Power System Analysis
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Lecture - 26
Symmetrical Component Analysis

Welcome to lesson 26, on Power System Analysis. In this lesson, we will learn about Symmetrical Component Analysis. If you recall in our previous lesson, we talked about, how to analyze faults on power systems. We will be continuing with that, as we have seen, most of the faults in power systems are single phase to ground faults. And say, line to line faults or double line to ground faults.

All these faults are basically asymmetrical faults, because they create an unbalance in the voltage and the currents in the three phase system. To analyze faults, which are unbalanced or asymmetrical? We need help of the symmetrical component transformation. In this lesson, we will discuss about how to use this symmetrical component transformation for analyzing asymmetrical faults.

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So, in this lesson, first we will start with an introduction, what symmetrical component transformation is. Then, we will go into the sequence network for power system components, how we develop the sequence networks. And we will also, talk about the relationship between phase and sequence components.

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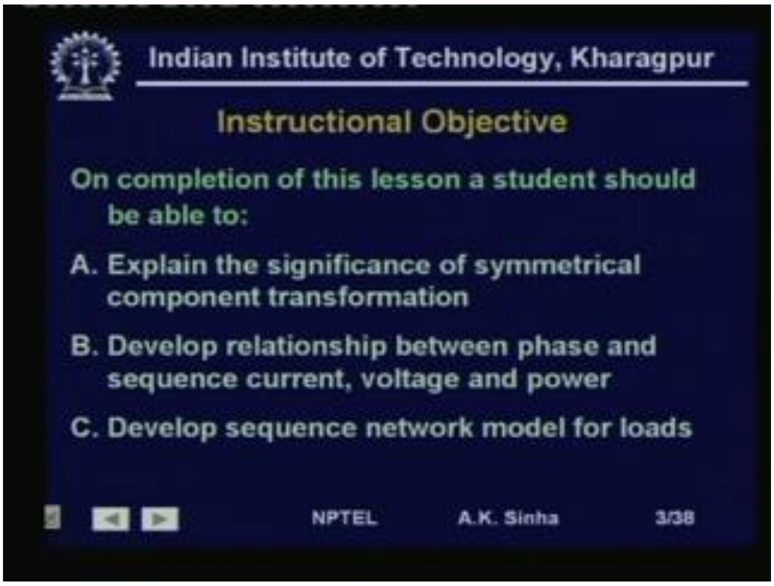

The instructional objective for this lesson is, that after this lesson, you should be able to explain the significance of symmetrical components transformation. You should be able to develop relationship between phase and sequence current voltage and power quantities. And you should be able to develop sequence network for load models. We will take up the other power system component models in the next lesson.

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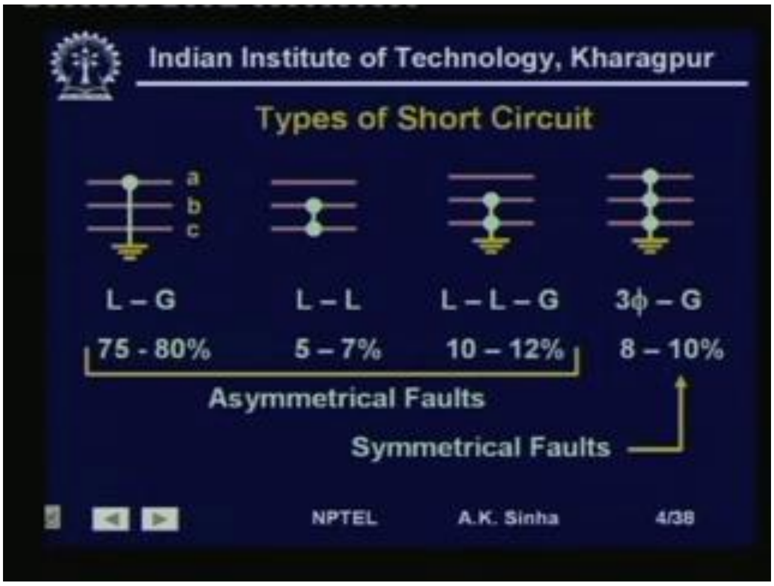

Well, as I said earlier, the different kinds of faults. That we have in power system are single line to ground fault. Single line to ground fault, line to line fault, line to line to ground fault are LLG fault and three phase to ground fault or three phase faults. Out of these types of faults, as we have indicated here, line to ground fault, line to line fault and line to line to ground fault or basically asymmetrical faults. As we said, these create asymmetry in the currents and voltages in the three phase of the three phase system.

Except for this three phase fault, which is asymmetrical fault, because the currents and voltages in all the three phases are going to be same or asymmetrical same in magnitude and which a phase displacements of 120 degrees asymmetrical with each other. So, to analyze faults, which are asymmetrical? We need to work on three phase phases. That is, in case of symmetrical faults we used a single phase basis of analysis, because all the three phases were identical.

That is, if you come to know of the current and voltages in 1 phase. You can always find out the currents and voltages in the other phases. Because, they will be of equal magnitude and displaced from this phase current and voltage by 120 degrees. The situation is not same, for these other type of faults and we need to represent the three phase system in it is completeness. That is all the three phases have to be represented.

Now, this creates complications, because there are mutual impedances involved and we have to work with 9 quantities as such. In order to make the analysis of three phase system and unbalance condition, we normally take help of, what we call symmetrical component transformations. So, now, we will see how this is done.

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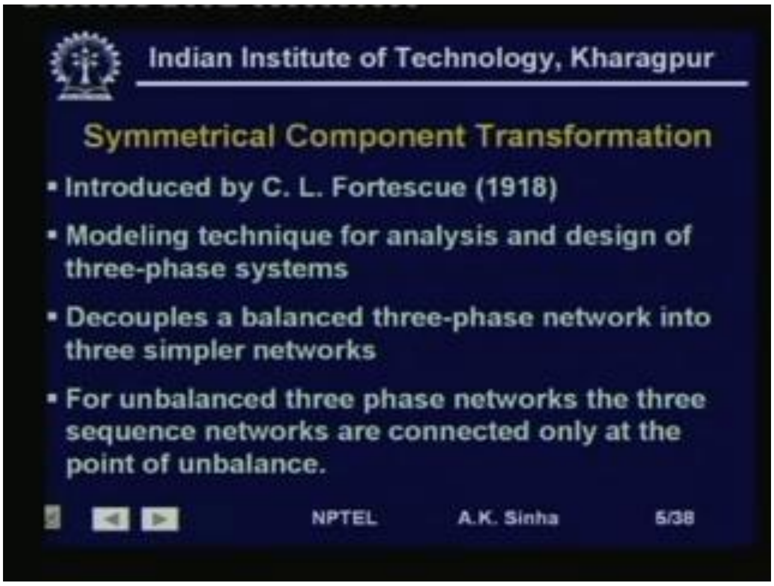

Well, symmetrical component transformation was first introduced by C.L. Fortes cue, in 1918 of what he suggested was, we can always transform these three phase quantities into three different sets of quantities, which are symmetrical. We will talk about that, what how does it help, when symmetrical component transformation as a modeling technique for analysis and design of three phase system, as I said earlier.

It the advantage of this transformation is, that it decouples a balanced three phase network into three simpler networks. Means, these three simple networks are not connected to each other. They are separated from each other and therefore, there are no mutual's involved here. For unbalanced three phase networks, the three sequence networks are connected, only at the point of unbalance. Because, we have in a power system most of the elements are designed to be asymmetrical three phase system.

All the generators are designed to work as a balanced three phase system, all the motors transformers all are designed to works as a balanced symmetrical, three phase component. Transmission lines, as we have learnt during the modeling of transmission lines are transposed, most of the times, especially for long distance lines. So, that, the three phase transmission line also becomes asymmetrical three phase system.

Therefore, as we have seen the three sequence symmetrical components, decouples a balanced three phase networks. So, for all these components, you are going to get decoupled sequence networks Only, at the point of fault are unbalance, there is going to be a coupling between these three sequence networks.

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Because of this particular property, it is a very powerful tool for analyzing three phase system. It makes the analysis, much more simpler to do. Another advantage is, it reveals the complicated phenomena during unbalanced operation in simple terms. That is in terms of sequence currents and voltages. And we can understand the unbalance phenomena in very simple terms of the sequence current and voltages.

However, this has one disadvantage; that you need to first transform the three phase system quantities into symmetrical component quantities. Once, you have done the analysis and got the symmetrical component values for currents voltages, power, whatever it is that we are interested in. Finally, we will have to again do an inverse transformation, because finally we would be interested in finding out the currents and the voltages in the actual system.

So, in terms of phase currents and voltages, so we need to do a inverse transformation to get this. That is sequence network results; have to be superposed to obtain the three phase network results.

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Now, what is this symmetrical component transformation? Well, as defined, it is defined as an unbalanced set of n phasors can be resolved into n sets of balanced phasors, which we call symmetrical components. The n phasors of each set of components are equal in magnitude and angle; angles between adjacent phasors of the set are equal. That is if we have a set of n phasors, which are unbalanced n phasors. Then we can resolve them into n sets of phasors of n phasors.

Basically n sets of n phasors, which are symmetrical with respect to each other. And this, if we try to use for our three phase system, can be said as unbalanced phasors. For a three phase system, can be resolved into three balanced systems of phasors, which we call as the positive negative and zero sequence systems. So, what we have done is, basically three phase quantities are now being converted into three different sets of quantities, which themselves are three phase quantities. So, one would think in terms of that, what we are doing is from three quantities, we are going to 9 quantities. But, actually this is not the case, as we will see later.

