Power System Analysis
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Lecture - 31
Bus Impedance Matrix

Welcome to lesson 31 on Power System Analysis. In this lesson we will discuss about formation of Bus Impedance Matrix.

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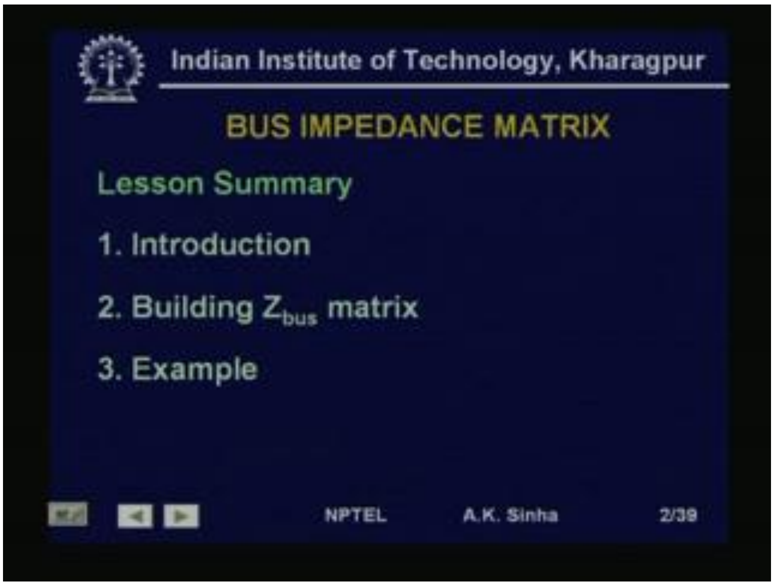

We will start with an introduction. Then, we will go to the method for building Z bus matrix. And finally, we will take up an example for building this Z bus matrix.

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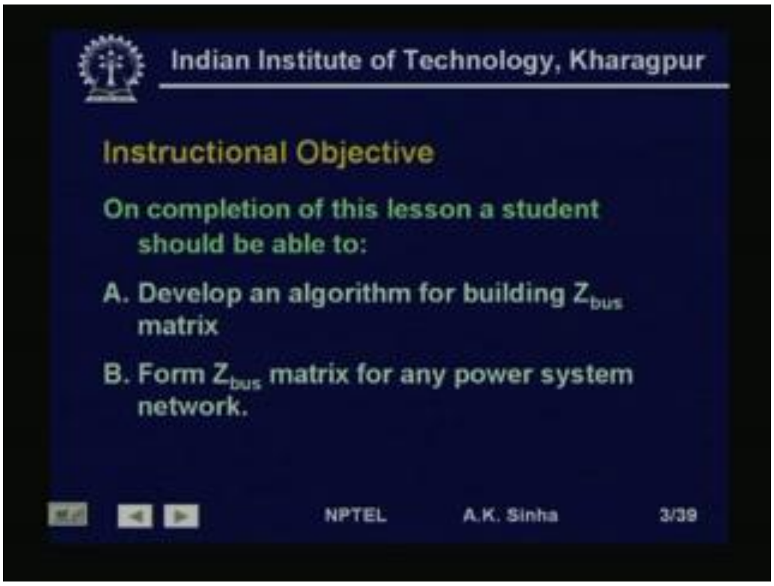

On completion of this lesson, you should be able to develop an algorithm for building Z bus matrix. And form Z bus matrix for any power system.

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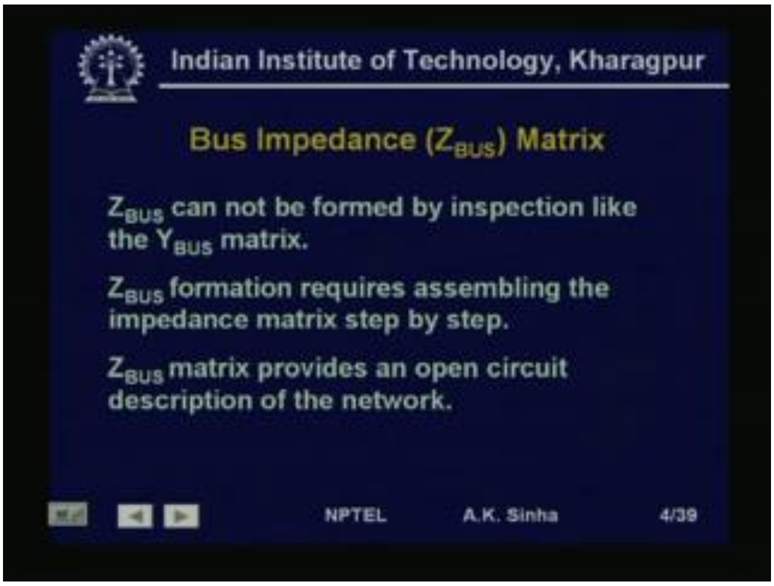

Well, as we had discussed in the earlier lesson, that we need to use Z bus matrix for short circuit or fault calculation for large power systems. We had seen in the earlier lesson that we can obtain this Z bus matrix by inverting the Y bus matrix. But, inverting this Y bus matrix for a large power system is computationally very expensive and time consuming. Therefore, we most of the time try to build this Z bus matrix by a step by step algorithm.

Just like Y bus, that we could build using or by just inspecting the network, we cannot do the same for Z bus. That is Z bus cannot be formed by inspection, just like the Y bus matrix formation that we had done earlier. Now, Z bus matrix requires building algorithm, where we add each element step by step to finally, form the Z bus for the complete system. Z bus matrix is basically describing the open circuit description of the network, we will see why this happens.

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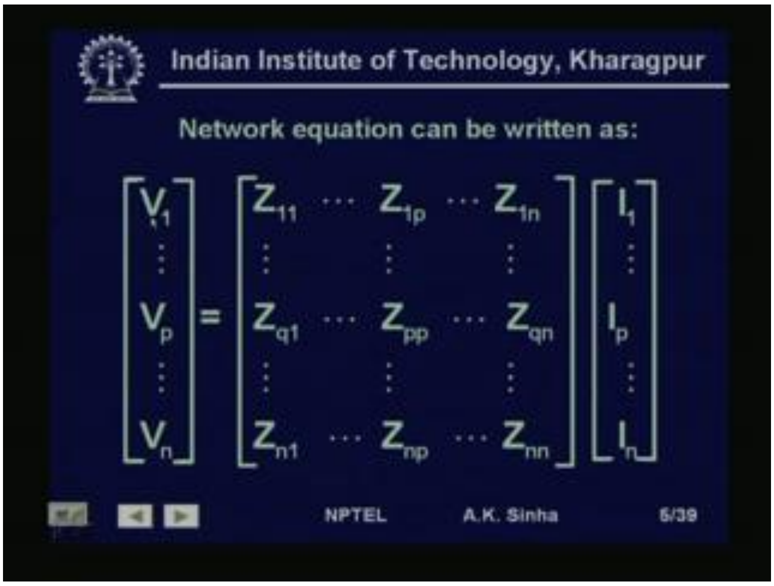

Now, if you recall the network equation we can write as V bus is equal to Z bus into I bus. So, we have written this equation in expanded form, where we have a N bus power system. So, we have voltage at various buses as V 1, V 2 up to V n. And we have the current injections as I 1, I 2 up to I n. Now, these voltages or the bus voltages, that we have are measured with respect to a common reference, which can be the common neutral of the generators or the ground.

So, all the bus voltages are referred to that common reference. Now, these elements Z 1 1 up to Z 1 n are for Z n 1 up to Z n n, these elements are the elements of the Z bus matrix. And we need to find the values for these elements for the system. And we do this by the Z bus building algorithm, we will see how we do that.

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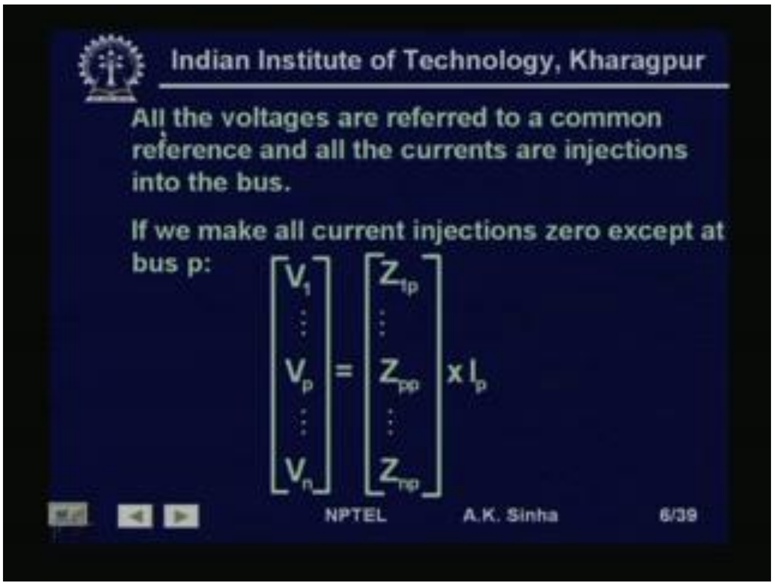

So, now what we can do is, this that if we make all current injections zero, except at the bus p. That is, if you see ((Refer Time: 04:42)) this relation set here we make I 1 up to I p minus 1 0 and I p 1 plus 1 to I n as 0. That is except I p we make all these current injections into the network as 0. Then, we will get V 1 will be equal to Z 1 p in to I p, because all the other terms are going to be 0s they are getting multiplied by 0.

So, they will be 0. So, V 1 will be Z 1 p into I p, V 2 will be Z 2 p in to I p, V p will be equal to Z p p in to I p and similarly V n will be equal to Z n p into I p. That is what we will get is only the pth column of the Z bus matrix, which is involved in this case. So, when the current is only I p current is non zero. Then, we are seeing the relationship that we get will be something like this V 1 up to V n is equal to the pth column of the Z bus matrix into I p.

So, what do we understand from this. That, if we want to compute any column of this Z bus matrix, what we need to do is open all the other bus voltages except, that is all the other bus injections are made 0. Means, we have to keep all the buses other buses all the buses except bus p will be open circuited. There is no current injection at those buses except the pth bus where we are injecting a current I p.

