Power System Analysis
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Lecture - 30
Fault Analysis for Large Power Systems

Welcome to lesson 30 on Power System Analysis. In this lesson we will discuss Fault Analysis for Large Power System.

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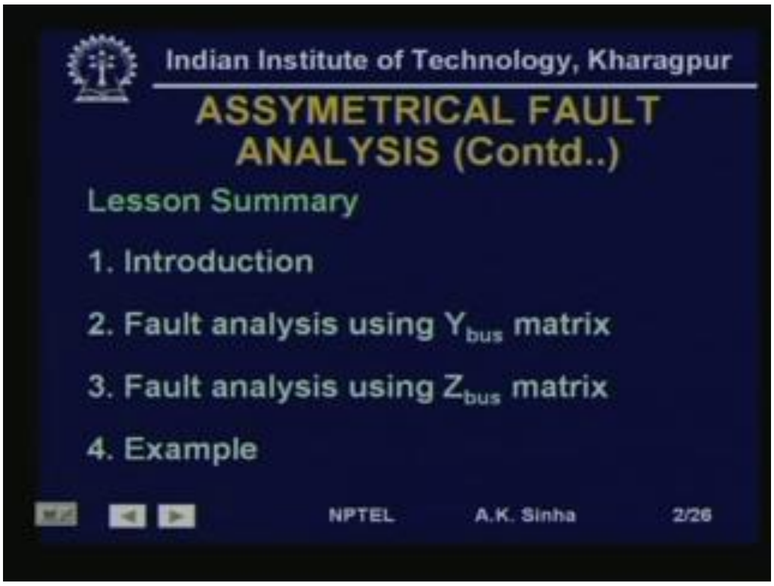

We will start with an introduction. Then we will discuss fault analysis for large power system using Y bus matrix. Then, we will talk about fault analysis using Z bus matrix. And then finally we will solve one example using Z bus matrix.

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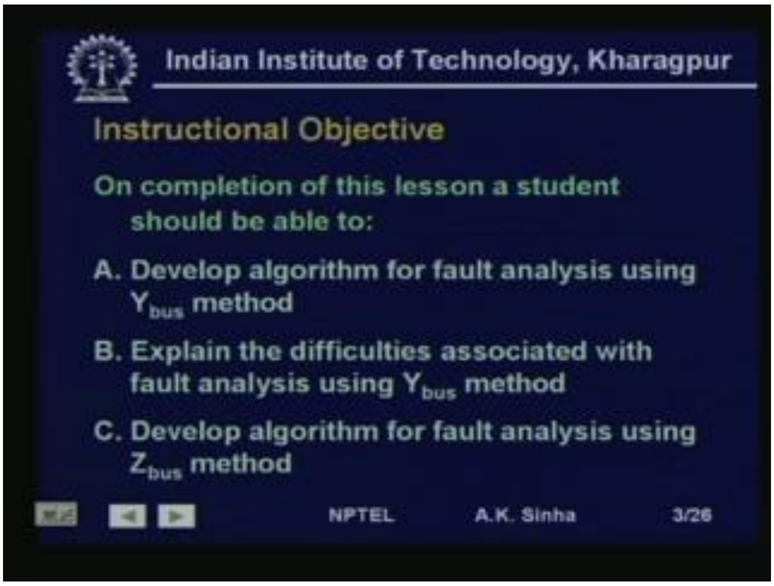

Well, the instructional objective for this lesson is that on completion of this lesson. You should be able to develop algorithm for fault analysis using Y bus method. Explain the difficulties associated with fault analysis using Y bus method. Develop an algorithm for fault analysis using Z bus method.

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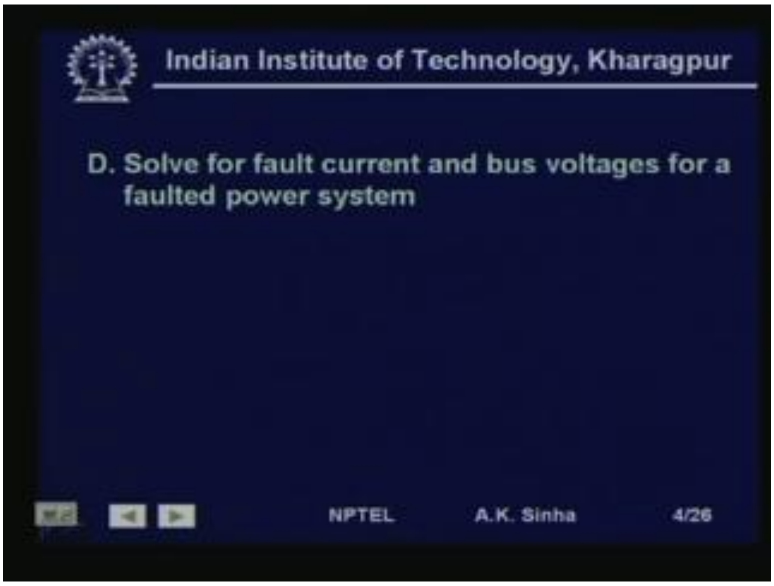

And solve for fault current and bus voltages for a faulted power system using the Z bus method.

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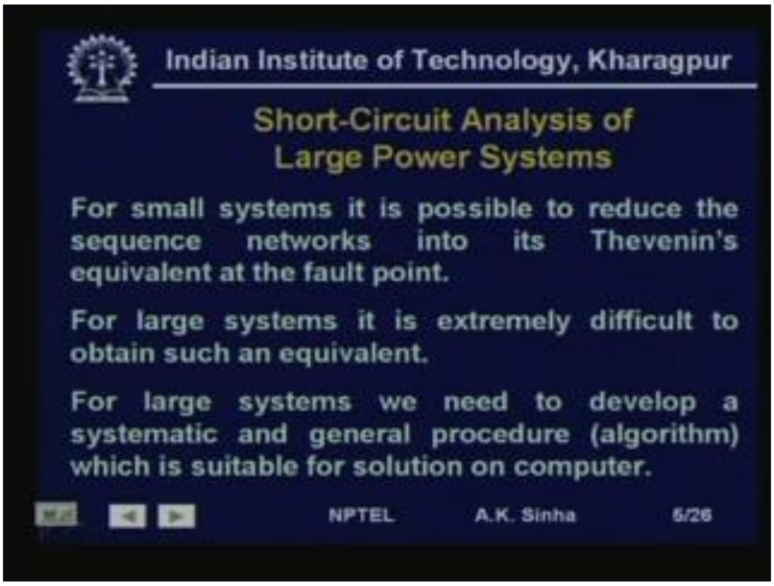

Well, as we had discussed in lesson 29, that we can do symmetrical as well as asymmetrical fault analysis using Thevenin's theorem. But, when we are trying to do it for a large system, trying to find out the Thevenin's equivalent of the network at the fault point is a very complex job. For small systems, it is possible to reduce the sequence networks into it is Thevenin's equivalent at the fault point.

For large network it is extremely difficult to obtain such an equivalent, because you need reduction for a very large network, with large number of voltage sources in the system. So, therefore for these large systems we need some kind of a systematic method to solve this problem using a digital computer. So, we need to develop a systematic and general procedure, which is suitable for solution on a computer.

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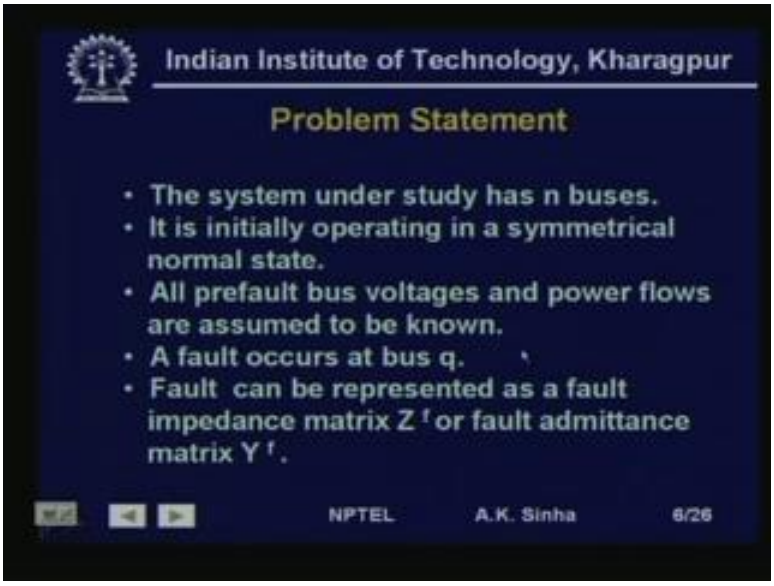

Now, how do we go about this. First, what we will do is we will define or we will state the problem for a large power system analysis. So, what we will do is, we will state a general problem. In which we will say that system under study has n buses, where n is pretty large. It is initially operating in a symmetrical normal state. That is, before the fault has occurred the system is working in a normal state, which is basically operating under a symmetrical loading condition.

So, voltages and currents in the system are normal they are not abnormal. So, load currents are there. The system voltages or the bus voltages at various places will be very near to their rated values. Then, we say that all prefault bus voltages and power flows have assumed to be known. Now, as we had said earlier also we can always make an assumption, that all the bus voltages are very nearly equal to 1 per unit. And the prefault currents are 0.

But, if we want to consider the operating condition as such, what we can do is we can do a load flow study and get the voltage at all the buses, as well as the power flowing in various lines. So, using the load flow we can get all prefault bus voltages and power flows. So, we assume that these are known, because we have already done a power flow analysis.

