Power System Analysis
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Lecture - 24
Review of Power Flow Study

Welcome to lesson 24 on Power System Analysis. In this lesson, we will Review the Power Flow Study that we had done earlier. We will start with how a power system operates. And then, we will go into what power flow study is. And then, what are the various techniques used for doing this power flow study.

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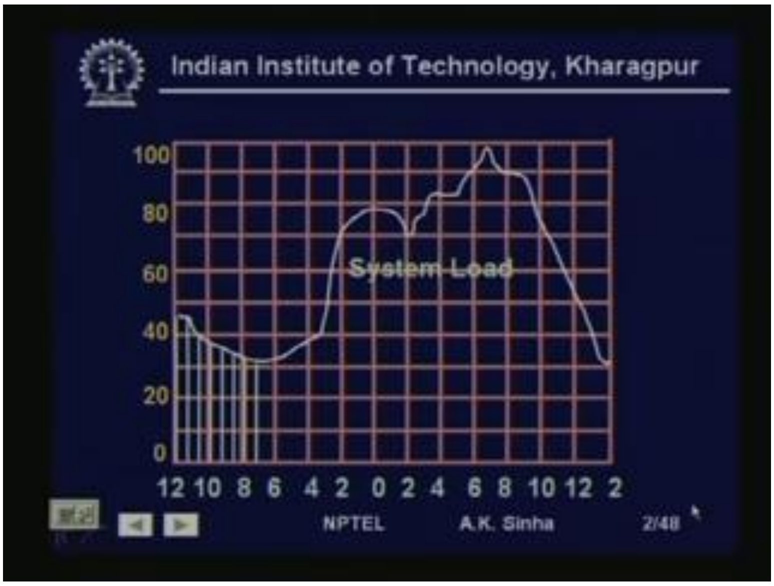

Well, as we can see from this diagram. The load on any system is not constant all the time, it keeps varying all the time. So, when we want to do a study, what we do is we take any time instant. And take the load at that time instant. In fact, what normally we do is, we take small time slices, in which we consider the load to be constant. And this we can keep the time slices every minute, or a couple of minutes.

Time in which period, we expect there is very little variation in load. And therefore, we consider it to be constant for that period. So, every couple of minutes, we can do this kind of a static analysis for the system. And therefore, we call this kind of analysis, as well as this operation. That we are considering time slices of small periods. Or which we are considering the operation of the system to be of steady state.

This kind of operation we call as Quasi-static operating condition. And for this kind of study, for the given time slice, we consider the system load to be constant. And we do this analysis, since the load is constant. We do a static analysis for this purpose. Now, this static analysis, which we do for the system is popularly known as the load flow or power flow analysis.

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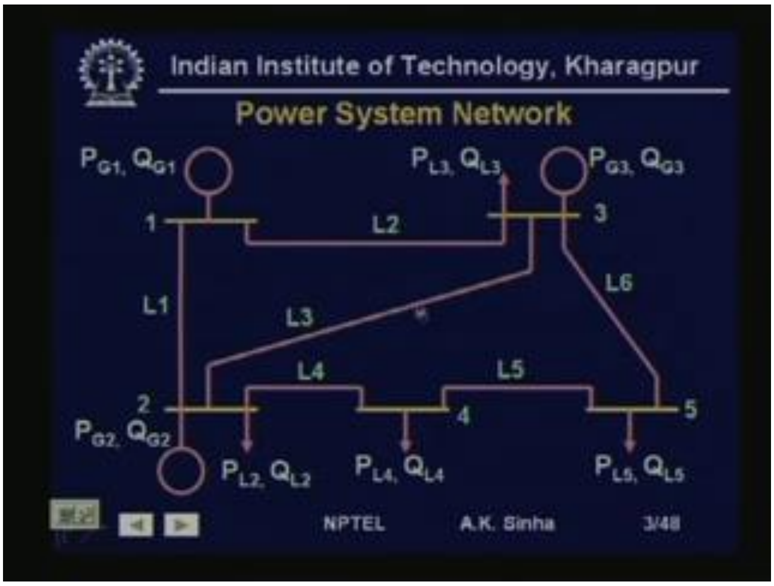

Now, let us take a very simple example of a power system network. Now, here this network, we have 5 nodes, 1, 2, 3, 4 and 5. Some of the nodes, we have generators associated with it. Like node 1 has a generator, node 2 has a generator, node 3 has a generator. Some of these nodes have loads on them, like node 2, node 3 has a load. Node 2 also has a load, node 4 has load, a node 5 also has load here.