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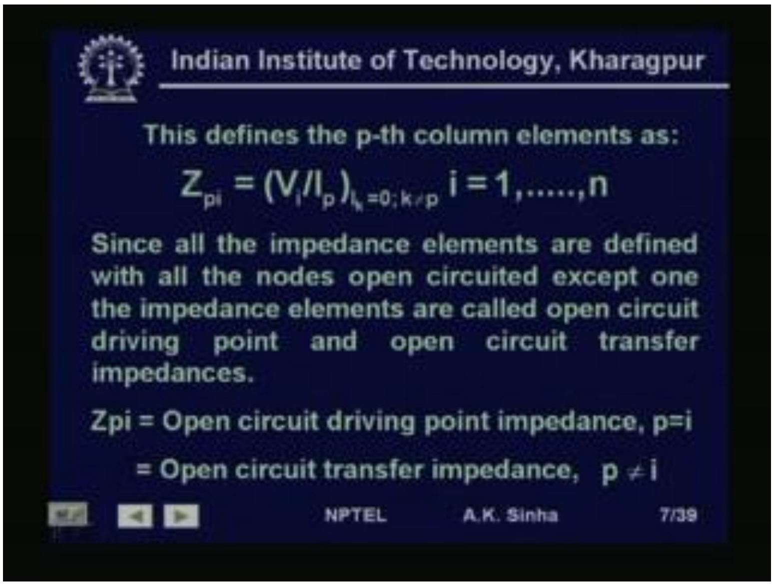

If we do this then we will get this Z p i as V i by I p i for where I k is equal to 0 with k not equal to p. That is, we will get all the elements of the pth column I is equal to 1 to n, that is Z p i. So, that is all the elements of the pth column we will get by keeping all the currents 0 except the pth current injection.

Since, all the impedance elements are defined with all the nodes open circuited, except one the impedance elements are called open circuit driving point and open circuit transfer impedances. That is, what we have done is except for the bus p, all other buses are open. So, the impedance that we are getting here, we called them as open circuit description. That is, we get the impedances these impedances are called open circuit driving point impedance or open circuit transfer impedance, when that is Z p i is equal to open circuit driving point impedance, when p is equal to i. That is, Z p p is basically what we call the driving point impedance for bus p. And Z p i for P naught equal to i is called the transfer impedance open circuit transfer impedance between buses p and bus i.

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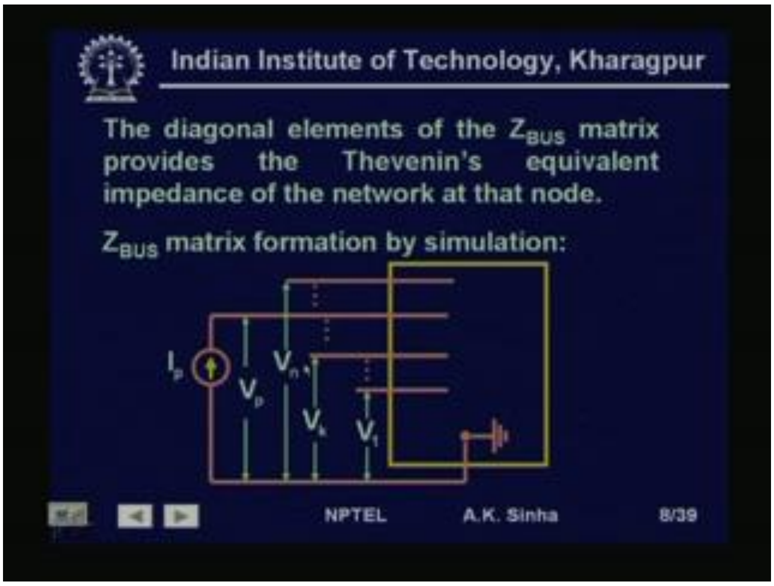

So, this is how we can do if we want to do this by simulation, what we need to do is in this n bus power system, we inject a current I p into bus p. And measure the voltage at all other buses. And then we can find out V n by I p as Z p n or V 1 by I p as Z p 1. So, we can Z 1 p or Z p 1 they will be same. So, this is the way we can do it, we can simulate this experiment also by writing down the Kirchhoff's law for the network.

That we have and do this though doing that reduction again will be pretty complex. And that is why we do not work this kind of a simulation for large systems. This is good for small systems, but for a large system we need a systematic algorithm, which we can program to run on a digital computer. Now, the diagonal element, that is V p by I p that is what Z p p is, what we call the driving point impedance of at point p or the bus p.

And this is also equal to the Thevenin's equivalent impedance of the network at node p or bus p. So, driving point impedance is same as the Thevenin's impedance that we will get by reducing the network up to point p.

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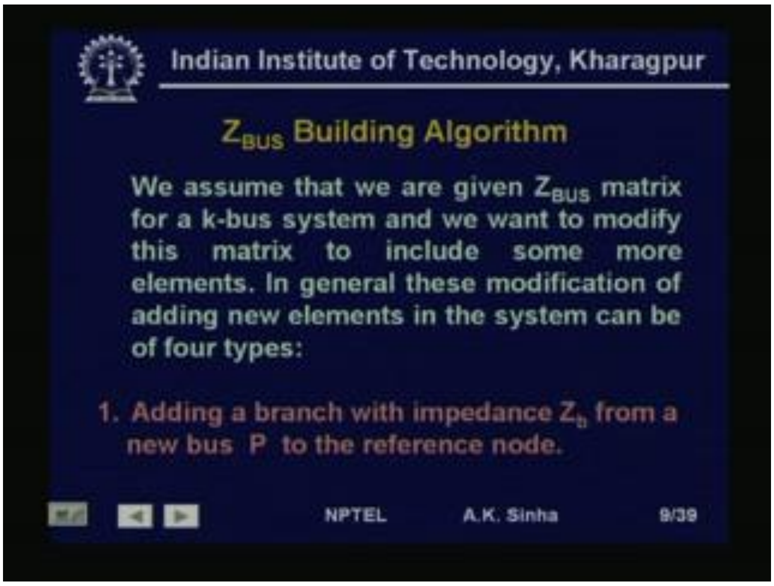

So, now we will get into the Z bus building algorithm. What we do is, we start with an assumption that we already have some Z bus matrix. For say a k bus system or a n bus system. And we want to modify this matrix to include some more elements. That is, what we are trying to do is, we already have a Z bus. Now, we are adding more elements or making it larger for the modified system.

So, what we are looking at is how we are able to modify this Z bus to include other elements in the system. In general the modifications that we can get for adding these new elements are of four different types. Type one is adding a branch with impedance Z b from a new bus P to the reference node. That is, we have now we are adding a new node to the already existing n bus system that we have. So, we are adding a new node p to this system and the impedance is connected from this new node to the reference node.

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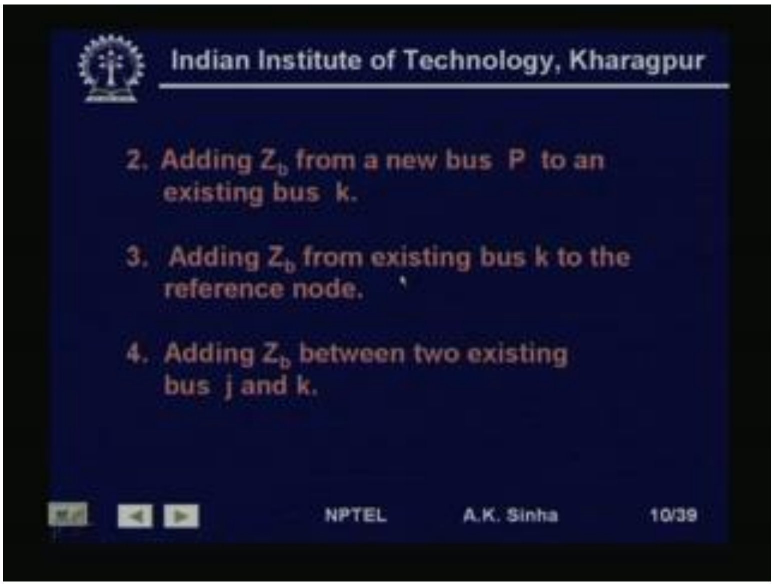

This is the type one kind of modification, that we are saying. Then, type 2 modification is adding an branch with impedance Z b from a new bus P to an existing bus k. That is, we are again adding a new bus p and from this bus P we have a branch connected to some other bus k, which is already in the existing network.

Type 3 modification can be adding a branch with impedance Z b from the existing bus k to the reference node. That is, we are adding this branch Z b from an existing bus k to the reference node. So, this can be another kind of modification, which is there we call this as type 3 modification. Type 4 modification is adding a branch with impedance Z b between two existing buses j and k.

So, the two buses j and k are existing in the already Z bus for which Z bus is available to us. Now, we are trying to add a new branch between bus j and k of this existing system. So, these four types of modification in general take care of all the kinds of additions or modifications, that we will require in most of the cases.

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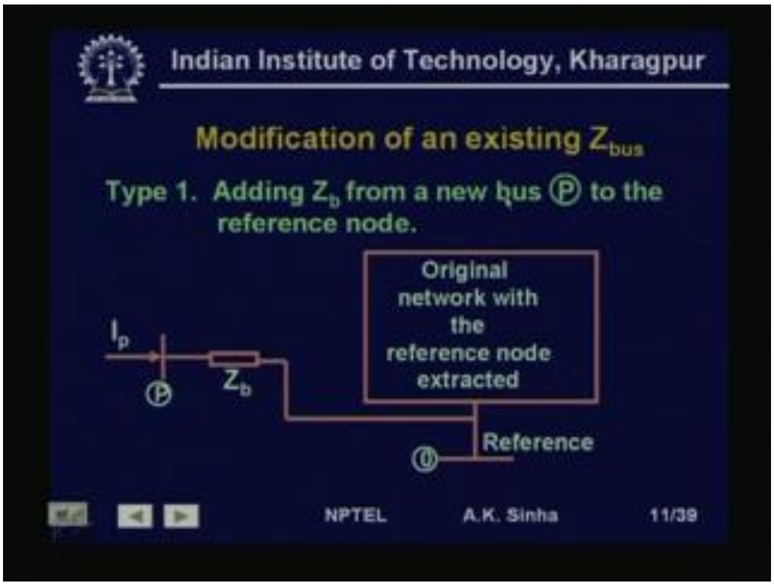

So, we will start with the type one modification. That is, adding a branch with impedance Z b from a new bus P to the reference node. Now, this is the original network that we already have this is the n bus work let us say. And we have added a new node P from which we have connected a branch with impedance Z b to the reference node. We have taken out the reference node here to show that we are making the connection to the reference node. And rest of the network and bus network is inside this rectangle. Now, we can write down the equations for this kind of a system. Now, here we will see V p will be equal to how much Z b into I p.

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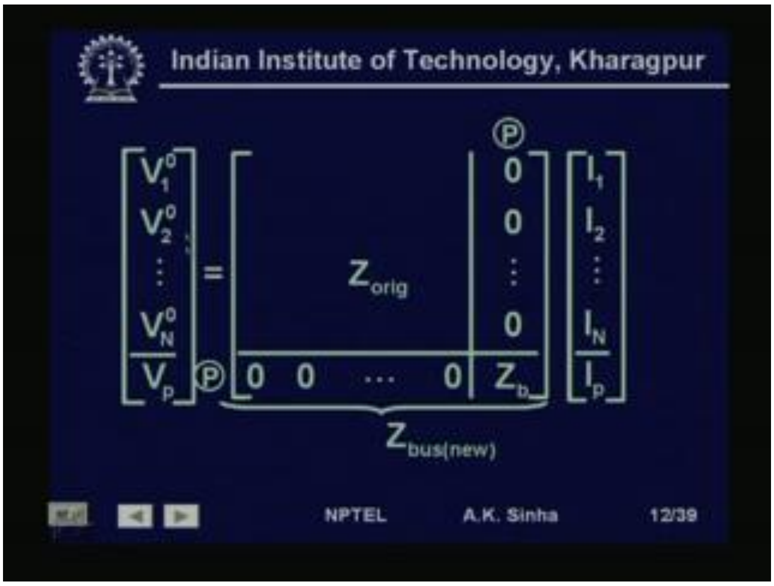

So, if we write down the equation in this case. Then, we have for the existing system V 1, V 2, V n which was already there. So, this 0s are basically indicating the previous values are already existing values. The Z original is the N by N original matrix that we have for the n bus system, that we already have. And I p I 1 to I N are the current injections in the existing system.

