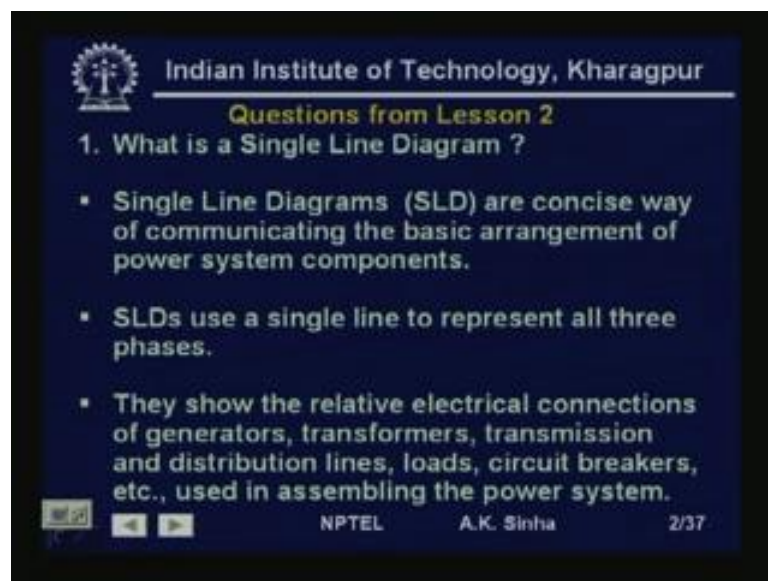


Power System Analysis
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Lecture - 3
Transmission Line Parameters

In this lesson, we will talk about Transmission Line Parameters, before I go into the transmission line parameters itself.

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The slide is a dark blue presentation slide with white text. At the top left is the IIT Kharagpur logo. The text on the slide reads: 'Indian Institute of Technology, Kharagpur', 'Questions from Lesson 2', '1. What is a Single Line Diagram?', and a bulleted list of three points. At the bottom, there are navigation icons, the NPTEL logo, the name 'A.K. Sinha', and the slide number '2/37'.

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Questions from Lesson 2

1. What is a Single Line Diagram ?

- Single Line Diagrams (SLD) are concise way of communicating the basic arrangement of power system components.
- SLDs use a single line to represent all three phases.
- They show the relative electrical connections of generators, transformers, transmission and distribution lines, loads, circuit breakers, etc., used in assembling the power system.

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We will first talk about the questions. That I ask at the end of lesson 2. Well, the first question was, what is a single line diagram? Well, the answer to that question is, single line diagrams are concise way of communicating the basis arrangement of power system components. Single line diagrams use a single line to represent all the three phases. And they show the relative electrical connections of various electrical components, which are used in assembling the power system.

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2. How base values of voltages are chosen on two sides of a transformer ?

- Ratio of V_b on either side of a transformer is selected to be same as the ratio of transformer voltage ratings

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The second question was, how base values of voltages are chosen on two sides of a transformer? Well, the answer of this question is, ratio of base values on either side of a transformer is selected to be the same as the ratio of the transformer voltage ratings. Well, this is necessary, because when we use this voltage basis on the two sides as the say as the voltage ratio of the transformer. Then, we find that the per unit impedance on the two sides of the transformer are same. And therefore, it eliminates the use of the ideal transformer in the process steps. This helps considerably in reducing the calculations for the power system circuit is...

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3. For a 500 Mw, 22 kV generator the per unit impedance is 0.8 pu on its own base. What is its pu impedance at 100 Mw and 33 kV base ?

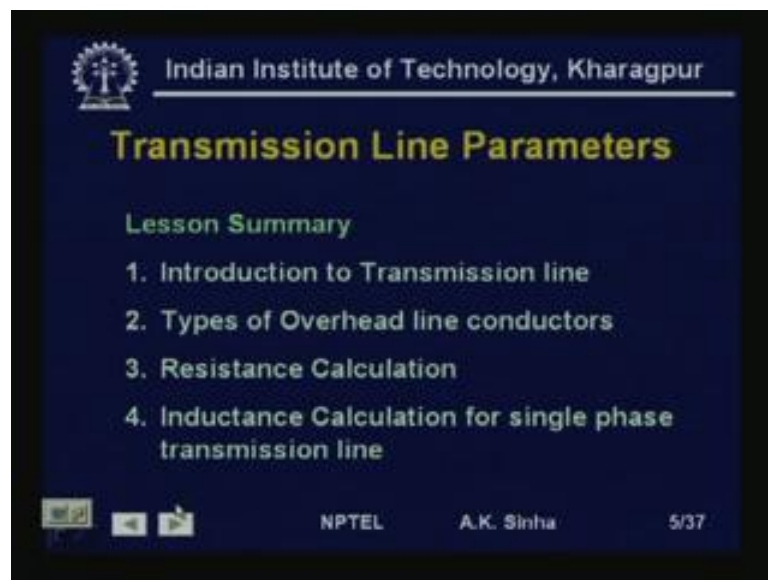
$$Z_{p.u.new} = Z_{p.u.old} \left(\frac{V_{bold}}{V_{bnew}} \right)^2 \left(\frac{S_{bnew}}{S_{bold}} \right)$$
$$Z_{p.u.new} = 0.8 \left(\frac{22}{33} \right)^2 \left(\frac{100}{500} \right) = 0.0711$$

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Well, the third question was, for a 500 Mega Watt 22 kV generator the per unit impedance is 0.8 per unit on its own base. What does its per unit impedance at 100 Mega Watt and 33 kV base? Well, as we had seen in lessons 2, the Z per unit on the new base is given as set per unit and the old base multiplied by old V base divided by new V base square multiplied by new m V a base divided by old m V a base.

So, using this relationship, once we substitute these values, we will get set per unit at the new base of 100 Mega Watt and 33 kV as pointed into 22 by 33 whole square multiplied by 100 by 500, which will come out to be equal to 0.0711 per unit. That is from 0.8 per unit, the values changes to 0.0711 per unit on the new base.

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So, after this, we will get into the lessons 3. In this, we will be talking about the transmission line parameters. Well, what we will do is, we will start with introduction to transmission line. Then, we will talk about the types of overhead conductors. We will talk about, how to calculate resistance of the transmission line. And we will talk about the calculation of inductance, for a single phase transmission line.

So, first we will go into the overhead transmission system. The overhead transmission system is mostly used at high voltage. Mainly, because it is much cheaper, compared to an underground system. Now, the type of conductance, which we use in overhead transmission are based on the kind of conducting material. That we are using as well as the strength, which we need for these overhead conductors.

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So, the conductors, which are used, are generally copper conductors. Nowadays, copper conductors are hardly chosen or they are almost non-existent. They are basically extinct; we do not use them at all. Mostly, because copper is very expensive and its supply is quite limited. Well, the type of conductors, which are used mostly, is called ACSR conductors. That is Aluminum Conductor Steel Reinforced. This is mainly because; aluminum is much cheaper as well as it is available in abundance.

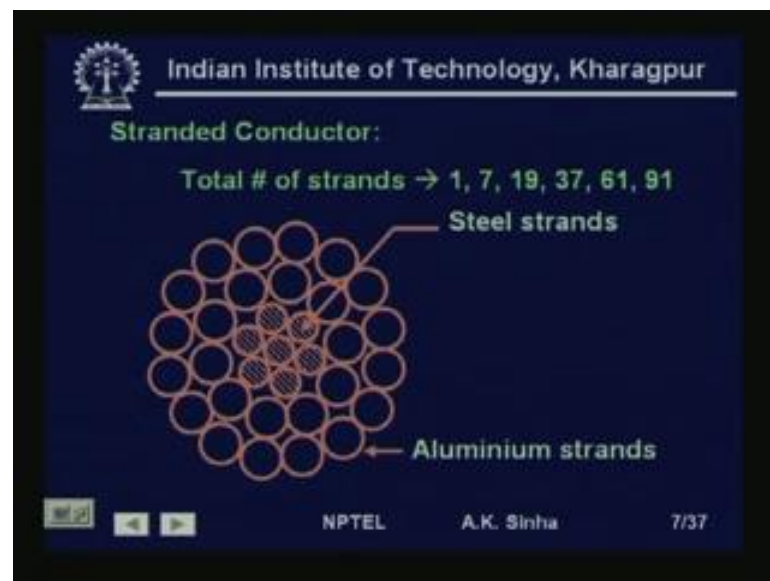
Aluminum has much lower conductivity as compared to copper, which means, that we have to use larger cross sectional area, for the same current carrying capacity of the line or the conductor. Another problem with aluminum conductor is, that its tensile strength is not very high. And therefore, it needs to be reinforced, that is it has to have larger tensile strength to take care of its own weight or to make it more robust. When, we string it between the towers.

