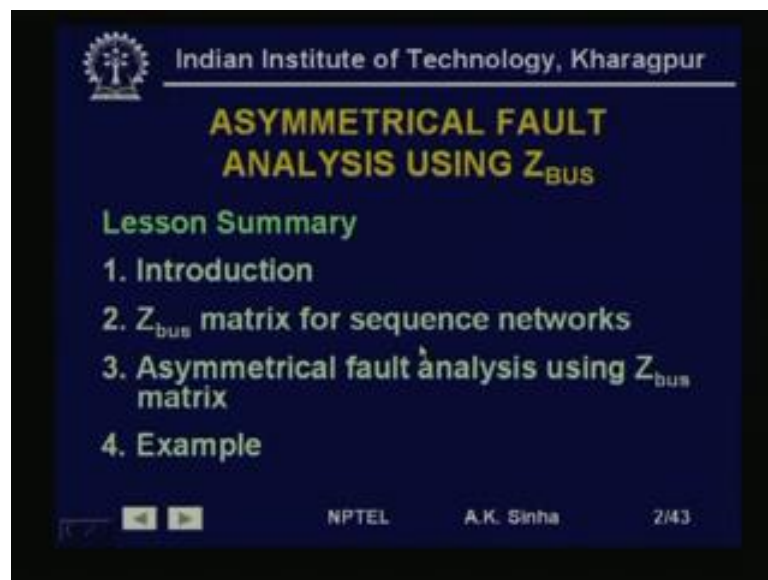


Power System Analysis
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Lecture - 32
Asymmetrical Fault Analysis Using Z Bus

Power System Analysis. In this lesson, we will discuss about Asymmetrical Fault Analysis Using Z Bus method, for large power systems. As we had discussed in the earlier lessons, we have seen how we can use the Z bus algorithm for solving symmetrical faults for large power systems. In this lesson, we will try to extend that method, solving for asymmetrical faults.

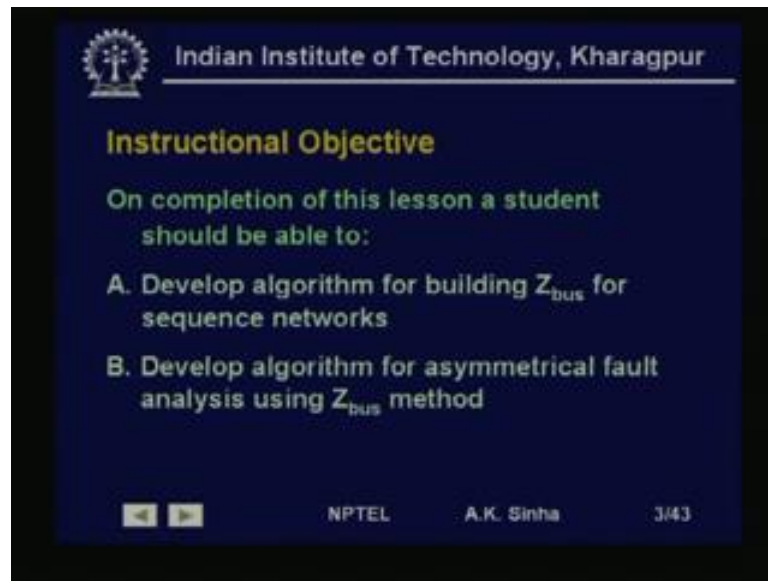
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Well, we will start with an introduction. Then, we will talk about Z bus matrix formulation for sequence networks. Because, we know that for symmetrical faults, we do the symmetrical component analysis. And we get the positive negative and zero sequence network, for the system. So, we will talk about formulating Z bus matrix for the sequence networks.

Then, we will talk about how we use this Z bus for the sequence networks. For finding out or for solving a asymmetrical faults, for large power systems. And finally, we will take up an example to show how this is done.

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Instructional Objective

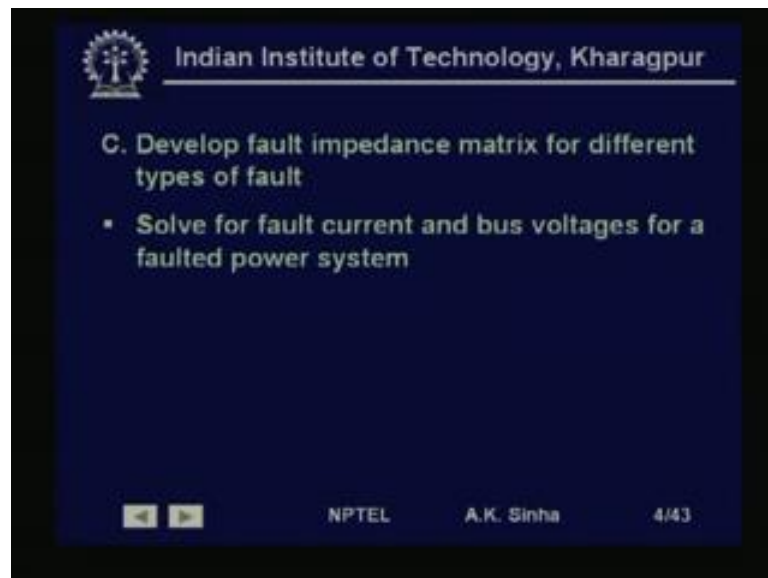
On completion of this lesson a student should be able to:

- A. Develop algorithm for building Z_{bus} for sequence networks
- B. Develop algorithm for asymmetrical fault analysis using Z_{bus} method

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Well, on completion of this lesson, you should be able to develop algorithm for building Z bus, for sequence networks. You should be able to develop algorithm for asymmetrical fault analysis, using Z bus method.

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- C. Develop fault impedance matrix for different types of fault
 - Solve for fault current and bus voltages for a faulted power system

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And you should be able to find out the fault impedance matrix, for different types of faults. And finally, should be able to solve for fault currents and bus voltages, for any power system, for any type of fault. That means, symmetrical as well as asymmetrical faults.

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Asymmetrical Fault Analysis of Large Power Systems

The system under study has n buses. It is initially operating in a symmetrical normal state.

All pre-fault bus voltages and power flows are assumed to be known.

A fault occurs at bus q .

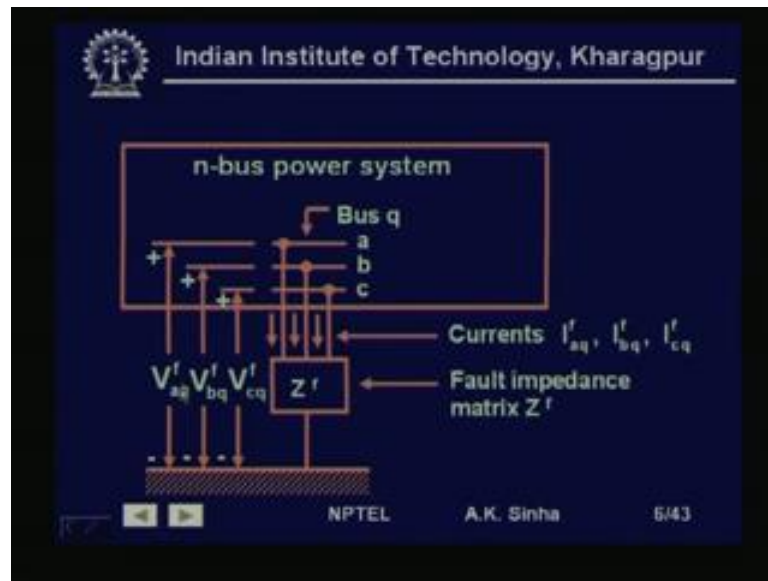
Fault can be represented as a fault impedance matrix Z^f or fault admittance matrix Y^f .

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Well, as we had discussed in the earlier lesson, when we talked about solving for symmetrical faults, using the Z bus method. Here also, we will consider the system and the study has n buses. It is initially operating in a symmetrical normal state. That is a balance three phase system state, is available before the fault. All pre-fault bus voltages and power flows are assume to be known.

That is either we do a load flow analysis to get these values. Or we assume that all pre-fault voltages are 1 per unit. And all pre-fault currents are 0 in all the elements. Now, we will assume that the fault has occurred at a bus q . This bus q can be any bus in the system. And we will ((Refer Time: 03:54)) type of fault, by means of a fault impedance matrix or a fault admittance matrix.

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So, this is the power system that we have. N-bus power system, in which we have just extracted this bus q. And the bus q has a, b, c phases. And there is a fault which has occurred, this fault we are representing by a fault impedance matrix. We will see later, how we do that. The voltages after fault for bus q in phase a, b and c are represented here as V_{aq}^f , V_{bq}^f or V_{cq}^f . Or we can call it as V_f , V_q , V_{faq} and V_{fcq} whatever way you want to call it. So, these are the post fault voltages.