Now, we have added a bus p and we have seen that V p is equal to Z b into I p. So, we will now get N plus 1 into N plus 1 matrix, where N into N original matrix is already there. And we have only one equation added V p is equal to Z b into I p. So, these elements of this N plus 1 nth column from 1 to n will all be 0. Similarly, for n plus 1 nth row from 1 to N this all these terms will be 0, because V p is equal to Z b into I p only.

So, the modification or the new modified matrix will be N plus 1 into N plus 1 matrix, which will look like this. Where, we have this Z b as the diagonal N plus 1 into N plus 1 element. So, this is what we need to do? So, this modification is very simple. What whatever matrix we have, we add one row and one column to it. And all the elements of these this extra row and column are 0s, except the last element or the diagonal element, which will be equal to Z b the branch impedance. That we are adding from the new node to the reference node.

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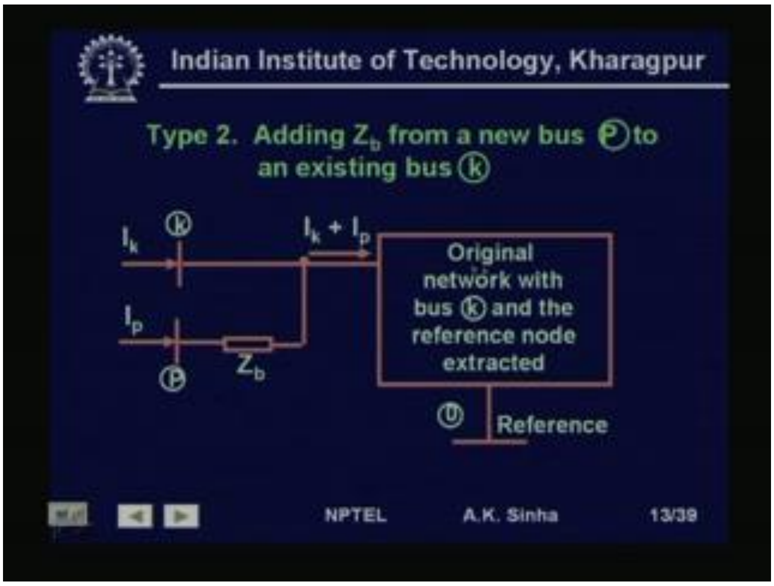

So, next we will go to type 2 modification. That is, adding a branch with impedance Z b from a new bus P to an existing bus k. Now, this what we have is the original network is there in this rectangle. What we have is, we have taken this bus k and the reference node outside to indicate the system situation. Since, we have added a new node p and we are adding this new node p by connecting it through a branch Z b to the node k.

So, this is what is indicated here to the node k, we are adding this branch from the new node p. Now, what happens if the current injection in this node is I p, then the current injection now the modified current injection from node k will be I k plus I p. Because, earlier it was I k now we have added this, this I p is flowing like this. So, here into the network at node k what will be entering? It will be I k plus p current which will be entering. And therefore, it is going to modify the voltages at all other buses. So, we will write down the equation for that.

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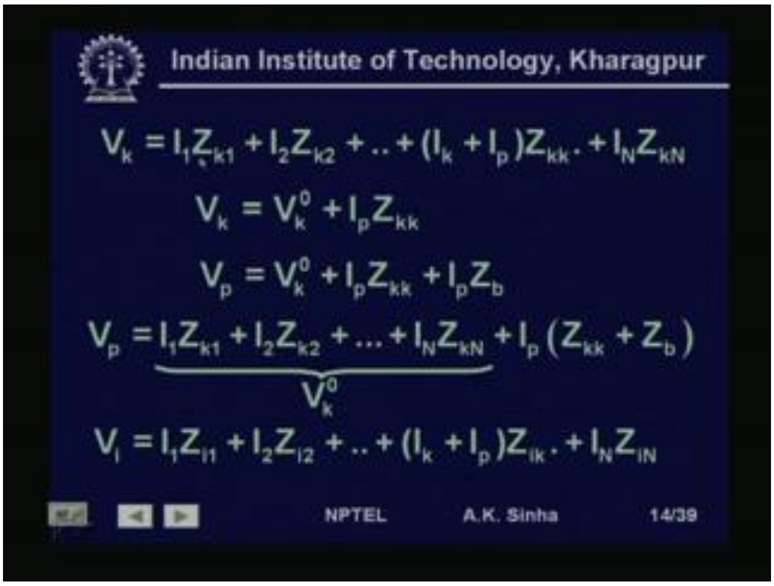

So, now V k as if you recall from here ((Refer Time: 17:53)) V k will be equal to Z k 1 into I 1 plus Z k 2 into I 2, Z k k into I k and plus up to Z k n into I n. So, we are writing that expression V k is equal to I 1 into Z k 1 plus I 2 into Z k 2 plus... Now, at the kth bus what is the current which is entering? The current which is entering now is I k plus I p. So, we will have I k plus I p into Z k k plus up to I N into Z k N.

That is what we are seeing here is, that from the previous value that we had where I p was not injected. So, we had V k is equal to now the change V k is equal to what was the previous V k voltage at that bus plus I p into Z k k I p into Z k k is what is getting added or delta V is at bus k.

Similarly, for bus ((Refer Time: 19:13)) P if we want to find out the voltage. What will be the voltage? The voltage will be whatever is the voltage now at bus K plus Z b into I

p. So, this because Z b into I p is the drop from V p to V k. So, we can write this as V p is equal to V k 0 plus I p Z k k. That is, we can write this whole term as V k plus I p into Z b.

Now, we can write this in expanded form where we are putting for V k 0, the values here. So, I 1 Z k 1 plus I 2 Z k 2 up to I N Z k N plus I p into Z k k plus Z b, so I p into Z k k plus Z b. Similarly, we can write down the voltage at any other bus i as I 1 into Z i 1 plus I 2 into Z i 2 plus up to... Now, the current at kth bus is I k plus I p, so I k plus I p in to Z i k plus plus up to I N Z i N. So, this is the way we can write down the voltage at any other bus. So, what is happening at the kth element impedance for that row is not multiplied by I k now. But is getting multiplied by I k plus I p, because now this is the modified current injection at bus k.

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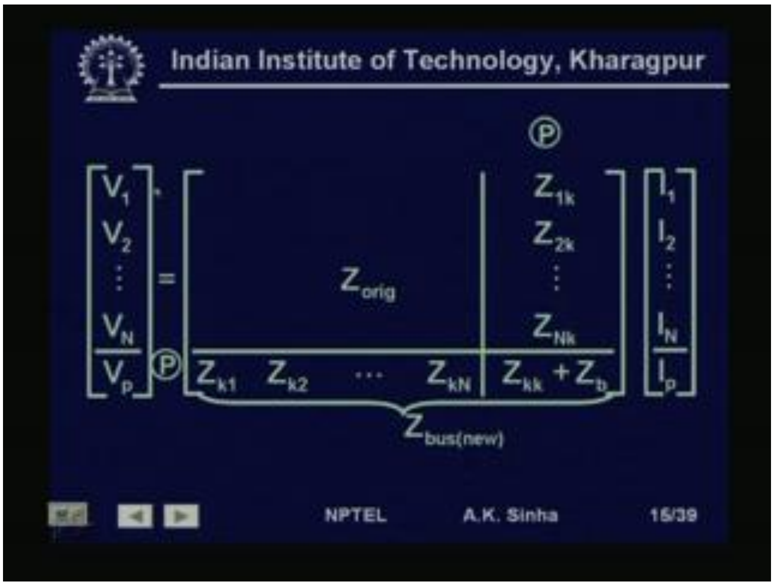

So, if we do that then we can write this whole thing as a matrix form as V 1 to V n Z original. Now, what is happening here for each one whatever was the previous value? That we will get from this N bus matrix. That is Z 1 1, Z 1 2 and so on Z 1 1 in to I 1, Z 1 2 into I 2 and so on plus Z 1 k into I p. So, the new voltages will get modified by addition of Z I k into I p.

And therefore, we have this column, that N plus 1 nth column. Where, we will have the elements which will be Z 1 k, Z 2 k and so on which is same as saying the kth column of the original Z bus matrix becomes the column here up to nth element. Similarly, we will write the expression for V p. Here, again what we will get? Z k 1 into I 1 plus Z k 2 into I 2. This is what is giving us the V k 0 term plus Z k k plus Z b into I p.

So, the N plus 1th row and column term, that is Z n plus 1 N plus 1 term will be Z k k plus Z b. And the row N plus 1th row will be nothing but the kth row of this original Z bus matrix. So, this how this gets modified. Now, again since this bus p is a new bus. So, we have now in the system N plus 1 nodes. So, we will get Z plus which will be N plus 1 into N plus 1. And this is the modified Z bus which will be N plus 1 into N plus 1. So, this is how we can take care of the type 2 modification. Where, we are adding a new bus by connecting it through a branch Z b to an existing bus k.

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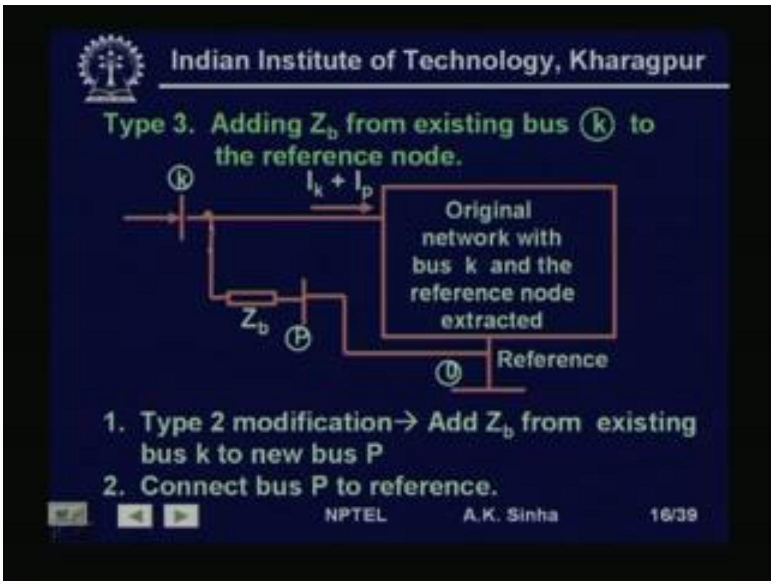

Now, we will take type 3 modification, where we are adding a branch with impedance Z b from existing bus k to the reference node. Now, this if we look at this as from an existing node k, which is already in the original network. We are connecting a branch Z b to the reference node. Now, this what we do is, we do this modification in two steps.