This is provided by steel reinforcement, which is done by putting steel strands at the core and the aluminum strands of conductor are put on top of this steel core. Other types of conductors, which are not so much in use, are All Aluminum-Alloy Conductor. Here again, aluminum alloy is used, because it gives you more strength to weight ratio. That is, it provides higher strength, for the same weight as compared to all aluminum conductors. So, this is also used some time, but it is somewhat more expensive. Other types are ACAR, that is Aluminum Conductor Alloy reinforced.

Now, this is again the same thing instead of steel reinforcement. We have aluminum alloy reinforcement provided. One type of conductor, which is used for extra high voltage lands is called expanded ACSR conductor. This is same as the aluminum conductor is steel reinforce, except that, between the steel core and the aluminum conductors. We provide some filler material, such as paper or fiber; this increases the effective diameter of the conductor, for same conducting cross sectional area.

This has an advantage that with the larger diameter, the electrical stress, for these high voltage conductors, extra high voltage conductors is reduced. And their by, it reduces corona losses. Also, this increase in the diameter reduces the inductance of the conductor also to some extent. We will talk about these later. Well, the conductors let we, generally use are always stranded conductors.

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Here, I am showing you a diagram or a cross section for an ACSR conductor, where we are using the steel reinforcement or the steel strands at the core or a center for the conductor. And the aluminum strands are put on top of this. Now, why do, we use stranded conductor. One of the reasons for this is, that manufacturing or making these stranded conductors is much easier, especially if the size is larger. Because, for larger sizes, what you need is keep on adding layers of strand. Therefore, manufacturing becomes much easier to build.

The other advantage is, that these stranded conductors, provide better mechanical strength as well as better handling. That is, they are much more flexible as compared to a solid conductor of the same diameter or the same cross sectional area. Therefore, they are prepared over single solid conductor.

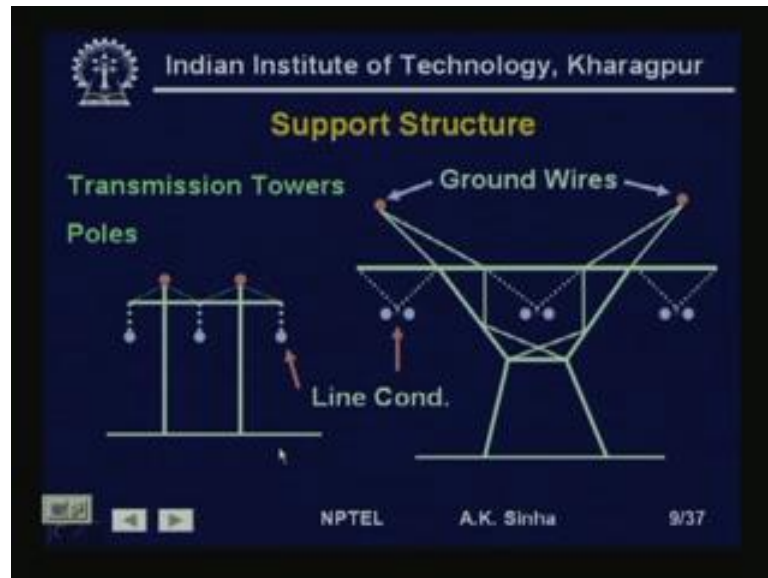
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In this diagram, we have taken the view of this stranded conductor. Again, here you find that the steel strands are at the center and we have layers of strands of conductor, aluminum conductor on top of it. Now, aluminum has much higher conductivity as compared to steel. And therefore, most of the current flows through the aluminum conductor and hardly any current flows through the steel strands.

Here, one more thing, you can notice, that is the strands or the layer of, there is conductors strands are spiral in opposite direction. This is only to provide a better binding for these conductors. That is the strands, do not open up easily. So, this provides a better binding and better mechanical strand. That is why, spiraling is there. Next, we talk about the support structure, which is normally transmission tower or poles.

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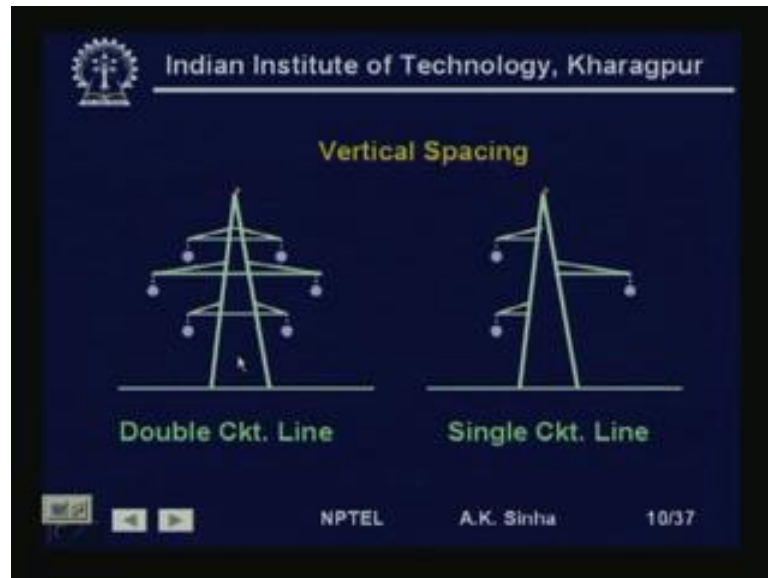


Now, here you can see, that we have we use different kind of structure depending on the voltage level. So, at lower voltage level, we can use this kind of a structure. Also, we see that for this, we are using a single conductor. Whereas, in this structure, we are seeing, we are using two conductors for each phase. This kind of an arrangement is called bundle conductors. This is generally used at high voltages of 220 kV and above.

The reason behind this is, this increases the effective radius of the conductors. Therefore, it reduces the electrical stress and so the corona losses. And this also reduces the inductance, which is very much useful in terms of reducing the voltage drop. Now, in this structure, we also find that at top of the structures are the towers or the poles, we use ground wires.

Now, these ground wires are provided mainly to protect the phase was from the direct lightening strike. What happens, when since, these are at the top, the lightening will normally strike these ground wires. And these wires are grounded at the tower potting with low resistance. Therefore, when a lightning strikes, it harmlessly flows to the ground without effecting or damaging the phase conductors at all.

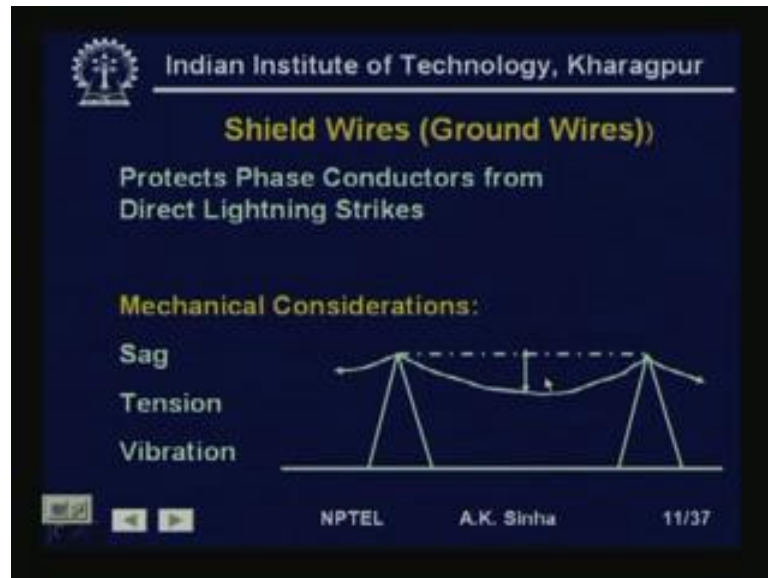
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Now, conductors can be placed in various arrangements. The earlier diagram was showing the arrangement of conductors in a horizontal plane. Many times, we arrange the conductors in different fashion, like in vertical plane or in a hexagonal plane. Here, this tower is showing a double circuit line. Where, the three phase conductor a, b, c of one circuit is on this side and a, b, c of the other circuit is on the other side.

This kind of arrangement is also used for especially putting this, a, b, c here. And putting a, b, c in this fashion, this again reduces the inductance of the system. Sometimes, when the load on the system has not grown, what we use is, we use a single circuit line. But, with a tower, which can be expanded to take care of double circuit line, when the load builds up. Now, when we are stranding these conductors on towers, we have to take care of certain mechanical considerations also.

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Whenever, we strand these conductors, we find that, there is some sag, which occurs at the middle of the span of the towers. Now, this sag depends on the tension. That these conductors can take. And therefore, we cannot provide very large amount of tension to reduce this sag. Another aspect of the sag is, that this governs, what is the minimum ground clearance. That is available.