The currents flowing into the fault are I_{faq} , I_{fbq} , I_{fcq} in the three phases. So, this is the system that we have. And we want to find out these voltages, and these currents for this system; when a fault occurs on the system. And the type of fault is governed or governs this fault impedance matrix Z^f . Now, since for asymmetrical fault we know that, we do the analysis using symmetrical component transformation.

That is we get into the positive negative and zero-sequence voltages, and currents. And we had seen for a balanced three phase system. These three networks are independent of each other. And when a fault occurs which is an asymmetrical fault. Then we find that these three networks do get connected, by means of at the fault point. By means of fault impedance and the type of connection depends on the type of fault.

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Network Modeling using Sequence Bus Impedance Matrices

- Network is separately modeled for each of the three sequence systems (i.e., there will be three different bus impedance matrices).
- The network matrices are assembled into a Sequence bus impedance matrix, $Z_{s.bus}$, of dimension $3n \times 3n$.

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So, what we will do first is, we will form the network for positive, negative, and zero-sequence networks separately. That is we will build the Z bus for positive sequence network, negative sequence network and zero-sequence network. And finally, we will assemble these three networks, which we have developed separately. For positive sequence, negative sequence and zero-sequence into a symmetrical component Z bus matrix.

Now, this Z s bus, which is denoting the symmetrical component matrix, that is positive, negative and zero-sequence impedance matrix. For the n bus system, will be of the order of 3 n into 3 n. Because, we have a n into n positive sequence matrix, n into n negative sequence matrix and n into n zero-sequence matrix.

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$$Z_{+bus} = \begin{bmatrix} Z_{+11} & Z_{+12} & \cdots & Z_{+1n} \\ Z_{+21} & Z_{+22} & \cdots & Z_{+2n} \\ \vdots & \vdots & \vdots & \vdots \\ Z_{+n1} & Z_{+n2} & \cdots & Z_{+nn} \end{bmatrix}$$
$$Z_{-bus} = Z_{+bus}$$

The zero-sequence network will be vastly different from both

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So, here as shown this positive sequence matrix Z_{+bus} . We are writing here, which is showing the positive sequence matrix. We have already seen, how we can build this matrix for the positive sequence network of the system. Same way we can build this matrix for the negative sequence, as well as zero-sequence. In general, the negative sequence matrix and positive sequence matrix are considered to be same.

Because, the only elements where we find, that the impedances will be somewhat different are the generators. As well as the loads, where we may have induction machine loads. Then we will find that positive and negative sequence matrix. As positive and negative sequence values for these loads and generators, are somewhat different. And therefore, there can be slight variation in these two matrices.

But, in general this is not large enough that we, that will warrant a separate building of the negative sequence matrix. Therefore, in most of the time, we say that positive sequence and negative sequence matrix are same. And we build only the positive sequence, we assume the negative sequence to be same. Zero-sequence network of course, will be quite different.

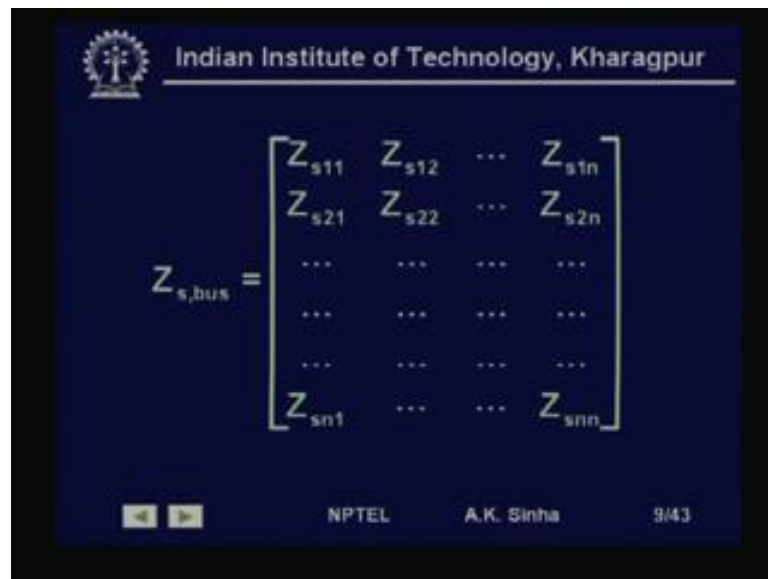
Because, it depends very much on the path for zero-sequence currents are available or not. That is if we have a system with star ungrounded zero-sequence current, cannot flow. So, that is an open circuit for zero-sequence currents. Similarly, if we have delta winding. Then, zero-sequence current can flow in the winding, but they cannot come outside. So, which means that winding will be shorted to the ground, showing that the

current flows in the zero-sequence part. Or zero-sequence current flows in the delta winding.

So, we have already seen all this. How to build positive, negative and zero-sequence network, for different types of connections in power system. So, we will build the zero-sequence network for the given power system, taking care of all star delta connections, that we have on different size of the transformer, as well as whether generator neutral is grounded or ungrounded.

So, all these are taken care of, while building the zero-sequence network. Once we have got the zero-sequence network, we build the zero-sequence Z bus for that network. Of course, since the network for positive sequence and zero-sequence are quite different. Therefore, zero-sequence network or zero-sequence bus matrix or Z bus 0 is going to be quite different. And we need to build this zero-sequence Z bus.

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The slide displays the zero-sequence bus impedance matrix $Z_{s,bus}$ as a square matrix with elements $Z_{s11}, Z_{s12}, \dots, Z_{s1n}$ in the first row, $Z_{s21}, Z_{s22}, \dots, Z_{s2n}$ in the second row, and Z_{sn1}, \dots, Z_{snn} in the last row. The matrix is presented on a dark blue background with the IIT Kharagpur logo and name at the top. Navigation icons and the text 'NPTEL A.K. Sinha 9/43' are visible at the bottom.

$$Z_{s,bus} = \begin{bmatrix} Z_{s11} & Z_{s12} & \dots & Z_{s1n} \\ Z_{s21} & Z_{s22} & \dots & Z_{s2n} \\ \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots \\ Z_{sn1} & \dots & \dots & Z_{snn} \end{bmatrix}$$

So, once we have got these, we have built these three sequence Z buses. That is positive sequence Z bus, negative sequence Z bus and zero-sequence Z bus. Then, we can assemble them together into a Z s bus, which is the sequence bus impedance matrix; where the elements of this matrix $Z_{s11}, Z_{s12}, Z_{s1n}$ and so on, up to Z_{snn} . All these elements are basically are 3 by 3 matrix, which will include the 1 1 element of positive sequence impedance, 1 1 element of negative sequence impedance and 1 1 element of zero-sequence impedance. From the positive negative and zero-sequence Z bus matrix of the system.

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Where each element of the $Z_{s,bus}$ matrix is a 3X3 matrix

$$Z_{s,ij} = \begin{bmatrix} Z_{+ij} & 0 & 0 \\ 0 & Z_{-ij} & 0 \\ 0 & 0 & Z_{0ij} \end{bmatrix}$$

and voltage and current vectors are given as :

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So, this is what this $Z_{s,ij}$ will look like. So, any element ij of $Z_{s,bus}$ matrix will be a 3 by 3 matrix, where we have the positive sequence element ij from the positive sequence Z_{bus} matrix. As a diagonal element placed in position one. And the negative sequence in position two and as the diagonal element and zero-sequence in position three in the diagonal. So, this will be a 3 by 3 diagonal matrix, where the positive negative zero-sequence impedance values are placed as the diagonals. So, this is how the Z_{bus} will look like. That is we will have this n by n $Z_{s,bus}$, where each element of this $Z_{s,bus}$ will be a 3 by 3 matrix. So, finally, we will land with $3n$ into $3n$ matrix.

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$$V_{s,bus} = \begin{bmatrix} V_{+1} \\ V_{-1} \\ V_{01} \\ \vdots \\ V_{+n} \\ V_{-n} \\ V_{0n} \end{bmatrix} = \begin{bmatrix} V_{s1} \\ \vdots \\ V_{sn} \end{bmatrix} \quad \text{and} \quad I_{s,bus} = \begin{bmatrix} I_{+1} \\ I_{-1} \\ I_{01} \\ \vdots \\ I_{+n} \\ I_{-n} \\ I_{0n} \end{bmatrix} = \begin{bmatrix} I_{s1} \\ \vdots \\ I_{sn} \end{bmatrix}$$

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The voltage and currents vectors in the sequence, terms will be given by as shown here. So, $V_{s, bus}$ will again be a $3 \times n$ vector, that is $3 \times n$ rows will be there for this and of course, one column for the voltage. So, V_1, V_{s1} will be basically a 3×1 vector, which will be consisting of V_{+1}, V_{-1} and V_0 . In general the i th element of this $V_{s, bus}$ will be consisting of a 3×1 vector, which will be V_{+i}, V_{-i} and V_0 .