That is, what we do is first we say that, we add this Z b to a new node p. That is, already we have seen how we do that. That is, from a new node we add this branch Z b to bus k, which is same as type two modification. And then we connect this node p to the reference. So, now this Z branch Z b is connecting the node k to the reference. This is what the type this type three modification is... So, we do it in two steps. So, type 2 modification add Z b from existing bus k to new bus p. And then connect bus p to the reference. So, let us see how we do that.

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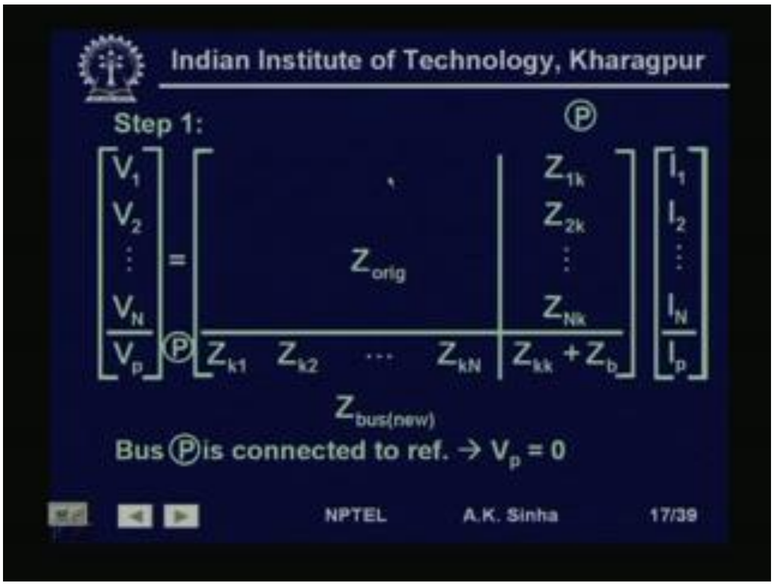

So, first part which is the type 2 modification is very simple we have already seen how do it. So, step 1 is to build this N plus 1 into N plus 1 matrix. Now, we in next step we are connecting this V p to the reference. So, the voltage V p becomes equal to 0.

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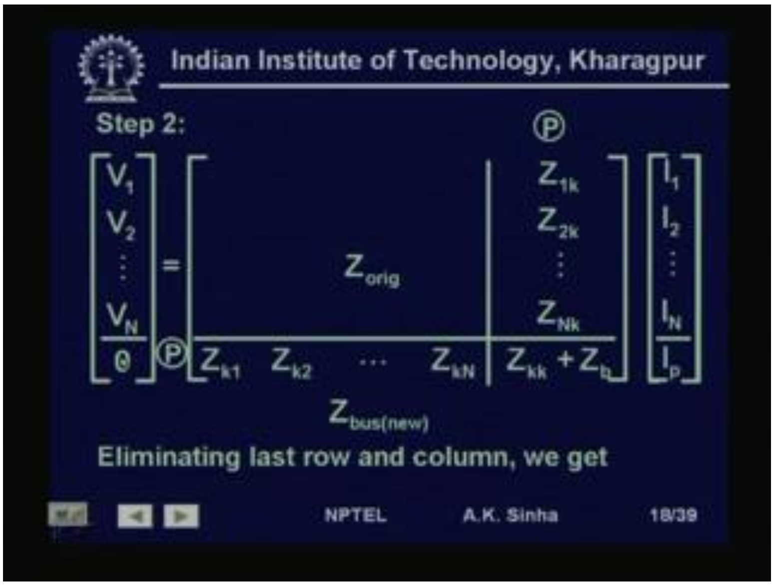

So, this is what we do in step 2, this voltage is made 0. So, now we have this as the modified equation for the system. Where, this is defining our system where a branch with impedance Z b has been connected from a bus k to reference bus. The only problem that here that we see is, that since we have not added any new bus. So, the network should be is having only N buses.

So, the impedance bus impedance matrix will also be a N into N matrix. But, this modified matrix that we have is N plus 1 into N plus 1. So, what we need to do is, we need to eliminate the last row and last column of this matrix. How do we eliminate this? This we do by what we call as the chrons reduction technique. That is, we try to use this expression and this expression we have 0 here. So, from this equation we try to find out the value of I p in terms of Z Z and all other currents.

Once, we get that value of I p we can substitute that value of this I p in this expression. So, everything will come in terms of only currents I 1 to I n. And we will get a reduced N into N matrix. Let us see how we do that.

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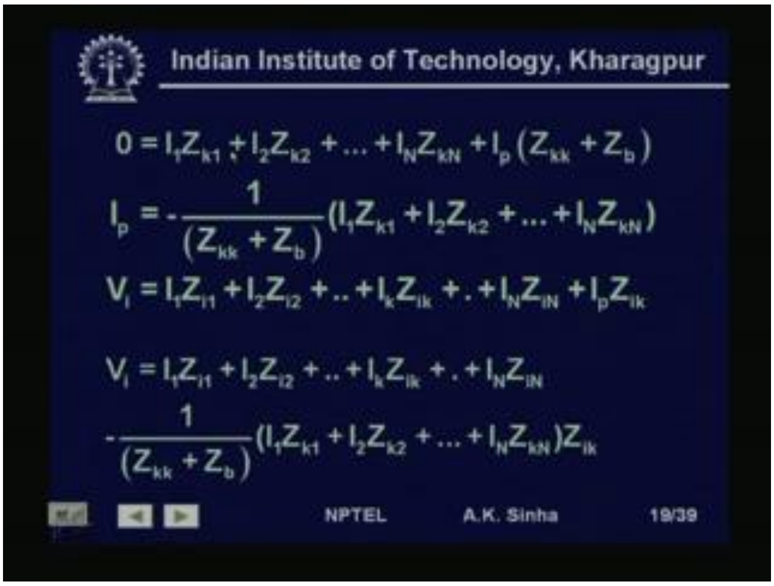

So, we write the last equation 0 is equal to I 1 into Z 1 k 1 plus I 2 into Z k 2 plus I N into Z k N plus I p into Z k k plus Z b. That is ((Refer Time: 27:16)) we are writing this 0 is equal to Z k 1 into I 1, Z k 2 into I 2 up to Z k N into I N plus Z k k plus Z b into I p. So, that is what we have written here I p into Z k k plus Z b.

Now, from this we can find out the value of I p, we can take this expression on the left hand side. So, we can write this as minus I p into Z k k plus Z b is equal to this term or we can write I p into Z k k plus Z b is equal to minus of all these terms minus of I 1 into Z k 1 minus I 2 Z k and so on minus I N Z k N.

And from that expression, we can find out the value of I p by dividing or bringing this Z k k plus Z b which we have taken on this side. On this side, that is dividing by Z k k plus

Z b this hole expression. So, we will get I p is equal to minus 1 by Z k k plus Z b into I 1 Z k 1 plus I 2 Z k 2 plus plus I N Z k N. So, this is the value of I p that we get.

Now, if you write down the expression for voltage at any bus i, then Vi is equal to I 1 into Z i 1 plus I 2 into Z i 2 plus plus up to I k into Z i k plus up to I N into Z i N plus I p into Z i k ((Refer Time: 29:02)). That is here if we see for any expression we will get I 1 into Z i 1, I 2 into Z i 2 and so on I N into Z i N plus Z i k into I p. That is what we are getting here, this is what I p into Z i k.

Now, we know the value of I p from here. So, we can write Vi is equal to all these term plus I p into Z i k this term is I p. So, I p into Z i k we can write like this. Now, we can simplify this by combining or arranging terms for I 1, I 2 and so on together.

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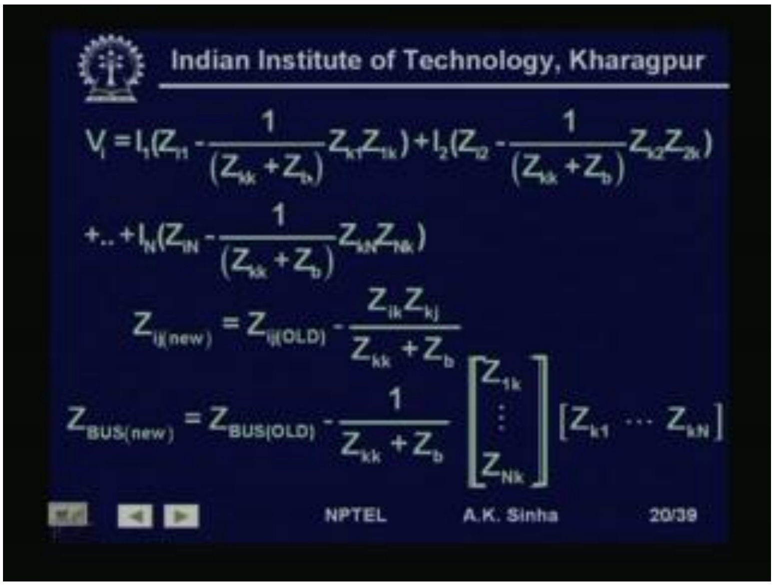

Therefore, we will get V i is equal to I 1 into Z i 1 minus 1 by Z k k plus Z b into Z k 1 into Z 1 k I 2 will plus I 2 into Z i 2 minus 1 by Z k k plus Z b Z k 2 into Z 2 k plus so on up to I N Z i N minus 1 by Z k k plus Z b Z k N into Z N k. So, this is what we will get by arranging terms for I 1, I 2 and all that separately.

So, finally we get this like this. That is, the new elements of the Z bus matrix can be seen as Z i j new values will be Z i j old values, which is this one here. Minus Z i k into Z k j divided by Z k k plus Z b. This will be the new value that we are going to get for any element I j. Say for I N if we are writing, then we will have Z i N which is the old value which is their minus Z N k or N k into Z k N, so same thing here that we are getting divided by Z k k plus Z b.

So, this is how we will get the new modified element. And if we want to write this thing in matrix form, then we have Z bus new is equal to Z bus old minus 1 by Z k k plus Z b into this column of Z 1 k to Z N k. That is the kth column of the Z bus matrix multiplied with the kth row of the Z old Z bus matrix.

This is, if we do multiplication this will be again a N by N matrix. So, this N by N matrix that we will get well is divided by this term Z k k plus Z b. That is all element of this matrix, that we will get N by N matrix will be divided by this term. And that is subtracted from the old N by N Z bus matrix.

So, this is how we take care of the type three modification that is we finally, get a N by N matrix by eliminating the last row and last column of the modified matrix, that we had obtained by including that bus P, and then connecting it to the reference.