The loads are complex nodes. So, real loads are like node 5 has a load P L 5, Q L 5. P L 5 is the real power demand and Q L 5 is the reactive power demand. Now, these nodes are connected by various transmission lines. So, we have these transmission lines, which carry power from the generator to various nodes. And what we want to do in the load flow analysis. Is, to find out the flows in various lines as well as the voltages or different busbars.

And this we find out for a given condition of loads and generation. So, this is basically what we do in power flow analysis. That is find out the voltage magnitudes at various nodes, which in power system terminology we call busbars. So, find out the voltage magnitude and angle, at all the busbars in the power system. And the flow of real and reactive power, on various transmission lines. Given the generation and loads at the various busbars.

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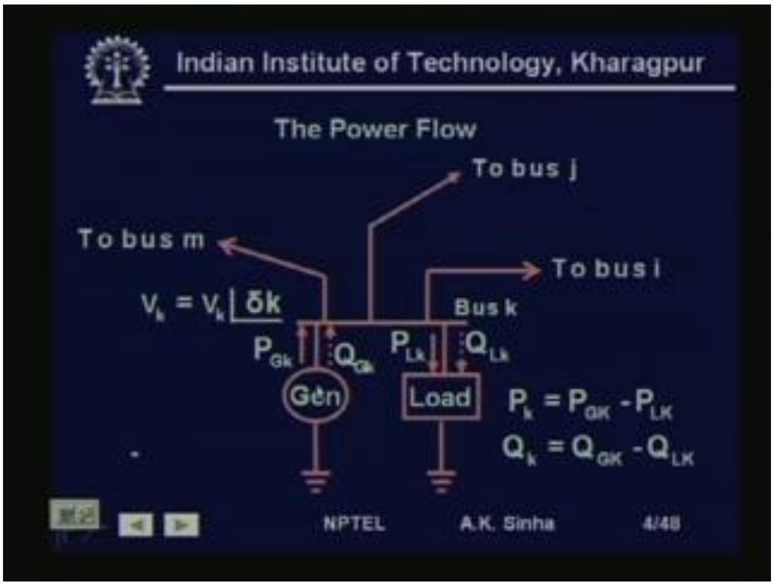

Now, if we look at a single bus here, which has a generation said we are taken a bus k, it has a generation P G k, and reactive power generation Q G k. And this bus has a voltage V k, which we write as V k magnitude and angle delta k. So, this is a complex voltage, which has a magnitude and a phasor angle delta. Now, this bus also has a load P L k and a reactive power load Q L k. So, real power P L k and reactive power Q L k are the loads at this bus.

Now, what we can do is. Now, from this bus if we have some power, which will be flowing out to some other bus m or bus j or bus I through this transmission lines. It may be some power may be coming this way, some power may be going out this way. So, there will be power flows, which will be going out or coming in, through various lines through this node.

And this node if you see we have generation, as well as load here. So, what we can do is, we can combine these two. And find out a real power and reactive power injection at the bus. Now, we say real power injection is equal to the real power generation minus the real power load, at that bus. Similarly, reactive power injection is the reactive power generation minus the reactive power load at that bus.

That means, what we are trying to say is. Whatever is the algebraic sum of generation and load, load is considered as negative generation. So, algebraic sum, here we get is going out from this bus. So, that is being injected here. And it will be going out from this bus. So, this is how we defined real and reactive injections at the busbars.

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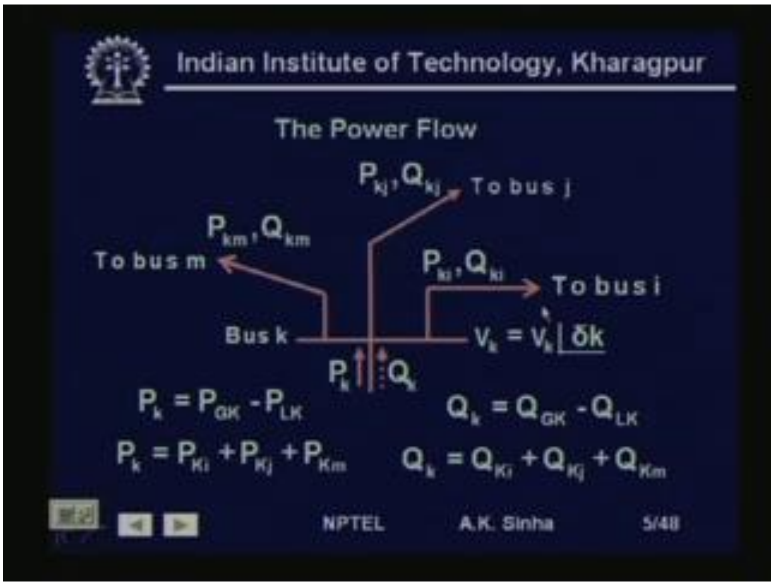

So, here we have done that. And we have put the real and reactive injection here. And we have shown various power flows, through the transmission lines to various other buses. Now, if we look at the situation at this node, then what do we find? We find that the injection here is going to be equal to how much, it will be some of all the power flowing out from this. So, this P k will be equal to P k i plus P k j plus P k m, these power flowing out.