So, this is how this voltage vector and sequence components will look like. Similarly, the current components will also look in the same fashion. Each element I_{si} will consist of the 3 elements I_{+i}, I_{-i} and I_0 . So, this is how it will look like, both the current and voltages vectors are now $3 \times n$ vectors, again as we had developed the algorithm for the symmetrical faults.

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$$V_{s, bus} = Z_{s, bus} I_{s, bus}$$

$$V_{si} = Z_{si1} I_{s1} + Z_{si2} I_{s2} + \dots + Z_{sii} I_i + \dots + Z_{sin} I_{sn}$$

Using the Thevenin's theorem the postfault Bus voltages are given by

$$V'_{s, bus} = V^0_{s, bus} + Z_{s, bus} I'_{s, bus}$$

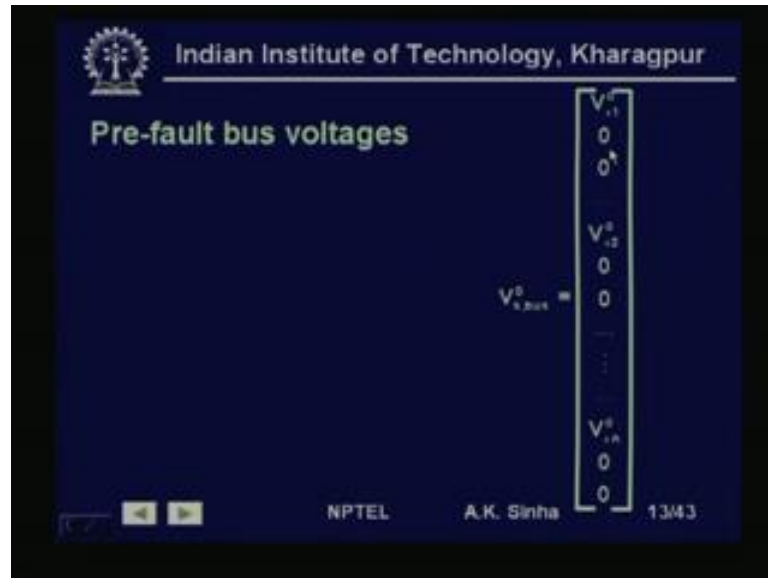
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In the same manner we are going to develop the algorithm here. We have $V_{s, bus}$ is equal to $Z_{s, bus}$ into $I_{s, bus}$. That is where all we have, these values as sequence components there. So, these are $3 \times n$ into 1 vector $3 \times n$ into $3 \times n$ matrix and $3 \times n$ into 1 vector. Now, for from this, we can write down the relationship for the i th element. So, V_{si} is equal to Z_{si1} into I_{s1} plus Z_{si2} into I_{s2} and so on, plus Z_{sin} into I_{sn} .

Now, these elements as we have seen are 3×1 vector. So, this will basically represent three equations. Now, using the Thevenin's theorem, the post fault bus voltages as we have seen earlier, can be written as $V_{s, bus}$ is equal to $V^0_{s, bus}$ plus $Z_{s, bus}$ into $I_{s, bus}$. That is this is the pre-fault voltage, plus the changes caused by the fault current flowing into the system; where we have already shorted all the generating sources. Or all

the voltage sources in the system, and represented them by their internal impedances. So, this we have already seen except that. Now, what we have here is this s coming here, which shows that we are dealing with the three sequence quantities for each one of them.

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So, the pre-fault bus voltage, similarly can be written as it is a $3 \times n$ into 1 vector, where we have V plus 1 0. And since, we have said that initially before fault, the system is working in normal state, which means a balance three phase system state. So, the negative and zero-sequence values for the voltages will be 0. And this is true for all the buses. So, we have negative and zero-sequence values for all the buses as 0, except positive sequence values which will be having 1 per unit. If we are not considering the pre-fault values or if we have done the load flow, then the bus voltages that we get from the load flow, will be put in these locations.

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Fault Current injection:

$$I_{s,bus}^f = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ -I_{s,q}^f \\ \vdots \\ 0 \end{bmatrix}$$

← qth component

$$I_{s,q}^f = \begin{bmatrix} I_{+q}^f \\ I_{-q}^f \\ I_{0q}^f \end{bmatrix}$$

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The fault current as we have seen earlier is again an injection of minus $I_{s,q}^f$. Since, the fault has occurred at bus q , that is if we go back and see this ((Refer Time: 16:40)) system. Then, only at this bus we have a fault, since the fault current is coming out like this which is $I_{s,q}^f$ in terms of sequence currents. This fault current is coming out of the bus and going into the ground, or the reference. Whereas, we have assumed that the faults currents or the current injections are into the bus.

So, we have a negative sign associated with this. And therefore, we can see that, at the q th element for this fault current bus matrix, vector. We have $-I_{s,q}^f$ term coming here, where $I_{s,q}^f$ term is consisting of these three currents I_{+q}^f , I_{-q}^f , I_{0q}^f . So, each element as we have seen consists of all the three sequence values. So, the fault current is injected into the bus which will be equal to $-I_{s,q}^f$, so this what we have...

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$V_{s,\text{bus}}^0$ and $Z_{s,\text{bus}}$ both are known.
We can, therefore, find I_{sq}^f as follows:

$$V_{s1}^f = V_{s1}^0 - Z_{s1q} I_{sq}^f$$

.....

$$V_{sq}^f = V_{sq}^0 - Z_{sqq} I_{sq}^f$$

.....

$$V_{sn}^f = V_{sn}^0 - Z_{snq} I_{sq}^f$$

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Therefore, we can write the for the post-fault voltages, using the pre-fault voltages. And the bus impedance matrix like this. That is V_{fs1} is equal to V_{0s1} minus $Z_{s1q} I_{sq}^f$. That is if we see the relationship, that we have is V is equal to $V_{fs\text{ bus}}$ is equal to $V_{0s\text{ bus}}$ plus $Z_{s\text{ bus}}$ into $I_{fs\text{ bus}}$. Now, $I_{fs\text{ bus}}$ since only one element is non-zero. That is the element at q th location.

So, we are getting only the q th column of $Z_{s\text{ bus}}$, which will be effective, because all other will be multiplied with 0. And they do not come into consideration at all. So, since all these other terms are 0 here. So, all the others $Z_{\text{ bus}}$ matrix columns will get multiplied with 0. And so they will not be coming into picture. So, what we get is only the q th column element for the $Z_{\text{ bus}}$.

So, V_s , V_{0s1} minus, minus is coming because the current is the fault current vector at the q th element. We have the injection as minus I_{fsq} . So, this will be V_{0s1} minus $Z_{s1q} I_{fsq}$ and so on, for all the other buses, we can write this. Now, let us take this at the faulted bus the post-fault voltage is given as V_{fsq} is equal to V_{0sq} minus Z_{sqq} into I_{fsq} .

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$$Z_s^f I_s^f = V_s^0 - Z_{sqq} I_s^f$$

$$I_s^f = (Z_s^f + Z_{sqq})^{-1} V_s^0$$

$$V_{si}^f = V_{si}^0 - Z_{siq} (Z_s^f + Z_{sqq})^{-1} V_{sq}^0 \quad \text{for } i \neq q$$

$$V_{sq}^f = Z_s^f (Z_s^f + Z_{sqq})^{-1} V_{sq}^0$$

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Now, we also know that at the fault bus we have the voltage V_{fq} is equal to Z_{fs} into I_{fs} . That is if we go back and see this. Then, the voltage at this bus is nothing but, this Z_{fs} into the current which is flowing into this. So, this is what we have written. So, if the current here is I_{sq} , which is flowing and Z_{fs} is the impedance of the fault. Then, we are going to get the post fault voltage as V_{fs} is equal to Z_{fs} into I_{fs} . And this is equal to V_{sq}^0 minus Z_{sqq} into I_{fs} .

This is what we have got from here. So, this we are replacing with this term Z_{fs} into I_{fs} , so this is the that we have. From this relationship we can calculate the fault current as I_{fs} is equal to $Z_{fs}^{-1} (V_{sq}^0 - Z_{sqq} I_{fs})$. Now, this relationship is very very similar to what we had got for the symmetrical faults. Except that, now we have this term s coming here into picture. Indicating that each term is basically a 3 by 3 matrix consisting of positive, negative and zero-sequence quantities.

So, this is what we are going to get. Once we have got this I_{fs} value, then we can calculate the V value for any bus. So, V_{fi} is equal to $V_{si}^0 - Z_{siq} I_{fs}$. So, I_{fs} , we are substituting here. So, for $I_{fs} = I_{sq}$, this is the that we have got for I_{fs} . That is post fault voltage at bus q is given by $V_{fq}^f = Z_{fs} I_{fs}$. So, I_{fs} , we have substituted here.