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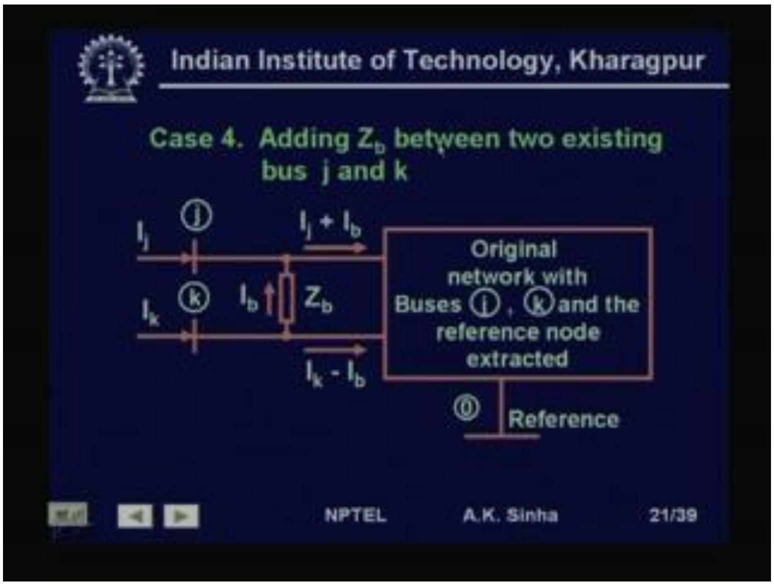

Now, we will go to type 4 modification, which is adding a branch with impedance Z b between two existing buses j and k. So, here we have taken out two nodes j and k. Outside to show the connection of a new branch Z b between these buses. That is, we are adding a branch Z b between these buses. And of course, we have taken out reference to indicate this.

So, this is the original network the N bus network. We have just taken out these two nodes. And we have shown that we are adding a new branch Z b here. Now, if we look at this, then what we have I j current injection at bus j I k current injection was at bus k.

Now, when we add this branch Z b, then we are assuming that the current direction I b is like this.

Then, we will get the current injection into the network at bus j. Now, will be equal to I j plus I b and at bus k now it will be I k minus I b. Because, I k is coming here out of which I b goes like this into the network like this. Whereas, the current which comes here is I k minus I b and this current becomes I j plus I b.

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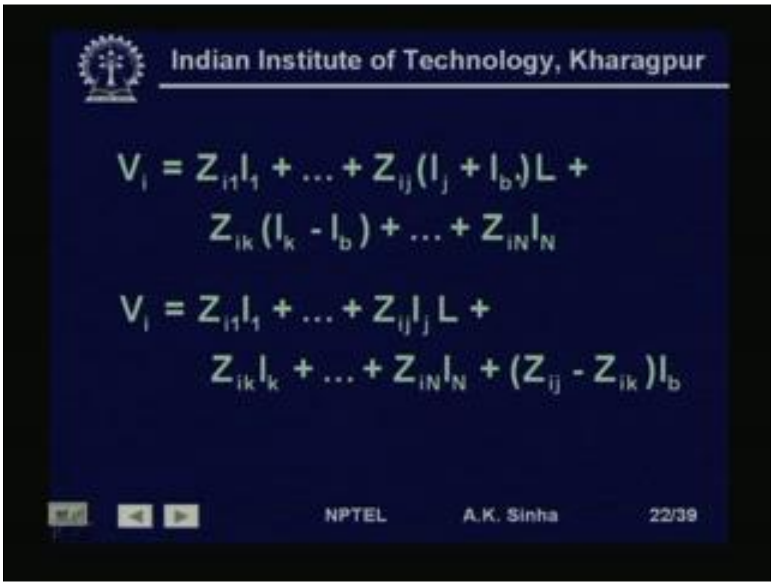

So, we can now write the expression for any voltage V i for this system as Z i 1 I 1 plus up to Z i j. Now, for bus j what is the current injection? This is I j plus I b this should be dot, dot, dot there is no this L is not there this should be dot, dot, dot plus Z i k at bus k what is the injection? I k minus I b. So, Z i k into I k minus I b plus dot, dot, dot Z i N into I N.

So, this is the voltage that we will get this L here is not actually L it should indicate dot, dot, dot. So, now we are writing the same expression here by taking the terms of I b together outside. So, we have V i is equal to Z i 1 into I 1 plus plus up to Z i j into I j again this L is dot, dot, dot, this has come wrongly typed. So, this should be dot, dot, dot not L. So, this dot, dot, dot plus Z i k I k plus dot, dot, dot plus Z i N I N plus what we will get Z i j I b minus Z i k I b. So, Z i j minus Z i k into I b, this is what we have got now for this V i.

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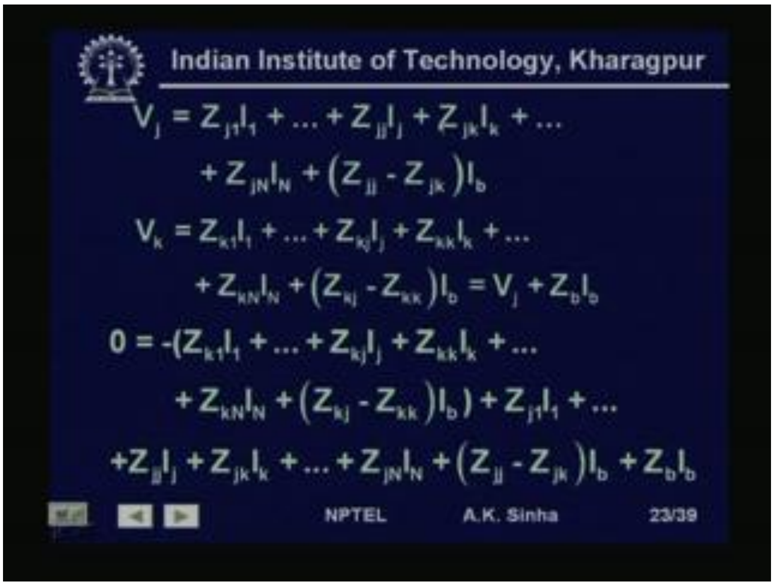

Now, we can write the same thing for V j, V j will be Z j 1 I 1 plus up to Z j j I j plus Z j k I k plus all this plus Z j j minus Z j k into I b. That is, we are saying at this bus what is the current injection at bus j ((Refer Time: 37:06)) the current injection at bus j is I j plus I b. So, we will get this I j plus I b. So, Z j j into I b and it at bus k what we have I k minus I b, so Z j k minus I b. So, this we have written as Z j j minus Z j k into I b in the end.

Similarly, for any bus other bus k we can write this in a similar way like this. Where, again the at bus k we will be having I j plus I b. So, we have Z k j into I b minus this will be at bus k we will have I k minus I b ((Refer Time: 38:11)) at bus k we have I k minus I b. So, we have Z k k into I k minus I b, so Z k k I b will be negative. So, we are adding it here in the end.

So, what we get is we are writing this as this term and what is V k equal to V k is equal to ((Refer Time: 38:40)) if we see from this diagram. This voltage will be equal to the voltage at this point. That is, V j plus the drop here which is Z b into I b. So, we can write this V k is equal to V j plus Z b into I b. What we can now do is? We can take all these terms. That is, on this side then we will get this V k minus V j minus Z b I b will be equal to 0 or we take this V k term on this side.

So, we will have V j minus V k plus Z b I b is equal to 0. So, we are writing 0 is equal to this is V j minus V k. So, we are writing minus V k here plus V j this is minus V k, this whole expression with a negative sign. So, minus V k plus V j, this is V j term that we have this expression that we have, so plus V j plus Z b into I b. So, this is what we are going to get 0 is equal to this much. This again we can combine the terms of currents I 1, I 2 and so on separately.

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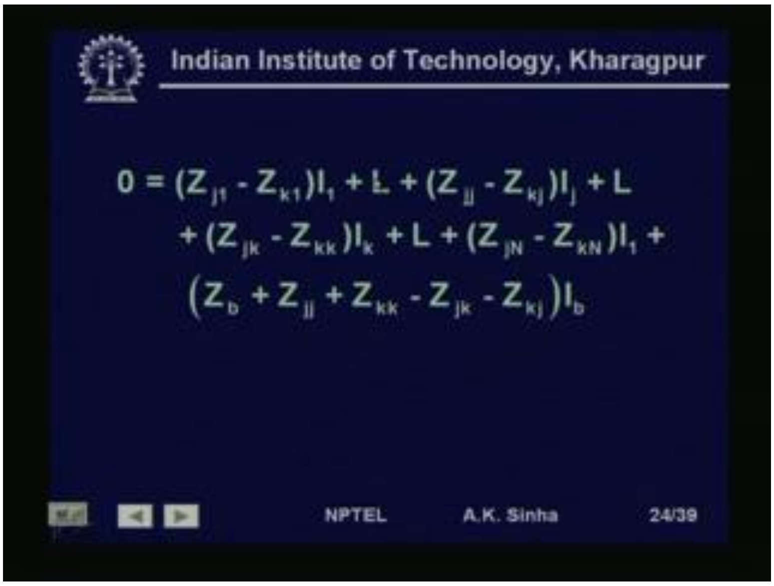

Then, we can write this as Z j 1 minus Z k 1 into I 1 plus this again L is not L this should be dot, dot, dot. Plus Z j j minus Z k j into I j plus dot, dot, dot plus Z j k minus Z k k into I k plus this will be again dot, dot, dot plus Z j N minus Z k N into I 1 plus this term that we are getting for I b. That is, when we add it for ((Refer Time: 40:50)) I b then we have this term for I b, this term for I b and this term for I b.

So, we have Z k j I b minus Z k k I b this is again a minus term. So, this will be Z k k I b minus Z k j I b plus Z j j I b minus Z j k I b plus Z b I b. So, this term is what we are getting here. We can write this whole thing in matrix form, where this expression goes as the N plus 1th expression.

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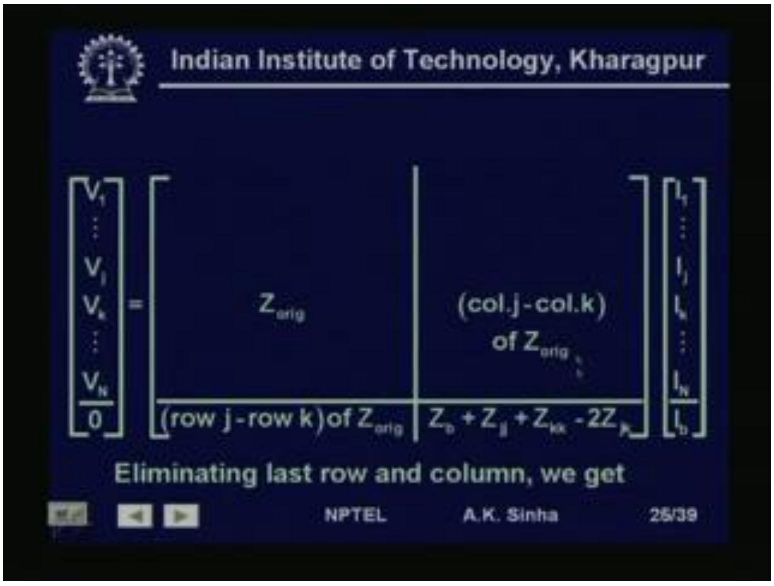

So, we have V 1 up to V 1, this is the original matrix. And we have I b terms for all these voltages are coming as multiplied by I b into ((Refer Time: 41:55)) Z k j for V k we are at getting a V i we can write this will be Z j j minus Z k j k. That is, what we are getting is the column of jth column minus the kth column is coming here. That is, multiplied by I b.