If the some power is flowing in, then that will be considers as negative power going out from this bus. Similarly, for the reactive power also, we have Q k. That is injected here must be equal to all the reactive power, which is going out from the bus. So, Q k i plus Q k j plus Q k m, this is what we will get. So, what we need to do is write down the equations, in terms of this. Then we can solve for those equations, that is what we do in power flow solution.

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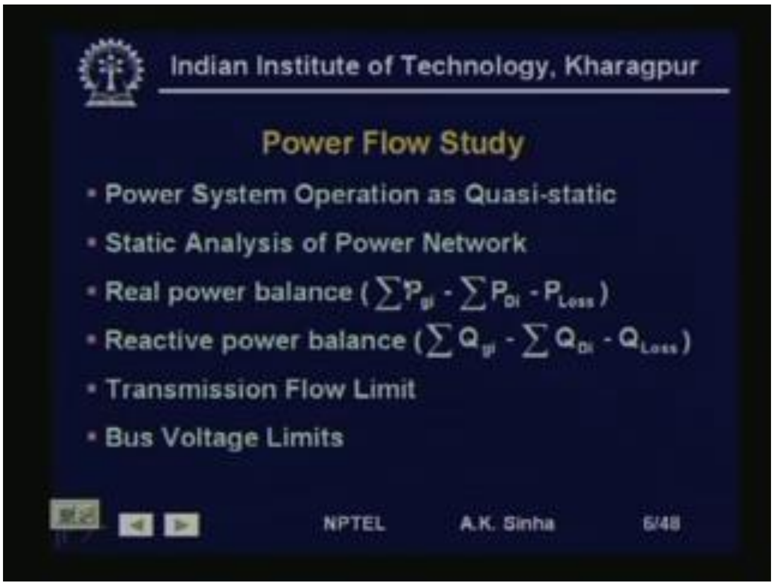

So, what we are trying do in power flow study. We are considering the power system operation as quasi-static operating condition. Power flow study is basically a static analysis of power network. We use the equations as real power balance at each bus. That is sum of the total generation, in the network is equal to the demand plus the losses. That sigma P g i minus P D i, this is the demand and minus P loss, the losses in the transmission system.

This must be valid for the network. Same thing for the reactive power also. Sigma Q g i minus sigma Q D i minus Q loss, is what gives us the reactive power balance. That is sigma Q G i minus sigma Q D i minus Q loss must be equal to 0. And what we also need to see is some time, that is the output of the power flow, will tell us how much power is flowing, and therefore if we know the transmission capacity of that transmission line.

Then, we can find out whether line flow limits are violated or not, that is lines or overloaded or not. Similarly, we can check the bus voltages. And find out whether the voltage is within the operating limits or not. So, this is what we do in power flow study, and how we do this we will see now.

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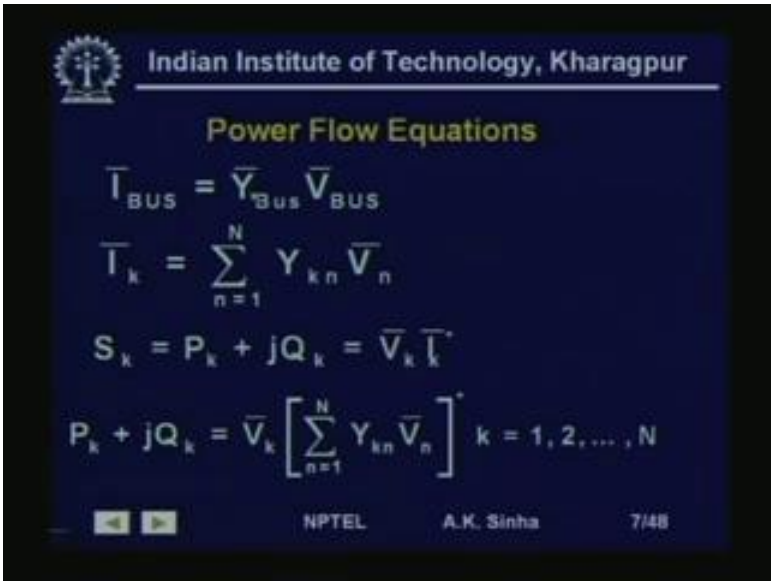

We can write the bus current injection at all the N buses in the system. As I bus is equal to Y bus into V bus. And where we have the current injection phasor at any bus k, is written as I k is equal to summation n is equal to 1 to capital N, where capital N is the number buses is equal to Y k n into V n, where Y k n is a complex admittance of. That is the k, k through and nth column element of the Y bus matrix. And V n is the voltage phasor at bus n.