Now, in this we know the pre-fault voltages, and we know the impedances. So, we can calculate the voltages at all the buses. And we can calculate the fault current also. So, this is how we try to solve for a symmetrical faults in a large power system using the Z

bus algorithm. Now, here there is only thing, this fault impedance matrix Z_{fs} , how do we define this matrix, we will see how we do that in the next.

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- All relations involve three-dimensional vectors and 3×3 matrices. The matrices Z_{ij} are diagonal, but the matrix Z'_s is not. In any case, all the involved matrix operations can be easily programmed on a digital computer.
- The pre-fault bus voltages V^0_i are either obtained from a load flow analysis or, are usually set equal to 1.0 pu; i.e.,

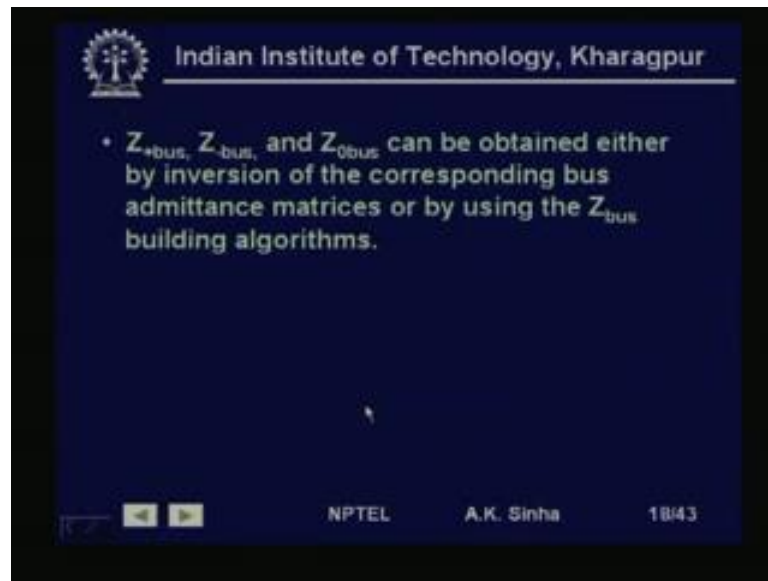
$$V^0_i = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad \text{for } i = 1, \dots, n$$

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Now, here what I have said earlier, I have just written this down that all relations involve three dimensional vectors, and 3 into 3 matrices. That is impedance elements are 3 into 3 matrices. The matrices Z_{ij} are diagonal, but the matrix Z_{fs} which is the fault impedance matrix is not. And this we will see how this matrix changes for different kinds of faults.

In any case all the involved matrix operations, can be easily programmed on a digital computers. So, for large systems, we can always write a program to solve for symmetrical, as well as asymmetrical faults. The pre-fault bus voltages V^0_i are either obtained from load flow analysis, as I said earlier. Or are usually set equal to 1 per unit, that is if we are assuming that pre-fault currents are 0. And all voltages are 1 per unit. Then we can set these values as 1 0 0. Otherwise, we use the load flow analysis to get the results for this.

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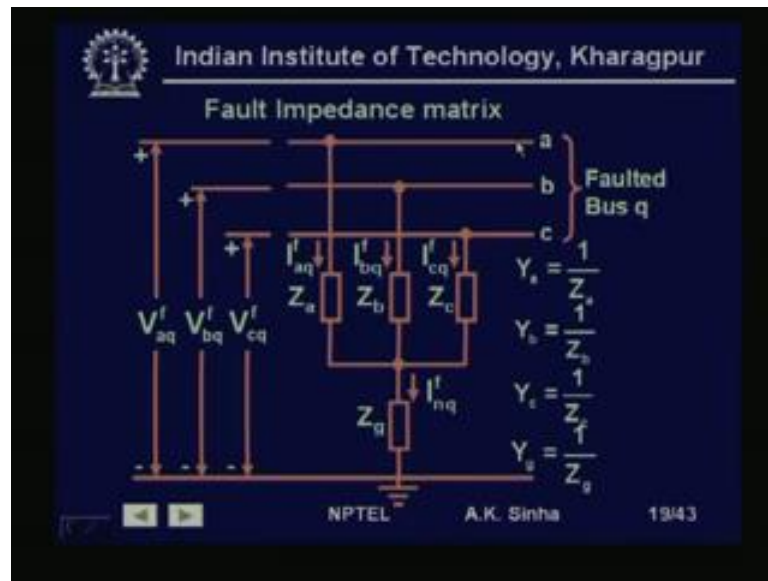


The positive negative and zero-sequence bus impedance matrix can be obtained either by inversion of the corresponding, bus admittance matrices or by using Z bus building algorithm. That is as we have seen earlier, we generally prefer to use Z bus building algorithm to trying to invert the Y bus matrix. Now, we will also sometimes what we do is, what we call partial inversion of the Y bus matrix, which is now preferred over building Z bus.

Because, for any particular fault, what we need is only one column of the Z bus matrix. That is the column corresponding to the faulted bus. So, we can generate this by using the Gauss elimination method using the Y bus for the system. So, positive, negative, zero-sequence Y bus we can build. And we can generate one column of this Z bus matrix corresponding to the faulted bus. And that is done by using gauss elimination method, or LDU factorization.

So, instead of doing this building, sometimes we prefer that, when we do not have to calculate the fault current, at all the buses in the system. But if you need to calculate the fault current at all the buses in the system. For fault at all the buses in the system, then it is better to build the whole Z bus matrix.

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So, next we will talk about the, how to get the fault impedance matrix, because this was the only term which we did not know about. So, let us take a general situation, where we have this faulted bus q and at this faulted bus q, three phases a, b, c are there. Now, we have impedance Z_a , Z_b and Z_c connected to these phases. And these are connected in star, and then connected by means of a grounding impedance Z_g to the ground.

Now, if you want to create any type of fault, we can use this kind of a system. Suppose, we want to create a line to ground fault on phase a, then what we need to do is make this Z_b and Z_c infinity. So, these become open. And we use appropriate values of Z_a and Z_g to indicate the fault impedance, for the single line to ground fault. If we want to make a line to line fault, then what we need to do is? We will between say line to line fault between bus phase b and c.

Then what we need to do is make Z_a as infinite, and Z_g as infinite. And choose appropriate value for Z_b and Z_c , then we have got a line to line fault between bus b and c. Similarly, if we have line to line to ground fault, then only Z_a is made infinite and Z_b , Z_c and Z_g are chosen proper values depending on the fault impedance, that we have. So, we can create all kinds of fault using this network.

Now, let us see that, how we build the impedance matrix for this network. So, now we have Z_a , Z_b , Z_c and Z_g , we can always find out the admittance by taking inverse of these impedances. So, Y_a is $1/Z_a$, Y_b is $1/Z_b$, Y_c is $1/Z_c$ and Y_g is $1/Z_g$. So, we can use either admittance or impedance. Now, let us write down the voltage for

this. Then, what is V_{aq}^f ? This will be equal to I_{aq}^f or I_{faq} , the current flowing in this branch. So, I_{faq} into Z_a , this will be the voltage drop up to this point, plus the voltage drop here. This current I_{fnq} will be equal to sum of all these three currents. So, plus I_{faq} plus I_{fbq} plus I_{fcq} into Z_g . So, this what we are going to get.

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The Fault Impedance Matrix Z^f

$$V_{aq}^f = I_{aq}^f Z_a + (I_{aq}^f + I_{bq}^f + I_{cq}^f) Z_g$$

$$V_{bq}^f = I_{bq}^f Z_b + (I_{aq}^f + I_{bq}^f + I_{cq}^f) Z_g$$

$$V_{cq}^f = I_{cq}^f Z_c + (I_{aq}^f + I_{bq}^f + I_{cq}^f) Z_g$$

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So, this is what we have written here, V_{faq} is equal to $I_{faq} Z_a$ plus I_{faq} plus I_{fbq} plus I_{fcq} into Z_g . Similarly, we can write for phase b and c. So, we get this three equations, for the post-fault voltages of the three phases at the faulted bus.

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$$\begin{bmatrix} V_{aq}^f \\ V_{bq}^f \\ V_{cq}^f \end{bmatrix} = \begin{bmatrix} Z_a + Z_g & Z_g & Z_g \\ Z_g & Z_b + Z_g & Z_g \\ Z_g & Z_g & Z_c + Z_g \end{bmatrix} \begin{bmatrix} I_{aq}^f \\ I_{bq}^f \\ I_{cq}^f \end{bmatrix}$$

Where fault impedance matrix

$$Z^f = \begin{bmatrix} Z_a + Z_g & Z_g & Z_g \\ Z_g & Z_b + Z_g & Z_g \\ Z_g & Z_g & Z_c + Z_g \end{bmatrix}$$

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Now, this in a matrix form we can write this as $V_{f a q}$, $V_{f b q}$, $V_{f c q}$ is equal to this matrix into $I_{f a q}$, $I_{f b q}$, $I_{f c q}$. And what is this matrix, now this is the fault impedance matrix in phase values. So, this is the fault impedance matrix in phase currents. And in terms of phase a, b, c not in sequence.