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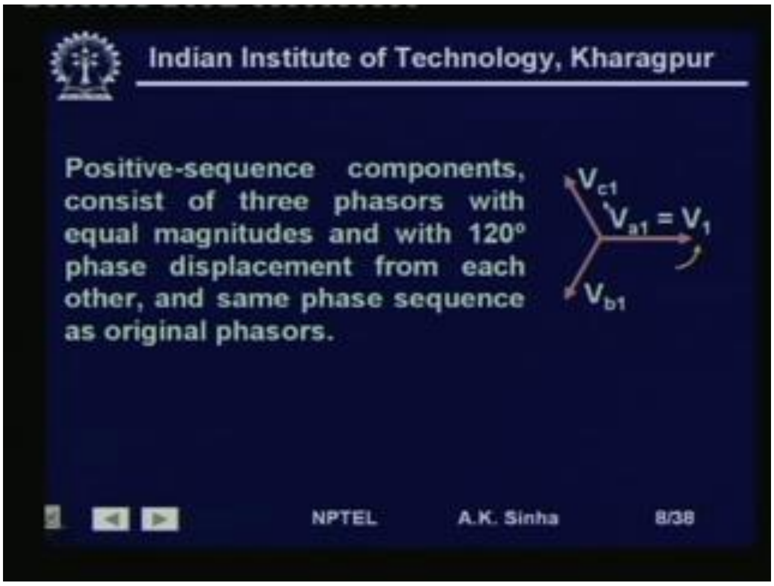

What are these sequence quantities, positive sequence components, consist of three phasors, with equal magnitudes and with 120 degree phase displacement from each other. This is shown here. That is, we have a phasor, which is a balance set, a symmetrical set, where we have a three phase quantities V a 1, V b 1, V c 2. And these are 120 degree out of phase, this is what happens in a balance three phase system. So, these positive sequence components are basically same, as the balanced three phase components that we have.

So, each that is these phasors are displaced from each other by 120 degree and have the same phase sequence as the original phasors. That is the phase sequences a, b, c, a, b, c, a. So, first a is here, then it moves after 120 degree, b will be in this position, then again another 120 degree, c will be in this position and so on. So, the phase sequence a, b, c, a, which is the same as the original phasors phase sequence.

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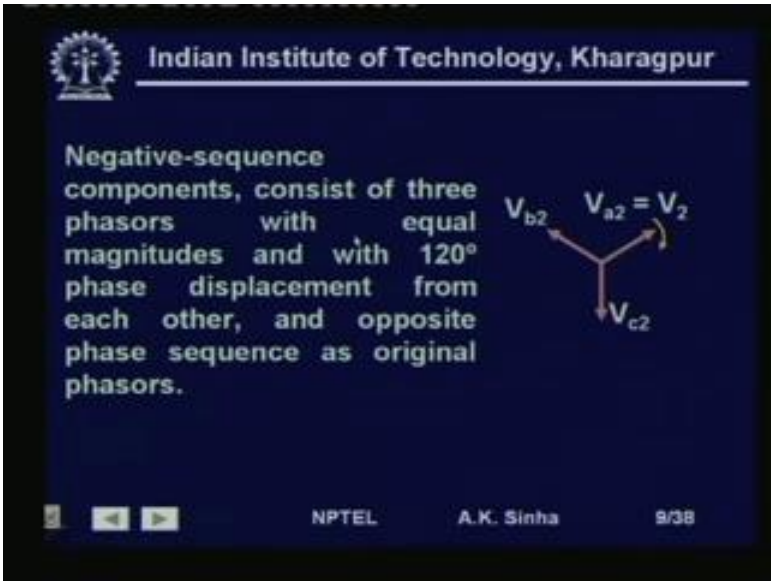

Negative sequence components consist of three phasors, with equal magnitudes and with 120 degree phase displacement from each other. Again, here if you see, these are three phasors, which are equivalent magnitude and have 120 degree phase displacement from each other. Again, this is a balanced set of quantities. So, balanced symmetrical set of three phasors, we have.

The only difference between the positive sequence and the negative sequence component is, that the phase sequence of negative sequence component is reversed to that of the positive sequence component. That is, if we see this, the rotation of this phasor will appear to be in the reverse direction. So, phase a is here, then phase b will come here. That is rotation will be like this.

Or, if we take the sequence direction in the same way, that is rotation, always as anticlockwise rotation. Then, the phase sequence for this can be seen as a, then c and then b. So, we call it as a, c, b, a phase sequence. So, basically what we are looking at is, that it is a system of quantities, which is symmetrical balance set, but rotates in the reverse direction to that of the positive sequence quantities.

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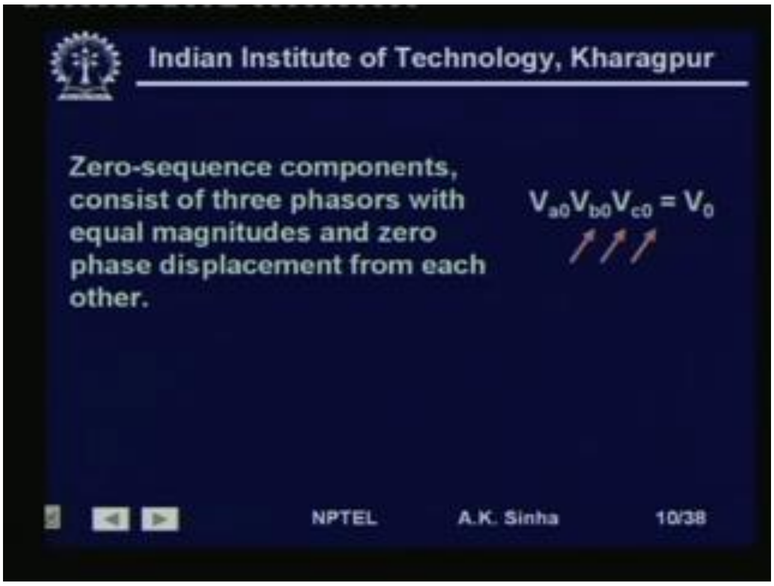

The third sequence components are called zero sequence components. And these consist of three phasors with equal magnitude and zero phase displacement from each other. That is, they are basically of the same phase. That is all the three phasors, have the same phase. So, there is no phase displacement between them and the magnitude of the three is equal. So, these are the three sets. That we use in symmetrical component transformation for a three phase system. Now, we will see how by adding these three sequence sets. We can create any unbalance set up three phasors.

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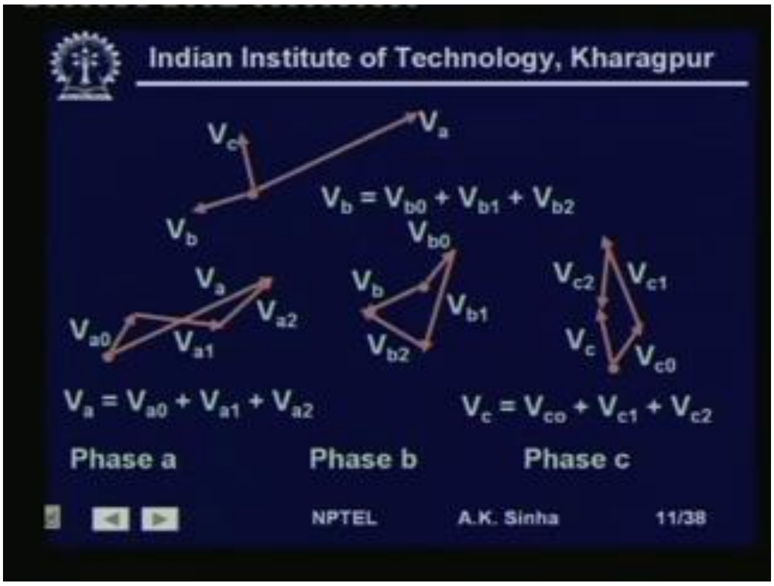

If you see here, we have an unbalanced set of phasors, V a, V b and V c, this certainly is a asymmetrical unbalance set. Now, if we look at this, V a is equal to V a 0 plus V a 1 plus V a 2. This phasor V a 0, if we see here, V a 0 is having this magnitude and this phase. So, here, we have this V a 0 and then V a 1 is, if we go back and see V a 1 is like this. So, again here, we have V a 1 and similarly V a 2 and when, we add these three phasors, we are getting this phasor V a.

Similarly, V b 0 plus V b 1 plus V b 2 will give us this phasors V b, which is this phasor here. Similarly, V c 0 plus V c 1 plus V c 2 will give me this phasor V c. So, you see by having different magnitudes of these phasors and angles of these phasors in symmetrical components, when we add them, we can generate any unbalance set of three phasors.

So, in reverse we can say that any unbalance set of three phasors can be resolved into these three sets of symmetrical components. That is positive sequence component, negative sequence component and zero sequence components.

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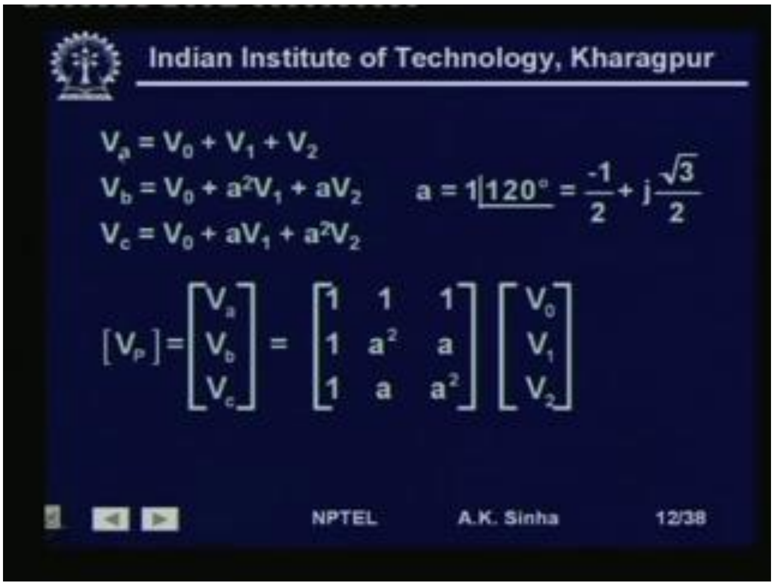

If we want to write this in mathematical terms, we can write this V a is nothing but V 0 plus V 1 plus V 2. Now, why I am writing here, I am not writing here, if you see I am not writing a 0, a 1, a 2. Because, we are assuming that, phase a, is taken as the reference. So, with respect to phase a, we are writing all the quantities. So, we are no longer writing that a. So, V 0 means basically V a 0, V 1 means V a 1 and V 2 means, V a 2.