A fault occurs at bus q. Now, we have said that the fault is occurring at any bus, which we have defined here as a bus q. So, out of these n buses in the system, it can occur at any bus q. And this q can keep on varying, because if we want to do a short circuit analysis. Then, basically we need to find out the currents and voltages under various fault conditions. So, we can say that analysis for a fault on any bus, we are assuming this bus as bus q.

Now, fault can be represented as fault impedance matrix Z f or a fault admittance matrix Y f. Now, this part we will take up later when we discuss about the asymmetrical faults. For symmetrical fault, we are assuming that the fault occurs at bus q with some impedance Z f, which is same in all the three phases, because this is a symmetrical fault. So, it is a three phase short circuit, which may be with some impedance to ground. So, this impedance can be Z f. However when we are talking about the asymmetrical faults, then we have to talk about positive negative and 0 sequence quantities. And therefore, in that case we will have a 3 by 3 matrix which will represent this fault impedance or fault admittance.

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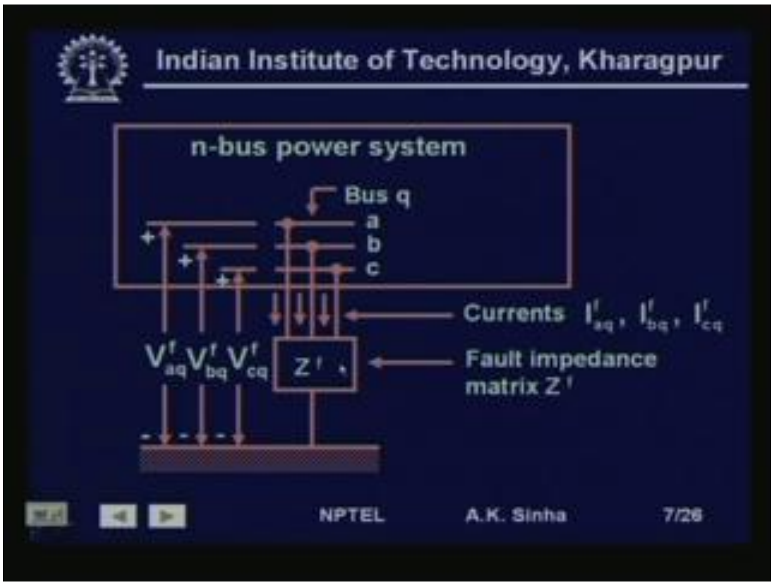

So, this is what we have said that we have a n bus power system, which is represented here in this rectangle. At a particular bus q, where we have shown all the 3 phases a, b, c. The a fault has occurred through an impedance Z f, this impedance Z f is common to all the 3 phases to the ground. In such a case we are going to have the voltages in all the 3 phases as V a q f. This sub superscript f indicates the voltage after the fault has occurred we will use a superscript 0 indicating before the fault or the prefault values.

So, we have voltages V a q f, V b q f and V c q f for all the 3 phases. The magnitude values of all these voltages are going to be same. Because, we are talking of symmetrical fault and the angle between them is going to be 120 degree out of phase from each other. So, that is these three voltages will be symmetrical voltages and the currents here I a q f, I b q f and I c q f all these also will have the same magnitude. And they will have a phase displacement of 120 degree from each other. As we have shown here as a fault impedance matrix will be basically Z f connected to all the three phases and then to the ground. So, this a symmetrical fault, that we have in the symmetrical fault case. And when we talk about a symmetrical fault, then we will have this Z f as 3 by 3 matrix.

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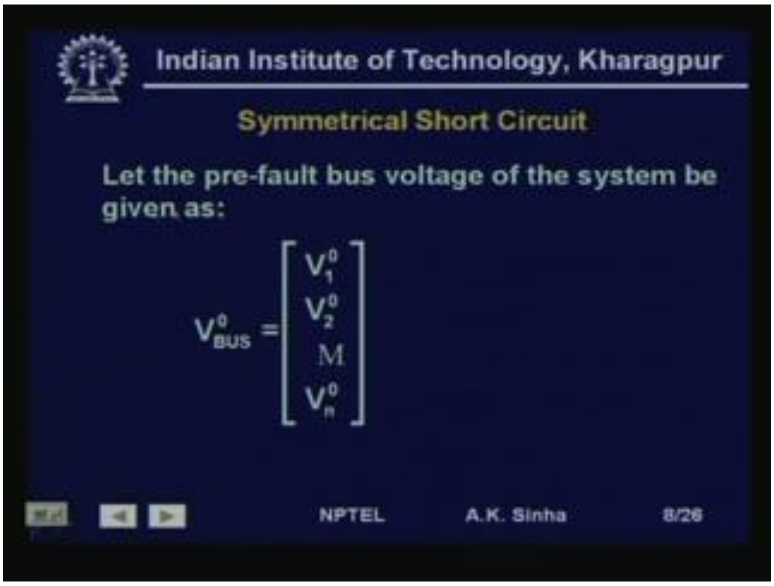

Now, let us start for the symmetrical short circuit, how we are going to work on it. Let the pre-fault bus voltages of the system be known or they are given as V 0 bus, which is basically a n vector I am sorry this should not be M this will be dot, dot, dot. So, V 1 0, V 2 0 up to V n 0. So, these are known I if they are not known, we can always assume them to be one angle 0. But, in most of the cases we can always do a load flow analysis and get these voltages.

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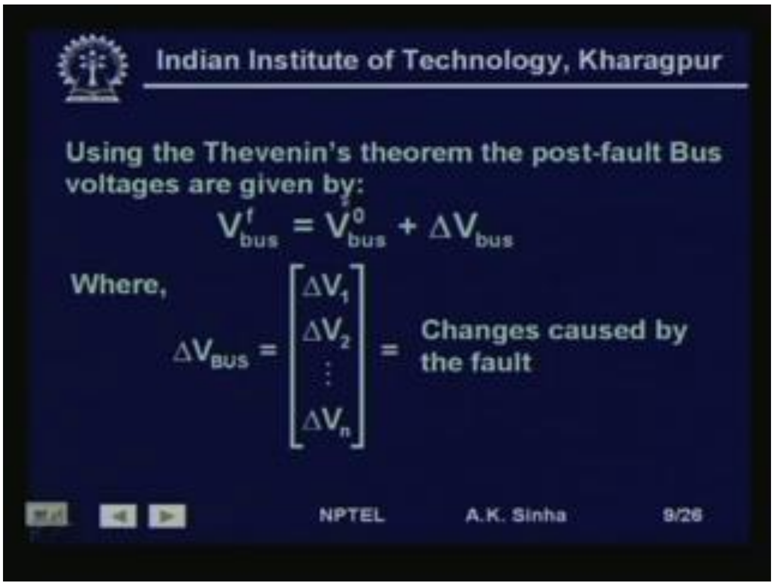

Now, using the Thevenin's theorem as we had done earlier, using the Thevenin's theorem. The post fault bus voltages can be written as V bus f sub superscript f indicating post fault voltages is equal to V bus 0, the pre-fault voltages plus delta V bus where delta V Bus is nothing but the change is in the bus voltages caused by the fault. So, here we have defined delta V bus as equal to delta V 1 up to delta V n, these are the changes in bus voltages caused by the fault.

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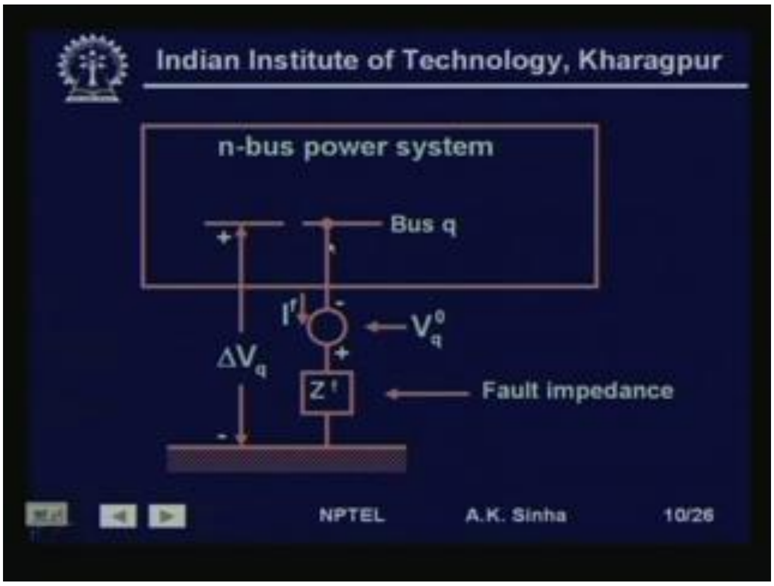

Now, we have as we had seen earlier, how we can represent or find out these changes caused by the fault. We had seen earlier, that we can have the same system, where we have replaced all the voltage sources. That is, we have shorted all the voltage sources. So, there is no voltage source in the system and we apply a negative voltage equal to the prefault voltage at that bus, and in series with the fault impedance.

This is what we had seen earlier. So, here also we are going to do the same thing. So, at bus q we are applying this voltage V q 0 with a negative sign. That is minus on this side plus on this side, which indicates that the fault current will be flowing into the ground from the bus. So, this voltage in series with this impedance Z f will is what we have introduced into the system.

And now with the system having no voltage sources. That is all the voltage sources or generators have been now their voltage has been made 0 and they are represented only by their internal impedances. So, with this condition on the system we will try to find out the change in voltages at various bus. So, at this bus we are going to get this voltage delta V q f. So, this delta V q is what we will find out for the qth bus and similarly for other buses.