Now, knowing this I we can write the complex power injection at bus k. As S k is equal to P k plus j Q k is equal to V k into I k conjugate, where V k is the voltage phasor at bus k. And I k conjugate is the conjugates of the current phasor injected at bus k. Now, this I k, we can replace by this relationship. So, P k plus j Q k is equal to V k phasor multiplied by I k conjugate. So, we have taken the conjugate here and I k we have written same as this one. This is applicable to all buses k is equal to 1 to n. That is from 1 to n all buses this expression can be used, where we will replace k by 1, 2, 3 and so on.

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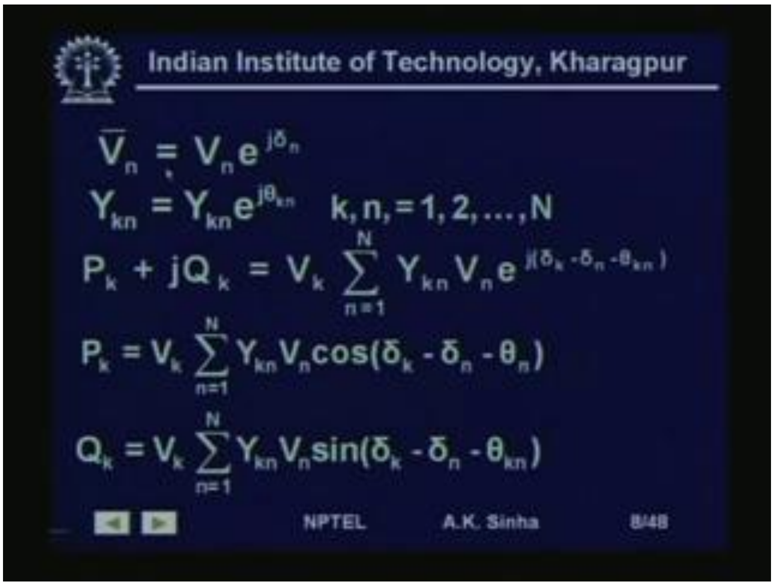

Where the voltage phasor at bus n, any bus n is V n is equal voltage magnitude at that bus, this V n e to the power j delta n, where delta n is representing the phase angle of the voltage phasor. Y k n is equal to, since Y k n is a complex quantity. This again in phasor form can be written as Y k n e to the power j theta k n, where k n n are from 1 to N. So, now in these terms, if we want to write this in terms of phase angles and magnitude for the voltage and the admittances.

Then, we will write this power injection or complex power injection at bus k as P k plus j Q k is equal to V k summation n is equal to 1 to capital N. Y k n V n, where Y k n and V n are the magnitude values of the admittances element k n. And the voltage phasor at bus n e to the power j delta k minus delta n minus theta k n. Now, these terms delta n and theta k n are negative. Because, we have to take the conjugate as shown here, we have to take the conjugate, so angle is negated.

So, we get this expression, phasor expression like this for the complex power injection, if we separate out the real and imaginary parts. Then for the real part, real power injection P k is equal to V k summation n is equal to 1 to capital N. Y k n V n cos delta k minus delta n minus theta k n, this should be k n. So, this is the real power injection. That is we are instead of we take the cos of this angle, then we get the real part. When we take the sign of this angle, we will get the reactive part.

So, reactive power injection Q k is equal to V k summation n is equal to 1 to capital N, Y k n V n sin of this angle. That is delta k minus delta n minus theta k n. So, this way we can see that, we have got the real power and reactive power injection, for any bus k. Now, let us see the characteristics of the power flow equation.

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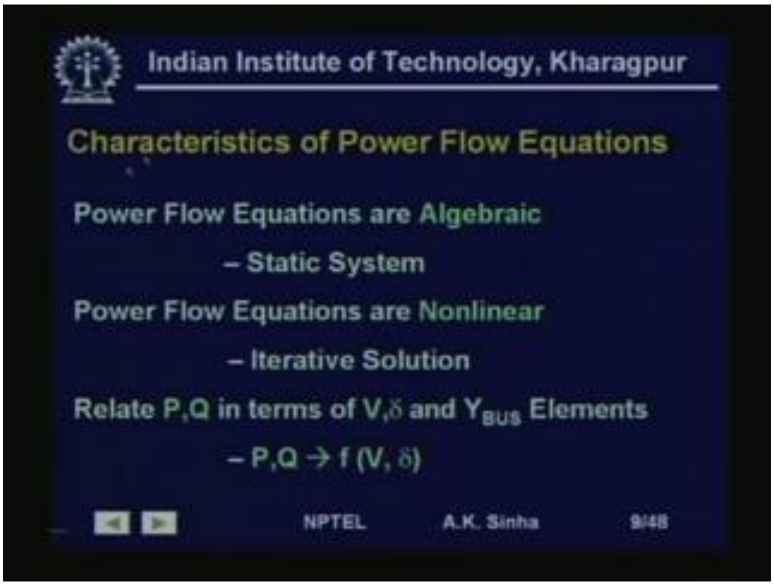

