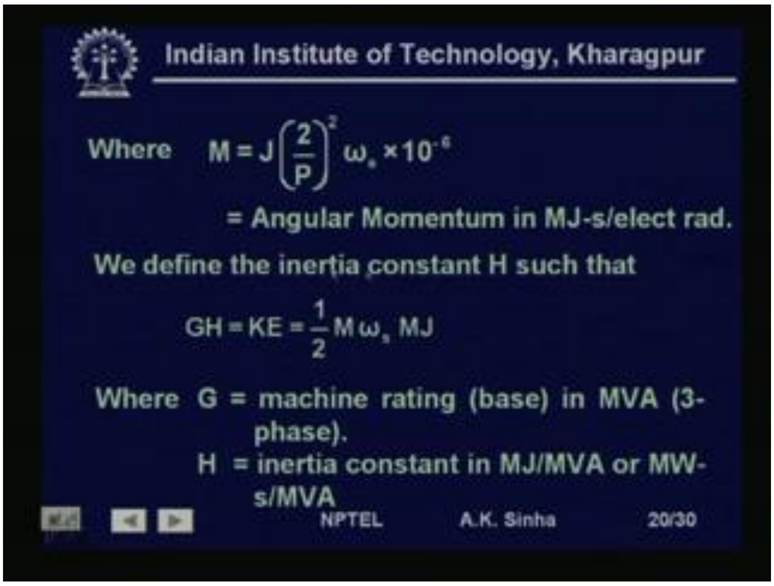
There is a difference between the electrical and mechanical speed. Because, if we have a 2 pole machine, then when it goes through one full revolution. The electrical voltage generated also goes through one full cycle. But, suppose we have 4 pole machine, then when this machine goes through one full revolution rotor, goes through one full revolution. By that time, the voltage generated would have gone through 2 full cycles. Because, the poles would have gone through North, South, then again North and then again South.

So, we would have got through 2 cycles. That means, the electrical revolution is now twice, that of the mechanical revolution. And therefore, we can write the speed, in terms of electrical radiance as the synchronous speed. In electrical radiance per second is equal to P by 2 into omega s m. That is the synchronous speed in mechanical radiance. Because, if P is now 4 in this case, 4 by 2 that is the electrical radiance are twice the mechanical radiance and so on.

So, depending on the number of poles. We have the relationship between the electrical speed and the mechanical speed. So, omega s is equal to P by 2 omega s m, which is the rotor speed in electrical radiance per second. where P is the number of machines poles. Therefore, kinetic energy in Mega Joules, we can write as half into J into we had omega s m square. So, omega s m, we are writing as equal to 2 by P into omega s. So, 2 by P in to omega s whole square of this.

So, whole square of this means 2 by P squared into omega s squared, we have taken this omega 1omega s outside. And so we have J 2 by P squared omega s into 10 to the power minus 6, because we are writing the kinetic energy in Mega Joules. So, half this into omega s is the expression that we get. Now, we can write this term in this bracket. That is J 2 by P square omega s into 10 to the power minus 6 as M. And this M we call as the angular momentum of the machine rotor. So, we can write kinetic energy is equal to half M omega s.

(Refer Slide Time: 50:17)