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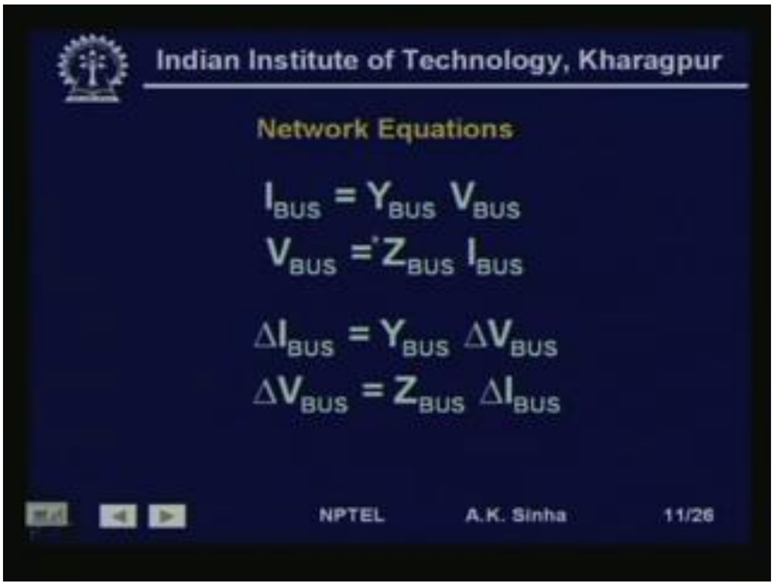

Now, how do we find these changes in voltages. Well, we know that the relation set for current and voltages. That is current injections into the network and the voltages at different buses is given by the relation set I bus is equal to Y bus into V bus. This is what we had used in the power flow or the load flow equations. Or it can be written as V bus is equal to Z bus into I bus. Now, where Z Bus is the bus impedance matrix and we can find out this Z bus by inverting Y bus or there are other ways in which we can form this Z bus.

Now, this is the relation set. So, we can write down this relation set for the changes also in the same fashion. So, delta I bus is equal to Y bus into delta V bus. And similarly, delta V bus is equal to Z bus into delta I bus. Now, we can use any of these two relations to find out the changes in bus voltages, that we need or the fault current that we want.

So, we will start first with the Y bus method. In fact, we will show that Y bus method is not very much suitable for short circuit analysis. Whereas, we had seen earlier that in power flow analysis, we always use the Y bus method. The one of the reason behind that is very simple that Y bus is are very easy to form plus it is a sparse matrix. Whereas, the Z bus matrix which is the inverse of this Y bus is difficult to form, as well as if we want to find out by using inverse, it involves quite large computation.

And another aspect which goes with Z buses, Z buses no longer sparse it is a full matrix. Therefore, the storage requirement also increases considerably. However, as we will see later that for a short circuit analysis Z bus is preferable to Y bus. Anyway, we will start first by seeing how we can solve this using the Y bus method.

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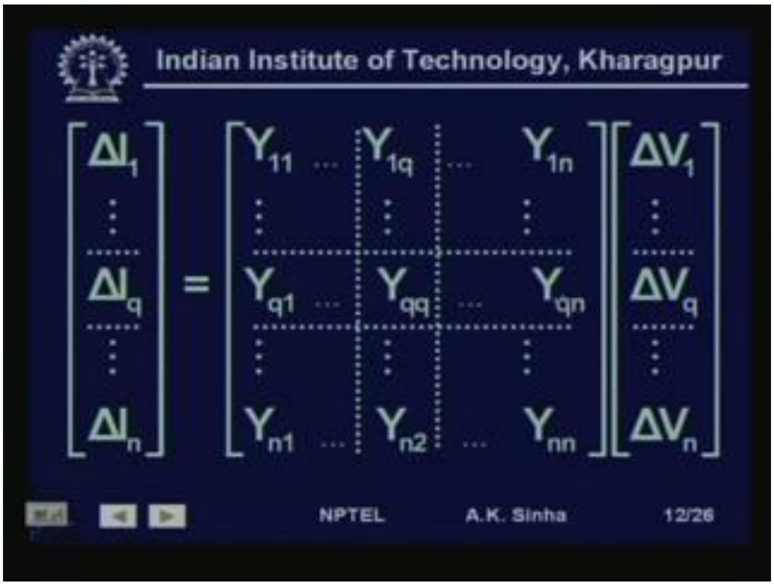

So, let us write down the equation relating the changes in voltage and the changes in the current injections at various places. So, we can write delta I as delta I 1 up to delta I n, this is a column vector is equal to this Y matrix, which is a Y bus matrix into delta V have from delta V 1 up to delta V n. Now, the fault has occurred at bus q. So, we have identified this qth row and qth column.

Now, what we will do is we will rearrange this whole matrix, in such a way that the qth element or the qth current and voltages are the first element. That is the qth row and qth column become the first row and first column of the Y bus matrix.

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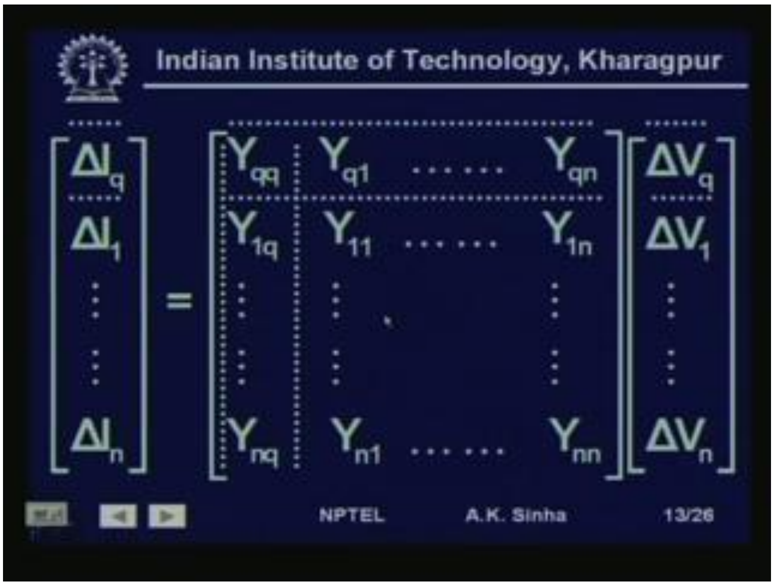

So, if we do that then we will have the system of equations written like this. Where the first row is Y q q, then Y q 1 up to Y q n, Y q q column has been shifted to column 1, similarly Y q q row has been shifted to row 1. So, this is the qth row of the Y bus matrix, that we had earlier. Now, we have arranged this qth row as row 1, similarly the qth column has been arranged as column 1.

So, therefore we have delta I q as the first element, in the I delta I vector and delta V q as the first element in the delta V vector. So, this how the equations will get rearranged if we do this. So, rearranging the equation in this form, we get the system of equations like this.

Now, we see we can divide this into 4 this matrix into 4 sub matrices, one is this matrix of Y q q element only. Another is this row of Y q 1 to Y q n of course, Y q q will not be there and this column Y 1 q to Y n q and we have a n minus 1 into n minus 1 matrix from Y 1 1 to Y n n.

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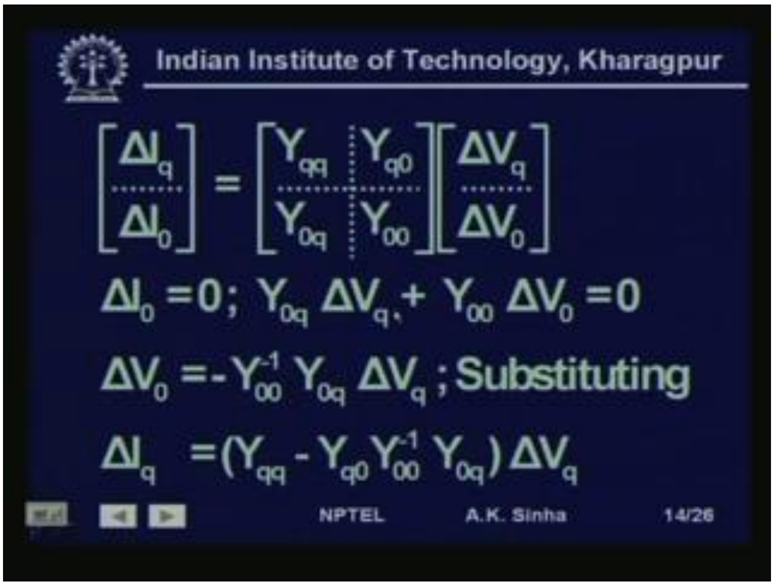

So, this is the form that we will have, which we can write in short as delta I q and delta I 0 I am writing this delta I 0 ((Refer Time: 16:47)) this delta I 0 for delta I 1 to delta I n of course, delta I q is not in that. So, this term that is all the terms current injection terms or changes in current injection terms, for all the other buses except the qth bus is termed as delta I 0. Similarly, this all the terms for change in voltage at all the buses except the qth bus is also defined as delta V 0.

So, in that form if you write, then we can write delta I q and delta I 0 is equal to Y q q, Y q 0 Y q 0 is this row of n minus 1 element Y q 1 to Y q n. Then, Y 0 q, Y 0 q is this column of Y 1 q to Y n q and Y 0 0 is this n minus 1 into n minus 1 matrix. So, in this form we will get delta I q is equal to Y q q into delta V q plus Y q 0 into delta V 0 and delta I 0 is equal to Y 0 q into delta V q plus Y 0 0 into delta V 0.

So, this is the matrix equation that we are going to get. Now, we have already said earlier that we have replaced all the voltage sources or all the generators by their internal impedances. And the volt there is no voltage, because voltage sources are shorted forgetting the Thevenin’s equivalent impedance.

So, we have no current injection at any bus except the qth bus. So, delta I 0 is basically 0, that is all the elements here ((Refer Time: 18:54)) delta I 1 to delta I n are all 0. There is no current injections at these buses. Except that at bus q a current I f is flowing from the bus into the ground. That is the fault current is flowing from the bus to the ground which is basically negative directions of current injection is going to be minus I f in that case.