So, here V a is equal to V a 0 plus V a 1 plus V a 2, which we are writing as V 0 plus V 1 plus V 2. V b is equal to V b 0, which is same as V a 0. So, we are writing at V 0 plus V b 1.

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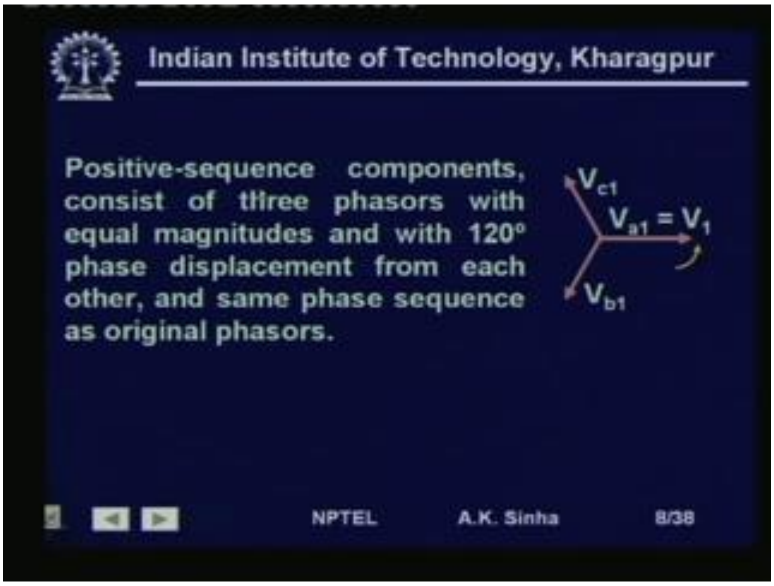

Now, V b 1, if we look into this set, V b 1 is lagging V a 1 by 240 degrees. Now, we here we have introduced an operator a, which is basically a unit phasor, with an angle of 120 degree in the positive direction. So, when I operate V a 1, with this phasor a, then what I get is, this phasor a, is now move to this position. And if I operate it again by a, then it gets moved to this position. So, basically V b 1 is nothing but a square V a 1.

So, if we see from that point of view, then we have V b 1 is same as a square V a 1. Since, we have taken a, is reference and we are not writing a, for this. So, we are writing this as a squared V a 1. Similarly, plus V b 2; V b 2, if you see here, this is V a 2 and if we multiply it by phasor or operate the operator a, on this. Then, this gives gets displaced to this position, therefore V b 2 is nothing but a times V a 2. So, we are writing here, as V b 2 as a times V a 2. Since, we are using a as reference, so it is a V 2.

Similarly, V c is nothing but V c 0, which is the same as V a 0 plus V c 1, which is again a times V 1, V a 1 or V 1 plus a square V a 2, which is basically V c 2. So, V c 2 is a square V a 2 or a square V 2. Here, as I said earlier, we are defining this operator a, as having a magnitude of unity. And a phase angle of 120 degree, which in rectangular coordinate version, you can write as 0.5 plus j root 3 by 2 or plus j 0.866.

So, this is the operator a, which is allowing us to write for all other symmetrical components, in terms of symmetrical component phase a. So, if we have V a 1, then we also know V 1 and we know V c 2 because V b 1 will be a square V a 1 and V c 1 will be a, V a 1.

So, similarly for the negative sequence components and zero sequence components, because of the symmetry from the 9 unknown quantities, we are basically brought it down to again three quantities only. That is, we need to know only V a 1, V a 0 and V a 2. If we now these, then V b 0, V b 1 and V b 2 is known V c 0, V c 2 and V c 2 are also known.

So, writing this set of equations, where we write a, b, c quantities as phase quantities. So, we write this as V p is equal to again the set of phase voltages V a, V b and V c is equal to this matrix, which is 1,1,1 for this 1, V 0 plus 1, V 1 plus 1, V 2. So, 1, V 0 plus V 1 plus V 2, V b, is equal to V 0 plus a square V 1. So, V 0 plus a square V 1 plus a, V 2 and V c is equal to V 0 plus a, V 1 plus a square V 2. So, this is what we write in matrix notation, where we write this 3 by 3 matrix, which is a symmetrical component transformation matrix as a matrix a.

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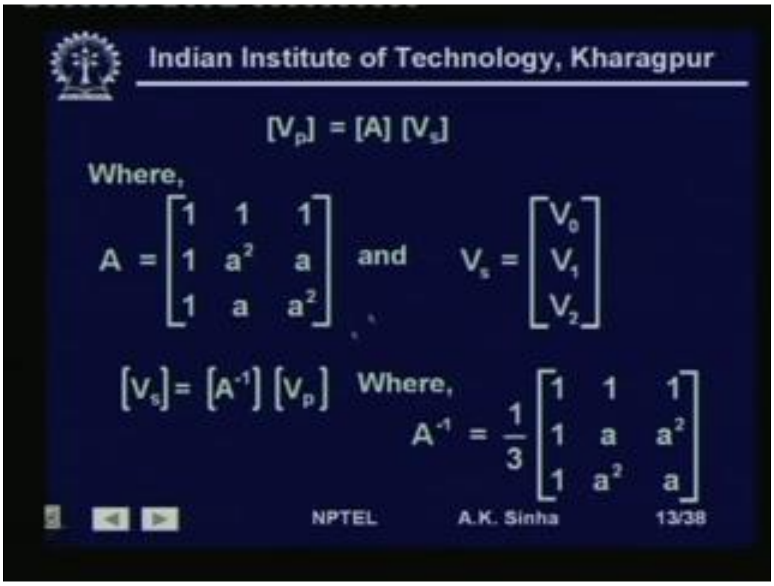

So, we write this expression in short form as V p is equal to A into V s, where this matrix A is symmetrical component matrix given by this 1, 1, 1, 1 a square a, 1 a, a square. And V s is the symmetrical component voltage in this case, so V 0, V 1, V 2; so vector of symmetrical component voltages. Now, from this relationship, if we pre-multiply both sides by a inverse, then we will get V s is equal to a inverse V p.

So, V s is equal to a inverse V p and if we take the inverse of this matrix, we will get a inverse is equal to 1 by 3 into 1, 1, 1, 1 a, a square, 1 a square a. This matrix is nothing but basically the transpose of this matrix, but 1 by 3 is coming here. So, it is multiplied by 1 by 3, with this and this is the relationship, that we get, a inverse is 1 by 3, 1, 1, 1, 1 a, a square, 1a square a. So, since we now a and a inverse, we can always convert the phase quantities to symmetrical component quantities and vice versa.

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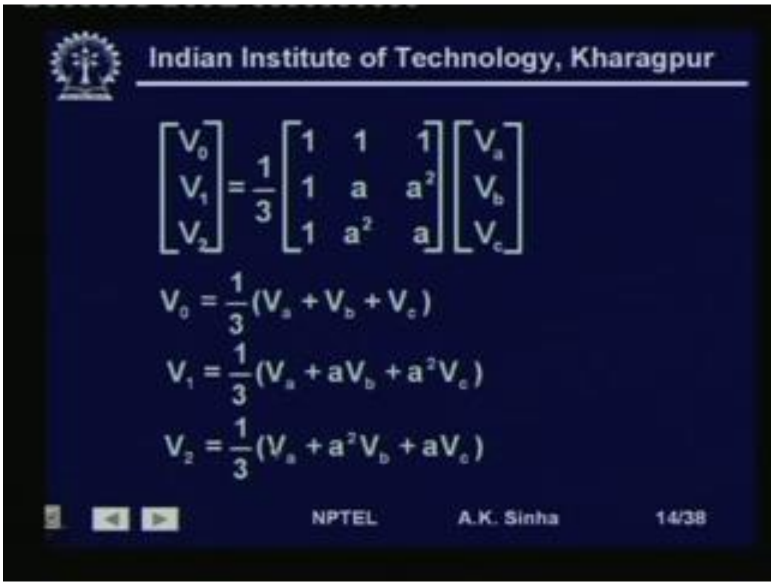

So, we can write V 0, V 1, V 2; that is the symmetrical component quantities. In terms of phase quantities, V a, V b, V c, this will be a inverse matrix, that we have 1 by 3 into this. So, we get, if we expand this we can write this as V 0 is equal to 1 by 3, V a plus V b plus V c. And V 1 is equal to 1 by 3, V a plus a V b plus a square V c and V 2 is equal to 1 by 3 V a plus a square V b plus a V c. So, this is the relationship between the sequence quantities and in terms of the phase quantities.