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Eq. can be written in compact form

$$V_{pq}^f = Z_{pq}^f I_{pq}^f$$

$$TV_{sq}^f = Z_{sq}^f T I_{sq}^f$$

$$V_{sq}^f = T^{-1} Z_{sq}^f T I_{sq}^f \triangleq Z_{sq}^f I_{sq}^f \quad T^{-1} = \frac{1}{3} \begin{bmatrix} 1 & a & a^2 \\ 1 & a^2 & a \\ 1 & 1 & 1 \end{bmatrix}$$

T = $\begin{bmatrix} 1 & 1 & 1 \\ a^2 & a & 1 \\ a & a^2 & 1 \end{bmatrix}$

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So, what we need to do is, we need to convert it into sequence terms. So, since we are working in powers, in terms of symmetrical components. So, we will change it to the positive negative and zero-sequence terms. So, now we can write down the phase relationship $V_{f P q}$, P is indicating the phasor values. Is equal to $Z_{f I f P q}$, this is the relationship as Z_{f} we have already seen is what.

Now, if we multiply it by the symmetrical component transformation matrix, on both sides. Then we have this $T V_{f s q}$, that is now we are saying $V_{f p q}$ is equal to T into $V_{f s q}$. This is what we had seen, when we talked about the symmetrical component. Similarly, $I_{f P q}$ can be replaced by T into $I_{f s q}$. So, we have put these values here. Now, pre-multiply both sides by T inverse. So, we will get $V_{f s q}$ is equal to T inverse Z_{f} into T into $I_{f s q}$.

Now, this term T inverse Z_{f} into T is again a 3 by 3 matrix, which we will get. And this is the symmetrical component or the fault impedance matrix, in terms of symmetrical component. Now, here that is what we have written $Z_{f s}$ into $I_{f s q}$, where $Z_{f s}$ is the fault impedance matrix, in terms of symmetrical component. Now, the transformation

matrix T as we have seen earlier is given like this, when we write the equations in terms of positive negative and zero-sequence voltages and currents.

And T inverse is given by this, which is basically 1/3rd into the transpose of T. So, with this we have already seen earlier, except that earlier we had written all the equations in terms of 0, positive, negative. We can write it any in any order we want. And just to show that, we can write it as positive negative 0, as well we have written here, as positive negative and zero-sequence terms. So, the transformation matrix in this case will be like this.

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$$= \frac{1}{3} \begin{bmatrix} Z_1 + Z_2 + Z_0 & Z_1 + a^2 Z_2 + a Z_0 & Z_1 + a Z_2 + a^2 Z_0 \\ Z_1 + a Z_2 + a^2 Z_0 & Z_1 + Z_2 + Z_0 & Z_1 + a^2 Z_2 + a Z_0 \\ Z_1 + a^2 Z_2 + a Z_0 & Z_1 + a Z_2 + a^2 Z_0 & Z_1 + Z_2 + Z_0 + 9Z_0 \end{bmatrix}$$

For some type of faults some of the impedances take on infinite values. Some elements of the Z_{fs} matrix become undefined. In such cases we must work with its inverse, the SC transformed fault admittance matrix.

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So, once we have got this T and T inverse. We know, then we after doing the multiplication, we will get the fault impedance matrix. In terms of sequence values will be like this, where we have a 3 by 3 matrix Z_{fs} given by this. Now, here these terms a a square all this as we know. A is given by one angle 120 degree. So, it is a vector, which rotates any phasor by 120 degree. So, this is the fault impedance matrix that we get.

Now, as we have seen for creating different kinds of faults, we have to make some of these impedance Z_a , Z_b , Z_c or Z_g as infinite. If we do that, then we find that this matrix will have terms, which are infinity. And therefore, this matrix will not be defined. In that case, we need to change over to fault admittance matrix. So, this is what I have mentioned here.

For some type of faults, some of the impedances take on infinite values. Some elements of Z_{fs} matrix becomes undefined in such cases, we must work with it is inverse the

symmetrical component transform fault admittance matrix. So, instead of fault impedance matrix, we work with fault admittance matrix. When fault impedance matrix is not defined means some of the elements of this become infinite.

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$$I_{sq}^f = (Z_s^f)^{-1} V_{sq}^f \triangleq Y_s^f V_{sq}^f$$

$$Y_s^f = \frac{1}{Y_a + Y_b + Y_c + Y_0}$$

$$\begin{bmatrix} \frac{1}{3}Y_a(Y_b + Y_c + Y_0) & \frac{1}{3}Y_a(Y_b + a^2Y_c + aY_0) & \frac{1}{3}Y_a(Y_b + aY_c + a^2Y_0) \\ + (Y_aY_b + Y_aY_c + Y_aY_0) & -(Y_aY_b + aY_aY_c + a^2Y_aY_0) & \\ \frac{1}{3}Y_b(Y_c + aY_a + a^2Y_0) & \frac{1}{3}Y_b(Y_c + Y_a + Y_0) & \frac{1}{3}Y_b(Y_c + a^2Y_a + aY_0) \\ (Y_bY_c + a^2Y_bY_a + aY_bY_0) & +(Y_bY_c + Y_bY_a + Y_bY_0) & \\ \frac{1}{3}Y_c(Y_a + a^2Y_b + aY_0) & \frac{1}{3}Y_c(Y_a + aY_b + a^2Y_0) & \frac{1}{3}Y_c(Y_a + Y_b + Y_0) \end{bmatrix}$$

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Now, how do we find out the fault admittance matrix. We have the relationship $I_{f s q}$ is equal to $Z_{f s}$ inverse into $V_{f s q}$. That is $V_{f s q}$ is equal to $Z_{f s}$ into $I_{f s q}$. Therefore, we pre-multiply both sides by $Z_{f s}$ inverse. So, we will get $I_{f s q}$ is equal to $Z_{f s}$ inverse into $V_{f s q}$, which shows that this part $Z_{f s}$ inverse is nothing but, the fault admittance matrix $Y_{f s}$.

And if we take this inverse of this 3 by 3 matrix, we are going to get the $Y_{f s}$ like this. This looks somewhat complicated here, but we know all these values of Y_a , Y_b , Y_c as well as a . And so we can calculate this for different kinds of faults. And we will be able to get this 3 by 3 matrix for the fault. So, fault admittance matrix can be computed for any type of fault. Sometimes the fault impedance matrix is not feasible. Then, we use fault admittance matrix.

Sometime fault impedance matrix is not feasible, then we use fault admittance matrix. So, different types of faults, we have to use the appropriate fault impedance matrix. Or fault admittance matrix, whichever is defined for that particular fault.

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$$V_{sq}^f = V_{sq}^0 - Z_{sqq} Y_s^f V_{sq}^f$$

$$V_{sq}^f = (U + Z_{sqq} Y_s^f)^{-1} V_{sq}^0$$

$$I_{sq}^f = Y_s^f V_{sq}^f = Y_s^f (U + Z_{sqq} Y_s^f)^{-1} V_{sq}^0$$

$$V_{si}^f = V_{si}^0 - Z_{siq} I_{sq}^f = V_{si}^0 - Z_{siq} Y_s^f (U + Z_{sqq} Y_s^f)^{-1} V_{sq}^0 \quad i \neq q$$

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So, once we have got this fault admittance matrix. Then, we need to write the equations in terms of fault admittance matrix. And we have to compute the voltages at various, post fault voltages at various buses and the fault current. So, V_{fsq} , we can write as V_{sq}^0 minus Z_{sqq} into we have I_{fsq} . Instead of I_{fsq} , we are writing as Y_{fs} into V_{fsq} . So, we are replacing I_{fsq} by this term. And if we do this, then we can write that V_{fsq} is equal to U is the unity matrix plus Z_{sqq} into Y_{fs} inverse into V_{sq}^0 .


So, this is what we will get. That is we take this part to the left hand side. And then we pre-multiply both sides by the inverse, this inverse that we will get, because we will get U plus Z_{sqq} into Y_{fs} into V_{fsq} . So, we pre-multiply by U plus Z_{sqq} into Y_{fs} inverse, then we will get this. So, this is U will be a 3 by 3 unity matrix, this is a 3 by 3 matrix, this is a 3 by 3 matrix.