So, we have delta I 0 is equal to 0. Therefore, we can write Y 0 q into delta V q plus Y 0 0 into delta V 0 is equal to 0, because this is delta I 0. So, from here we can eliminate this delta V 0. So, we can write delta V 0 is going to be equal to minus of Y 0 0 inverse into Y 0 q into delta V q from this equation, we can take this term on this side. So, it will become minus Y 0 q into delta V q. And then pre multiply both side where Y 0 0 inverse. So, you get minus Y 0 0 inverse into Y 0 q into delta V q.

So, this is the value of delta V 0. Now, we can substitute this delta V 0 for the upper equation, that is the equation for delta I q. So, delta I q is equal to Y q q into delta V q plus Y q 0 into delta V 0, now this delta V 0 we are replacing by this. So, what we have is Y q q into delta V q, Y q q in to delta V q plus Y q 0, Y q 0 into delta V 0. For delta V 0 we have minus of Y 0 0 inverse into Y 0 q into delta V q. So, minus Y 0 0 inverse into Y 0 q into delta V q. So, this is the equation that we get for delta I q.

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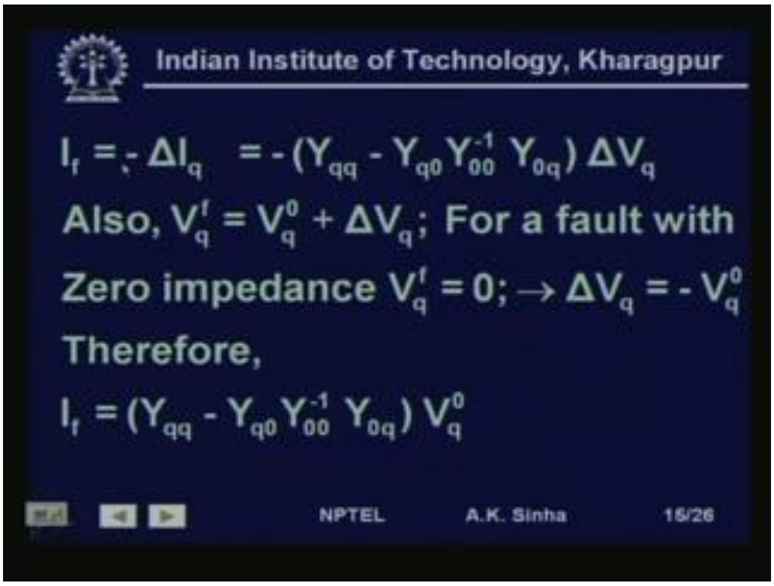

Now, what is I f the fault current. Fault current is basically a current, which is coming out from bus q into the ground that is if you see this diagram.

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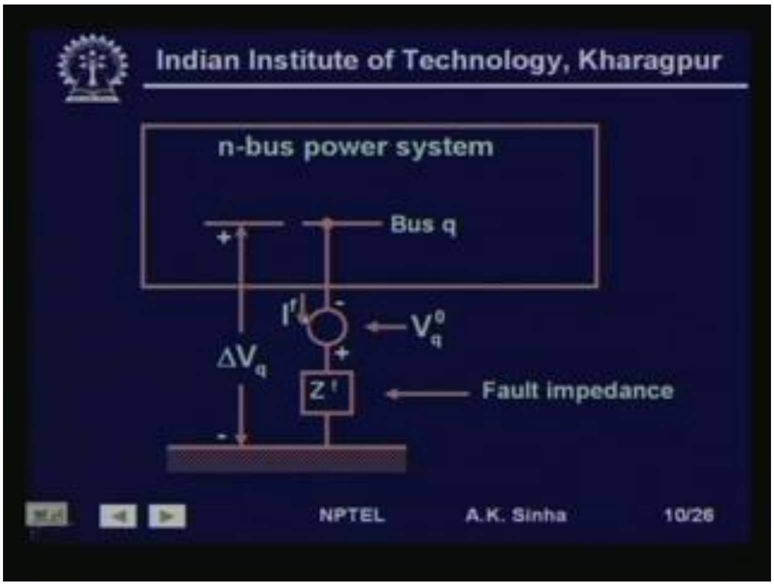

This is, I f the current which is coming from bus q into the ground, this is the fault current. And this current is occurring, because of this V q 0 source connected here, rest all other sources have been eliminated. So, there is no other source in the system except this one. So, this is I f flowing in the system like this ((Refer Time: 21:55)).

So, here we have I f is equal to minus delta I q. And therefore, we can write this equal to minus Y q q minus Y q 0 into Y 0 0 inverse into Y 0 q into delta V q, ((Refer Time: 22:14)) that is the same this thing with a minus sign. Because I f is minus delta I q ((Refer Time: 22:20)). Now, we have V f or V q f the voltage at post fault voltage at bus q is equal to V q 0 plus delta V q. So, this is the change and this is the pre-fault voltage. So, the post fault voltage is equal to pre-fault voltage plus the change in the voltage due to the fault.

Therefore for a fault with 0 impedance, what will be the value V q f will be equal to 0, because you have got a short circuit with 0 impedance. That is, ((Refer Time: 22:58)) here if you see this Z f is 0, then the voltage of this bus is going to be equal to 0 ((Refer Time: 23:09)). So, therefore, delta V q is nothing but, equal to minus V q 0, the change which has occurred is going to be equal to minus V q 0, that is exactly what we have seen here ((Refer Time: 23:25)).

That is what we have put here. So, delta V q is nothing but, this minus V q 0 which is put. So, at this point the bus voltage for the changes is going to be equal to this much. So, since it is equal to minus V q 0 ((Refer Time: 23:4)), so delta V q is equal to this.

Therefore, we can write I f is equal to Y q q minus Y q 0 into Y 0 0 inverse into Y 0 q into V q 0. Now, this minus term is no longer 0, because there because delta V q we are replacing with minus V q 0.

So, this is the way we can calculate the fault current. Now, the algorithm is very simple, we know the values of all these elements Y bus is known, V q 0 is known. Once, we know I f or I delta I q we can find out the voltages at all other buses as well. So, we can very easily find out the voltages at all the buses ((Refer Time: 24:35)) by using this equation. That is, we know that this is 0 and writing this equation with delta I q is known.

And so we can solve this and we will be able to get the values of voltages at all the buses, if we want to find that out. But, here what we are finding is that though this algorithm looks very straight forward, the problem that we face is if we need to compute the short circuit currents for faults at different buses. Then, what we need is for each bus we will have to solve for this Y 0 0 inverse, because Y 0 0 is going to be different ((Refer Time: 25:23)). If you recall here for the qth bus we have got this we have arranged it like this.

So, you have the Y 0 0 matrix like this. If instead of qth bus, the fault would have occurred at rth bus. Then, rth row and column would have been shifted and this matrix will be different. Therefore, again since this matrix is different ((Refer Time: 25:47)) we will have to get it is inverse which will be different. So, for every fault at every bus we need to compute this n minus 1 into n minus 1 matrix inverse, which is very computation intensive.

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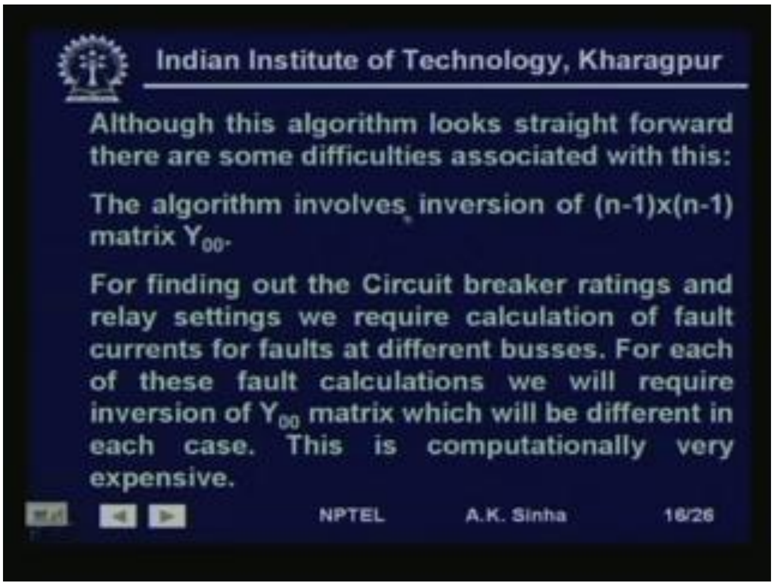

So, this is what we will we have written here. That, although this algorithm looks straight forward there are some difficulties associated with this. The algorithm involves inversion of n minus 1 into n minus 1 matrix Y 0 0. For finding out the circuit breaker ratings and relay settings, we require calculation of fault currents for faults at different buses. That is, what we said at various buses, we need to compute create the fault and find out the fault currents and the voltages.

For each of these fault calculations we will require inversion of Y 0 0 matrix, which will be different in each case. This is computationally very expensive and this is the main reason why, Y bus based algorithm for fault analysis is not much preferred, instead we prefer the Z bus based algorithm. And we will see how this Z bus based algorithm works.