So, we are jth column minus kth column of the original Z bus matrix, this multiplied by I b. So, N plus 1th term is jth column minus kth column of the original matrix will be coming here. Similarly, in the last row that we have jth row minus kth row is coming for this ((Refer Time: 42:43)) last expression, this is jth row terms are here j minus k. So, jth row term minus kth row terms are coming here.

So, the N plus 1th row here we have 0 is equal to these terms will be row j minus row k terms N term n number of such terms will be there. Plus N plus 1 and N plus 1 term which will be there will be for I b and that term is this 1. So, we have now this term here now again as we have seen we have a N plus 1 into N plus 1 matrix. Where, we have last term as 0 here.

So, using this last we need to eliminate last row and column. Because, there is no additional bus in this system the total number of buses in the system still remains as N. So, the Z bus matrix is going to be a N by N matrix only therefore we need to eliminate this N plus 1th row and N plus 1th column, which we again do using the trans reduction.

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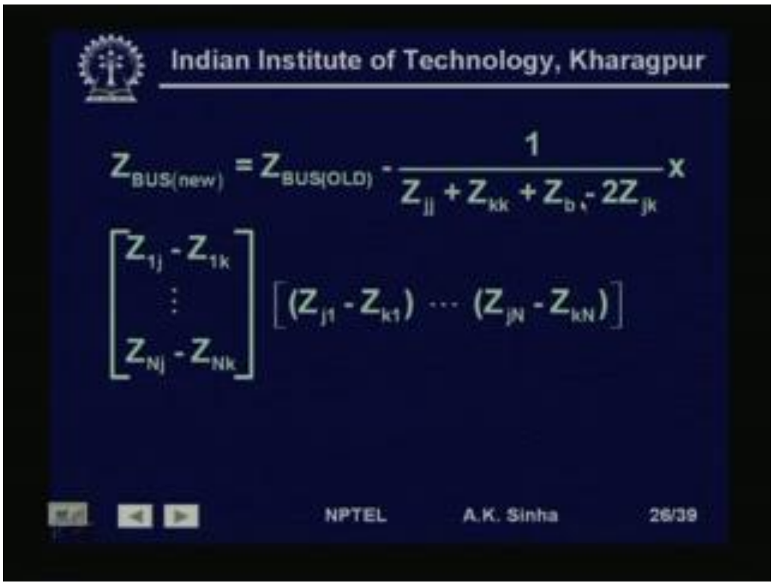

So, if we do that, then again we will get the new Z bus matrix will be equal to old Z bus matrix minus 1 by this term Z j j plus Z k k plus Z b minus 2 Z j k ((Refer Time: 44:15)) we are writing here. Because, we are saying that Z j k and Z j Z k j are same they are equal. Therefore, this minus 2 j k is coming here, as well as here we are writing the same thing. This multiplied by the N is to 1 column, which is the jth column minus the kth column terms multiplied by the N is to 1 row, which is jth row minus kth row for each term.

So, this again this whole term is going to be an N into N matrix, which we are subtracting from the old N into N matrix. And therefore, the new Z bus is going to be a N into N matrix.

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So, all this modifications that we have done are we have summarized here. So, the type 1 modification is, reference node to a new bus P. That is, direct from the reference node we are connecting a branch with impedance Z b to new bus. And this modification results in a N plus 1 into N plus 1 matrix. Where, the N plus 1th row and column are all 0s and the diagonal element is equal to Z b.

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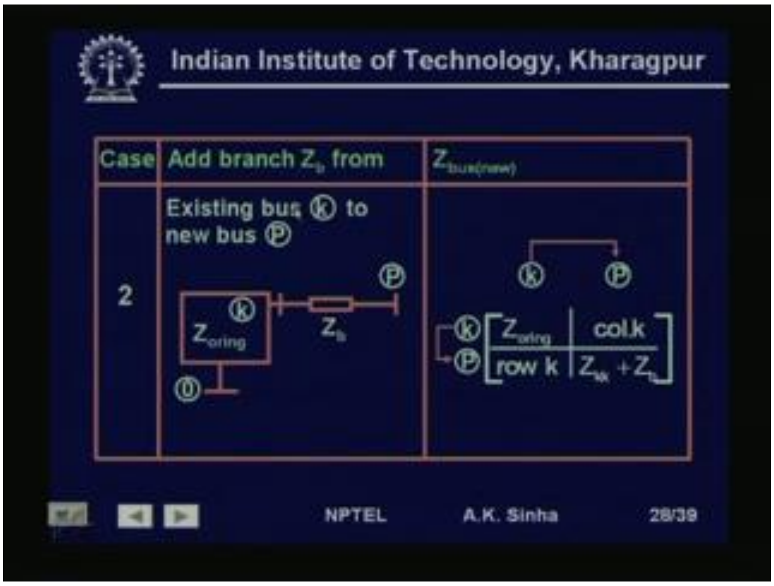

Similarly, for type 2 modification, where we are connecting an branch with impedance Z b from a new bus P to an old bus K. So, here we will get this kind of a network. Which we have seen results in this kind of a modification Z original. The N plus 1th column is the kth column. That is, the bus to which the new branch is getting added N plus 1th row is the kth row. And the diagonal element for N plus 1th row and column is Z k k plus Z b.

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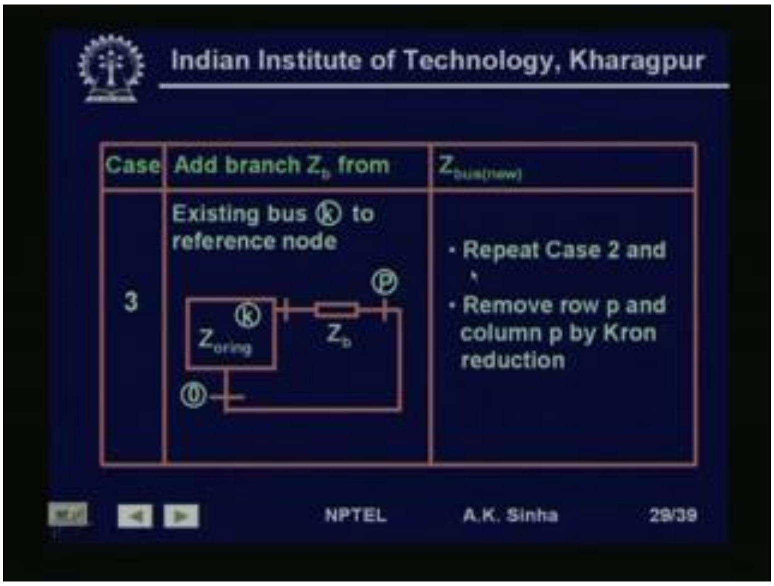

Similarly, for type three modification, we are adding a branch with impedance Z b to reference node. And this is shown here we do it in two steps, first connect this branch from bus k existing bus k to a new bus p. And then connect that bus p to the reference bus.

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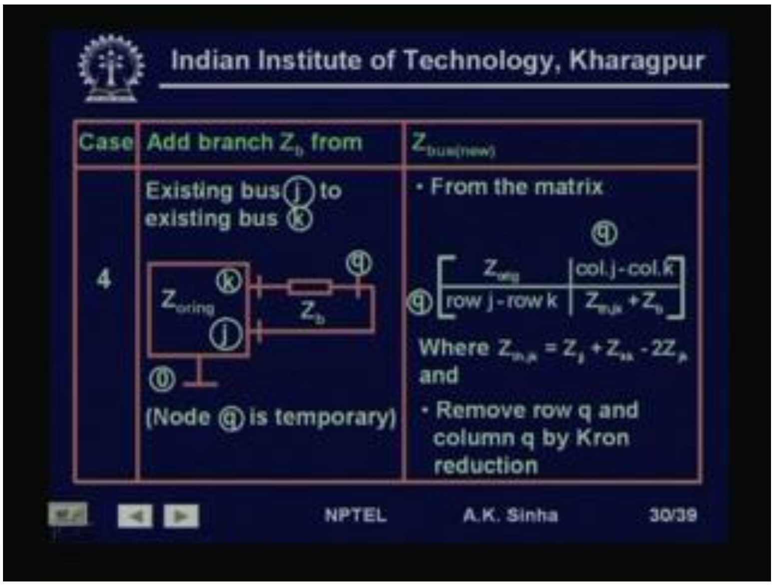

The fourth type of modification is existing bus j and existing bus k are connected by a new branch with an impedance Z b. This is what it is we have two existing buses and Z b branch is connecting this. So, we get this new matrix like this. And then we do the Kron reduction to eliminate the N plus 1th row and column. And get finally, the N is to N modified Z bus matrix.

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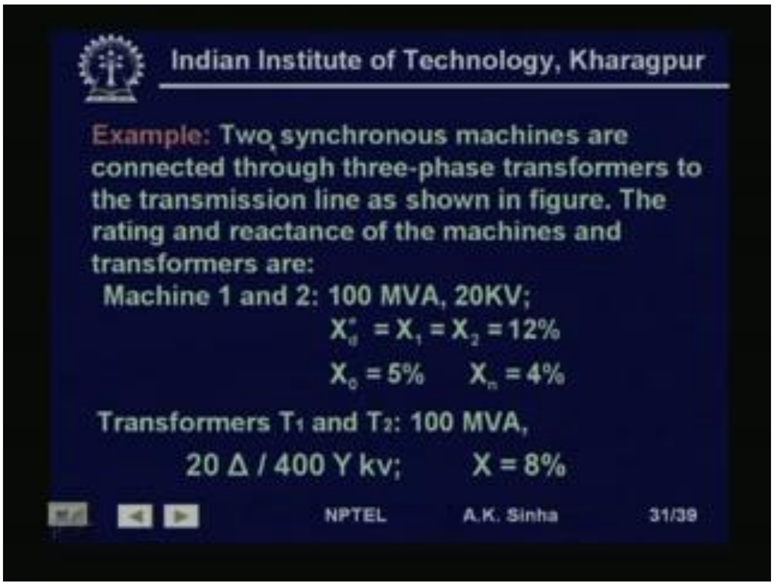

So, now we will take up a small example to see how we can build the Z bus matrix for any power system. So, again we are taking an example which is two synchronous machines connected through a three phase transformers to a transmission line as shown in figure. The ratings and the reactance's of the machines and transformers are machine 1 and 2 100 MVA, 20 KV, the X d double dash, which is same as X 1 that is the positive sequence reactance, which is same as negative sequence reactance is equal to 12 percent or 0.12 per unit X 0 is 5 percent. And X n the grounding reactance is 4 percent for this generator. The transformers T 1 and T 2 are 100 MVA transformers. That is the base we will choose as 100 MVA only. The transformers are 20 KV delta to 400 KV star and the reactance is 8 percent.