So, we are getting the three post fault voltages for the bus q , as positive sequence voltage, negative sequence voltage and zero-sequence voltages. So, once we have got this, we can calculate the fault current as I_{fsq} is equal to Y_{fs} into V_{fsq} . And V_{fsq} , we have already calculated here. So, substituting it we will get this value. So, if Z_{fs} is not defined, then using Y_{fs} , we can calculate the fault current like this.

And once we know the fault current, for the voltage at any bus V_{fsi} , can be calculated by substituting the value of fault current, from here into the equations. So, V_{fsi} is equal to V_{si}^0 minus Z_{siq} into I_{fsq} and I_{fsq} , we are putting this value here. So, this is what we do. So, this is what we are going to get for i not equal to q , this is $i \neq q$ is not

there, this is i naught equal to q . So, this is the way we calculate all the values of post-fault voltages, at all the buses; as well as the fault current. Once we know the post-fault voltages at all the buses, we can find out the post-fault currents in any branch in the system. So, this how we try to or we compute the post fault voltages, and currents for any type of fault on a power system.

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Example: Two synchronous machines are connected through three-phase transformers to the transmission line as shown in figure. The rating and reactance of the machines and transformers are:

Machine 1 and 2: 100 MVA, 20KV;
 $X_d'' = X_1 = X_2 = 12\%$
 $X_0 = 5\% \quad X_n = 4\%$

Transformers T₁ and T₂: 100 MVA,
 20Δ/400Ykv; $X = 8\%$

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So, we will take again the same example, two synchronous machines are connected to three phase transformers to the transformation line as shown in the figure. The rating and the reactances of the machines. And transformers are given as, it was given before machine 1 and 2 100 MVA to 20 KV. X_d'' , which is equal to X_1 is equal to X_2 which is equal to 12 percent.

We are taking sub transient reactance to so that, what we are trying to do is. We are trying to calculate the fault current almost immediately after the fault. And the value of X_0 is 5 percent and X_n is 4 percent. This is the reactance through, which the generator neutral is grounded. Transformer T₁ and T₂, the base MVA is 100 MVA. And their voltage ratios are given as 20 KV on the delta side and 400 KV. This is not delta, this will be star, this case we have taken both sides as star grounded. And the impedance for this is given as 8 percent on this voltage bases.

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On a chosen base of 100 MVA, 345 KV in the transmission line circuit the line reactances are $X_1=X_2=15\%$ and $X_0=50\%$. Draw each of the three sequence networks and find fault current and post fault bus voltages for a bolted L-G fault on bus 2 using the Z_{bus} algorithm.

Machine 1 (1) T_1 (2) (3) T_2 (4) Machine 2

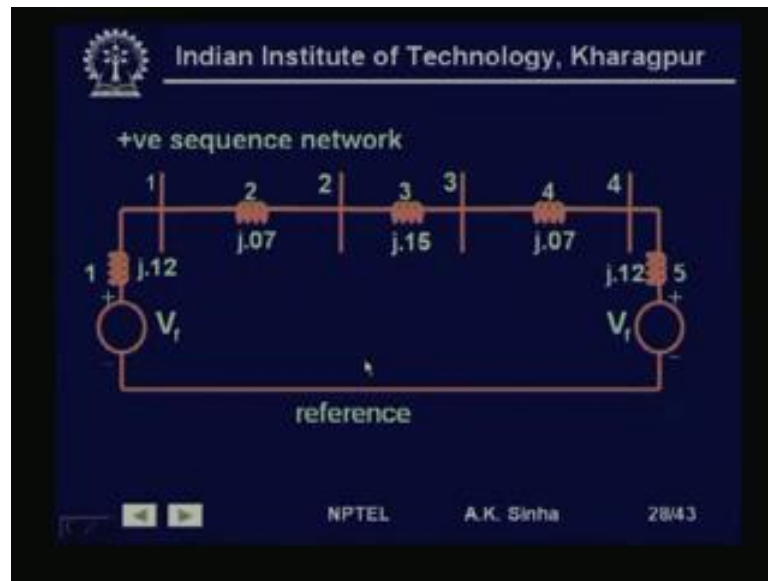
Single line diagram of the system

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Now, we have chosen a base voltage as 345 KV, which is for the line. So, we will convert the impedance for the transformer, transformer impedance positive, negative, 0 are same. So, 8 percent impedance given at 400 KV, which will be transformed to 345 KV in per unit system. Then, we will get this value as around 7 percent or 0.07. Transmission line has a reactance of X_1, X_2 as 15 percent.

And the zero-sequence reactance, which is generally higher for the transmission line is 50 percent. Draw each of the three sequence networks and find the fault current. And post fault voltages for a bolted line to ground fault on bus 2, using the Z bus algorithm. So, this is the system that we have. We have both the transformers, which are star grounded. And we have the generators also where the neutral is grounded by means of some impedance. So, this is the system that we have been working on, in other lessons also.

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So, we work on this, the positive sequence network we have seen earlier. Is we have the voltage here voltage source. And the positive sequence impedance is given as $j0.12$, 12 percent impedance X_d double dash or X_1 . And this is transformer impedance, as we said comes out to be 7 percent or 0.07. The positive sequence impedance of the transmission line, is given as 0.15.

Positive sequence impedance of transformer T 2 is again 7 percent. And positive sequence impedance of the generator is again 0.12. These elements as we have seen, we have numbered them 1, 2, 3, 4, 5. And we have already shown, how to build the Z bus matrix for this. So, positive sequence matrix for this we have built, in the previous class. So, we will not do it again.

Similarly, we will write the zero-sequence network, because positive and negative sequence values are all same in this case. So, both positive sequence Z bus matrix and negative sequence, Z bus matrix will be same. Now, the zero-sequence we do not have a voltage source. We have $3X_g$, so X_g is given as 0.04, that is 4 percent. So, this becomes 0.12, and the zero-sequence reactance of the machine is given as 5 percent.

So, these two are in series here, connected to bus 1. Between bus 1 and 2, we have the transformer whose zero-sequence reactance, is same as positive negative, which is 0.07. The line, this should read 5, 0. Because, the line has as we have shown here, the line has X_0 is 50 percent not 15 percent. So, that should read 50, this is a mistake here, it should read 50. So, $j50$, this same this has been taken care in the building algorithm.

So, that Z_{bus} , Z_0 bus is here only there is mistake in writing this one, should not have been there it is $j 0.5$ And the T 2 has a reactance 0.7, similarly for the generator 3 X_g that is $j 0.12$ and X_0 that is 0.5 Now, these are the elements, that we have element number 1, consisting of these two element number 2, 3, 4 and 5. We can use this in the same way and build the zero-sequence impedance matrix.

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Z_{bus} for the positive sequence network was already built in the previous lesson. Also we have

$$Z_{bus}^{(1)} = Z_{bus}^{(2)} = \begin{matrix} \textcircled{1} & \textcircled{2} & \textcircled{3} & \textcircled{4} \\ \begin{matrix} \textcircled{1} \\ \textcircled{2} \\ \textcircled{3} \\ \textcircled{4} \end{matrix} & \begin{bmatrix} j.0928 & j.0769 & j.043 & j.0271 \\ j.0769 & j.1218 & j.0681 & j.0430 \\ j.0430 & j.0681 & j.1218 & j.0769 \\ j.0271 & j.043 & j.0769 & j.0928 \end{bmatrix} \end{matrix}$$

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So, positive sequence impedance matrix, which we had build in the previous class is shown here.

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Z_{bus} for the zero sequence network using the building algorithm is given by

$$Z_{bus}^0 = \begin{bmatrix} j.1406 & j.1284 & j.0417 & j.0295 \\ j.1284 & j.1813 & j.0588 & j.0417 \\ j.0417 & j.0588 & j.1813 & j.1284 \\ j.0295 & j.0417 & j.1284 & j.1406 \end{bmatrix}$$

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And in the similar way we can build the zero-sequence matrix, which comes out to be like this. So, we have build both the positive and zero-sequence, bus impedance matrices. Now, we will see, how we will use them for our single line to ground fault at bus 2.

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For single line to ground fault,

- (1) $Z_b = Z_c = \infty$ ($\therefore Y_b = Y_c = 0$)
- (2) $Z_g = 0$ ($\therefore Y_g = \infty$)
- (3) $Z_a = Z^f$ ($\therefore Y_a = \frac{1}{Z^f} \triangleq Y^f$)

Since $Y_g = \infty$, Y_a^f Reduces to

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Now, for single line to ground fault Z_b is equal to Z_c is infinity. That is we have seen earlier Z_b and Z_c will be infinity, these are open only phase a is grounded through Z_a and Z_g . So, Z_b Z_c is infinities, therefore Y_b Y_c is equal to 0. Z_g we have made equal to 0. So, Y_g is equal to infinity, that is the fault has occurred only with impedance Z_a . Z_a is equal to Z^f and therefore, Y_a is equal to $1/Z^f$, which is we are defining as Y^f . Since, Y_g is infinity Y^f s reduces to this value.