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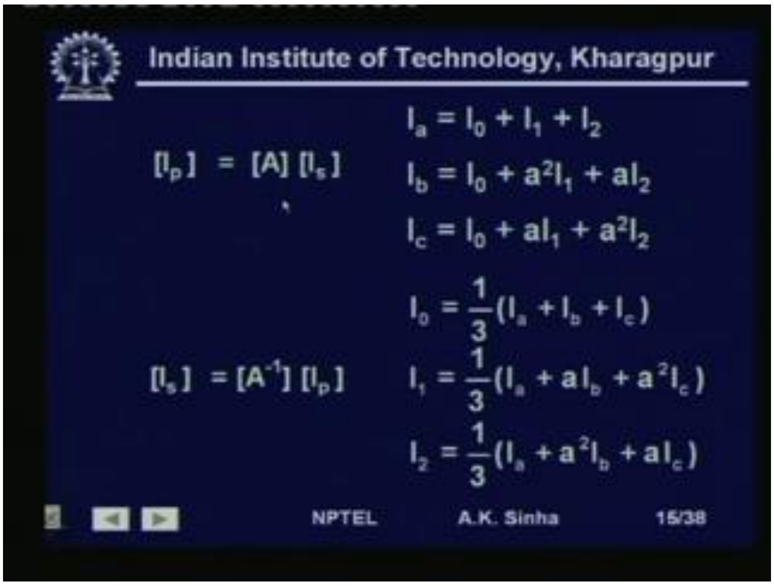

Same relationships can be obtained for the currents, just like we obtained for the voltage. So, here if you look at this write I p is equal to A I s, which can be written as I a is equal to I 0 plus I 1 plus I 2. I b is equal to I 0 plus a square I 1 plus a I 2, I c is equal to I 0 plus a I 1 plus a square I 2. Again, here the phase, since we are not writing a, because a is the reference phase.

And same thing, if we want to write the symmetrical component currents, in terms of phase currents, we will write I s is equal to A inverse I b, which in expanded form. Can, again be written as I 0 is equal to 1 by 3, I a plus I b plus I c. I 1 is equal to 1 by 3, I a plus a I b plus a square I c and I 2 is equal to 1 by 3, I a plus a square I b plus a I c.

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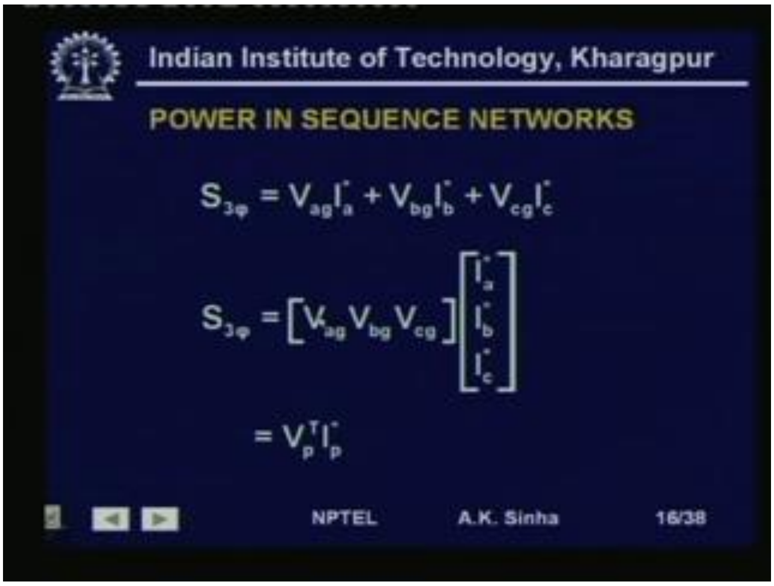

So, this is what we do, that is we have the voltage and current transformation from phase quantities to symmetrical component or from symmetrical components to phase quantities using a and A inverse matrices. Now, we will talk about, how to compute power, in terms of sequence quantities. So, power in sequence networks, can be written as three phase power S, can be return as V a into I a conjugate, where we are writing this V a in terms of voltage from phasor a to ground or neutral. That is phase voltage.

So, V a g into I a plus V b g into I b, this is I b conjugate, because we have seen earlier, that B plus J Q is given by V i conjugate. So, we are writing this complex power, as three phase power, as the some of the power in each of the three phases a, b and c. So, a phase power is V a g into I a conjugate, V phase power is V b g into I b conjugate and C phase power is V c g into I c conjugate.

This n matrix form can be written as S 3 phase is equal to this is a row vector V a g, V b g, V c g and this is a column vector I a conjugate I b conjugate I c conjugate. This if we write this, also has a column vector, which is the normal way of writing the vectors. Then, we can write this V p transpose.

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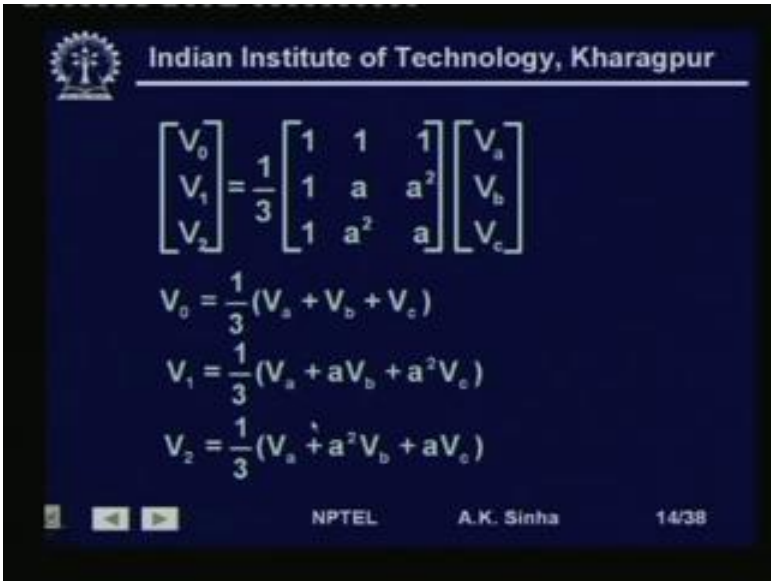

Because V p, if you remember, we had written this as a column vector V a, V b, V c. So, V b is like this, therefore, we are writing this as V b transpose into I b conjugate.

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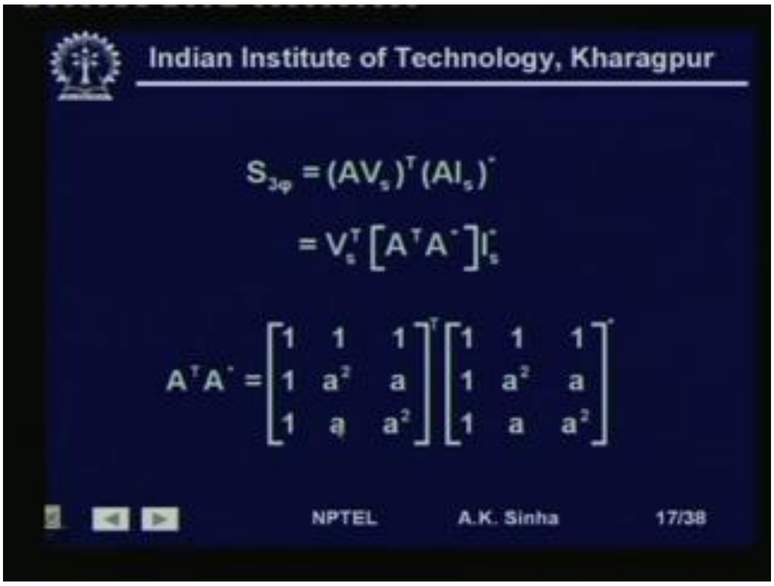

Now, if we see that, then in terms of, if we write this in terms of symmetrical component, then V b is equal to a into V s. So, it is A into V s transpose and I b is A into I s. So, it is A into I s conjugate. Now, if we do this, then if we take the conjugate of this, then we will get this, as this if we do take the transpose, then this is Vs transpose, A transpose. So, V s transpose A transpose into A conjugate, I s conjugate.

So, we have done this A transpose, A conjugate, together. So, if we write that, A transpose, A conjugate. Then, we will get this A transpose as this 1, 1, 1, 1 a square a, 1 a, a square and the conjugate of a is this is a and it is conjugate.

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So, if we do this conjugate, then what happens is a becomes a square and a squared becomes a. So, here, when we do that and then multiply, then we will find that this comes out to be a diagonal matrix. With, 3 in the diagonal terms or this can be written as 3, multiplied by unity matrix. That is A transpose, A conjugate is 3 times unity matrix, 3 into unity matrix.

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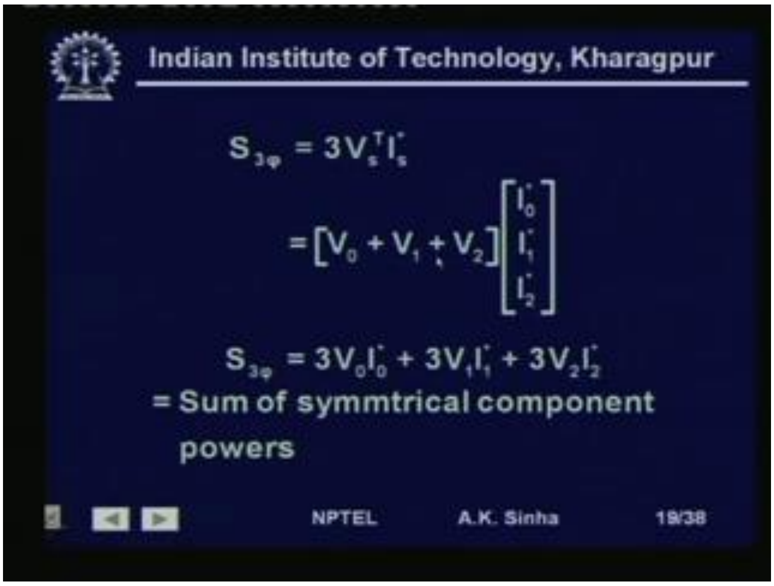

So, if you substitute that, then we get three phase complex power as 3 times V s transpose into I s conjugate. So, if we write here, again this, then this is V s transpose and I s conjugate, three is missing. So, this should be 3 times, this V s transpose and this is I s conjugate. And therefore, this is three phase power is equal to 3 times V 0, I 0 conjugate plus 3 times V 1, I 1 conjugate plus 3 times V 2, I 2 conjugates.