Now, this ground clearance has to be there to avoid the voltage stress for to any person or any object, which is on the ground. It also has to be there, if people are any vehicle is moving, if that is also will get stress by this electrical field. And therefore, a certain minimum clearance is required. This is given by certain standards, like for a forward kV line. You need around 12 meters of ground clearance. That is the minimum ground clearance should be around 12 meters.

This clearance again governs, what is going to be the length between the two towers or the span of the conductor. Another mechanical aspect that we need to consider is vibration. Because, when flowing across the conductors, there will be vibrations. And therefore, we that introduces fatigue in the conductors as well as tower. And they become brittle and can get damage.

Therefore, we need to damp these vibrations and for this, it is also done by adjusting the tension. That is one way of doing it and the other is putting vibration dampers in the on the conductors, which absorbs the energy. Also, when these transmission conductors are

subjected to across air flow, then they vibrate. And because of this vibration, one has to see, that the minimum distance between the two phase conductors is always maintained. And this governs, the spacing between the conductors, like for a 400 kV line. The conduct spacing between two phase conductors must be of the order 10 meters or so.

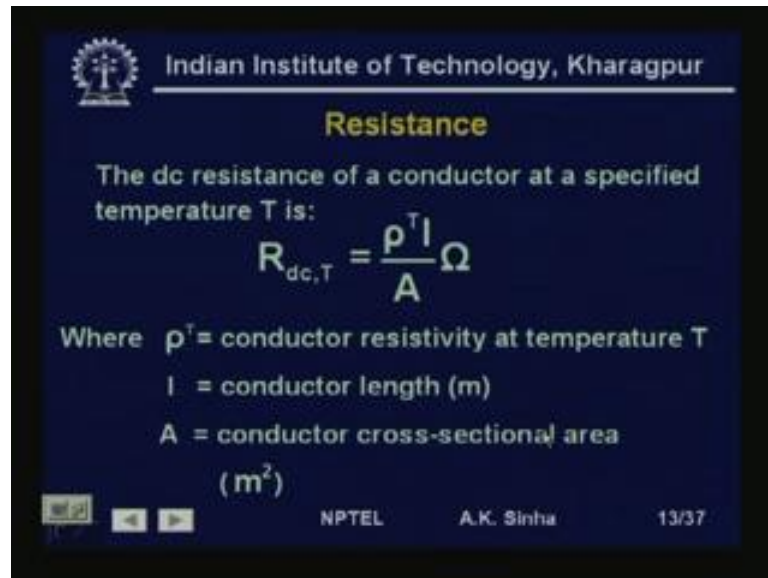
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Let us we will talk about the electrical parameters. Now, we are using conductor, over a conductors and these conductors has certain amount of resistance to the current flowing through them. So, depending on this resistance, there is going to be power loss in these conductors. Therefore, we must find out, what is the resistance of the conductor. Then, we will also talk about inductance.

This comes because, once the current flows in the conductor, it sets of magnetic field. Because, of which there is certain amount of inductance. Since, these conductors are at high voltage. There is voltage difference between the two phase conductors as well as between phase conductors and ground. Therefore, there is capacitance involved between them. So, we will also talk about capacitance. So, these three parameters are the most important parameters for a transmission line electrical modeling.

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Resistance

The dc resistance of a conductor at a specified temperature T is:

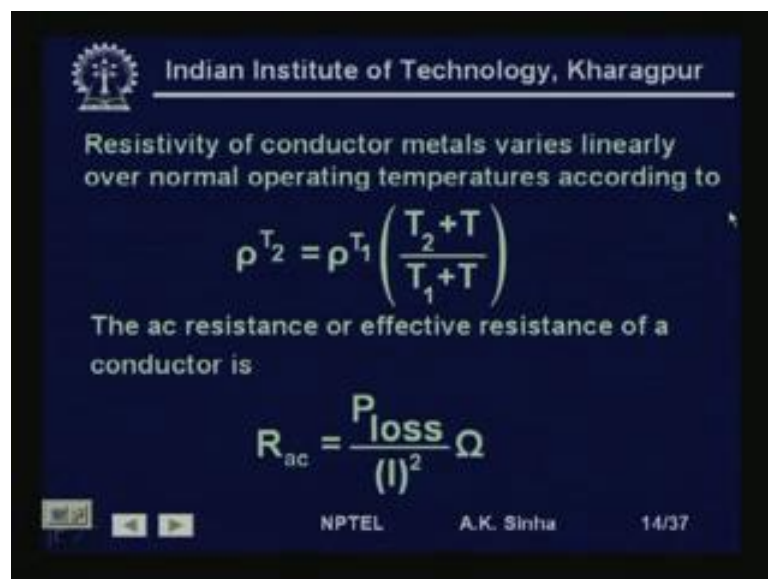
$$R_{dc,T} = \frac{\rho^T l}{A} \Omega$$

Where ρ^T = conductor resistivity at temperature T
 l = conductor length (m)
 A = conductor cross-sectional area (m^2)

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Well, as per as resistances concerned, the dc resistance of a conductor at any temperature is given by R_{dc} is equal to ρl by A . Where, ρ is a function of time, sorry, ρ is a function of temperature. And that is the resistivity of the conductor changes with temperature. If the temperature goes up, resistivity will also increase. l is the length of the conductor and A is the conductor cross sectional area. If we take length in meter and cross sectional area in meter square. And then we get this resistance in ohms. Where, ρT is given in ohm meter.

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Resistivity of conductor metals varies linearly over normal operating temperatures according to

$$\rho^{T_2} = \rho^{T_1} \left(\frac{T_2 + T}{T_1 + T} \right)$$

The ac resistance or effective resistance of a conductor is

$$R_{ac} = \frac{P_{loss}}{(I)^2} \Omega$$

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Now, resistivity of a conductor material varies linearly over normal operating temperature, according to the relation. ρ at temperature T_2 is equal to ρ at temperature T_1 into $T_2 + T$ divided by $T_1 + T$. Where, this T is a constant or a temperature constant. And this value is different for different material as we will see in the table shown.

This is about the d c resistance. But, the ac resistance is somewhat higher than the dc resistance. This is mainly, because at higher frequency, the current density in the conductor is no longer uniform. In fact, higher the frequency less and less current will be flowing through the center of the conductor. And more will be flowing through the outer part of the conductor; this is termed as skin effect. And this reduces the effective conductor cross sectional area. And therefore, the resistance of conductor and a alternating current is somewhat higher.

Normally, we find out the ac resistance from the relationship. That the power loss divided by I^2 , gives us the resistance, ac resistance of the conductor. That is, you can measure the power loss and you can measure the current and from that, we can find out the ac resistance of the conductor. In fact, most of the manufactures, who manufacture these conductors, who they manufacture them in standard sizes. And there are table available, which provide the ac resistance of the conductor per kilo meter length.

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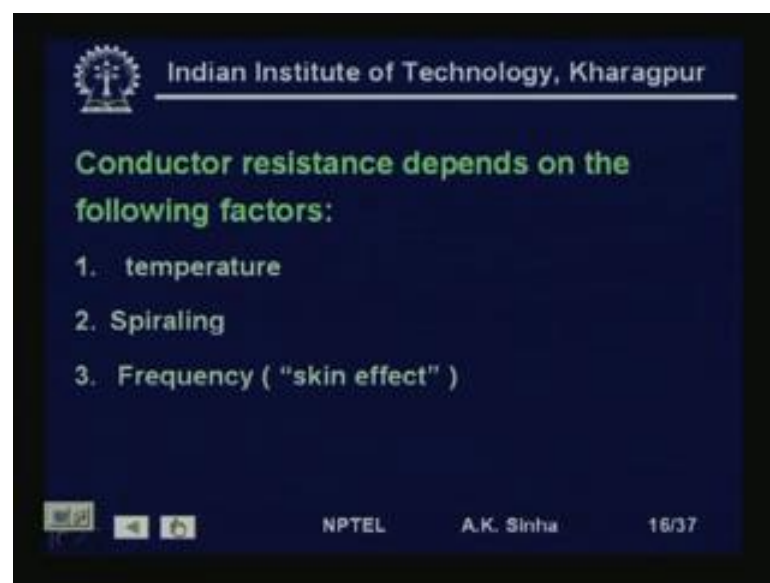
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Material	% Conductivity	20°C		T
		$\Omega \cdot m$	$\Omega \cdot \text{cmil/ft}$	°C
Copper:				
Annealed	100%	1.72	10.37	234.5
Hard-drawn	97.3%	1.77	10.66	241.5
Aluminum:				
Hard-drawn	61%	2.83	17.00	228.1
Iron	17.2%	10	60	180
Silver	108%	1.59	9.6	243

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As I said, this table shows the temperature constant values. It is, if we can see from this table, which provides also the conductivity. The copper, annealed copper has the input as 100 percent conductivity. Then, the hard-drawn aluminum, which is used for a conductor has 61 percent conductivity as compared to annealed copper. For finding out, the resistance, dc resistance of the conductor at different temperatures, we use this temperature constant T. Now, conductor resistance depends on certain factor as we have seen. It depends on temperature, because the resistivity increases, when the temperature increases.