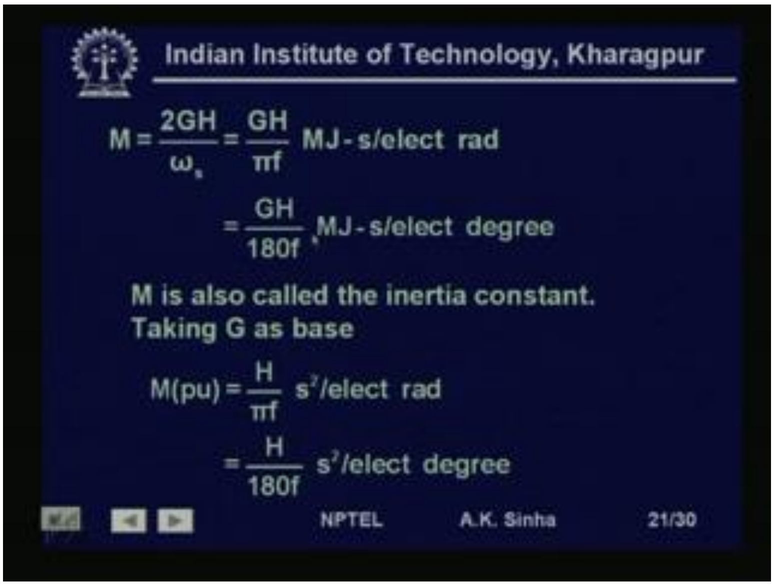


Where, the angular momentum M is equal to J into 2 by P square, omega s into 10 to the power minus 6. And M, the unit for M will be mega joule second for electrical radiant. Instead of working with this angular momentum M or moment of inertia J. We normally use a term which we call inertia constant. So, we write inertia constant H as G H is equal to kinetic energy, where G is the rating of the machine.

So, H is basically K E by G. That is kinetic energy stored in the machine rotor, at synchronous speed divided by the MVA rating of the machine. So, kinetic energy in

Mega Joules divided by the MVA rating of the machine. And this kinetic energy, we are calculating at synchronous speed. So, if we write that way, then we have G H is equal to half M omega s, which we are simply putting this expression. So, where G is the machine rating in MVA, three phase base and H is inertia constant or mega volt, M mega volt. Second per MVA, that is mega joule per MVA or mega watt second per MVA.

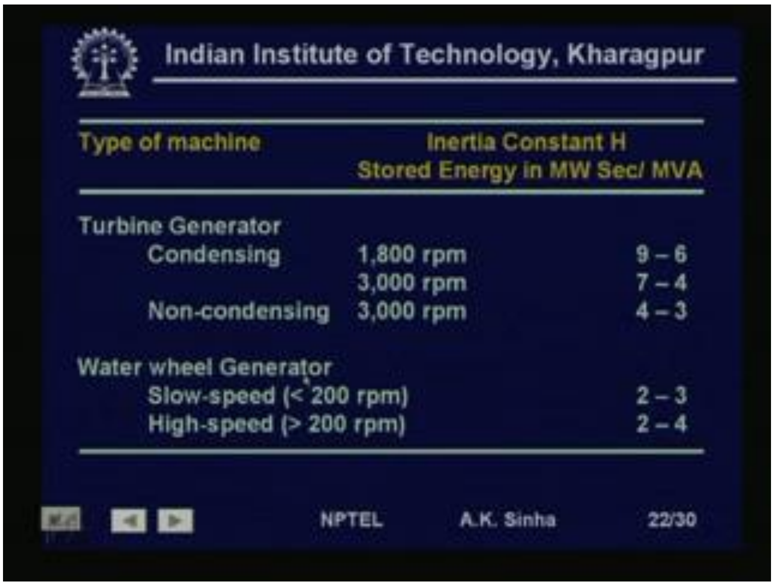
(Refer Slide Time: 51:52)



Therefore, we can write M is equal to 2 G H by omega s, which we can write as G H by pi f. Because, omega s is twice pi f, this is mega joule second by electrical radiance. If we want to write instead of electrical radiance in terms of electrical degrees, then we will write this as G H by 180 f, because pi is 180 degrees. So, 180 f mega joule second per electrical degree.

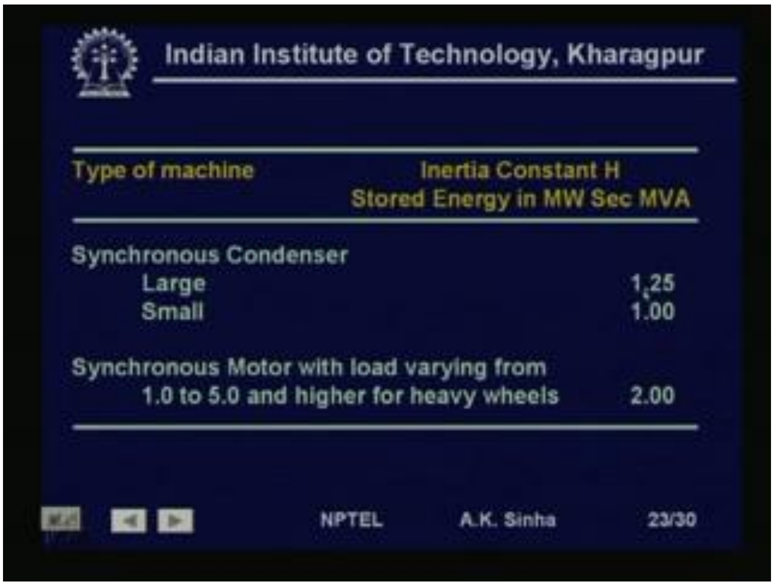
Sometimes, M is also called the inertia constant. And we can find out the value of this inertia constant M, also in per unit as M per unit is equal to H by pi f, because we are diving it by G. So, G is the base value for this machine. That is it is rating, then we have M per unit on the machine rating base is equal to H by pi f second square into per electrical radiant. This is equal to H by 180 f, when we are writing it in electrical degrees. So, H by 180 f seconds square per electrical degree.