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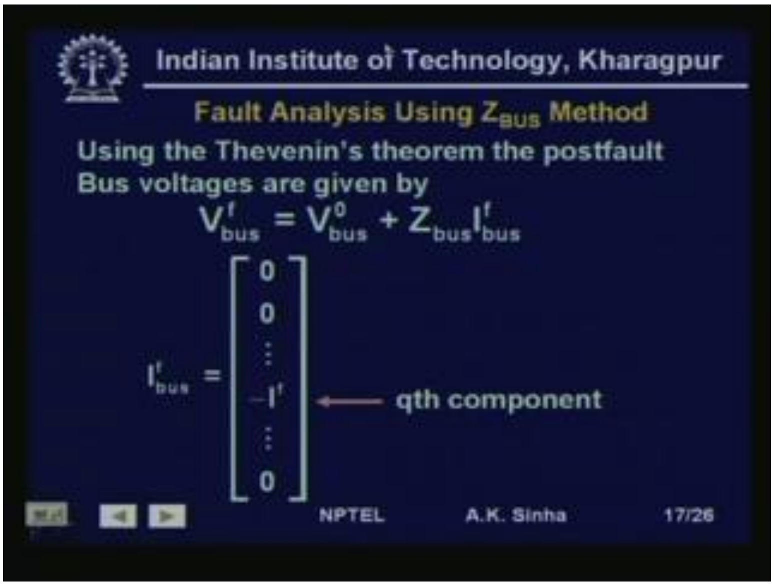

So, fault analysis using Z bus method. Now, again we have seen using the Thevenin's theorem, the post fault bus voltages can be written as V bus f is equal V bus 0 plus Z bus into I bus f. Now, what is this I bus f? I bus f is basically the current injections at various points, when the fault has occurred. Now, as we have seen for Thevenin's equivalent, what we have written is that this delta V involves, this delta I this is because of the changes which have occurred.

And in case of finding out the changes we have already eliminated all generating sources. The only source which is available in the system is, the voltage source at the fault point. And that is a current I f flowing from the bus towards the ground. So, if we see in terms of current injection at all the other buses except the qth bus is going to be 0. And at qth bus the current injection is nothing but, the negative of the fault current, because fault current is flowing away from the bus to the ground. And since we write, these equations in terms of current injections into the bus. Therefore, we write this as minus I f. So, all the elements of this I f bus are going to be 0 except for the qth element, that is the element corresponding to the faulted bus, which will be equal to minus I f. So, this is what we have ((Refer Time: 28:55)).

And therefore, once we write down the equations in terms of the Z bus. That is delta V is equal to Z delta I. Now, delta I we have already seen is having only one element, which is non zero and that is the qth element equal to minus I f. This is the Z bus, the question for finding out Z bus as we have already said. Once, we know the Y bus we can always invert this Y bus and get the Z bus.

Inversion I n into n Y bus matrix, where n is large is a computationally intensive task. But, still it is much less computation intensive as compared to the Y bus method, where for each fault at each bus, you need to invert n minus 1 into n minus 1 matrix. So, anyway, so we write this equation as delta V equal to Z into delta I, where delta I we have only this qth element, which is non zero and we need to find these values. So, what we will do is we will expand this relationship and write this like this.

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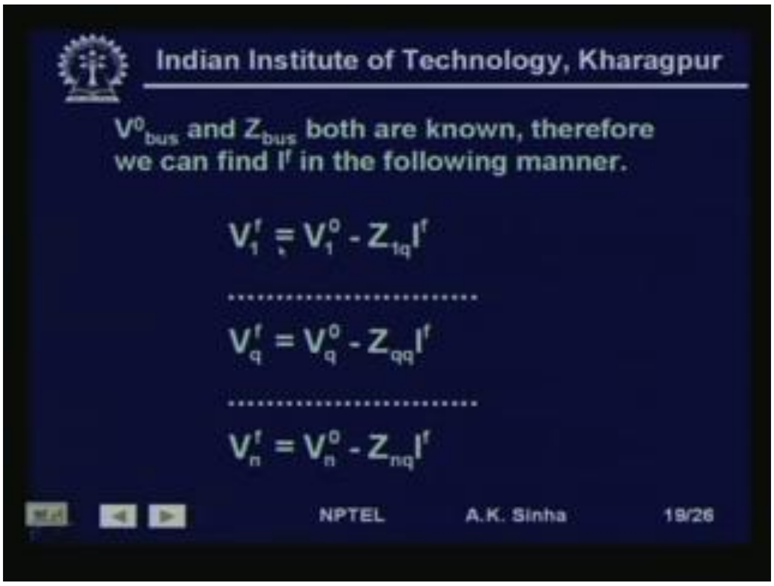

So, V 0 bus and Z bus both are known therefore, we can find I f in the following manner. What we will do? We will ((Refer Time: 30:21)) expand this relationship, we can write delta V 1 is equal to Z 1 1 into 0, Z 1 2 into 0 and so on. Only except for this column elements, that is Z 1 q into I q delta I q which is equal to minus I f is going to be present, rest others are again going to be 0 ((Refer Time: 30:43)).

So, we have V 1 f is equal to V 1 0 minus Z 1 q into I f. Because, this V 1 f is equal to V 1 0 plus delta V 1 and delta V 1 is equal to Z 1 q into I f with a minus sign. So, minus Z 1 q into I f, so this is what we write for this. Similarly, we will write for all other buses we for the faulted bus we have delta V q is equal to Z q q into minus of I f. So, we can write this as V q f is equal to V q 0 minus Z q q into I f.

Similarly, for other buses till the nth bus which is V n f is equal to V n 0 minus Z n q into I f ((Refer Time: 31:38)). That is what we are seeing is only the qth column elements are the one, which are used when a fault at bus q is simulated. So, we have this expression for all the bus voltages. After fault that is post fault bus voltages in this term.

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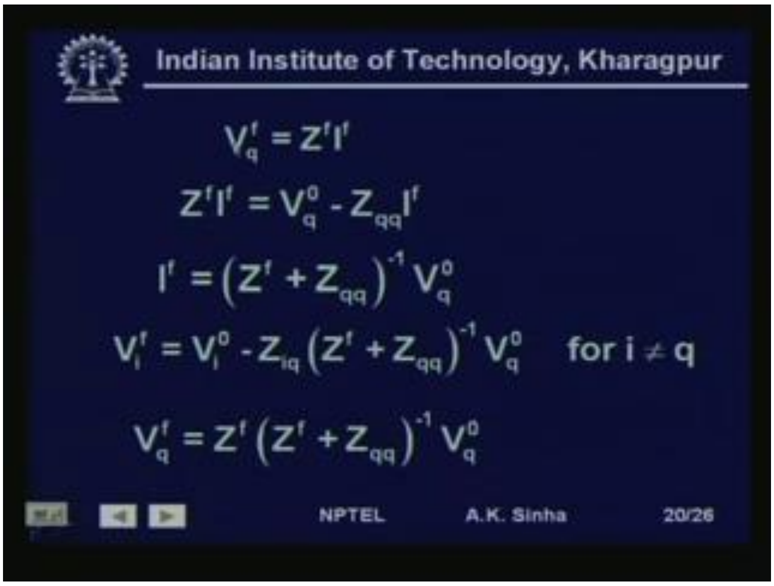

Now, V q f is equal to how much ((Refer Time: 32:15))? If we go back to this point V q f is going to be equal to Z f into I f. So, I f is the current flowing and Z f is the impedance. So, what is going to be the voltage at the bus after the fault. So, that voltage is going to be equal to Z f into I f only. So, using that we have V q f is equal to Z f into I f.

And we also have written here V q f is equal to V q 0 minus Z q q I f. So, we can write Z f into I f is equal to V q 0 minus Z q q into I f. From here we can take this term on this side. And therefore, we can write I f is equal to Z f plus Z q q inverse by into V q 0. So, this as we have seen for symmetrical fault, these are scalar quantities Z f is a scalar quantity Z q q is a single element.

So, we have I f is equal to Z f plus Z q q inverse into V q 0 which we can simply write as V q 0 divided by Z f plus Z q q. And once, we have calculated this I f, then we can always substitute this I f ((Refer Time: 33:40)) into these equations to find out the post fault voltages at all other buses. So, we can write V i f is equal to V i 0 minus Z i q into I f we are writing as this term.

So, Z f plus Z q q inverse into V q 0 for i not equal to q, when i is equal to q of course, we know that V q f that is post fault voltage at the faulted bus is equal to Z f into I f. So, substituting for I f we will get Z f into Z f plus Z q q inverse into V q 0, which we can write as Z f divided by Z f plus Z q q into V q 0. So, this way we can find out the post fault voltages at all the other buses as well the fault currents. Once, we know these post fault values we can calculate current flowing in any branch in the system.

And thus we can set our relays properly we can find out the ratings for the circuit breakers. So, this is how we create or we calculate the fault current and post fault voltages. And thereby study the condition of the system under fault.

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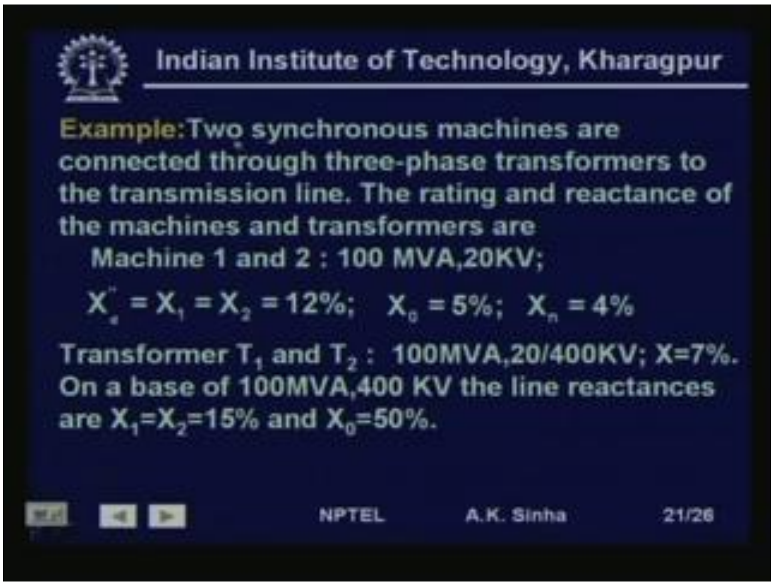

Now, let us take a small example to try to understand this how we do this. So, we have taken the same example, which we have worked out earlier. This is a small system example only, just to illustrate how we work out this Z bus algorithm. So, here we have two synchronous machines connected through three phase transformers to the transmission line.