Now, power flow equations as we see from here. These equations are algebraic equations. Since, they do not have they are not a function of time. So, this these equation represent the a static system of equations. That is these equation are representing a static system. We have already said earlier, that power flow is equations basically provide a snap shot value for the voltage. And power flows at various buses.

So, this is a static system analysis, that we are doing. Another thing that we see from ((Refer Time: 15:44)) these equations, are that we have trigonometric functions involved in these equations. So, these equation are non-linear equations. So, power flow equations are non-linear. And since, these are non-linear algebraic equations, they need an iterative solution. Another aspect of these equations we can see, is they involve the voltages and the admittance matrix element for the system.

So, we are writing the real and reactive power, in terms of voltage phases. And the admittance matrix elements of the system. So, we can see that, the power flow equations relate P and Q, the real and reactive power injection at any bus, in terms of V n delta. That is the voltage magnitude angle and it is phase angle and the Y bus elements of the system. So, we can write P Q as a function of V and delta.

That is the real and reactive power are related, in terms of the voltage magnitude. And it is phase angle in terms of course, the system admittance matrix element, which are going to remain constant.

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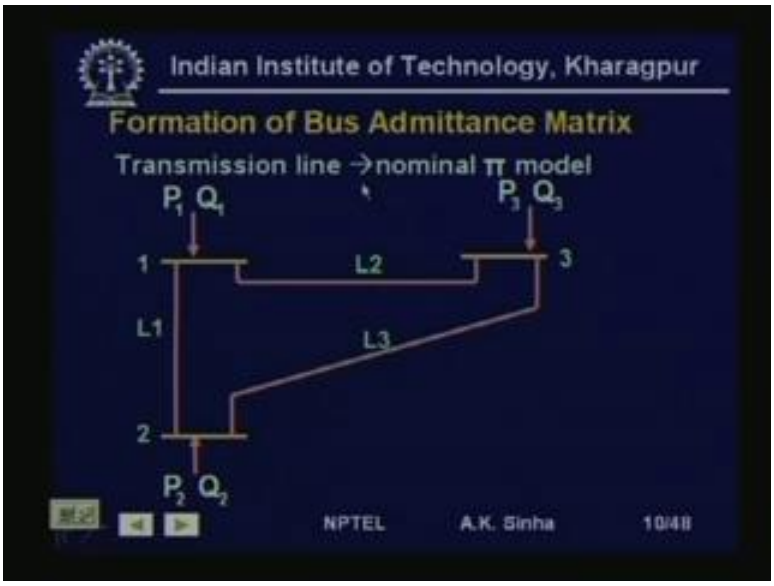

Now, let us take a small system to show, how we can find out the Y bus elements for the system. Now, this is a simple 3 bus system, where we have power injections P 1 Q 1 at bus 1, P 2 Q 2 at bus 2, P 3 Q 3 at bus 3. And we have line L 1 connecting bus 1, 2, L 2 connecting bus 1, 3 and L 3 connecting bus 2, 3. Now, for this system these transmission lines as we have seen earlier are modeled as nominal pi model.

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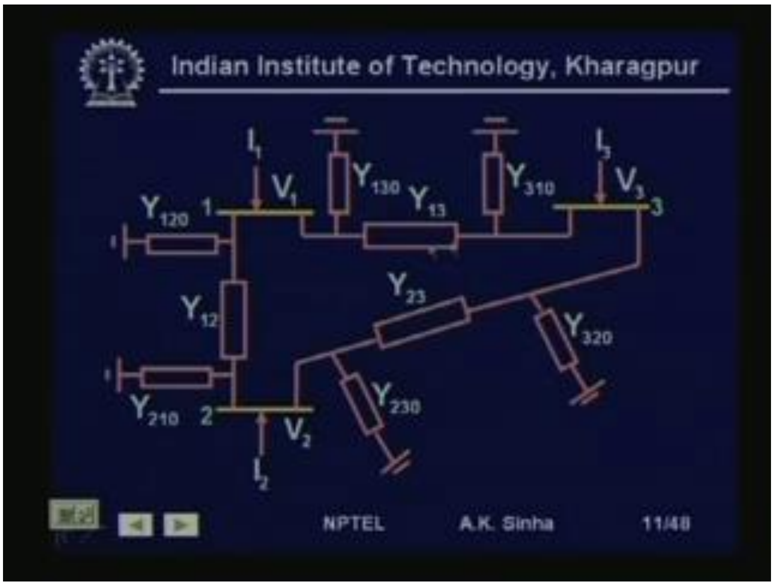