This is nothing but the total power in the complex power in the sequence networks or in the sequence components, zero sequence component, positive sequence components and the negative sequence component. That is, since we have three phasors, there. So, the total power will be sum of the 3 and since, they are symmetrical. So, the power n 1, that is in a, will be same as in b and c. So, that is why, it is multiplied by 3 here.

So, three phase power is given by this, which is nothing but the sum of the symmetrical component powers, which tells us one very important property of symmetrical component, that this component transformation symmetrical components transformation is power invariance. So, power in phase quantities and power in sequence component quantities are same.

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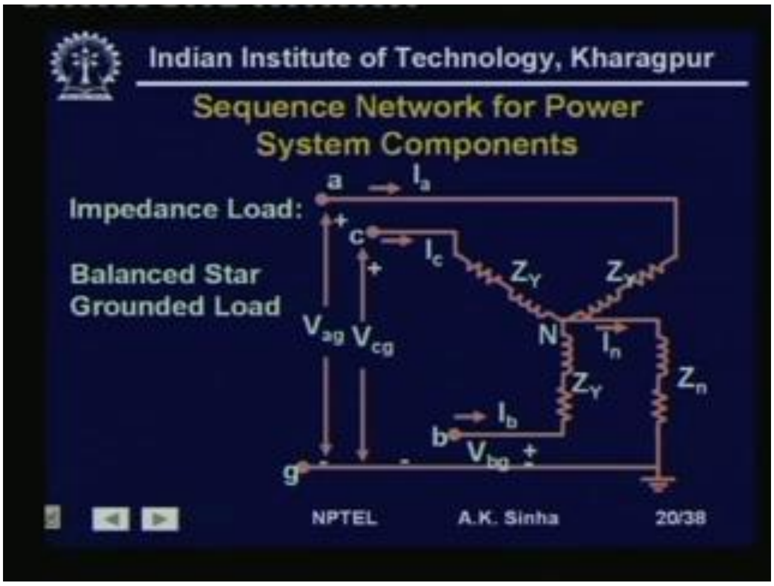

Now, let us go into trying to find out, the sequence component model for three phase loads. We will start first, with a balanced star connected load, which is having it is neutral grounded. So, here we have an impedance load, which is a balanced star grounded load. So, this is a balance star connected load Z y, Z y and Z y in the three phasors and the ground or the neutral is connected to ground through an impedance Z n.

The currents flowing in this load are I a, I b in phase b and I c in phase c. The voltage to with the respect to ground for phase a is V a g, for phase b is V b g and phase c, it is V c g. So, this is the system that we have and this is a balanced three phase star connected load, with neutral grounded. We want to find out, it is sequence component network or when we transform this, in terms of the symmetrical components, what is the kind of network that we are going to get.

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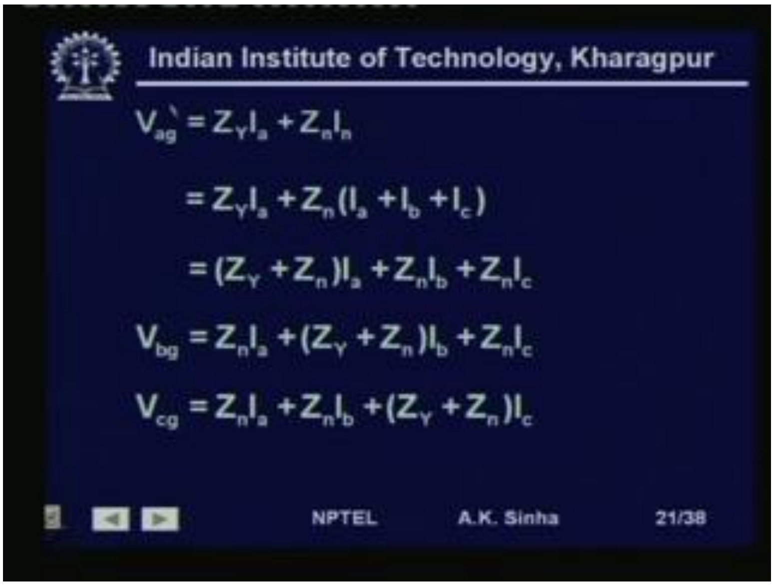

So, let us see, how we do that. If we write the voltage relationship V a g, sorry V a g, what is this voltage? This voltage is going to be equal to I a into Z y this drop, which is taking place plus I n into Z n. So, from here, if we see this I n into Z n is the voltage rise here and I a into Z y is the voltage rise here. So, from here to here, the voltage V a g is nothing but sum of these two voltage rises or the voltage drop, if we are seeing the current flowing from phase a.

So, the same thing, that we have written here V a g is Z y into I a plus Z n into I n. now, what is this current I n? If I a is flowing like this I b is flowing like this and I c is flowing like this. Then, I n is nothing but sum of these three current I a plus I b plus I c. So, I n is I a plus I b plus I c. So, that is what we are writing here Z n into I a plus I b plus I c. Now, if we arrange this in such a way, that currents I a, I b, I c are separated. Then, we have Z y plus Z n into I a plus Z n into I b plus Z n into I c.

In the same way, we can find out the voltages for phase b and c, V b g will be equal to Z n into I a plus Z y and plus Z n into I b plus Z n into I c. And V c g will be Z n into I a plus Z n into I b plus Z y plus Z n into I c.

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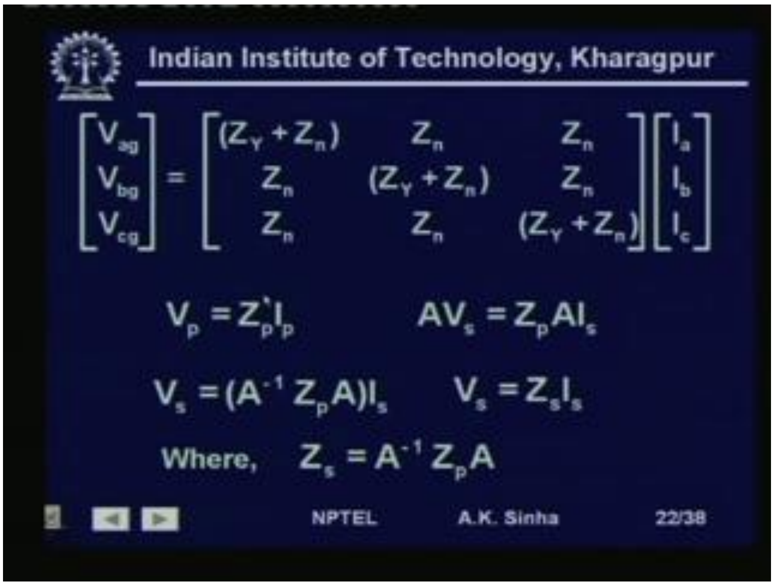

In matrix form we can write this as V a g is equal to Z y plus Z n into I a plus Z n into I b plus Z n into I c, in the same way for V b g and V c g. So, we have V b g is equal to Z n into I a plus Z y plus Z n into I b plus Z n into I c and V c g is Z n into I a plus Z n into I b plus Z y plus Z n into I c. Now, this in short form, we can write as V p is equal to Z p into I p, but this where this 3 by 3 matrix, we are writing as Z p.

Now, we multiply both sides, with a or we convert this phasor quantities in terms or write these phasor quantities, in terms of symmetrical component quantities. Then, V p can be written as AV s and I p can be written as A I s. So, AVs is equal to Z p into A I s. Now, from here, if we pre-multiply both sides by A inverse, then we have here A inverse A will become unity matrix. So, we have V s is equal to A inverse will pre-multiplying this side. So, A inverse Z p into A into I s.

This we can write as V s is equal to Z s into I s and what is this Z s, this is nothing but A inverse Z p into A. So, the symmetrical component form positive negative and zero sequence, impedances from the phasor quantities, can be found using this relationship Z s is equal to A inverse Z p into A.

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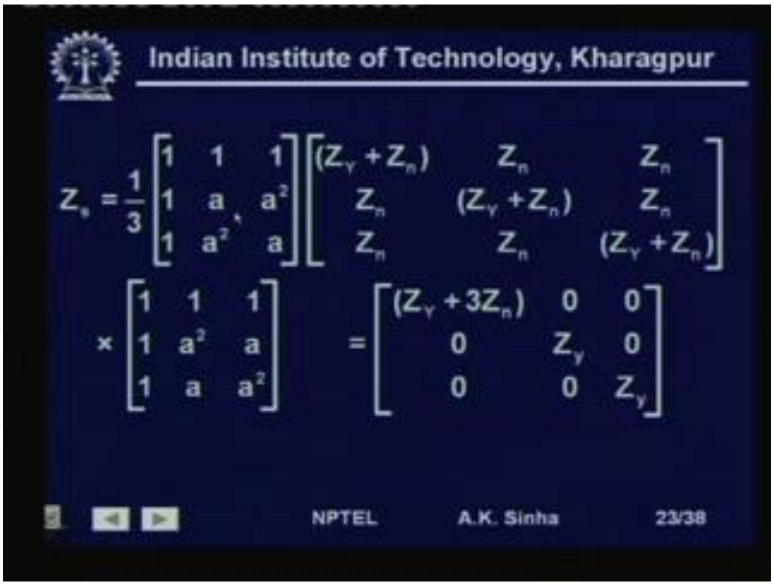

So, now if we do this multiplication then we have Z s is equal to A inverse is 1 by 3, this matrix 1, 1, 1, 1 a, a square, 1 a square a this is Z p matrix. That we have Z y plus Z n, Z n, Z n, Z n, Z y plus Z n, Z n and Z n, Z n, Z y plus Z n, so Z y plus Z n in the diagonal elements and Z n, with the off-diagonal elements. So, this is A inverse Z p into A. So, A is this matrix. When, we do this multiplication, we will get this as Z y plus 3 Z n, 0, 0, 0, Z y, 0, 0, 0, Z y.