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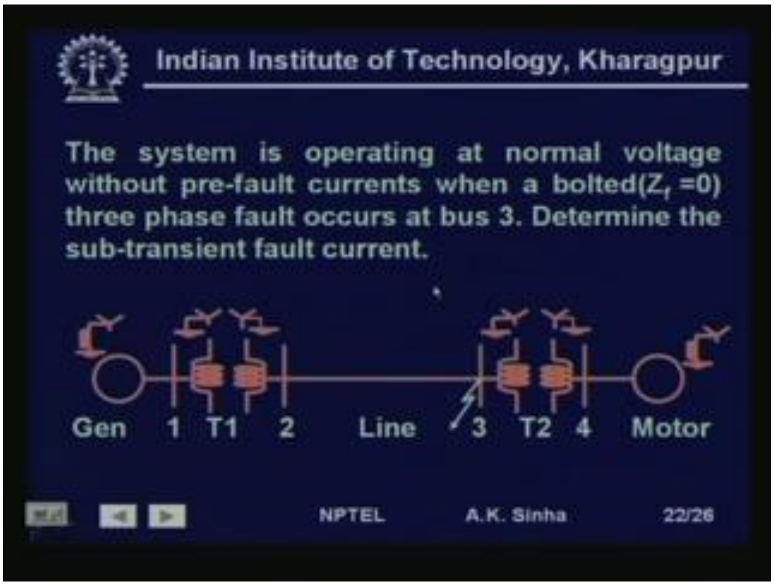

That is we have two generators connected through transformers to a transmission line ((Refer Time: 35:54)). The rating and the reactance's of the machines and the transformers are, that is here what we are saying is the resistance is very small. So, we have neglected the resistance and we are talking only of the reactance's. The ratings and reactance's of the machines and transformers are machine 1 and 2 100 MVA, 20 KV. That is, generating voltage is 20 KV and the rating of the machines are 100 MVA.

There are sub transient reactance for this machine is given as 12 percent or 0.12 per unit, which is all same at as the positive sequence and negative sequence reactance of the machine. In fact, negative sequence reactance may be a little less. But, if it is a cylindrical rotor machine they will be more or less same. The zero sequence reactance is given as 5 percent and the grounding reactance is given as 4 percent.

However, we do not require these other values, because these will be required only for asymmetrical faults. For symmetrical faults it is only the positive sequence current, which flows in the system. Now, let the transformer T 1 and T 2 have a rating of 100 MVA and the voltage rating is 20 KV to 400 KV. The reactance is given as 7 percent. So, the reactance is 0.07 per unit and the rating is same 100 MVA and the voltage ratings are 20 and 400 KV.

On a base of 100 MVA, 400 KV that is on the high voltage side of the transformer we have the base we have the rated value as 400 KV. So, the base which we have chosen on the transmission line side is 400 KV. And of course, 100 MVA base we have already chosen for the generators as well as transformers. So, we are using the same base MVA.

So, on a base of 100 MVA, 400 KV the line reactances are X 1 is equal to X 2 equal to 15 percent. That is, the positive sequence and negative sequence reactance of the line is equal to 15 percent or 0.15 per unit. And the 0 sequence reactances is 50 percent or 0.5 per unit, as we as we are working on a symmetrical fault. So, we need only the positive sequence reactance ((Refer Time: 38:32)).

So, now what we have to do, the system is operating at normal voltage without pre-fault currents. That is, we are considering unloaded system. That is, we have assumed pre-fault currents to be 0, when a bolted three phase fault, bolted means a fault with zero impedance. That is at that short circuit, a short circuit occurring without any impedance. So, occurs at three phase fault occurs at bus 3, that is at this bus we have a three phase short circuit occurring.

Determine the sub transient fault current. So, what we need to do? We have to find out the sub transient fault current. So, first what we are going to do? We are going to draw the positive sequence network or the impedance diagram for this system. And then find out the Y bus matrix for the system.

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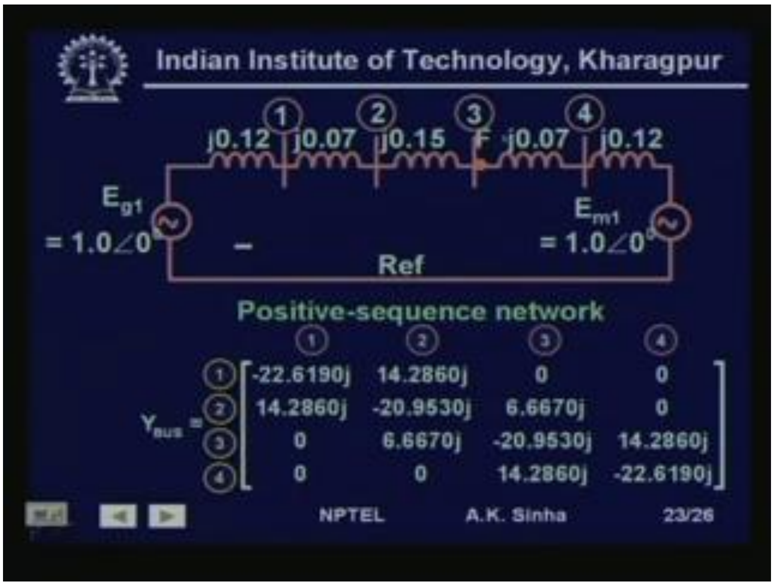

So, this is what we have done we have created the positive sequence network diagram. We have this generator here, generator has is voltage is one angle 0, because pre-fault currents are neglected. That is initially it is operating without any current. So, the rated value of the generator voltage is taken as one angle 0 it is impedance ((Refer Time: 40:08)) is the positive sequence impedance or the sub transient impedance is 12 percent that is 0.12 per unit.

So, that is what we have got and then we arrive at this bus 1. Between bus 1 and bus 2 we have a transformer shown here. So, this transformer T 1 has a reactance of 7 percent. So, we have a reactance of 7 percent 0.07 per unit between the bus 1 and 2((Refer Time: 40:44)). Between 2 and 3 we have a transmission line, the positive sequence reactance of which is 15 percent or 0.15 per unit. So, we have between 2 and 3, this transmission line with a reactance of 0.15 ((Refer Time: 41:02)).

Between 3 and 4 again we have transformer T 2 which has a reactance of 0.07, that is 7 percent on the chosen base. So, we have a reactance of 0.07 between bus 3 and 4. At bus 4 ((Refer Time: 41:22)) we have this generator connected and this generator has a reactance of 12 percent. So, we are showing here a reactance of 12 percent, in series with the voltage source with voltage equal to 1 per unit.

Since, the currents are initially 0. So, the angle for both the voltage sources are chosen as 0 degree. Now, using this network we can calculate the admittances by taking 1 by 0.12 1 by j by 0.12 for this element 1 by j 0.07 1 by j 0.15 and so on. We can do that find out the admittances. Now, finding out for Y 1 1 what we need we add these two admittances if we do that, we will get this value between 1 2 Y 1 2 will be what is this would 1 by this term.

So, we are going to get negative of that. So, it is coming out to be 14.2860 j between 1 and 3 there is no impedance connected directly. So, for the admittance matrix we have a 0 here. Similarly, between 1 and 4 we have no connection therefore, the admittance is 0 here. Similarly, 2 1 element, 2 2 element, 2 3 element we have 2 1 is again same as 1 2 we will get 2 2 will be 1 by this plus 1 by this which comes out to be minus 20.9530.

Similarly, 2 3 will be 1 by this element, which comes out to be this much and 2 4 we do not have any connections. So, this is zero again we have for bus 3, when we are writing 3 1 is 0, there is no connection between 3 and 1 directly, 3 2 again 1 by this. So, this is here 3 3 will be sum of 1 by this plus 1 by this. That is coming out to be this much, that is minus 20.953 j and 3 4 is 1 by this.

So, that comes out to be 14.2860 of course, when I am saying 1 by this, you will get j as negative and we take negative of the admittance connected between 2 buses. So, this becomes positive and so on. So, for 4 also we will get between 1 and 4 there is no admittance. So, 0 4 and 2 there is no admittance. So, this is 0 4 and 3 this is the admittance, so negative of this that is 1 by minus j 0.07 which comes out to be this much. Similarly, 4 4 will be 1 by j 0.07 plus 1 by j 0.12 that comes out to be minus 22.6190 j. So, this way we will form the Y bus matrix, once we have form the Y bus matrix we can invert this matrix.