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$$Y'_s = \frac{1}{3} \begin{bmatrix} Y_a + Y_b + Y_c & Y_a + a^2 Y_b + a Y_c & Y_a + a Y_b + a^2 Y_c \\ Y_a + a Y_b + a^2 Y_c & Y_a + Y_b + Y_c & Y_a + a^2 Y_b + a Y_c \\ Y_a + a^2 Y_b + a Y_c & Y_a + a Y_c + a^2 Y_b & Y_a + Y_b + Y_c \end{bmatrix}$$

Upon substitution of
 $Y_a = Y'_s$ and $Y_b = Y_c = 0$, simplifies to

$$Y'_s = \frac{Y'_s}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

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That is if you recall this matrix, now here you have Y_g , and in all these we have Y_g . So, what we can do is, we can divide this by Y_g . Then this we will have Y_a plus Y_b plus Y_c divided by Y_g plus 1. So, this whole term becomes 1 by 1 and because, Y_g is infinite. So, this term will come out to be 0. So, this whole will come out to be 1. And this Y_g will not be there, because we have already taken it out and divided here. So, this is what we are going to get.

So, the matrix reduces like this. And upon substitution of Y_a is equal Y_f and Y_b is equal to Y_c is equal to 0. If we substitute here, then we this matrix Y_f s simplifies to Y_f by 3 and 1 1 1, 1 1 1, 1 1 1. So, single line to ground fault, this is the sequence fault impedance matrix that we get.

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$$V_{s,q}^f = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \begin{bmatrix} Z_{+qq} & 0 & 0 \\ 0 & Z_{-qq} & 0 \\ 0 & 0 & Z_{0qq} \end{bmatrix} \frac{Y^f}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}^{-1} \begin{bmatrix} V_q^d \\ 0 \\ 0 \end{bmatrix}$$

$$V_{s,q}^f = \frac{V_q^d}{1 + \frac{Y^f}{3}(Z_{+qq} + Z_{-qq} + Z_{0qq})} \begin{bmatrix} 1 + \frac{Y^f}{3(Z_{-qq} + Z_{0qq})} \\ -Z_{-qq} \frac{Y^f}{3} \\ -Z_{0qq} \frac{Y^f}{3} \end{bmatrix}$$

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Once we have got this fault impedance matrix like this. Then, we can calculate the post-fault voltage at the faulted bus like this, where we are assuming the pre-fault voltages to be 1 per unit, because we have not been given any pre-fault bus voltages, from load flow. So, we are assuming it to be 1 per unit. So, we have this relationship here, as $V_{s,q}^f$ is equal to U plus $Z_{s,q,q}$ into Y^f inverse into $V_{s,q}^0$. I made this mistake, this is U .

So, we have a unity matrix here, unity matrix here plus $Z_{s,q,q}$. So, $Z_{s,q,q}$ is 3 by 3 matrix here into Y^f by 3 into this. So, this is Y^f matrix this inverse into $V_{s,q}^0$. Because, the pre-fault voltages negative and zero-sequence are 0; and the only the positive sequence value is here. So, if we simplify this, we are going to get $V_{s,q}^f$ as $V_{s,q}^0$ divided by $1 + \frac{Y^f}{3}(Z_{+qq} + Z_{-qq} + Z_{0qq})$ term coming here, multiplied by this $1 + \frac{Y^f}{3}(Z_{-qq} + Z_{0qq})$ minus $Z_{-qq} \frac{Y^f}{3}$ into Y^f by 3 minus $Z_{0qq} \frac{Y^f}{3}$. So, this is what we are going to get for the post fault voltage at the faulted bus.

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For the short circuit at the faulted bus,

$$I_{sq}^f = Y_s^f V_{sq}^f = V_q^0 \frac{Y^f/3}{1 + \frac{Y^f}{3}(Z_{+qq} + Z_{-qq} + Z_{0qq})} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

The post fault voltages other than the faulted one are obtained as

$$V_{si}^f = V_{si}^0 - Z_{siq} I_{sq}^f$$

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And similarly, we can get the current I_{fsq} as Y_{fs} into V_{fsq} since V_{fsq} , we have already seen how the relationship. So, substituting it we will get this relationship. So, the post-fault voltages for buses, other than the faulted one are obtained as V_{sif} is equal to V_{si0} minus Z_{siq} into I_{fsq} and I_{fsq} , we can calculate like this.

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$$= \begin{bmatrix} V_i^0 \\ 0 \\ 0 \end{bmatrix} - \begin{bmatrix} Z_{+iq} & 0 & 0 \\ 0 & Z_{-iq} & 0 \\ 0 & 0 & Z_{0iq} \end{bmatrix} I_{sq}^f$$

Substituting I_{sq}^f

$$V_{si}^f = \begin{bmatrix} V_i^0 \\ 0 \\ 0 \end{bmatrix} - V_q^0 \frac{Y^f/3}{1 + \frac{Y^f}{3}(Z_{+qq} + Z_{-qq} + Z_{0qq})} \begin{bmatrix} Z_{+iq} \\ Z_{-iq} \\ Z_{0iq} \end{bmatrix}$$

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So, if we want to do that, this comes out to be, this is the pre-fault voltage for that bus i . Minus Z_{siq} into I_{fsq} and substituting for I_{fsq} , we will get this relationship for post fault voltage at bus i .

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For $Z_f = 0$ above equations simplifies to,

$$V_{s,q}^f = \frac{V_q^s}{1 + \frac{Y_f}{3}(Z_{+qq} + Z_{-qq} + Z_{0qq})} \begin{bmatrix} Z_{-qq} + Z_{0qq} \\ -Z_{-qq} \\ -Z_{0qq} \end{bmatrix}$$

$$I_{s,q}^f = Y_f V_{s,q}^f = \frac{1}{Z_{+qq} + Z_{-qq} + Z_{0qq}} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

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So, this is what we are going to get for Z_f is equal to 0. The above quantities, since we are assuming Z_f to be 0, that is the bolted fault. We are the above equation will simplify like this. So, $I_{s,q}^f$ is equal to $V_{s,q}^f$ into Y_f which comes out to be this much.

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$$V_{s,i}^f = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} - \frac{1}{Z_{+qq} + Z_{-qq} + Z_{0qq}} \begin{bmatrix} Z_{+iq} \\ Z_{-iq} \\ Z_{0iq} \end{bmatrix} \text{ for } i \neq q$$

Since short circuit occur at bus 2, $q = 2$

$$Z_{s,qq} = Z_{s,22} = \begin{bmatrix} Z_{+22} & 0 & 0 \\ 0 & Z_{-22} & 0 \\ 0 & 0 & Z_{022} \end{bmatrix}$$

$$= \begin{bmatrix} j.1218 & 0 & 0 \\ 0 & j.1218 & 0 \\ 0 & 0 & j.1813 \end{bmatrix} \text{ p.u.}$$

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So, we have got $V_{s,i}^f$, V in terms of Z is like this. Since, the short circuit occurs at bus 2. So, q is equal to 2, so $Z_{s,qq}$ is equal to $Z_{s,22}$, which is equal to this positive negative, 0. The values are seen from the positive negative zero-sequence matrices, that we have calculated zero-sequence Z_{22} is this.

And positive sequence Z_{22} is this. So, we use these values here, we get this like this. And then we can calculate V_{f2} as from by substituting all the values, this comes out to be like this. And using V_{f2} , we can calculate I_{f2} as this one. So, we have got the values, this should be I_{f2} . So, this is the value that, we will get for the fault current positive negative zero-sequence. Now, we can transform it into the phase values.

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$$Z_{12} = \begin{bmatrix} Z_{+12} & 0 & 0 \\ 0 & Z_{-12} & 0 \\ 0 & 0 & Z_{012} \end{bmatrix} = j \begin{bmatrix} .0769 & 0 & 0 \\ 0 & .0769 & 0 \\ 0 & 0 & .1284 \end{bmatrix}$$

$$Z_{32} = \begin{bmatrix} Z_{+32} & 0 & 0 \\ 0 & Z_{-32} & 0 \\ 0 & 0 & Z_{032} \end{bmatrix} = j \begin{bmatrix} .0681 & 0 & 0 \\ 0 & .0681 & 0 \\ 0 & 0 & .0588 \end{bmatrix}$$

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Similarly, if we want to calculate the values current flowing in different branches. We can use Z_{s12} , Z_{s32} ; and all that for calculating the values at different branches.