What we find is, we have got a diagonal matrix, where the first element, which is corresponding to the zero sequence terms, is Z y plus 3 Z n, whereas, the positive and negative sequence quantities are Z y and Z y. Now, what is the difference, that we have found here from the phasor quantities and the in the sequence quantities. If you see here Z b is a 9 by a 3 by 3 matrix, with all off-diagonal elements present.

Whereas, this Z s is a diagonal element no off-diagonal terms, what does it mean? The three sequence networks are uncoupled with each other. There is no coupling between the networks. Whereas, here phase a and b has a coupling of Z n phase a and c has a coupling of Z n. That is, Z a c is equal to Z n, Z a b is equal to Z n, same way for other terms. Whereas, here there is no coupling, there is no terms as Z s 10 or Z 02 or Z 12 those kind of terms are not coming there at all. That is for a balanced three phase network. That we have is, now transformed into three independent sets of network, which is much simpler.

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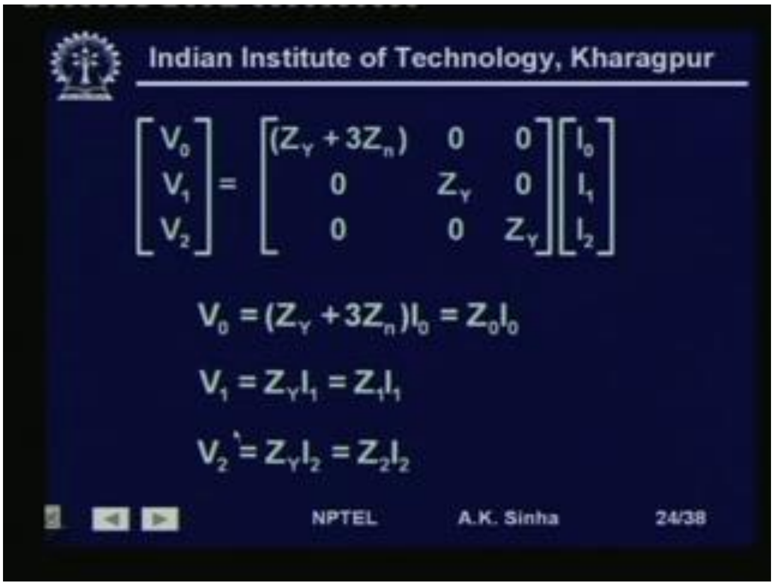

So, if we are putting this voltage relationship, then we have this V s, which is V 0, V 1, V 2 is equal to this Z s, which is a diagonal matrix into I s, which is I 0, I 1, I 2. So, what we see is, we have three independent equations, V 0 is equal to Z y plus 3 Z n, I 0, which we can write as Z 0, I 0. V 1 is equal to Z y into I 1, which we can write as Z 1, I 1 and V 2 is equal to Z y into I 2, this is equal to Z 2 into I 2. So, we have three equations, which are independent of each other, they are not coupled equations, which is the case when we work with the phasors quantities.

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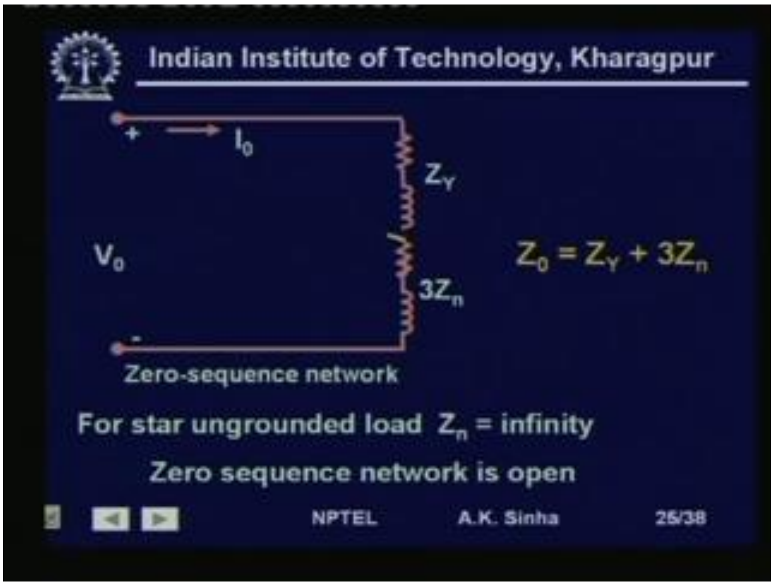

So, if we see in terms of the network. Then, we have the zero sequence network, like this, which is Z y plus 3 Z n, V 0 is the voltage applied to this and I 0 is the current flowing in this. Now, in case of this network, we are finding the Z 0 is equal to Z y plus 3 Z n. Now, if the neutral in this was not grounded then what would have happened, this Z n. Since, this is an open circuit; this Z n would be infinity, which is same as same that this will be open.

So, what does it tell us, this will be open in case the neutral point is not grounded? So, in case of circuit is, where we have do not have a ground return available. Then, this zero sequence network will be open, means no zero sequence current can flow. This is a very, very important relationship that we obtain. So, for star ungrounded load Z n is infinity and zero sequence networks is open. So, no zero sequence current can flow.

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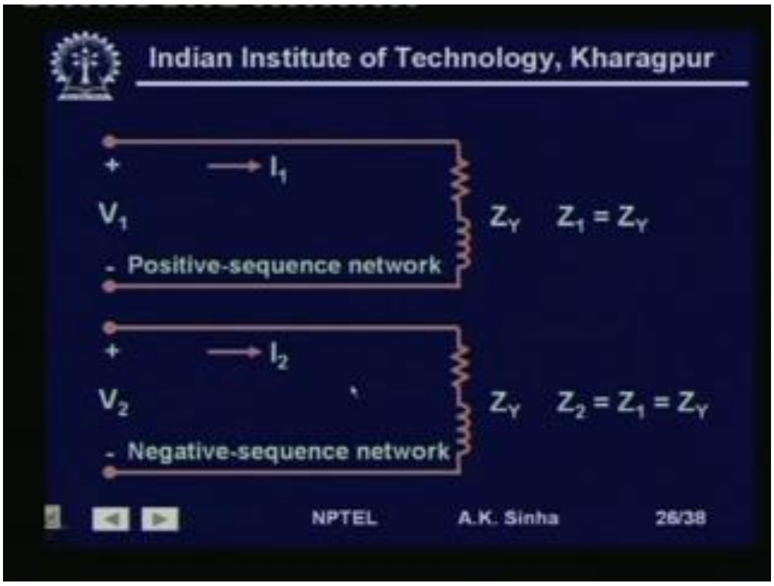

Similarly, positive sequence network, we find is same as Z y and negative sequence network is impedances same as Z y voltage is V 2 and current is I 2. So, this is how we get three separate networks, for a balanced three phase load system.

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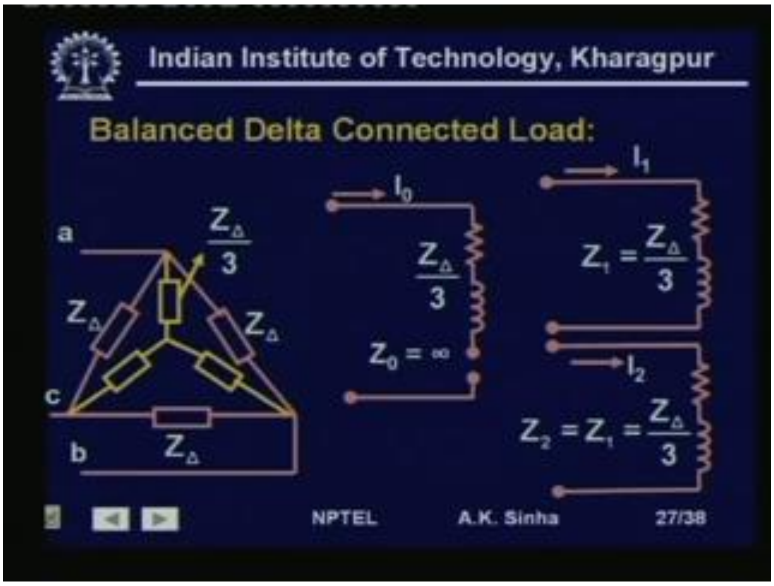

Now, if we have a balanced delta connected load, what will happen? Well, it is very easy to analyze this, because we can always convert the balanced three phase delta load into an equivalent balanced three phase star connected load. Where, if we have the Z delta as the impedance value of the delta branches. Then, the equivalent star branch impedance will be Z delta by 3. So, this will be Z delta by 3, this will be Z delta by 3 and this will be Z delta by 3, this is the equivalent star for this balance delta network.

Now, if we see this, again what we find, that Z 0 will be equal to Z delta by 3 and Z n since, there is no Z n connected, because this is neutral ungrounded. So, this is open; that means, Z 0, which is this Z delta by 3 plus Z n. Since, Z n is infinity, so Z 0 is infinity. This is an open circuit. That is in case of delta connected load, there will be no zero sequence current flowing in the lines.

Similarly, positive sequence and negative sequence networks for this can be obtained, where positive sequence impedance will be Z delta by 3 and negative sequence impedance will be Z delta by 3. The currents in them will be positive sequence current; voltage will be positive sequence voltage across the terminals. These three networks again are independent from each other.