So, if we do that then for each line we have a nominal pi model associated with it. Where these elements Y 2 3 0 and Y 3 2 0 are basically the half line charging susceptances, associated with the line 2, 3, a same thing for the other lines. Here, what I have done is instead of writing the injections P 1 Q 1 and P 2 Q 2, P 3 Q 3, we have converted them as currents for the time being. So, we have a current injection I 1. We have a current injection I 2 here, and a current injection I 3 here.

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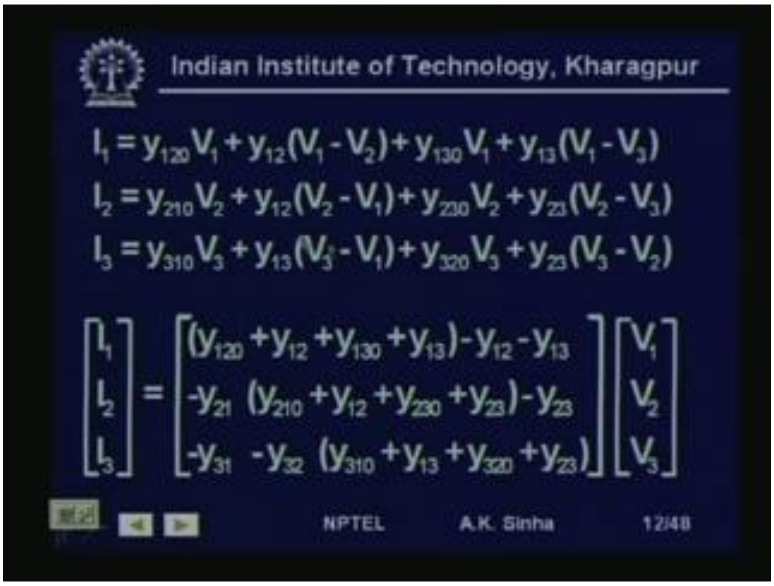

Now, we can write on the relationship for the current injections. In terms of voltages at the various nodes or the busbars. So, we can simply write using the law, the relationship as this. I is equal to Y into V, where Y is a 3 by 3 matrix for a 3 bus system. And we have the diagonal elements are basically coming out to be nothing but sum of all the elements connected to this particular busbars.

So, if you look at busbar 1, what are the elements connected to it. Y 1 3 0 is connected, Y 1 2 0 is connected, Y 1 2 is connected to it and Y 1 3 is connected to it. So, these elements are directly connected to it, and so if we sum up all these elements are what we get as the diagonal element y 1 1 1 for this busbar.

What are the off-diagonal elements, this y 1 2 element for this matrix is nothing but the negative of the value of the element which is connected between bus 1 and 2. Between bus in 1 and 2 what we have connected is this small y 1 2. So, negative of small y 1 2 is basically our capital Y 1 2 which is the Y bus element. Similarly, so off-diagonal elements are nothing but negative of the admittance of the element connected between those busbars.

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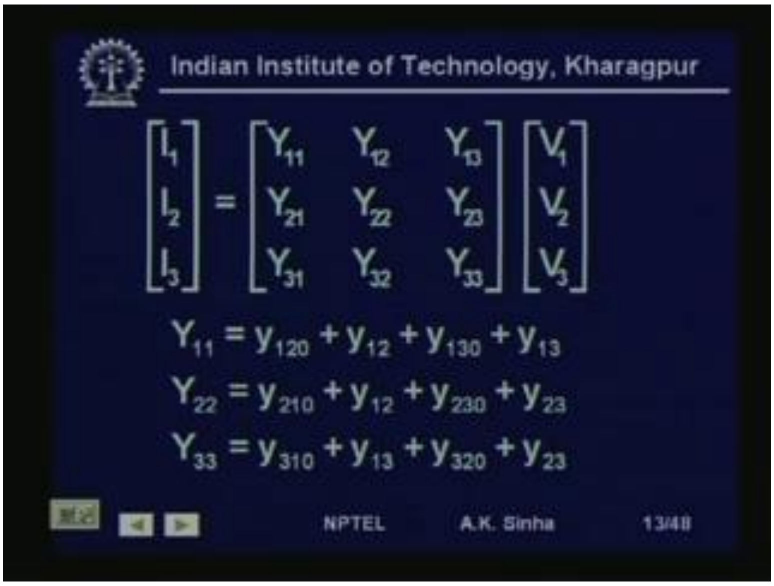

So, finally, we can write this as I is equal to this Y into V, where we as we said earlier the diagonal elements are nothing but sum of all the elements incident to that busbar.