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It also depends on spiraling, because we are using standard conductors. And we are seen, that these conductors has spiral in opposite direction to provide a better binding from the strands. And therefore, this spiraling effectively increases the actual length of the conductors. That is, the length of the conductor and actual length of the strand is not same. The length of strand will be one or two percent more than that of the conductors as such.

And therefore, we have to take care of this larger length and that increases the resistance. That is resistance, due to spiraling will also increase in the same order of one or two person. We have also said that, because of high frequency, the current density does not remain same, because more and more current flows, through the outer part rather than

the core. And therefore, the effective cross sectional area is reduced and this also increases the resistance of the conductor.

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Inductance

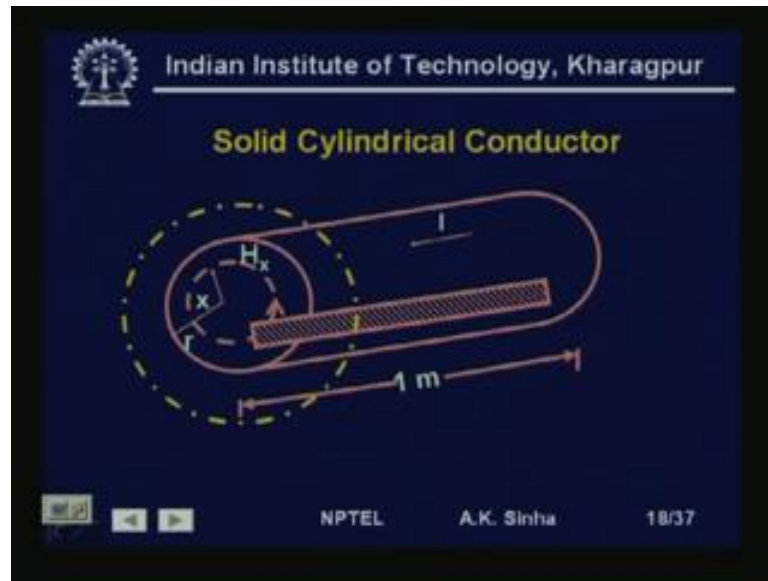
1. Magnetic field intensity H, from Ampere's law
2. Magnetic flux density B ($B = \mu H$)
3. Flux linkages
4. Inductance from flux linkages per ampere
($L = \lambda / I$)

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Next, we will talk about the inductance. Well, for calculating inductance, we need to go through four steps. First is, to find out the magnetic field intensity H using Ampere's law. Second step is, from H, we can find out the flux density, B is equal to mu H. Now, since, we are using aluminum or copper conductors mostly, mu of this is going to be equal to mu 0, because these conductors are non magnetic. So, permeability for these conductors is same as that of air, which is equal to 4 pi into 10 to power minus 7.

Now, flux linkages once we have calculated flux density, we can calculate the flux linkages for the current, which is flowing in that conductor. And finally, we can find out inductance from the flux linkages as inductance is equal to flux linkage per unit current.

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So, let us take first a solid conductor. Here, we have taken one conductor, which is of cross sectional area with a radius r and 1 meter in length. Now, in this conductor is carrying current I . Since, the current cannot flow in just one conductor, there has to be a returned conductor. Here, we are assuming the return conductor is at a very far away distance. So, it does not affect the magnetic field produce by this conductor. That is at a infinite distance from this conductor.

So, that is one assumption that we are making. Another assumption that we make here in finding inductance is that the current density inside the conductor is uniform. Though, we have seen that, the current density at higher frequencies is not uniform. But, at lower frequency is a 50 hertz or 60 hertz, it is more or less uniform. So, this assumption is generally made.

Now, we can see that, for this conductor, since the current is flowing inside the conductor. The current will produce magnetic field inside the conductor also. And because of the current flowing, there will be magnetic field outside the conductor also. Therefore, for calculating the inductance, we will have to find out the flux linkages, for the magnetic field, which is inside the conductor. And flux linkages for the magnetic field, outside the conductor. That is, we need to do two parts and add them up to get the final flux linkage.

So, first we will take the internal flux linkages. Now, here, what we have done is, we have taken circular contour at a distance x , from the center. And let H_x be the intensity of the magnetic field at any point on this contour. Since, this H_x will be, this contour is concentric, it is uniform with respect to this conductor. This H_x will also be same, everywhere on this contour and its direction will be tangential to this contour.

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Internal Flux Linkage

$$\oint H_{\tan} dl = I_{\text{enclosed}}$$

$$H_x (2\pi x) = I_x \quad \text{for } x < r$$

$$H_x = \frac{I_x}{2\pi x} \text{ A/m}$$

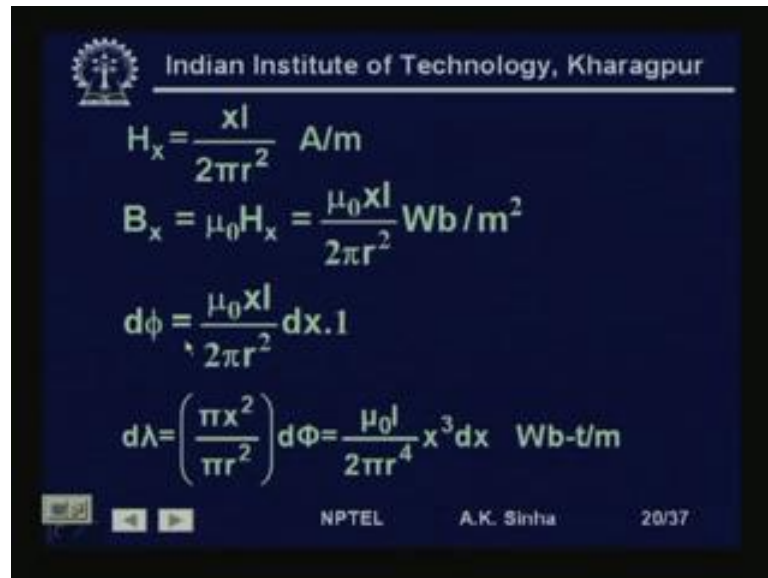
$$I_x = \left(\frac{x}{r}\right)^2 I \quad \text{for } x < r$$

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Therefore, from Ampere's law, we know that tangential component of H into dl integral, over a close contour is equal to the current in closed. So, using this relationship for this contour, we get H_x into twice πx , which is the contour length, when we integrated. So, H_x , x the tangential component of H is H_x and integral dl over the contour is twice πx . This is equal to I_x , for x less than r . From this relationship, we can calculate H_x is equal to I_x by twice πx , ampere per meter.

Now, this H_x field intensity H_x is enclosing current I_x . And what is this current I_x ? This current I_x , since we are assumed uniform current density throughout the conductor is x by r whole square into I fraction of the total current I .

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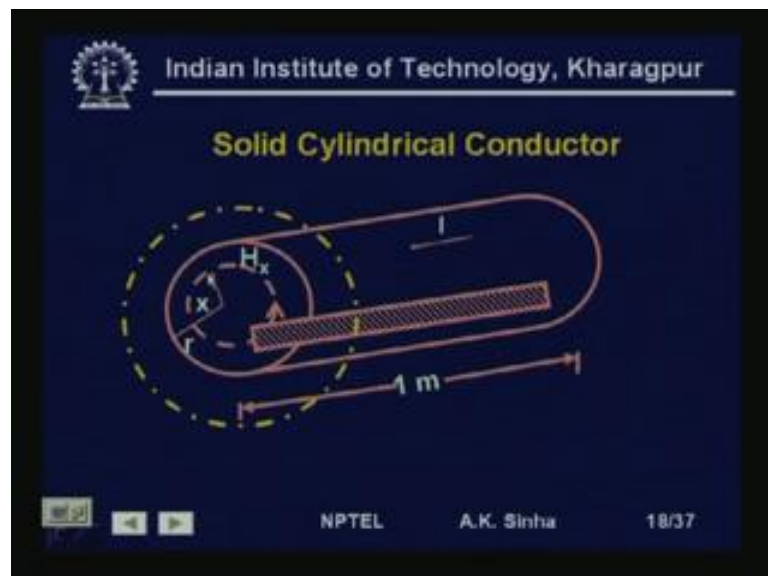
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$$H_x = \frac{xI}{2\pi r^2} \text{ A/m}$$
$$B_x = \mu_0 H_x = \frac{\mu_0 xI}{2\pi r^2} \text{ Wb/m}^2$$
$$d\phi = \frac{\mu_0 xI}{2\pi r^2} dx \cdot l$$
$$d\lambda = \left(\frac{\pi x^2}{\pi r^2} \right) d\phi = \frac{\mu_0 I}{2\pi r^4} x^3 dx \text{ Wb-t/m}$$

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Therefore, we can write H_x is equal to x into I by twice πr square. Because, for substituting for I_x , x by r whole square into I and in this position, we will get H_x is equal to $x I$ by twice πr square ampere per meter. And from this field intensity, we can find out the flux density B_x , which will be equal to $\mu_0 H_x$, as we are using non magnetic conducting material. So, $\mu_0 H_x$, this is equal to $\mu_0 x$ into I by twice πr square Webber per meter square.