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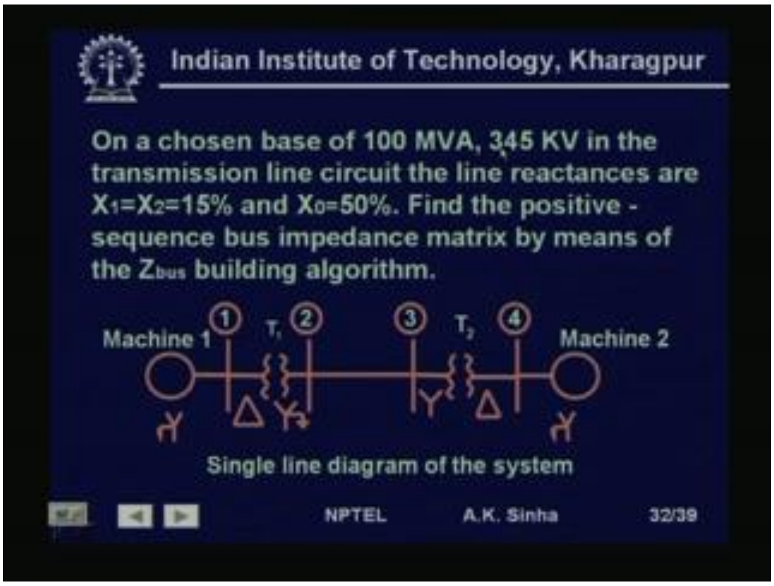

On the chosen base of 100 MVA, 345 KV in the transmission line circuit, the line reactance's are X 1 is equal to X 2 is equal to 15 percent and X 0 is 50 percent. Find the positive sequence bus impedance matrix by means of Z bus building algorithm. Now, since we have chosen here, the base KV as 345 KV. So, we need to modify the reactance of the transformers to this base. And if when we do that then instead of 8 percent it will come out to be 7 percent.

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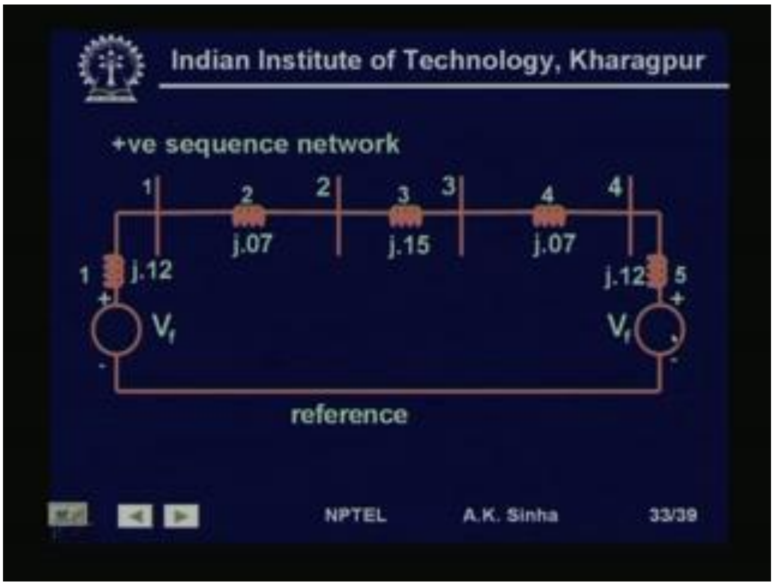

So, we have got the positive sequence matrix for this system shown here, where this is the synchronous machine one. This is as positive sequence reactance. This is the transformer T 1 it is reactive positive reactance is 7 percent, that is 0. 07 per unit. This is the transmission line it is positive sequence reactance is 15 percent. This is the transformer T 2 it is positive sequence reactance is 7 percent. And this is the synchronous machine 2 it is positive sequence reactance is 12 percent. Now, we have put these elements as 1, 2, 3, 4 and 5. And sequentially we will build the Z bus matrix for this system.

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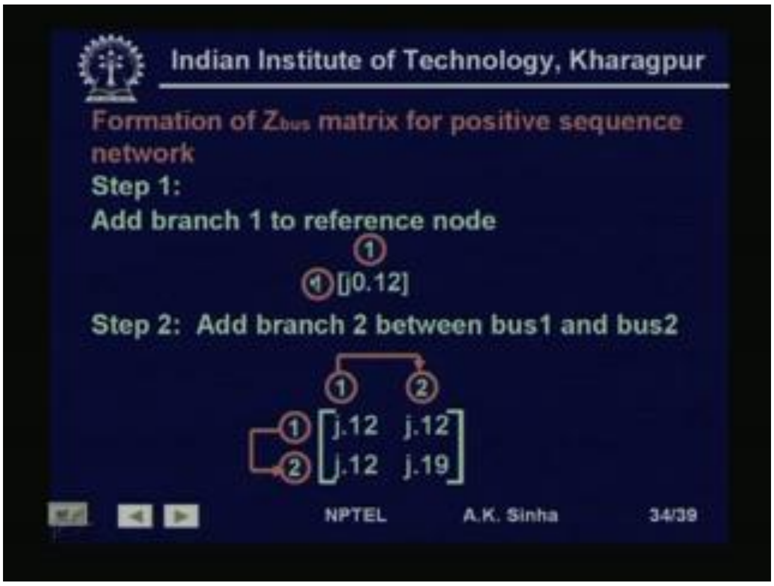

So, what we do is initially we will always start building the Z bus matrix. Where, we have one element, which is getting connecting one bus to the reference bus. So, always start with an element which is connected to the reference. So, if we see that ((Refer Time: 50:28)) we have this element one, this voltage source will be shorted. So, with this element one is connected to the reference.

So, we start with this element which is connected to the reference we can also start from this side, this element connected to the reference. So, either of the two we can start with. So, we have started with this. So, we have first element that we have got is connecting element number 1 is connecting node 1 with the reference. So, we will get a 1 is to 1 matrix for this which will be simply Z b.

Because, initial matrix that we have here is 0 is to 0. And the row and column for this first will all be those will be not there. So, we get start with j 0.12, that is if we go back ((Refer Time: 51:36)). This is the thing that we have, this row and column. Since, in the original matrix itself is 0, there is no elements. So, we have only this Z b coming into picture.

So, first is j 0.12 which is the impedance of Z b matrix. Next step is add branch 2 between bus 1 bus 2. So, again this is adding a new bus to an existing bus. So, this is type 2 modification, again we do the same thing. That is the row N plus 1 row and column will be whatever was the value for row and column of bus 1 which is to which we are connecting this node. And this term will be equal to Z b plus Z 1 1. So, Z 1 1 was j 0.12 and Z b is 0.07. So, this is the value we will get in this case.

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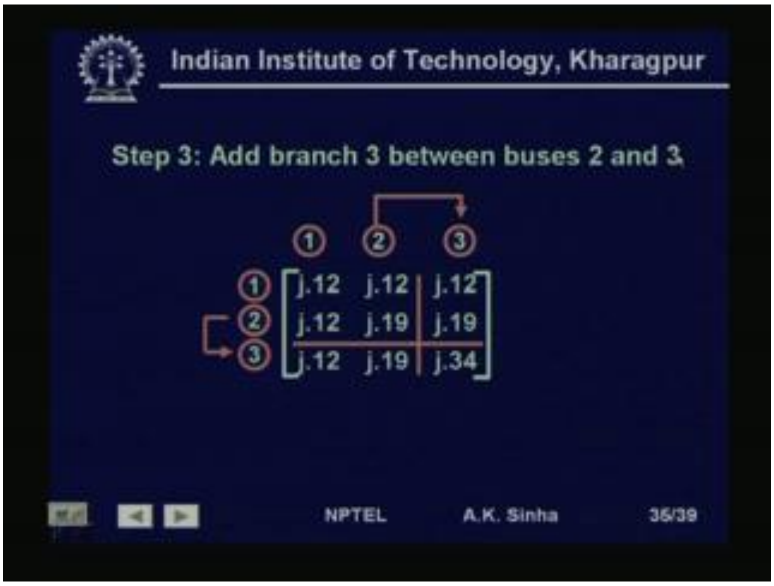

Again when we add branch 3 which is between bus 2 and 3, again we are adding a new bus, bus 3 connecting this branch to an existing bus 2. So, again the same type 2 modification. So, the column row and column N plus 1th row and column, that is third row and column here are same as the row 2 column row and column. And this element will be again Z b plus Z 2 2. That is 0.19 plus 1.15 that is equal to j 0.34.

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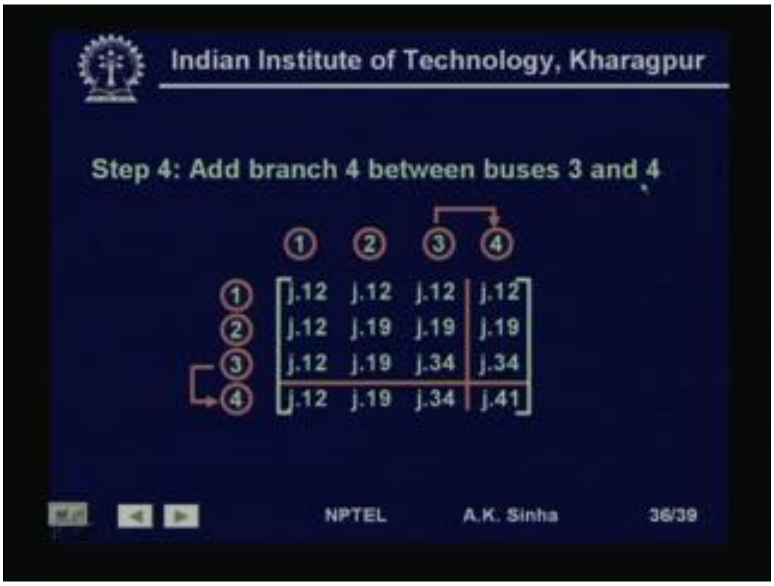

Again, add branch 4 between bus 3 and 4 a new bus is added connecting to an existing bus again type 2 modification. So, again the 4th row and column are same as the row and column for the 3rd bus. So, this is the column for 3rd bus, which is coming here. This is the row for 3rd bus which is coming here. And this will be Z b plus Z 3 3, Z 3 3 is 0.34 Z b is 0.07. So, we get j 0.41.