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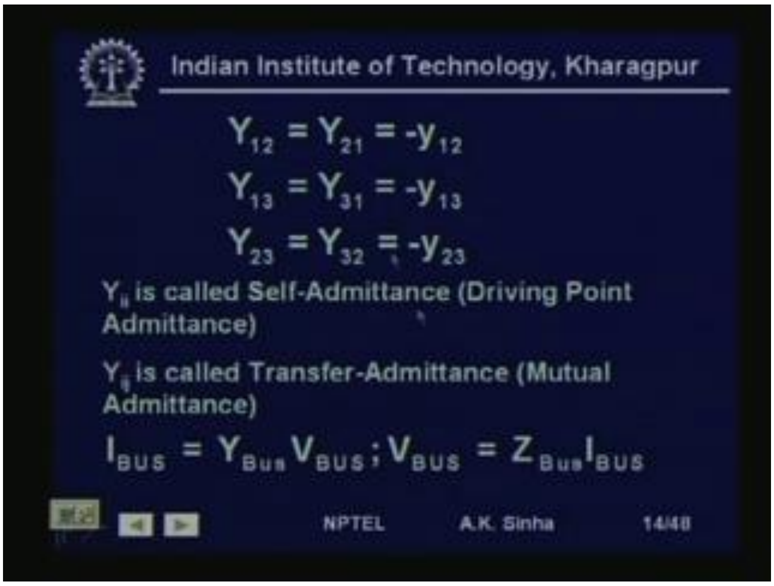

And the off-diagonal elements are nothing but negative of the admittance of the element connected between those 2 busbars. So, Y i i is called the self admittance or driving point admittance. And Y i j is called the transfer admittance or the mutual admittance. And we can write in short form this expression, as I is equal to Y into V. So, these are buses current injections. So, we write this as I bus is equal to this is the Y matrix, which denotes for the nodes. So, this is Y bus matrix and these are the voltages at various busbars. So, this is V bus, so we write I bus is equal to Y bus into V bus.

We can write this same expression also as V bus is equal to Z bus into I bus. But, there is most of the time, we do not use this relationship. Rather we use this relationship, the reason begin that is, that Y bus is much easier to find or compute. We have seen how easy it is to compute, Y bus for any system. Also the other advantage of this is that Y bus is a very sparse matrix. Because, if node I and k are not connected, then Y i k is 0.

And in a very large network say 1000 bus network. What we will find is that, on an average each bus will be connected to just 2 or 3 buses. So, you will find that is in particular row, you will have only 3, 4 elements there, the diagonal element and 2 or 3 elements of the other buses which are connected to it. Rest of the elements are going to be 0, so Y bus is a very sparse matrix.

However, if we invert this matrix to get Z bus, we find Z bus is generally a full matrix. So, the sparsity structure cannot be exploited in that case. And this is one of the reasons why we normally work with the kind of relationship. But, again as I said we do not have current injections, we know all the values in terms of powers. So, we have to write these equations, in terms of power only not in terms of I and V. So, we have to have here, P plus j Q, in terms of Y and V. This is this was just to show you how to formulate Y bus.

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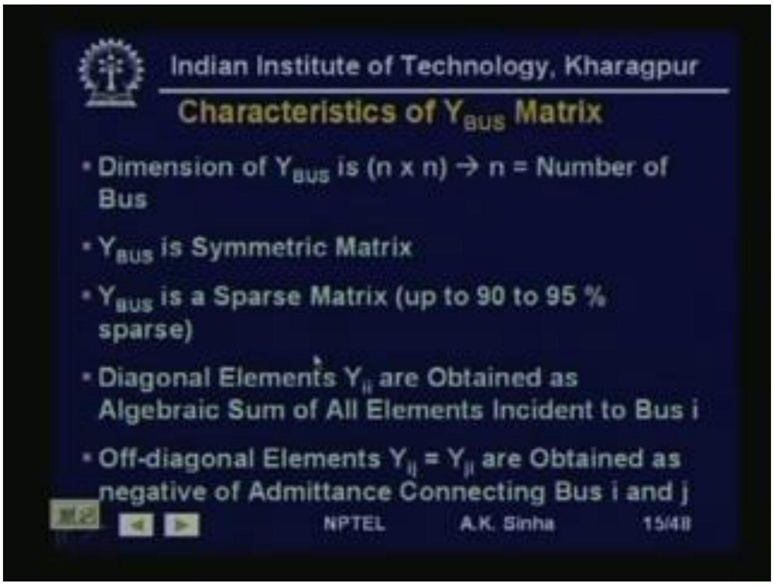