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Now, if we look at this strip, which is 1 meter long and of thickness dx , at this point x from the center. How much flux will be passing through this strip? Since, the flux lines are concentric. And we know the flux density the area, which will be involved for the flux lines will be $B \times 1$. Therefore, $d\phi$ is equal to $\mu_0 I$ by twice πr^2 into dx , which is the area.

And this flux is linking how much current? This flux is linking only the current which is enclosed by x . And that current is equal to how much? That is πx^2 by πr^2 fraction of the total current. Therefore, we have the flux linkage is equal to πx^2 by πr^2 into $d\phi$. This is equal to $\mu_0 I$ by twice πr^4 $x^3 dx$ Weber turns per meter.

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$$\lambda_{int} = \int_0^r d\lambda = \frac{\mu_0 I}{2\pi r^4} \int_0^r x^3 dx$$

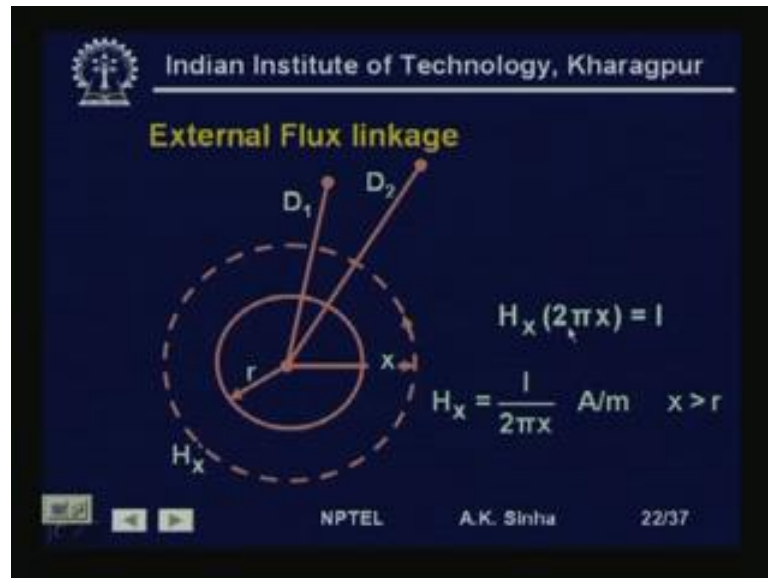
$$= \frac{\mu_0 I}{8\pi} = \frac{1}{2} \times 10^{-7} I \text{ Wb-t/m}$$

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Now, if we integrate from the center to the tip of the conductor. That is from 0 to r , this flux linkage in this strip, we will get the total internal flux linkage of the conductor for current flowing inside it. And this is equal to $\mu_0 I$ by twice πr^4 integral from 0 to r of $x^3 dx$, which finally comes out to be half into $10^{-7} I$ Weber turns per meter.

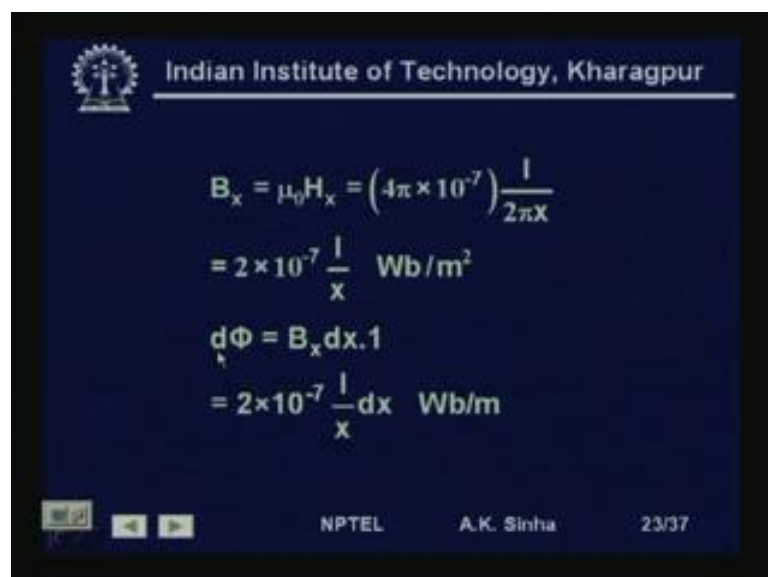
So, the internal flux linkage is constant that is half into 10^{-7} into I . So, depends on the current, which is flowing and it has nothing to do with the radius of the conductor. Now, next we have to find out the flux linkages outside the conductor.

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Now, again for the same conductor, we have taken a point, which is at a distance x from the center of the conductor, where x is greater than r . Now, again we can find out take a concentric contour and using Ampere's law, we will get H_x into twice pi x is equal to I . Because, now, this contour is enclosing what current, all the total current, which is flowing in this conductor? Therefore, H_x into twice pi x is equal to I , from there we will get H_x is equal to I by twice x ampere per meter for x greater than r .

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Again, we can find out the flux density at that point, this will be equal to $\mu_0 H x$, which is equal to $4\pi \times 10^{-7} I$ by twice πx . This will be equal to $2 \times 10^{-7} I$ by x Weber per meter square. Now, once, again we take a small strip of a 1 meters length and $d x$ thickness. Then, we can find out the flux, through this strip and this will be equal to $B x$ into $d x$ into 1. So, again substituting the value, this comes out to be $2 \times 10^{-7} I$ by x into $d x$ Weber per meter.

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$$\lambda_{12} = \int_{D_1}^{D_2} d\lambda = 2 \times 10^{-7} I \int_{D_1}^{D_2} \frac{dx}{x}$$

$$= 2 \times 10^{-7} I \ln\left(\frac{D_2}{D_1}\right) \text{ Wb-t/m}$$

$$\lambda_p = \frac{1}{2} \times 10^{-7} I + 2 \times 10^{-7} I \ln \frac{D}{r}$$

$$\frac{1}{2} = 2 \ln e^{1/4}$$

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Now, if we want we can find out the total flux linkages between any two points D_1 and D_2 . From the center of the conductor, which we can write as λ_1 to is equal to integral D_1 to D_2 of $d\lambda$, which will be equal to $2 \times 10^{-7} I$ integral D_1 to D_2 dx by x . And which after integrating comes out to be $2 \times 10^{-7} I \log n \frac{D_2}{D_1}$ Weber therms per meter.

So, if we want to find out the flux linkage, total flux linkage up to any point P , for this conductor carrying current I . Then, we need to add the flux linkages internal to the conductor and the flux linkage out to outside of the conductor. From the conductor surface up to a distance D of this point p , from the center of the conductor.

This will be equal to half into $10^{-7} I$, which is the internal flux linkage. Plus $2 \times 10^{-7} I \log n \frac{D}{r}$, which is the external flux linkage up to a distance D from the conductor.