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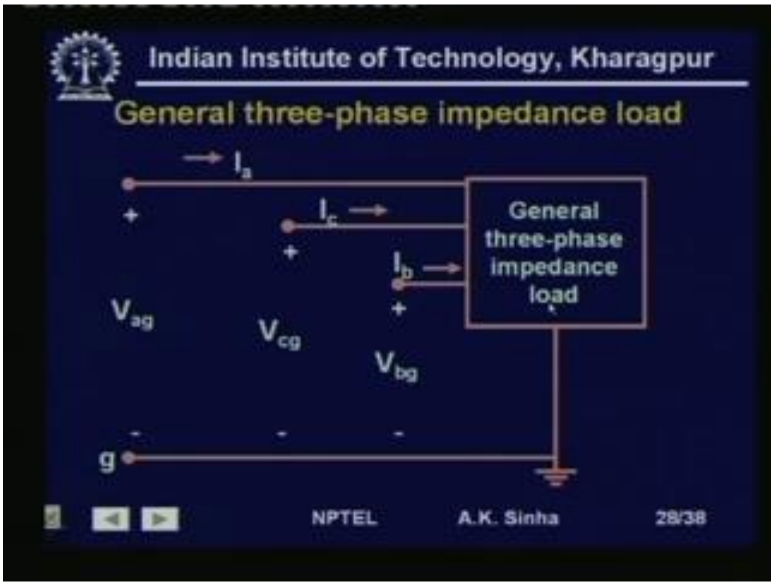

Now, let us take the case of a general three phase impedance load. It means a load, which is not a balanced load. It can be a star connected load the delta connected load ungrounded or grounded, whatever it is. We can write down the relationship for this general situation. The currents are I a, I b and I c. The voltages are V a g, V b g and V c g between the phase and the ground and the current in the ground will be this neutral current or I g flow.

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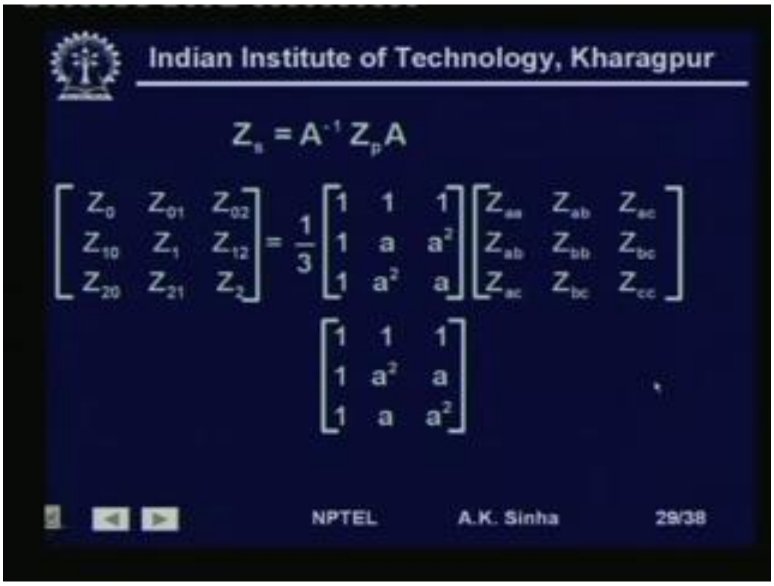

Now, we have for this system, the phase values as Z a a, Z a b, Z a c. That is the mutual terms Z a b between a and b and a and c will be there. Similarly, this will be Z a b, this is

Z b b, Z a a, Z b b and Z c c, are basically the self impedance of this three branches and a b, b c and c a, are basically the mutual impedances, which are involved. So, again, we can write Z s is equal to A inverse Z p into A.

So, Zs is nothing but Z 0, Z 01, Z 02, these are the mutual terms, which we are writing for zero sequence to positive sequence zero sequence to negative sequence. Similarly, positive sequence to zero sequence positive sequence and positive sequence to negative sequence. So, we are writing the nine terms for the 3 by 3 matrixes, including the coupling terms between the sequence networks.

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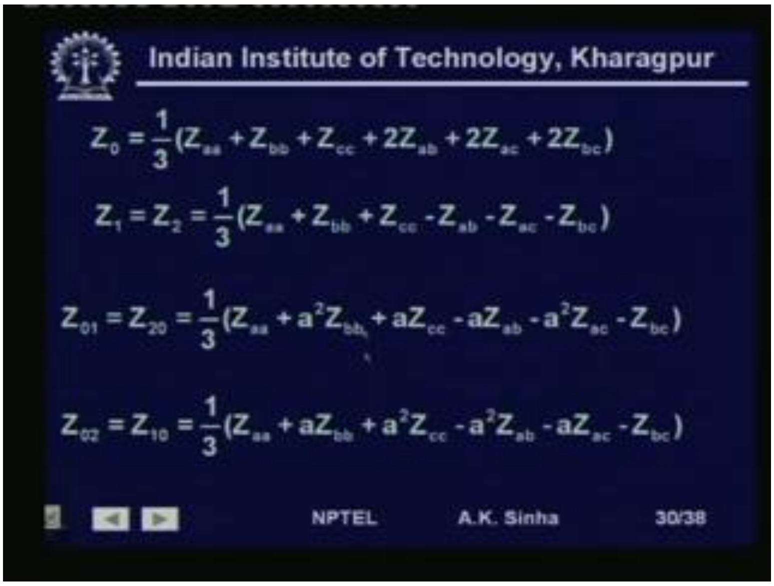

So, if we do this inverse A inverse Z b a, then we have 1 by 3 this matrix, which represents A inverse this is Z b and this is a. And if we do this all multiplication, then we get Z 0 is equal to 1 by 3, Z a a plus Z b b plus Z c c plus 2, Z a b plus 2, Z a c plus 2, Z b c. Z 1 will be equal to Z 2, will be equal to 1 by 3, Z a a plus Z b b plus Z c c minus Z a b minus Z a c minus Z b c. Z 01 will be equal to Z 20 and this will be equal to 1 by 3, Z a a plus a square Z b b plus a Z c c minus a Z a b, a minus a square Z a c minus Z b c. And Z 02 is equal to Z 10 will be equal to 1 by 3, Z a a plus a Z b b plus a square Z c c minus a square Z a b minus a Z a c minus Z b c.

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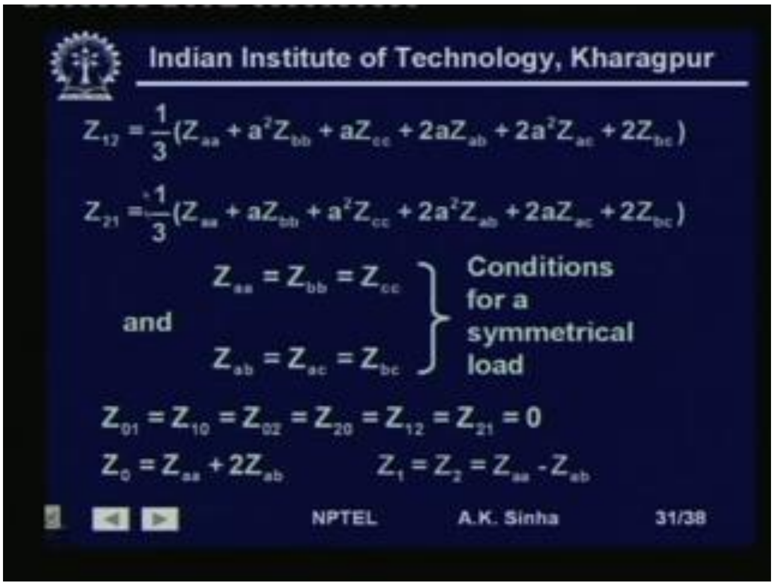

And Z 12, similarly will be equal to 1 by 3, Z a a plus a square Z b b plus a Z c c plus twice a Z a b plus twice a square Z a c plus twice Z b c. And Z 21, sorry Z 21 is equal to 1 by 3, Z a a plus a Z b b plus a square Z c c plus twice a square Z a b plus twice a Z a c plus twice Z b c.

Now, if we see in this case, all the nine terms, that is the coupling terms between positive and zero sequence and negative sequence will be present for a general three phase case. That is, for a unbalanced three phase load there is going to be coupling between the three sequence networks. If we put Z a a is equal to Z b b is equal to Z c c; that means, the self impedances are same. And similarly, the mutual impedance Z a b is equal to Z a c is equal to Z b c.

Then this is, what we get will be a symmetrical three phase load and in that case, we will find that Z 01, Z 10, Z 02, Z 20, Z 12, Z 21 all becomes 0. That is all off-diagonal terms becomes 0 and Z 0 becomes Z a a plus twice Z a b and Z 1 is equal to Z 2 is equal to Z a a minus Z a b. So, this is what happens, in case, we have the symmetrical network and then the three networks become uncoupled.

If the network is unbalanced, then there is this decoupling, does not take place and mutual terms will be available, which means that the three sequence networks will be connected with each other. There are going to be mutual terms between them. Since, most of the network elements are designed to be symmetrical networks. Therefore, we are going to get three uncoupled networks.