(Refer Slide Time: 53:23)



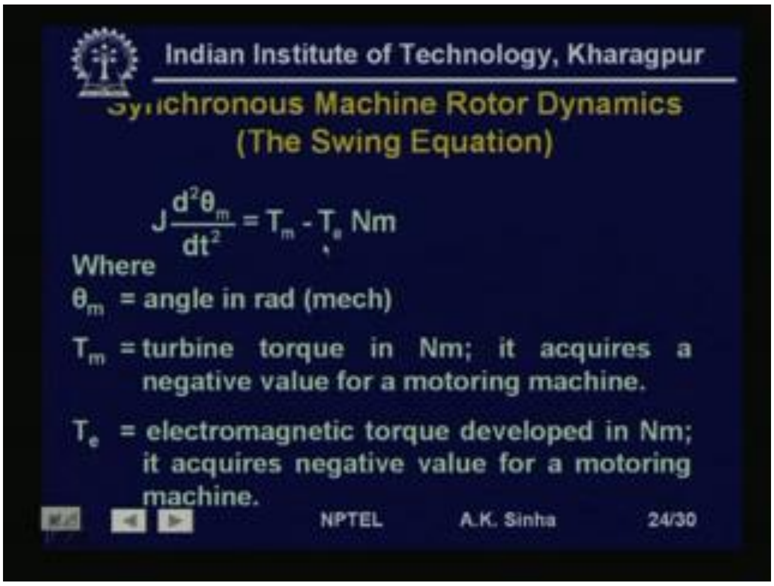
We use this H instead of M or J. Mainly because, the value of H for different kinds of machine falls within a certain range. That is for a turbine generator system, which has a condenser running at 1800 rpm. It is H value will be between 9 to 6 seconds or Mega Joules per MVA. At 3000 rpm it is around 7 to 4, that is between 7 to 4. And for non-condensing it is much less between 4 to 3, whereas for hydroelectric generators, that is water wheel generators. For slow speed it is between 2 to 3. And for high speeds it is between 2 to 4.

(Refer Slide Time: 54:11)



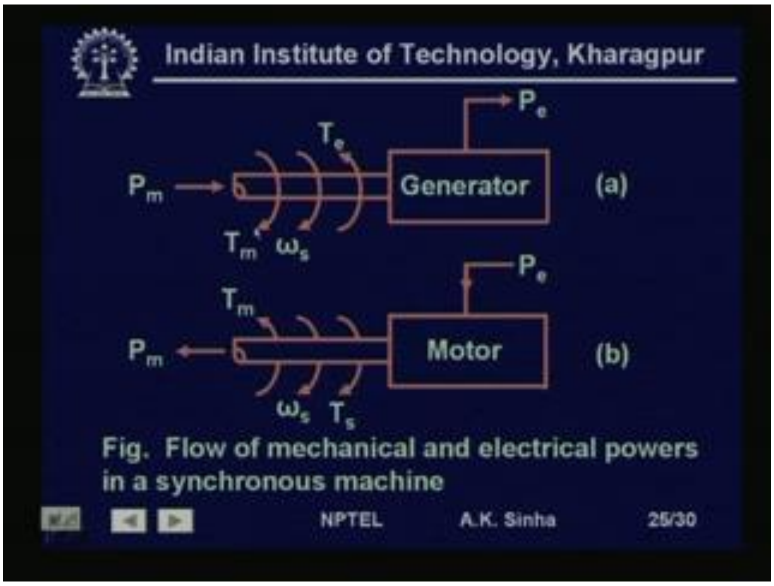
Similarly, for synchronous condensers for large ones it is value is 1.5. Whereas, for small one it is value is 1 second or so. For synchronous motors the value is around 2 seconds or 2 Mega Joules per MVA.