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$$\begin{aligned}\lambda_p &= 2 \times 10^{-7} I (\ln e^{1/4} + \ln \frac{D}{r}) \\ &= 2 \times 10^{-7} I \ln \frac{D}{e^{-1/4} r} \\ &= 2 \times 10^{-7} I \ln \frac{D}{r'} \quad \text{Wb-t/m} \\ r' &= e^{-1/4} r = 0.7788 r\end{aligned}$$
$$L_p = \frac{\lambda_p}{I} = 2 \times 10^{-7} \ln \left(\frac{D}{r'} \right) \text{ H/m}$$

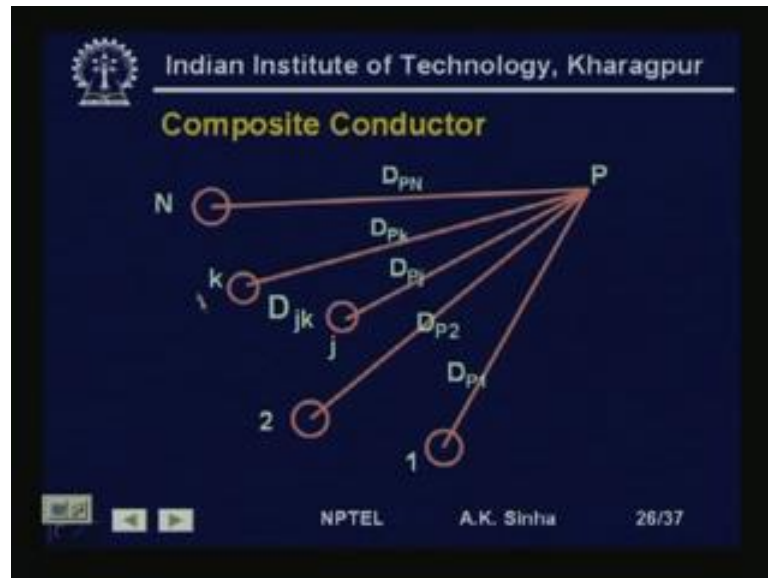
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Now, using half is equal to $2 \log n e$ to the power 1 by 4 . Substituting this, we get the flux linkage λ_p is equal to $2 \times 10^{-7} I \log n e$ to the power 1 by 4 plus $\log n D$ by r , which can be written as $2 \times 10^{-7} I \log n D$ by e to the power minus 1 by 4 r . This we can write as $2 \times 10^{-7} I \log n D$ by dash. Where, r dash is e to the power minus 1 by 4 into r , which is equal to $0.7788 r$.

That is, what we are seeing is, because of the internal flux linkages, the effective radius of the conductor is now r dash or which is $0.7788 r$. That is the effective radius has got somewhat produced as per as the flux linkages are concerned. That is, if we are not considering any internal flux linkage, then we have to use the relationship goes use as r dash, which is the effective radius in that case.

So, therefore, once we have calculated the flux linkages up to that point P . Then, we can also calculate the inductance, due to flux linkage is up to that point P . That will be λ_p by I . And this is equal to $2 \times 10^{-7} \log n D$ by r dash Henry's per meter.

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Well, we have now been able to find out the inductance of a single conductor with a return, which is very far away from it. That is not a normal practical case. Normally, we will have conductors, which will be nearby only and so we can now take up a case, where we have a system, where we have large number of conductors. Here we have assumed n number of conductors is forming this system in which current is flowing.

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The slide displays the following equations:

$$I_1 + I_2 + \dots + I_N = \sum_{j=1}^N I_j = 0$$
$$\lambda_{kPk} = 2 \times 10^{-7} I_k \ln \frac{D_{Pk}}{r_k'}$$

The slide is part of a presentation from the Indian Institute of Technology, Kharagpur, titled "Composite Conductor". It includes the NPTEL logo, the name A.K. Sinha, and the slide number 27/37.

Now, since these conductors form the total number of conductors in the system, one thing that we will see is that, sum of all the currents in these conductor must be equal to

0. That is some conductors are carrying current in one direction and some conductors are acting as the return conductors. So, this is, what we will get, that is $I_1 + I_2 + \dots + I_N$ or $\sum_{j=1}^N I_j$ will be equal to 0. That is sum of the all the currents in the conductors is equal to 0.

Now, using the earlier relationship, we can find out the flux linkage for the current flowing in for the conductor k up to a point P, due to current flowing in conductors k, which will be given by $2 \times 10^{-7} I_k \log \frac{D_{Pk}}{r}$. That is the distance of the conductor k from P divided by r dash, which is the radius of this conductor. Similarly, if we want to find out, the flux linkage with this conductor, due to current flows in some other conductor. We can write down this similar relationship, except that, instead of this r dash, we will have to write D_{Pj} , sorry D_{kj} .

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$$\lambda_{kP} = \lambda_{kP1} + \lambda_{kP2} + \dots + \lambda_{kPN}$$

$$= 2 \times 10^{-7} \sum_{j=1}^N I_j \ln \frac{D_{Pj}}{D_{kj}}$$

$$D_{kk} = r_k$$

$$\lambda_{kP} = 2 \times 10^{-7} \sum_{j=1}^N I_j \ln \frac{1}{D_{kj}} + 2 \times 10^{-7} \sum_{j=1}^N I_j \ln D_{Pj}$$

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Therefore, if we use that, we can find out the total flux linkage up to point P, for the conductor k, will be equal to λ_{kP} . That is flux linkage of conductor k up to point P, due to current flowing in conductor one and so on, for all the currents flowing in different, all the N conductors. Therefore, this is equal to $2 \times 10^{-7} \sum_{j=1}^N I_j \log \frac{D_{Pj}}{D_{kj}}$.

Now, here, we have used this relationship D_{kj} and for the same conductor, that is D_{kk} , basically it is own distance will be equal to r dash. Therefore, lambda flux linkage of conductor k up to point P can be written as $2 \times 10^{-7} \sum_{j=1}^N I_j \log \frac{1}{D_{kj}} + 2 \times 10^{-7} \sum_{j=1}^N I_j \log D_{Pj}$.

by D_{kj}^{+2} into $10^{-7} \sum_{j=1}^N I_j \log \frac{1}{D_{kj}}$. And what we have done is, this term we have separated into two parts as two summations. One is $I_j \log \frac{1}{D_{kj}}$ plus $I_N \log D_{PN}$.

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$$\lambda_{kP} = 2 \times 10^{-7} \left[\sum_{j=1}^N I_j \ln \frac{1}{D_{kj}} + \sum_{j=1}^{N-1} I_j \ln D_{pj} + I_N \ln D_{PN} \right]$$

$$I_N = -(I_1 + I_2 + \dots + I_{N-1}) = -\sum_{j=1}^{N-1} I_j$$

$$\lambda_{kP} = 2 \times 10^{-7} \left[\sum_{j=1}^N I_j \ln \frac{1}{D_{kj}} + \sum_{j=1}^{N-1} I_j \ln D_{pj} - \sum_{j=1}^{N-1} I_j \ln D_{PN} \right]$$

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We will see why we do this. Therefore, λ_{kP} is equal to $2 \times 10^{-7} \sum_{j=1}^N I_j \log \frac{1}{D_{kj}}$ plus. Now, the term that we had used here $D_{PN} \log \frac{1}{D_{kj}}$ plus for the N th conductor, we are separating it a plus $I_N \log \frac{1}{D_{PN}}$. Now, we know, I_N will be equal to minus of the sum of $N-1$ conductor, because the sum of all the currents is equal to 0. That is I_N is equal to minus of $\sum_{j=1}^{N-1} I_j$.

So, substituting this, here we get λ_{kP} is equal to $2 \times 10^{-7} \sum_{j=1}^N I_j \log \frac{1}{D_{kj}}$. This plus should not be their plus $\sum_{j=1}^{N-1} I_j \log \frac{1}{D_{pj}}$ minus for this term $\sum_{j=1}^{N-1} I_j \log D_{PN}$.

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$$= 2 \times 10^{-7} \left[\sum_{j=1}^N I_j \ln \frac{1}{D_{kj}} + \sum_{j=1}^{N-1} I_j \ln \frac{D_{pj}}{D_{PM}} \right]$$

$P \rightarrow \infty$

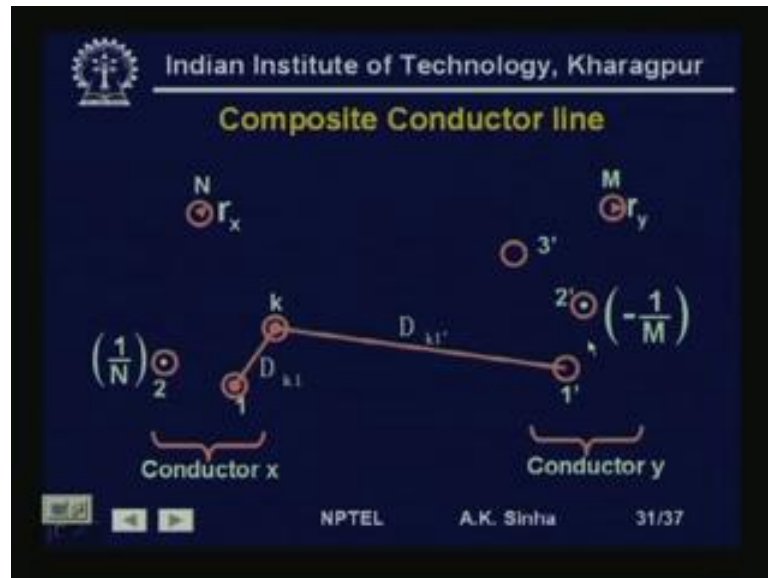
$$\lambda_k = 2 \times 10^{-7} \sum_{j=1}^N I_j \ln \frac{1}{D_{kj}} \quad \text{Wb-t/m}$$

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So, minus $\sum_{j=1}^{N-1} I_j \ln \frac{D_{pj}}{D_{PM}}$ is equal to $(N-1) \log \frac{D_{PN}}{D_{PM}}$, which can be written as $2 \times 10^{-7} \sum_{j=1}^{N-1} I_j \ln \frac{1}{D_{kj}} + \sum_{j=1}^{N-1} I_j \ln \frac{D_{pj}}{D_{PM}}$, sorry, this should be N . This $\frac{D_{PN}}{D_{PM}}$ term is coming. So, that will be $\frac{D_{PN}}{D_{PM}}$, this is $\frac{D_{PN}}{D_{PM}}$. Now, if we take this point P very, very far away, then what happens is this $\frac{D_{pj}}{D_{PM}}$ and $\frac{D_{PN}}{D_{PM}}$ will be almost same.