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$$Z_{s42} = \begin{bmatrix} Z_{+42} & 0 & 0 \\ 0 & Z_{-42} & 0 \\ 0 & 0 & Z_{042} \end{bmatrix} = j \begin{bmatrix} .043 & 0 & 0 \\ 0 & .043 & 0 \\ 0 & 0 & .0417 \end{bmatrix}$$

$$V'_{f1} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \cdot \frac{1}{(.1218 + .1218 + .1813)} \begin{bmatrix} j.0769 \\ j.0769 \\ j.1284 \end{bmatrix}$$

$$= \begin{bmatrix} .819 \\ -.181 \\ -.302 \end{bmatrix}$$

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So, and for finding out the values of post fault voltages at different buses. We can use this relationship. So, again this is $V_s - V_0$ minus this relationship, that we had got there. So, this is the sequence values that we have got.

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$$V'_{a3} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} - \frac{1}{(.1218 + .1218 + .1813)} \begin{bmatrix} j.0681 \\ j.0681 \\ j.0588 \end{bmatrix} = \begin{bmatrix} .839 \\ -.160 \\ -.138 \end{bmatrix}$$

$$V'_{a4} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} - \frac{1}{(.1218 + .1218 + .1813)} \begin{bmatrix} j.043 \\ j.043 \\ j.0417 \end{bmatrix} = \begin{bmatrix} .898 \\ -.101 \\ -.098 \end{bmatrix}$$

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Similarly, for 3 and 4 also, we will get the values, by substituting the values from the impedance matrix. And the fault current that we have calculated, we will get these sequence values. Once we have got the sequence values, we can transform them to get the phase values. So, this is how we can calculate the fault current; and the post fault voltages at all the buses. And knowing the impedances of the lines, or the other equipments, we can find out the fault current flowing through each element of the system.

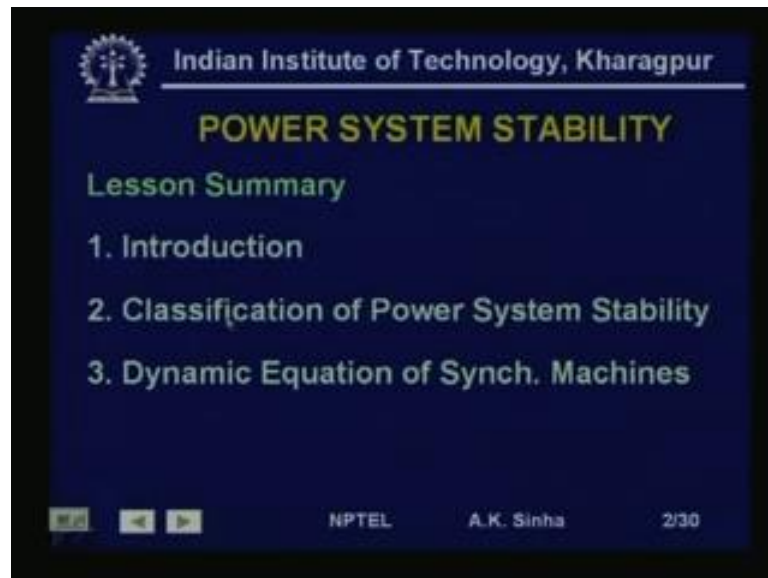
So, with this we have completed our short circuit analysis. Or the analysis of a faulted power system, where we have considered both the symmetrical faults. Asymmetrical faults for small systems by using the sequence network, or using the Z bus algorithm. And Y bus algorithm for large systems. So, with this we finish today. And we will talk about the power system stability in the next lessons.

Thank you very much.

Preview of next lecture

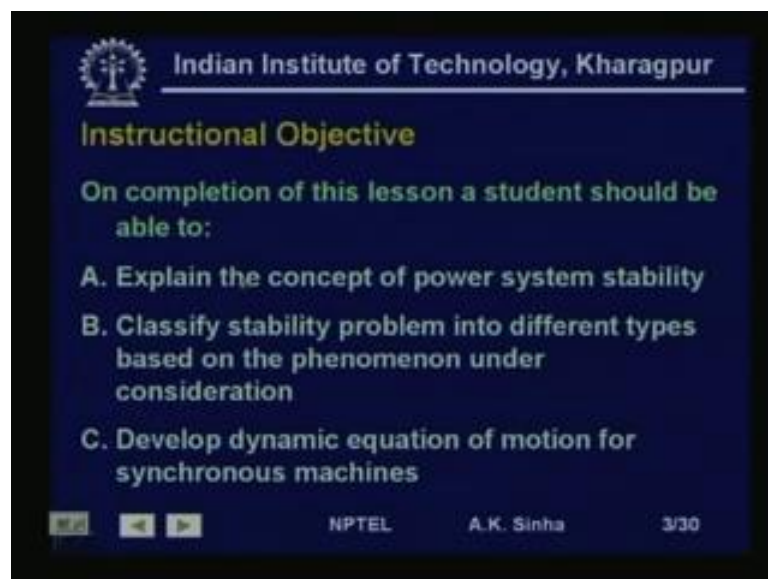
Welcome to lesson 33, on Power System Analysis. In this lesson, we will discuss the stability of power system, when it is subjected to disturbances. We will start with an introduction to the problem of stability.

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Then, we will go into the classification of different types of stability, in case of power systems. And finally, we will try to develop the dynamic equation for synchronous machines in the power system.

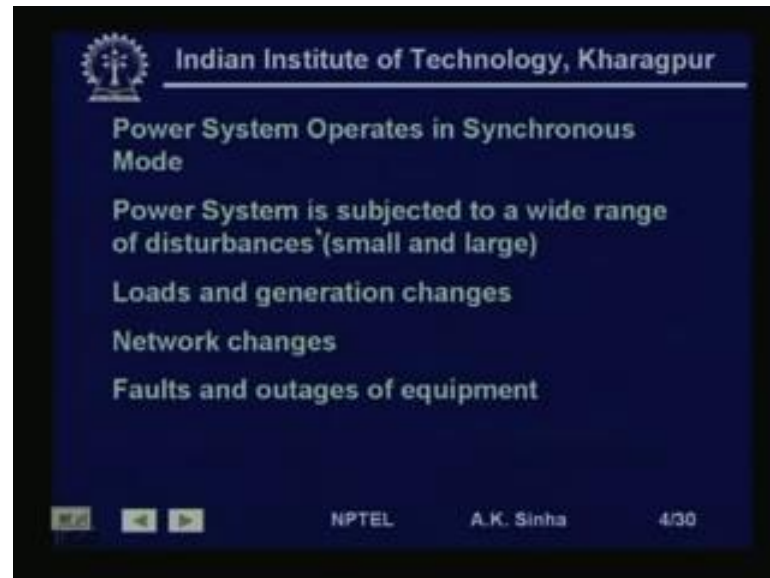
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On the completion of this lesson, you should be able to explain the concepts of power system stability. You should be able to classify stability problem, into different types,

based on the phenomenon under consideration. And you should be able to develop dynamic equation of motion, for the synchronous machines.

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Well, as we all know power system in its normal operating state, works as a synchronous system. That is all the machines in the power system, are synchronized with each other. That is they run at a common frequency, as we have talked about in our earlier lessons, when we dealt with short circuits. We had seen that short circuits or faults, create a condition, which is very different from the normal operating condition.

In terms, that a short circuit will make very heavy currents flow in the circuit, as well as the voltages at different parts of the system, will go to very low values. These are abnormal operating conditions. So, a fault does create a disturbance, in the power system operation. And due to these disturbances will the power system remain in synchronism, or not, is the kind of study. That we would be taking up in power system stability.

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CIGRE-IEEE Def. Of Power System Stability:-

Power system stability is the ability of an electric power system, for a given initial operating condition, to regain a state of operating equilibrium after being subjected to a physical disturbance, with most system variables bounded so that practically the entire system remains intact.

Stability → involves study of dynamics of the system about an equilibrium- initial op. cond.

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So, it says power system stability is the ability of an electric power system, for a given initial operating condition, to regain a state of operating equilibrium. That is to regain a state of operating equilibrium which means, where the real and reactive power balance is maintained. That is the system is working in synchronism. And the voltages at various points in the system, are ready in the normal operating limits, to regain a state of operating equilibrium, after being subjected to a physical disturbance. So, all this we are saying after the system has been subjected to a disturbance. Whether there is going to be, that is system is going to regain a state of operating equilibrium or not, with most system variables bounded, that is the frequency, the voltage, all these are within the limits. None of these are exceeding, or keep on increasing all the time.

So that, practically the entire system remains intact, which again says that most of the part or the major part of the system still remains intact, works in synchronism with voltages within normal limits. That is a normal operating condition for the system, is maintained even after the disturbance. Then, we call the system is stable. So, with this we will end today. Thank you, in the next class we will talk more about this, equation of rotor dynamics. And we will talk about how we work in multi machines systems, how we write the sync equation for each machine.

Thank you.