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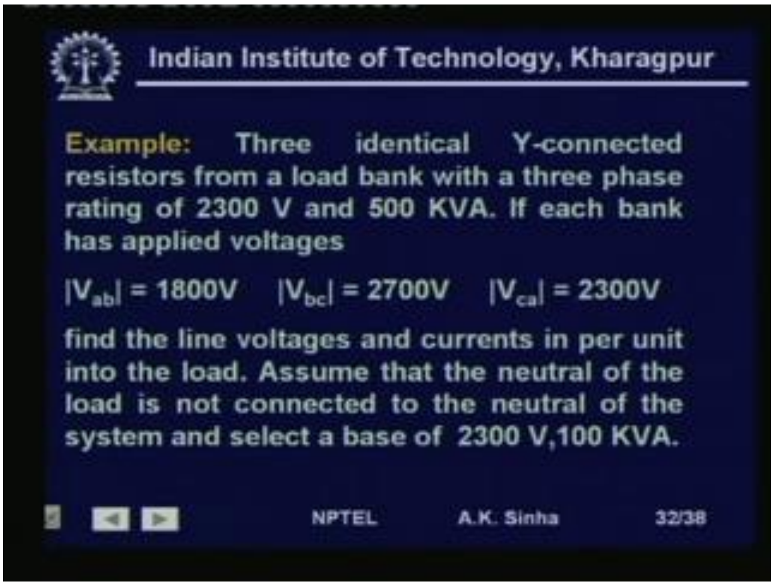

Now, let us take a simple example, to illustrate this point. We have for example, we have taken a three identical star connected resistors, from a load bank, with three phase rating of 2300 and volt and 500 KVA, I am sorry this is form, not from. Three phase, three identical star connected resistors, form a load bank. Means, we have a star connected load, with three resistors of equal resistors, the rating of the three phase load that the voltage rating is 2300 volt and the KVA phase is taken as 500 KVA. If each bank, has applied voltages of V a b is equal to 1800 volts V b c is equal to 2700 volt and V c a is equal to 2300 volt. That is the applied voltage is asymmetrical or unbalanced the load itself is the balance impedance.

Now, since we have seen that for balanced impedance load. We get the three sequence networks, which are uncoupled. So, here, even though the voltage applied is unbalanced, we will get an uncoupled network, for the three sequences. Find the line voltages and currents in per unit into the load. Assume that, the neutral of the load is not connected to the neutral of the system and select a base of 2300 volt, 100 KVA.

So, this is, what we have, that is we are using a base of 2300 volt, 100 KVA, whereas the load values are given on a 500 KVA base and 2300 volts. So, voltage base is same as what we have chosen, but the KVA base for the load is 500 KVA, whereas our chosen base is 100 KVA. We could have chosen 500 as well, but this is just to illustrate, how we can have different bases. And we can convert them to per unit quantity, for any given base MVA or voltage level.

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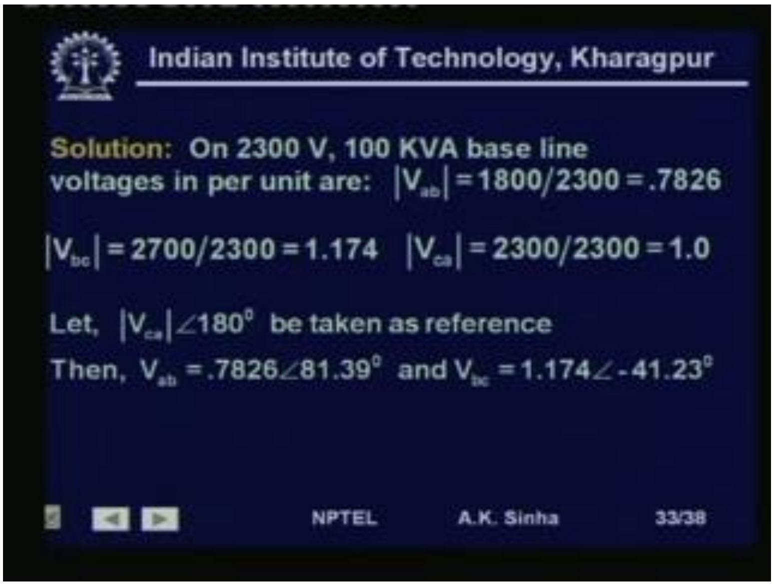

Now, we would like to solve this. So, on 2300 volt, 100 KVA base, line voltages in per unit are V a b is given as 1800. So, 1800 by 2300 comes out to be 0.7826. V b c is given magnitude is given as 2700. So, 2700 by 2300 is 1.174. V c a is equal to 2300. So, it is by 2300, 2300 which is 1 per unit. Now, let us choose one base, that we can use as a reference is one of the voltages, we can choose it is angle as a reference. The others will come out with respect to that. So, let us take the reference of voltage V c a, with 180 degree to be taken as a reference value. Then we will get V a b is equal to 0.7826 angle 81.39 and V b c is equal to 1.74 angle minus 41.23.

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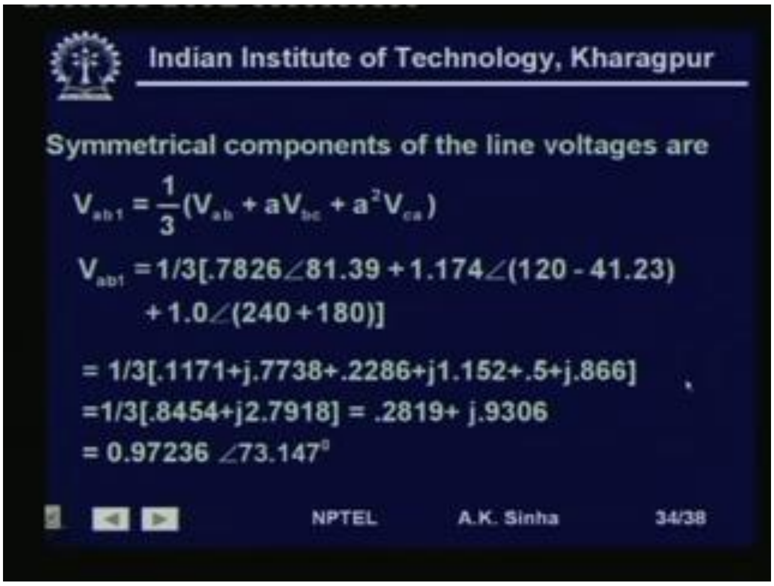

Now, symmetrical components of the line voltages will be, if we write this V a b 1 will be equal to 1 by 3 V a b plus a V b c plus a square V c a. This is the same relationship that we have written earlier. So, now substituting the values, we will get V a b 1 is equal to finally, putting all these values and solving as 0.97236 angle 73.147 degrees. So, we know the values of V a b, V b c and V c a and a and a squared are known.

So, substituting that, here a is equal to 1 angle 120, a square is 1 angle 240. So, this is what we have done. So, adding them together by converting them to rectangular coordinates we get this result for V am b 1. Similarly, we can write V a b 2, in the same way as 1 by 3, V a b plus a square V b c plus a V c a. And then substituting the values, we will get this as equal to 0.2272 angle 223.627 degrees.

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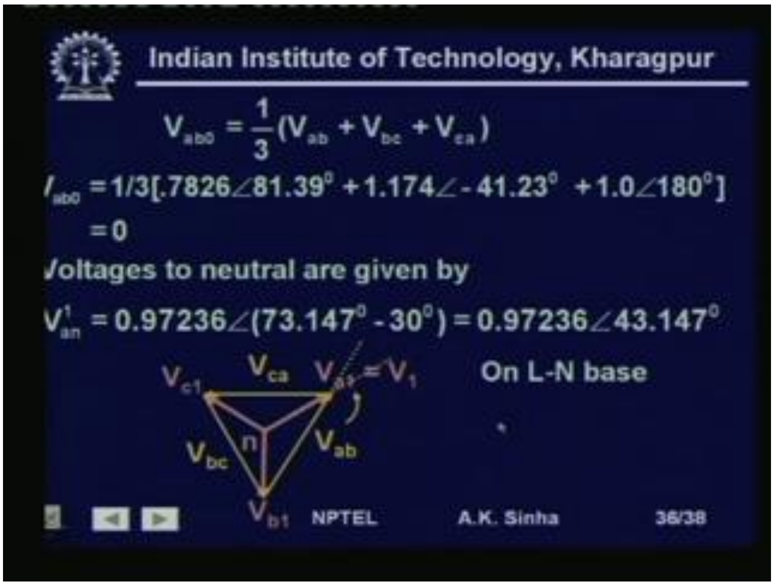

Now, similarly, we will get V a b 0 as 1 by 3, V a b plus V b c plus V c a. If we do that, we find that this comes out to be 0. This one need not compute, because the neutral is ungrounded. So, we know that V a 0, V a b 0 is going to be 0, because there is no 0 sequence current, which can flow. So, instead of the line voltages, if we want we can calculate the voltages to the neutral.

So, V a n 1 is equal to the same value, we are writing, because we are taking it on a line to neutral base. If we take it on a line to line base, then we will have to divide it by root 3. Now, here what we see is V a b 1, whatever is there, if we look at this diagram V a b is in this direction, V a n is in this direction. That is Van is locking V a b by 30 degrees. So, this is V a b, which we had and we have subtracted 30 degrees from that. So, we get V a n. If we are writing it on a line to line base, then we will have to divide it by root 3.

So, otherwise, if we are taking it on a line to neutral base, then this remain same, so 0.97236, with an angle of 43.147. Same thing for the negative sequence, we will find that V a b 2 was this value and because the sequence is rotation is in the revere directions. So, instead of subtracting the thirty degree in this case V a n 2, will be leading V a b 2 by 30 degree. So, we are adding these 30 degrees. So, this gives us this value on a line to neutral base.

Resistance value in per unit can be calculated since we have been given at 500 and we have to calculate it on 100 So, it comes out to be 0.2 unit. And therefore, we can calculate I a 1, which will be nothing but I a sorry, V a 1 divided by the resistance or the impedance. So, V a 1 by this, resistance or impedance, so this comes out to be 4.8618 angles 43.170 per unit.

Similarly, I a 2 is nothing but V a 2 divided by the resistance, that is 0.2. This comes out to be 1.136 angles 253.627 per unit and I a 0 is going to be 0, because there is no neutral connection in this case. So, this is all we are going to do in today's lesson. In the next lesson, we will talk about how to find out the sequence network for other components, such as transmission lines, transformers, generators and how to build sequence network, for a given power system. So, with this we end today.

Thank you very much.