And therefore, this will $\frac{D_{pj}}{D_{PM}}$ will become equal to 1 and the $\log \frac{D_{pj}}{D_{PM}}$ will be equal to 0. And therefore, finally, we will get the flux linkages, total flux linkages for the conductor k equal to $2 \times 10^{-7} \sum_{j=1}^N I_j \ln \frac{1}{D_{kj}}$. This is the final expression that we get and this is a very important relationship for finding out the total flux linkage for any conductor carrying current in a system of conductors carrying current.

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Now, we are try to use this, for a single phase transmission line, which is consisting of composite conductors. So, we have one conductor x and return conductor is y. Conductor x is consisting of total N number for conductors each with a radius r_x . That is same radius conductors, but they are a total N number of conductors. And each conductor carries $\frac{1}{N}$ of the total current. So, each conductor is the current is I, each conductor is carrying $\frac{I}{N}$.

Similarly, conductor y is consisting of total M number of sub conductors and each of these conductors has sub conductors have a radius r_y . And they are carrying $\frac{1}{M}$ part of the total current coming through. So, if the current is flowing is I, then the return conductor will have currents $\frac{I}{M}$ in each of these sub conductors.

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$$\Phi_k = 2 \times 10^{-7} \left[\frac{I}{N} \sum_{m=1}^N \ln \frac{1}{D_{km}} - \frac{I}{M} \sum_{m=1}^M \ln \frac{1}{D_{km}} \right]$$

$$\lambda_k = \frac{\Phi_k}{N} = 2 \times 10^{-7} I \left[\frac{1}{N^2} \sum_{m=1}^N \ln \frac{1}{D_{km}} - \frac{1}{NM} \sum_{m=1}^M \ln \frac{1}{D_{km}} \right]$$

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So, now we can find out the flux for any conductor k in this system. In the conductor n is sub conductor k in the system of conductor x. This will be given by $2 \times 10^{-7} I \left[\frac{1}{N} \sum_{m=1}^N \ln \frac{1}{D_{km}} - \frac{1}{M} \sum_{m=1}^M \ln \frac{1}{D_{km}} \right]$. That is using this relationship for this set of conductors. That is, what we are finding out for this conductor.

We are finding out the flux linkage, due to current in all the conductors in x as well as all the conductor sub conductors in y. So, we are adding all those fluxes. So, the total flux, which is linking this conductor k will be given by $I \sum_{m=1}^N \ln \frac{1}{D_{km}}$, where, D_{km} is the distance of all the conductors from the conductor k.

Similarly, the current I/M is flowing in each sub conductor. So, the flux linkage is or the flux produce by them, which will be linking this conductor will be given by $I/M \sum_{m=1}^M \ln \frac{1}{D_{km}}$. Now, what is the flux linkage of this conductor? The current flowing in this conductor is I/N . So, total flux linkage of this conductor will be $\frac{1}{N} \left[I \sum_{m=1}^N \ln \frac{1}{D_{km}} - \frac{I}{M} \sum_{m=1}^M \ln \frac{1}{D_{km}} \right]$. So, Φ_k/N is the flux linkage of this conductor, this will be equal to $2 \times 10^{-7} I \left[\frac{1}{N^2} \sum_{m=1}^N \ln \frac{1}{D_{km}} - \frac{1}{NM} \sum_{m=1}^M \ln \frac{1}{D_{km}} \right]$. And when we divide this expression by N we get $\frac{1}{N^2} \sum_{m=1}^N \ln \frac{1}{D_{km}} - \frac{1}{NM} \sum_{m=1}^M \ln \frac{1}{D_{km}}$. So, $\frac{1}{N^2} \sum_{m=1}^N \ln \frac{1}{D_{km}} - \frac{1}{NM} \sum_{m=1}^M \ln \frac{1}{D_{km}}$.

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$$\lambda_x = \sum_{k=1}^N \lambda_k$$

$$= 2 \times 10^{-7} I \left[\sum_{k=1}^N \frac{1}{N^2} \sum_{m=1}^N \ln \frac{1}{D_{km}} + \frac{1}{NM} \sum_{m=1}^M \ln \frac{1}{D_{km}} \right]$$

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Now, what will be the total flux linkage of the conductor x, that will be the summation of all the flux linkages of each sub conductor in that system of conductor x. Therefore, lambda x is equal to summation K is equal to 1 to N lambda K, which will be equal to 2 into 10 to power minus 7 I, sigma K is equal to 1 to N of this terms. That is 1 by N square sigma log n 1 by D k m minus 1 by NM sigma log n 1 by D k m. This is putting the expression lambda K here. And the summation is from is equal to 1 to N, because we are talking for conductor x.

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$$\lambda_x = 2 \times 10^{-7} I \ln \prod_{k=1}^N \frac{\left(\prod_{m=1}^M D_{km} \right)^{1/NM}}{\left(\prod_{m=1}^N D_{km} \right)^{1/N^2}}$$

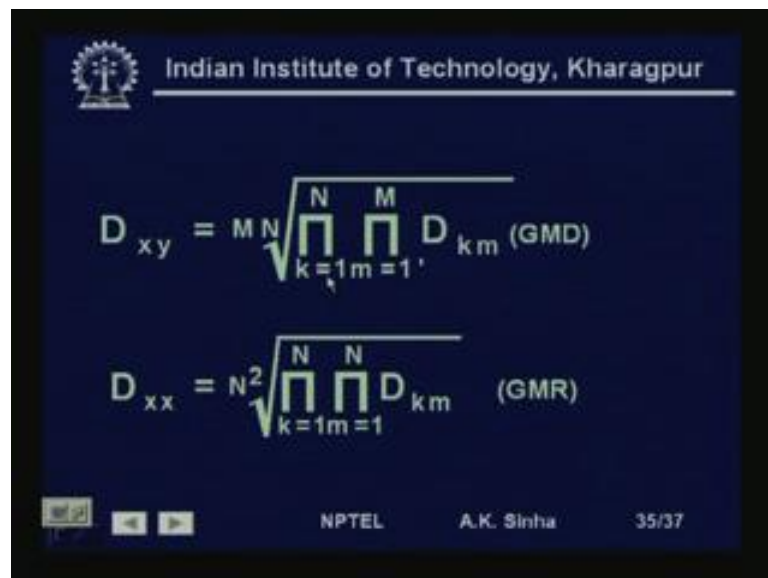
$$L_x = 2 \times 10^{-7} \ln \frac{D_{xy}}{D_{xx}}$$

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So, λx will be equal to $2 \times 10^{-7} \ln n$. Now, here, what we have done is, summation of $\ln n$ by D_{km} , can also be written as $\ln n$ multiplication of 1 by D_{km} . So, we have used that relationship and therefore, we have written $2 \times 10^{-7} \ln n$ has been taken out side. Therefore, it is multiplication K is equal to 1 to N into multiplication of m is equal to 1 dash to M D_{km} divided by multiplication m is equal to 1 to N of D_{km} .

This will be to the power 1 by N square and this will be to the power 1 by $N M$, because we had that 1 by $N M$ and 1 by N square. Therefore, we can calculate L_x , which will be equal to $2 \times 10^{-7} \ln D_{xy}$ by D_{xx} . Where, this term we are writing as D_{xy} and this terms, we will we are writing as D_{xx} .

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$$D_{xy} = \sqrt[MN]{\prod_{k=1}^N \prod_{m=1}^M D_{km}} \quad (\text{GMD})$$

$$D_{xx} = \sqrt{N^2 \prod_{k=1}^N \prod_{m=1}^N D_{km}} \quad (\text{GMR})$$

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As we will show D_{xy} is equal to $M N$ th root of sigma's product k is equal to 1 to N and product m is equal to 1 dash to M D_{km} . This total term D_{xy} is called the $G M D$ or Geometric Mean Distance between the conductor x and y . Similarly, the term D_{xx} , that is the self $G M D$ or Geometric Mean Radius is equal to the N square th root of product k is equal to 1 to N into product m is equal to 1 to N of D_{km} .