(Refer Slide Time: 54:28)



Now, we will try to see, how we can obtain the synchronous machine rotor dynamics, which we call as the swing equation. Now, the difference in mechanical torque and electrical torque. The mechanical torque input to the synchronous machine. And the electrical torque output from the synchronous machine, will give us the dynamics of the system. That is J into d 2 theta m by d t 2. That is J into acceleration mechanical rotor acceleration, will be equal to the difference in the torque, where, theta m is the angle in radiance. T m is the turbine torque in Newton meter. And T e is the electromagnetic torque. That is the output torque developed in Newton meter.

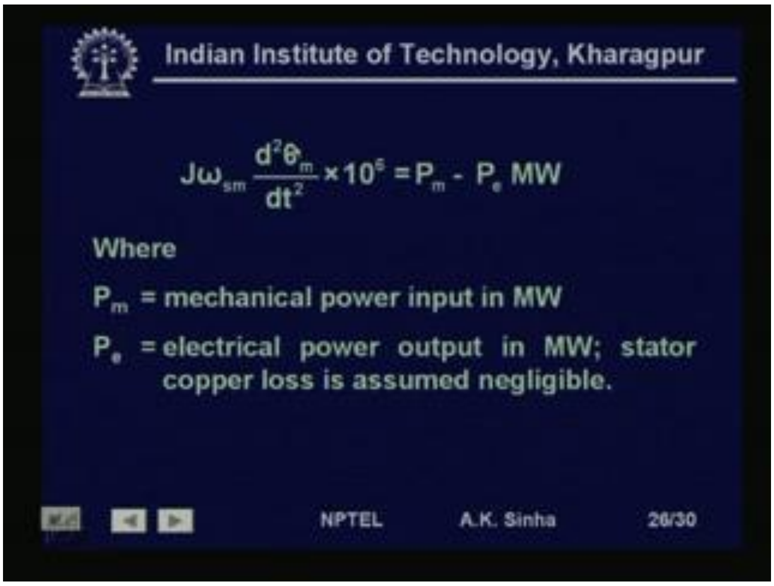
(Refer Slide Time: 55:27)



This we can see from this, where we have this as a generator. The mechanical input is put to the turbine generator system, which is rotating this in the clock wise direction. And there is an output electrical output, which goes from the generator, which is going to produce a torque, which is going to oppose. This mechanical torque, which is trying to accelerate this generator. So, that will be in the opposite direction, that is anti clock wise direction.

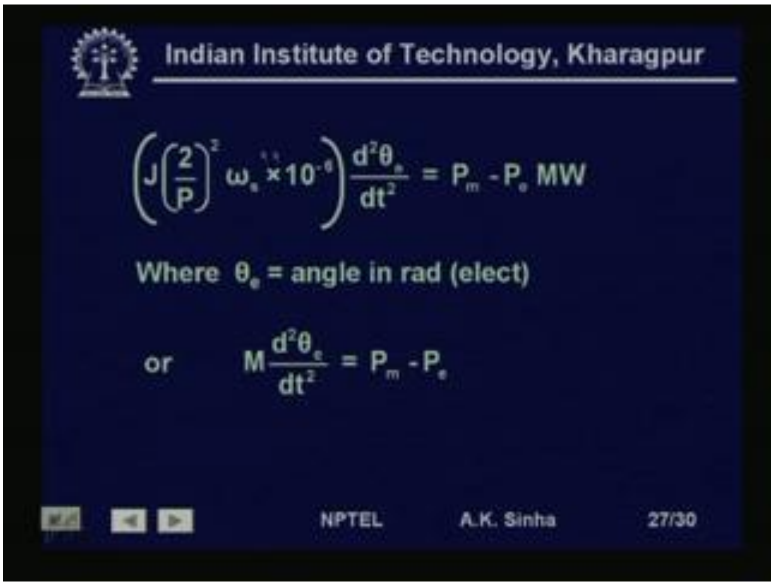
When these two torques are equal, then the speed will be constant, at that is synchronous speed. So, we run the machine at synchronous speed. We put more input, as when we put more load, when load the input to the machine and the output is equal. Then, the speed remains at synchronous speed. The same thing for the case of a motor will be opposite in the sense that, the input will be electrical, the output will be mechanical.