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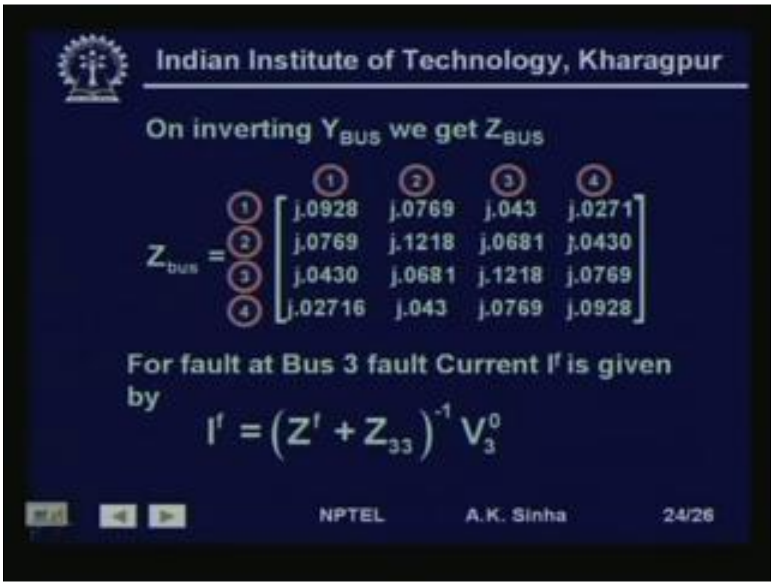

And then we will get the Z bus matrix. Now, you see the, whereas Y bus matrix has 6 elements, which are 0, Z bus matrix is a full matrix. That is we see that Y bus is a sparse matrix, where as Z bus is a full matrix. This is a major disadvantage of Z bus method. Because, here the matrix is going to be in most cases a full matrix. So, your storage requirement is going to be much higher.

Now, for a fault at bus 3, the fault current I f is given by I f is equal to equal to Z f plus Z 3 3 inverse into V 3 0. That is if you remember ((Refer Time: 45:53)) I f is equal to Z f plus Z q q inverse into V q 0. Now, q is bus 3, so Z f plus Z 3 3 inverse into V 3 0. So, that is what we are writing here.

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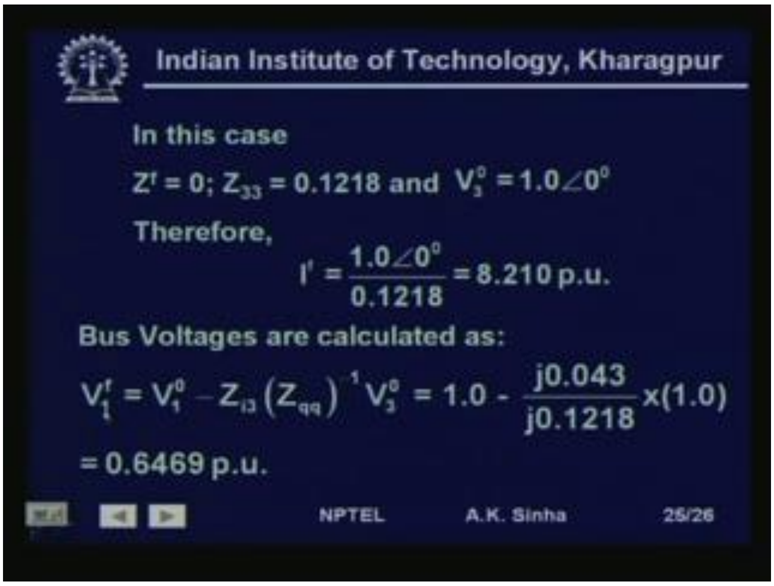

Now, substituting the value since Z f here is 0, Z 3 3 is 0.1218 and V 3 0 is 1 angle 0 degrees. That is, if we see Z 3 3 is this value. So, substituting we get I f is equal to 1.0 angle 0 degrees divided by 0.1218, this comes out to be 8.210 per unit. So, this is 8.210 per unit current that we will get here, what we are seeing is these are the values of this.

So, here j will be there, so minus j will be there. So, the current will be lagging by 90 degrees, which is case for a purely reactive circuit, that is the current lacks the voltage by 90 degrees. So, we should have here minus j volt here. The magnitude is of course, 8.210 per unit. Now, we can calculate the bus voltages in the same way V 1 f, if we want to write ((Refer Time: 47:26)) V i f is equal to V i 0 minus Z i q into Z f plus Z q q inverse into V q 0. This is the for post fault bus voltages at any bus i where i is not the faulted bus.

So, using this we can write for bus 1 as V 1 f is equal to V 1 0 minus Z 1 3 this should not be i Z 1 3 into Z q q inverse. Because, here Z f is 0, so Z f plus Z q q inverse was the thing. So, here Z f is 0, so it is Z q q inverse into V 3 0. Now, substituting the value we have V 1 0 as 1, Z 1 3 is this j 0.043. So, this is j 0.043 divided by Z 3 3 in this case, this is Z 3 3. So, Z 3 3 is j 0.1218, so j 0.1218 into V 3 0, which is 1 angle 0 degree.

So, if we do this we get the voltage at bus 1 as equal to 0.6469 per unit. So, this way we have calculated the voltage at bus 1. Same way we can calculate the voltage at bus 2 and bus 4 also. Once we know these voltages, then we can calculate the current flowing in each of these elements where easily. Because, the voltages are known the impedance is known. So, we can calculate the current flowing through this.

So, this is systematic procedure through which we can calculate all the bus voltages and the fault current. And after we have calculated this we can find out the current flowing through each element. And therefore, we will be able to find the ratings for the circuit breakers. As well as the settings for the relays, which we need to protect the system.

So, with this we will finish today, what we will do is in the next class, we will discuss about how we can avoid ((Refer Time: 50:18)) taking inverse of this Z bus. Rather, we would try to see, how we can build this Z bus matrix from scratch. That is, from the network how we are going to build this Z bus using E adding one element at a time.

And this method which we call Z bus building algorithm is one, which is very much used for building Z bus. Rather than, you getting this Z bus by inverting the Y bus matrix. After that what we will do is, we will try and see how we can introduce this into the for faults which are asymmetrical faults. And we will see how we can use the sequence quantities for... And use this positive negative 0 sequence Z bus matrices for computing fault currents and the asymmetrical faults. So, with this we will finish today and we will in the next lesson talk about building of Z bus matrix.

Thank you.

Power System Analysis

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Lecture Number - 31

Bus Impedance Matrix

Welcome to lesson 31 on power system analysis. In this lesson we will discuss about building Z bus matrix or the Bus Impedance Matrix.

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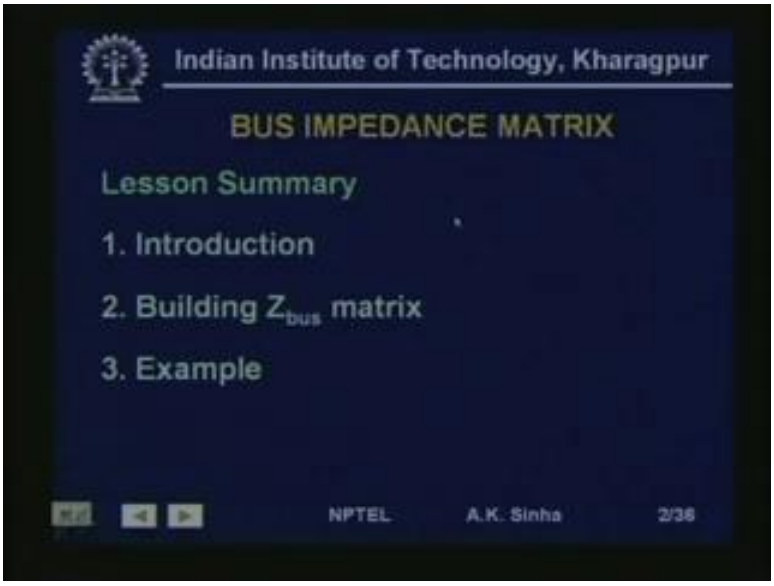

Well, we will start with an introduction. Then, we will go into developing an algorithm for building the Z bus matrix. And then we will take up an example of how to build Z bus matrix for a power system network.

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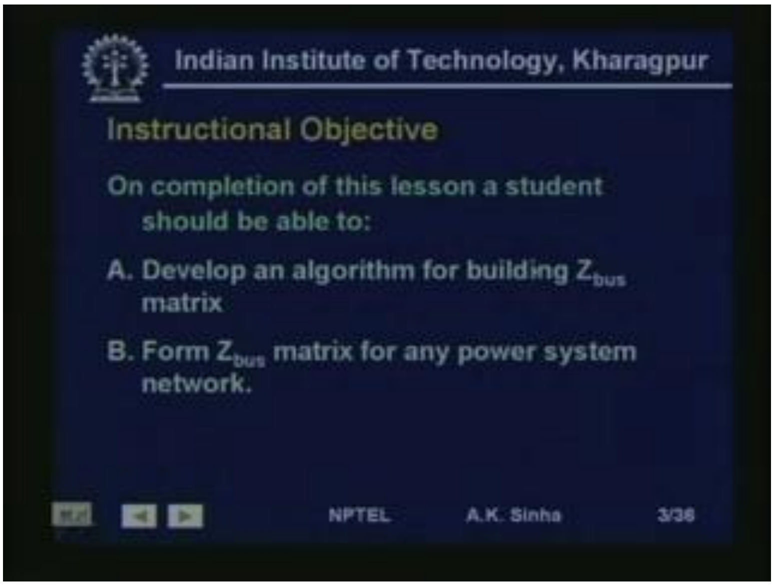

Well, on completion of this lesson, you should be able to develop an algorithm for building Z bus matrix and form Z bus matrix for any power system network.

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Well, when we had talked about short circuit calculation for large systems. We had seen, that if we use the bus admittance matrix. That is Y bus, then we have certain difficulties mainly we need to get an inverse of N minus 10 into N minus 1 matrix, for each short circuit calculation for each bus that we need to do. Whereas, if we formulate the Z bus matrix, then we can calculate the short circuit currents for faults on any bus using the same Z bus matrix.

We also discussed that we can form the Z bus matrix by inverting a N into N Y bus matrix. However, since the computational requirement for inversion of a large N into N matrix is very large or it is a very expensive computationally. So, we generally do not go by inverting the Y bus, what we do is we try to build this Z bus by a step by step formation of the Z bus matrix or we built this Z bus step by step. And we will see how we can develop an algorithm for building the Z bus in this lesson.