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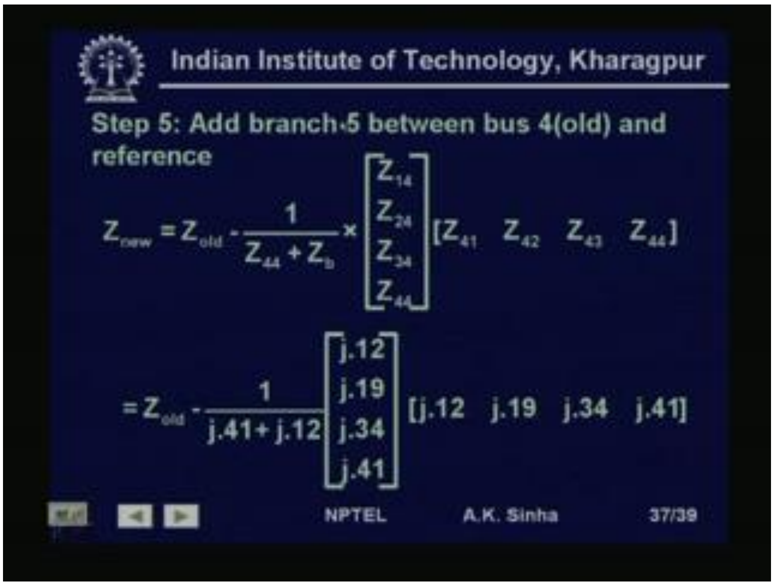

Now, we are adding branch 5, which is between bus 4 and the reference. That is from an old bus to the reference. So, this is the type 3 modification, that we have to do that is from an existing bus to the reference. So, using this relationship that we had written earlier. We substitute the values here Z old minus this is Z 4 4 is j 0.41 and Z b is 0.12 and this is the 4th row 4th column and 4th row like this put the values here.

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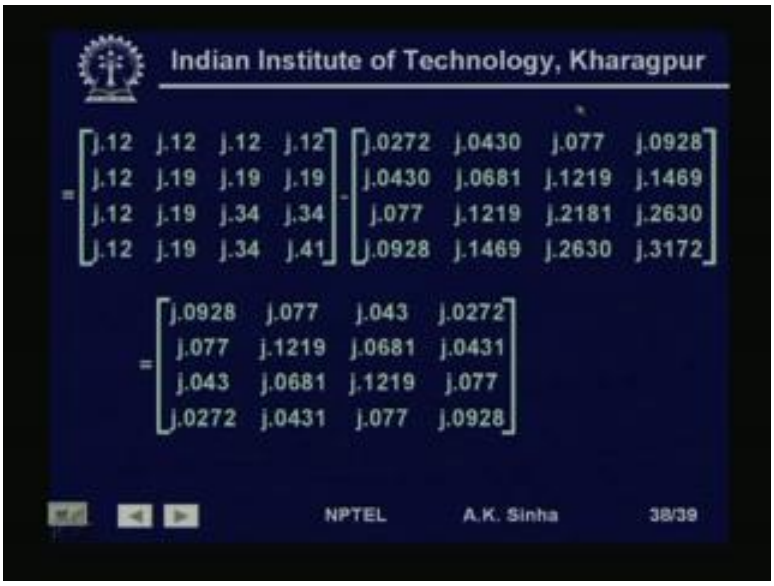

If when we do that, we get this is the old Z bus minus this term, which gives us the final matrix as this. So, this is the way we build the Z bus matrix for any power system. We go step by step adding one element at a time. And as we have seen to start with we always start with an element, which is connected to the reference. So, with this we finish this lesson today. And in the next lesson we will see how we use this Z bus algorithm for asymmetrical fault analysis.

So, thank you.

Preview of next lecture.

Power System Analysis

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Lecture Number - 32

Asymmetrical Fault Analysis Using Z bus

Welcome to lesson 32 on power system analysis. In this lesson we will discuss about Asymmetrical Fault Analysis Using Z bus method for large power systems. As we had discussed in the earlier lessons, we have seen how we can use the Z bus algorithm for solving symmetrical faults for large power systems. In this lesson we will try to extend that method for solving for asymmetrical faults.

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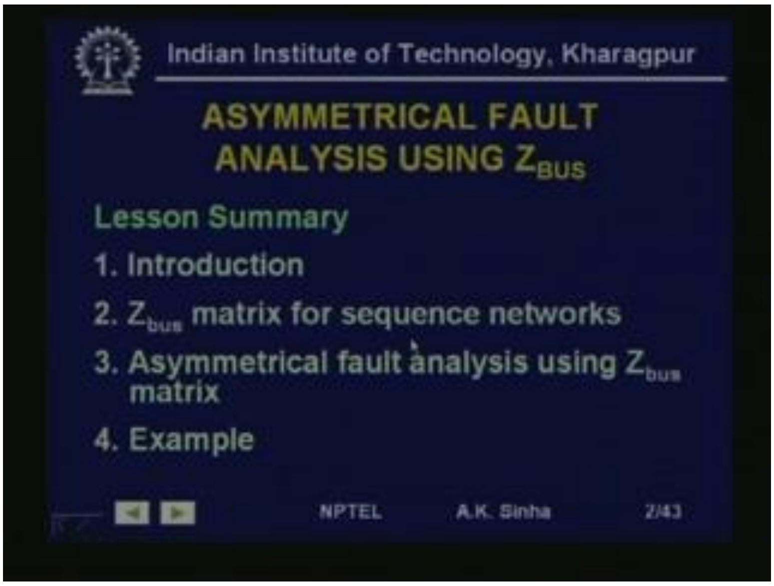

Well, we will start with an introduction. Then, we will talk about Z bus matrix formulation for sequence networks. Because, we know that for asymmetrical faults we do the symmetrical component analysis. And we get the positive negative and 0 sequence network for the system. So, we will talk about formulating Z bus matrix for the sequence networks. Then we will talk about how we use this Z bus for the sequence networks for finding out or for solving the asymmetrical faults for large power systems. And finally, we will take up an example to show how this is done.

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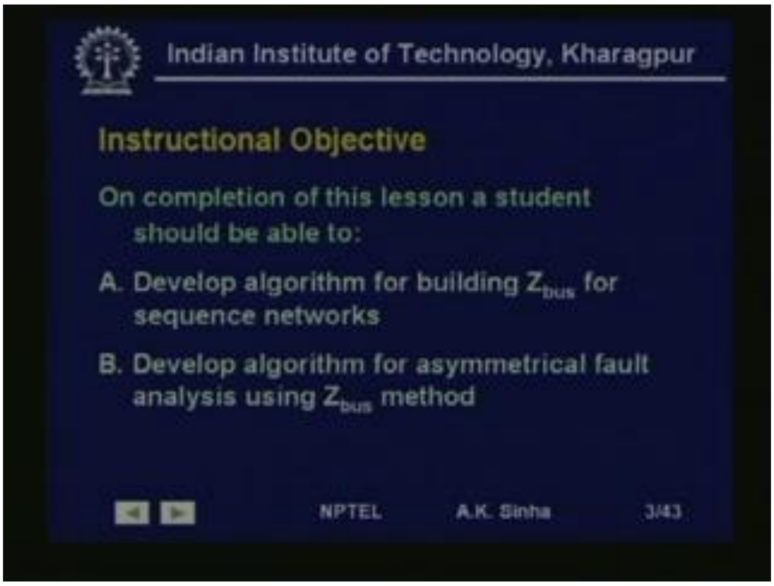

Well on completion of this lesson. You should be able to develop algorithm for building Z bus for sequence networks. You should be able to develop algorithm for asymmetrical fault analysis using Z bus method.

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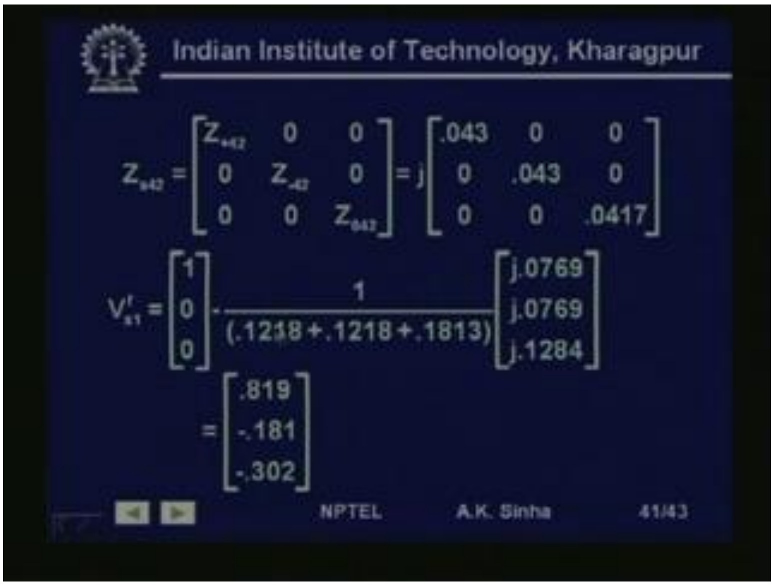

So and for finding out the values of post fault voltages at different buses, we can use this relationship. So, again this is V 0 V f V 0 S 1 minus this relationship that we had got there. So, this is the sequence values that we have got.

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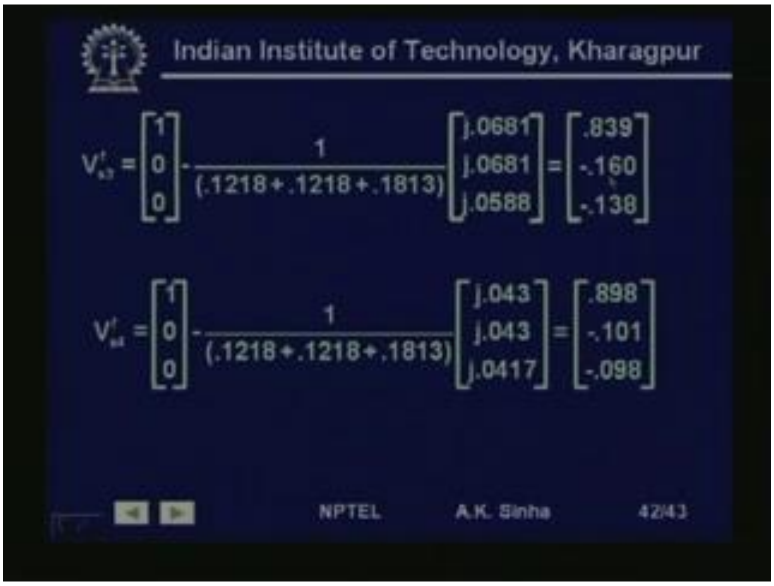

Similarly, for 3 and 4 also we will get the values by substituting the values from the impedance matrix, and the fault current that we have calculated. We will get these sequence values. Once we have got the sequence values, we can transform them to get the phase values.

So, this is how we can calculate the fault current and the post fault voltages at all the buses. And knowing the impedances of the lines or the other equipments, we can find out the fault current flowing through each element of the system. So, with this we have completed our short circuit analysis or the analysis of a faulted power system. Where, we have considered both the symmetrical faults. Asymmetrical faults for small systems by using the sequence network or using the Z bus algorithm and Y bus algorithm for large systems. So, with this we finish today. And we will talk about the power systems stability in the next lessons.

Thank you very much.