That is product of each sub conductor's distance to all the other distances. And this is done for all the conductors, so you are getting N square distances. And so we have the N square th root of this.

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$$L_y = 2 \times 10^{-7} \ln \frac{D_{xy}}{D_{yy}} \quad \text{H/m per conductor}$$
$$D_{yy} = M^2 \sqrt{\prod_{k=1}^M \prod_{m=1}^M D_{km}} \quad (\text{GMR})$$
$$L = L_x + L_y \quad \text{H/m per circuit}$$

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Similarly, we can calculate the inductance of conductor system y, which will be equal to $2 \times 10^{-7} \ln \frac{D_{xy}}{D_{yy}}$ Henry's per meter per conductor. And here, D_{yy} will be equal to M^2 square root of product k is equal to 1 dash to M and multiplied by product m is equal to 1 dash to M into D_{km} . That is the product of all the distances from each sub conductor to other sub conductor for all the sub conductors and conductor y. And the circuit inductance will be the inductance of conductor x plus the inductance of conductor y. Because, that is the total circuit, that we have consisting of both the conductors in which the current is flowing.

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Answer the following questions

- What type of conductors are used for overhead transmission lines ?
- How does conductor diameter affect inductance of transmission line ?
- Why bundled Conductors are used in Extra High Voltage lines ?

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So, once, we have calculated the inductance. Now, we will close here and before, we close, I would you to answer the follow questions. First is, what type of conductors is used for overhead transmission lines. Second question is, how conductor diameter affects inductance of transmission line. And the third question is, why bundled conductors are used in extra high voltage lines. So, I hope after going through this lesson, you should be able to answer these questions.

Thank you.

Power System Analysis

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Department of Electrical Engineering

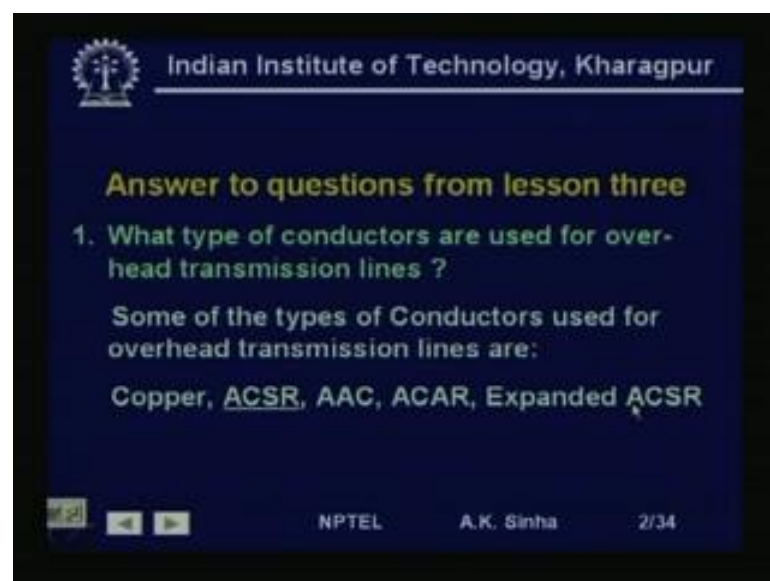
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Lecture - 04

Inductance Calculation

Welcome to lesson 4, on power system analysis. Before, we start this lesson 4, first I would like to take up the questions, that we asked in lessons 3.

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The slide features the IIT Kharagpur logo and name at the top. The main text is on a dark blue background with yellow and white text. It lists the question number and type, followed by the answer listing conductor types: Copper, ACSR, AAC, ACAR, and Expanded ACSR. The bottom of the slide contains navigation icons, the NPTEL logo, the name A.K. Sinha, and the slide number 2/34.

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Answer to questions from lesson three

1. What type of conductors are used for overhead transmission lines ?

Some of the types of Conductors used for overhead transmission lines are:

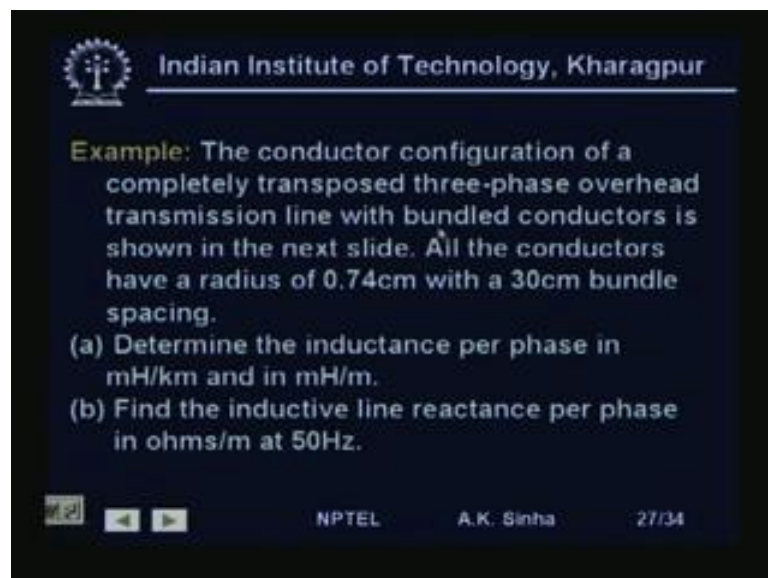
Copper, ACSR, AAC, ACAR, Expanded ACSR

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First question was, what are the types of conductors used for overhead transmission line? Well, some of the types of conductors used for overhead transmission lines are copper conductors, which are very rarely used nowadays. ACSR, that is aluminium conductors steel reinforced conductors. Then, all aluminum conductors are ACAR or expanded ACSR conductors, which are used.

As I have already said in lesson 3. Sometimes, we want to increase the effective radius of the conductor and for that, we used expanded ACSR conductor in EHV transmission lines. Now, Let us take an example for finding out the inductance of a three phase transmission system.

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Example: The conductor configuration of a completely transposed three-phase overhead transmission line with bundled conductors is shown in the next slide. All the conductors have a radius of 0.74cm with a 30cm bundle spacing.

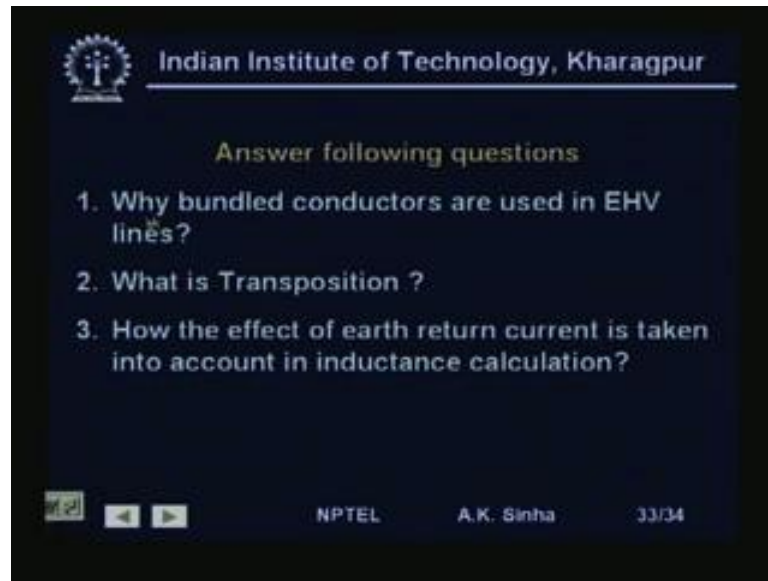
(a) Determine the inductance per phase in mH/km and in mH/m.

(b) Find the inductive line reactance per phase in ohms/m at 50Hz.

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The example, that we are taking is for, the conductor configuration of a completely transposed three phase overhead transmission line, with bundle conductor is shown. All the conductors have a radius of 0.74 centimeter with a 30 centimeter bundle space.

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The slide features the IIT Kharagpur logo and name at the top. Below this, the text 'Answer following questions' is centered. Three numbered questions are listed: 1. Why bundled conductors are used in EHV lines? 2. What is Transposition? 3. How the effect of earth return current is taken into account in inductance calculation? At the bottom, there are navigation icons (back, forward, search), the text 'NPTEL A.K. Sinha', and the slide number '33/34'.

First is, why bundle conductors are used in EHV lines. Second question is, what is transposition. And third question is, how the effect of earth return current is taken into account in inductance calculation for a three phase line with ground return system. Especially, when the system is carrying, unbalanced current, so with this, we finish this lesson.

Thank you very much.

We will meet again for lesson 5. In which, we will talk about calculating the capacitance of the transmission line.

Thank you.