(Refer Slide Time: 56:34)



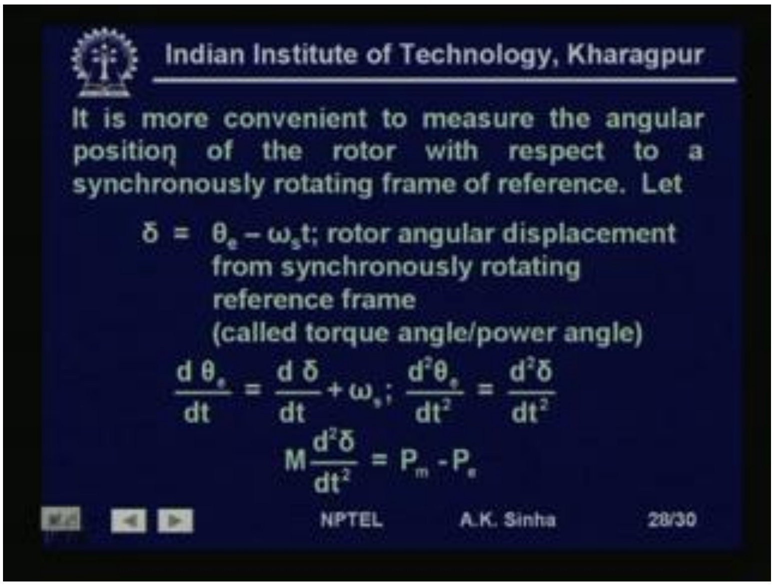
Therefore, we can write this equation, as if we multiply this equation. The earlier equation that we had with omega s m, that is mechanical speed. Then, instead of writing in terms of torque, we can write now in terms of power. So, J omega s m d 2 theta m by d t 2 into 10 to the power minus 6, because we are writing power in mega watts is equal to mechanical input power minus the electrical output power. So, this is the equation that we get.

(Refer Slide Time: 57:07)



Now, instead of J omega s m. For omega s m, in terms of omega s we can write. Then, we will get this 2 by P into omega s. And theta m, if we write in terms of theta e then again we will get 2 by P into theta e. So, this 2 by P 2 by P will go here. So, we have J into 2 by P square omega s into 10 to the power minus 6 into d 2 theta e by d t 2 is equal to pm minus p e in mega watts; where theta e is the angle in electrical radiance. This term as we have seen, we call it M the angular momentum. So, M d 2 theta e by d t 2 is equal to P m minus P. This is the equation that we get.

(Refer Slide Time: 57:59)



Now, instead of using the rotor angle theta e, whether electrical or mechanical whatever it is. We generally measure the rotor angle, in terms of a synchronously rotating reference frame, because if the rotor goes through a rotation of one rotation it has gone through 360 degree. So, we keeping track of the rotation will be very difficult. And therefore, what we do is we measure this rotor position, with respect to a synchronously rotating reference frame.

So, we write instead of theta, we write delta as the angle with respect to rotating reference frame. Delta is equal to theta e minus omega s t and substituting for this. So, if we take a derivative of this, then d theta e by d t is equal to d delta by d t plus omega s. And we differentiated again, it becomes d 2 theta e by d t 2 is equal to d 2 delta by d t 2. And therefore, substituting for d 2 theta e by d t 2, we get the expression as M d 2 delta by d t 2 is equal to P m minus P e.

So, this is the equation, that we call the swing equation or the equation which governs the motion of the synchronous machine. So, with this we will end today. Thank you, in the next class, we will talk more about this equation of rotor dynamics. And we will talk about how we work with in multi machines systems, how we write the swing equation for each machine.

Thank you.