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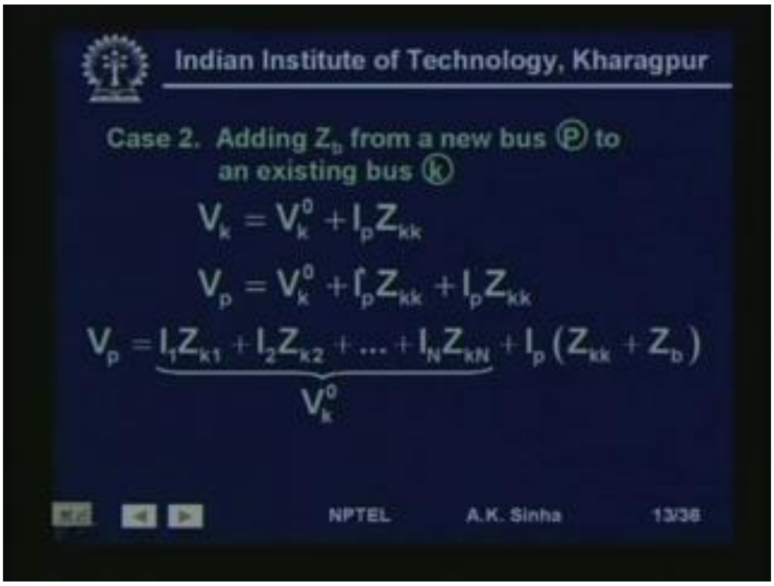

So, therefore we will get here V k the new voltage will be equal to V k 0 the existing or the previous voltage plus I p into Z k k. We can write for the voltage at bus p or the new bus as V p is equal to V k 0 plus I p into Z k k actually this is the V k voltage here.

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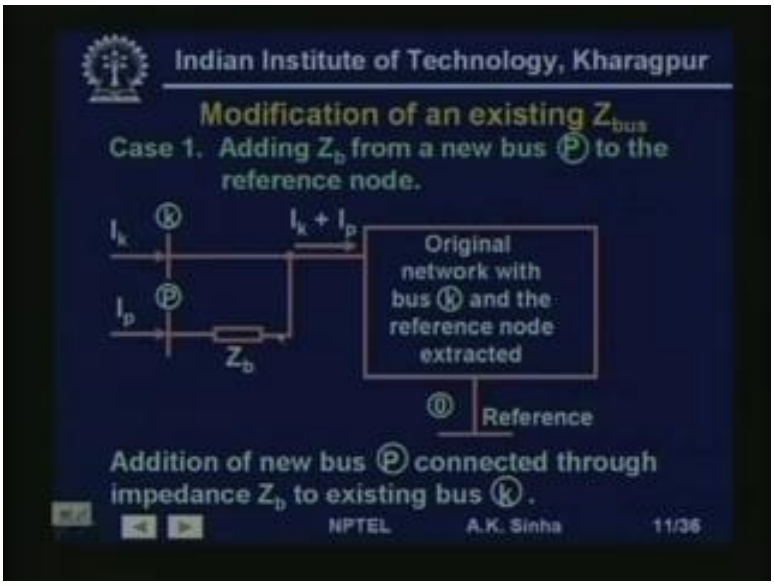

If you see, the voltage here is V K plus this voltage. Because, this I p current is flowing like this here. So, this is the voltage V k, so on this voltage drop has to be added. So, I p in to Z b has to be added ((Refer Time: 55:17)). So, plus I p into Z b therefore, V p we can write as now V k, if we go back here.

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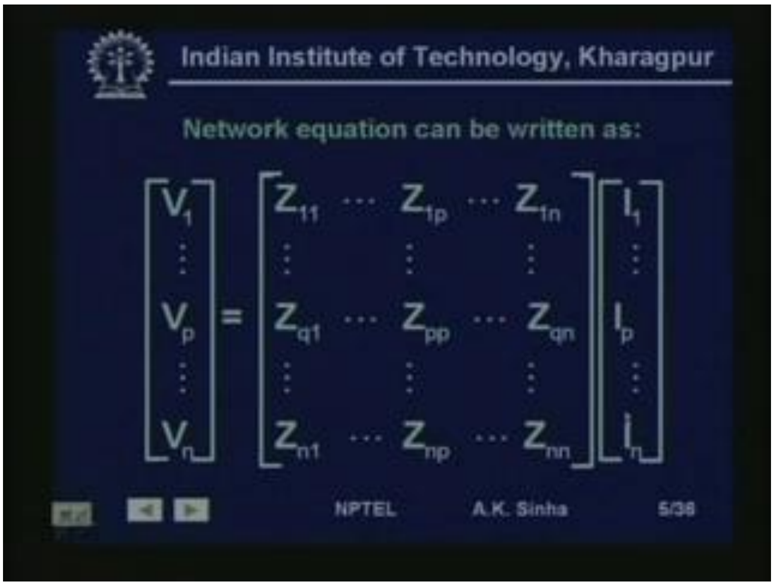

Z p 1 Z p p Z p 1 into I 1 plus Z p 2 into I 2 and so on up to Z p n into I n. So, we will have this should be P 1 and P n Z p 1 and Z p n here. So, we will have V p is equal to Z p 1 into I 1, Z p 2 into I 2 and so on like this. So, we can write here ((Refer Time: 56:14)) for V k we can write here as V k 0 is equal to I 1 into Z k 1, I 2 into Z k 2 plus In into Z k n plus I p into Z k k plus Z b, this is Z b, so I p into Z k k plus Z b. So, this is what we will get this term is basically V k 0.

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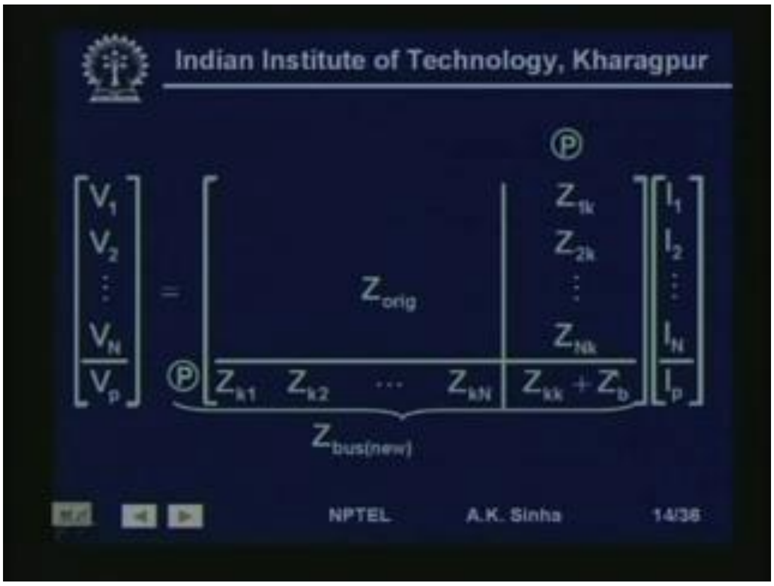

So, we can write V p now like this, V p is equal to Z k 1 into I 1, Z k 2 into I 2 up to Z k n plus Z k n into I n plus Z k k plus Z b into I p. So, this is what we will get as the new impedance matrix. And similarly for V 1 there is going to be a change in voltage, which will be given by Z 1 k into I p. Like here, we had ((Refer Time: 57:17)) written for any bus other bus I also we will have that is the change will be I p into Z 1 k.

So, Z 1 k into I p will give for V 1 Z 2 k into I p will be added to V 3 and so on, because of this current I p flowing into the network at this bus. So, now here we see we have a new N plus 1 into N plus 1 matrix. Because of this additional node p added to the system.

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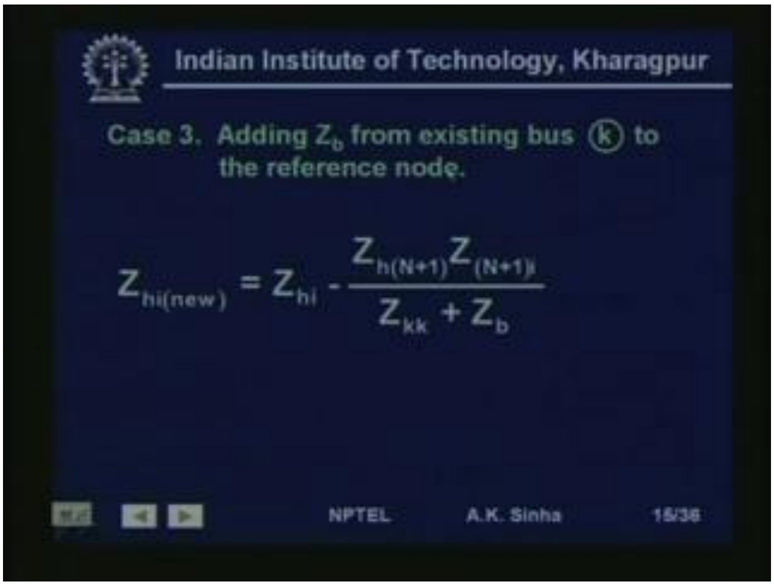

Now, we will take the case 3 or the type 3 modification, which is adding branch with impedance Z b from the existing bus k to the reference node. Now, what we have in this case, we are adding a branch from an existing bus to the reference. So, what we can do is? We can consider this similar to the case 2 modification. That is adding Z b from a new bus p to an existing bus k ((Refer Time: 58:38)). What we are doing is? We are putting this bus p a new bus p as. And then what we do is we connect this bus p to the reference. Then what happens if we connect this to the reference, then this branch is basically connecting this bus k with reference.