So, what is the characteristics of Y bus matrix. The dimension of Y bus is n into n, where n is the number of busbars. So, for a 3 bus system, we had a 3 by 3 matrix. If we have a 1000 bus system we will have a 1000 by 1000 matrix. Another aspects is Y bus is symmetric. This is mainly because the admittance Y 1 2 is same as Y 2 1, because it is the admittance of the element, which is connecting bus 1 and 2. And since this is a bilateral element, therefore the matrix will be a symmetric matrix. Y bus is a sparse matrix, as I said earlier that the elements, that is the nodes which are not connected, between each other. Those elements I j for those buses are 0. Diagonal elements of Y i i are obtained as algebraic sum of all elements incident to bus i. Off-diagonal elements Y i j is equal to Y j i are obtained as negative of the admittance connected between bus i and j, these all matrix what we have seen.

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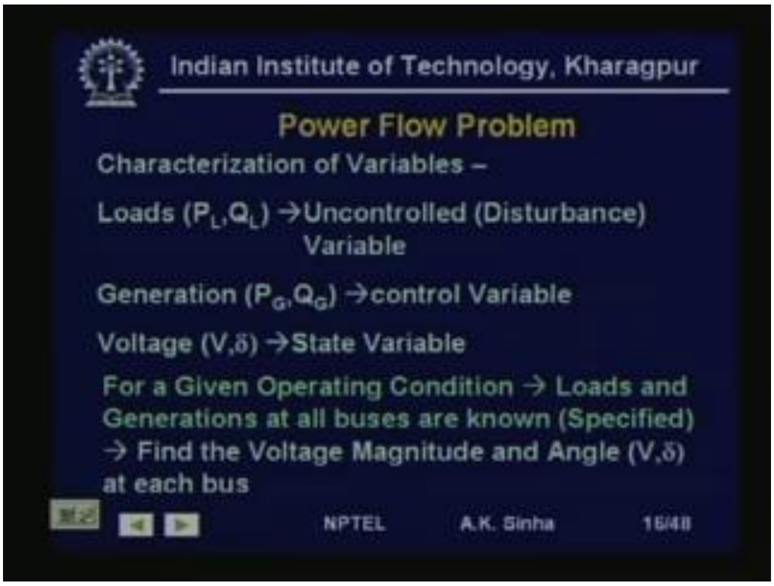

Now, we will go into the power flow problem itself. Now, let us see what we have. Now, we have loads P L and Q L, which are uncontrolled or disturbance variables. Because, loads are not controlled by us, it is controlled by the consumers. So, the power system operator has no control on the consumers. They can switch on or switch off their loads, as they feel like. So, these are controlled or disturbance variables.

Generation is of course, in the hand of the power system operator. He can control the generation, at various generating stations. So, generation P G and Q G are control variables. Now, voltage magnitude and phasor angle V delta at all buses are considered as state variables, because once we know these voltage magnitude and phasor angle delta, at all the buses, then knowing the admittance of the transmission network. We can calculate the power flow, we can calculate power injections; we can calculate all the other variables.

And this is the minimal number of variables. If we know this, then we can calculate all other variables. That is why we call this as state variables. So, V and delta are basically are state variables. So, for a given operating condition, that is loads and generation all buses are given. That is they are specified or known, that is what we say that, we want to find out what are the flows, for what condition. When the generation is such loads are such at various busbars.

Find out what are the flows in various lines, what are the voltages at various busbars, this is the power flow problem. So, for a given operating condition, that is loads and generations at all buses are known or specified. Find the voltage magnitude and angle at each bus. Because, once we know this, we can calculate all other quantities of our interest. So, this is what the power flow problem is.

Now, what is the problem in trying to find a solution, for this kind of situation. If we see here, what we had stated is all the loads and generation at all buses are known. Now, this condition is not true, why it is not true? Because, if we may know all the loads. But, we do not know all the generations, for the simple reason that, we cannot know the losses. If we see the real power or the reactive power, the equality is sum of all the generation. Real power generation or reactive power generation is equal to the real power load plus the losses. Now, how do we know the losses beforehand.

We can know the losses, only after we get to know the voltage at all the busbars. Then, we can know the power flow in each line and then, we can calculate the losses. So, we do not know the losses. And therefore, we cannot specify all the generations. This is one of the major problems of power flow analysis, but we can always handle this.

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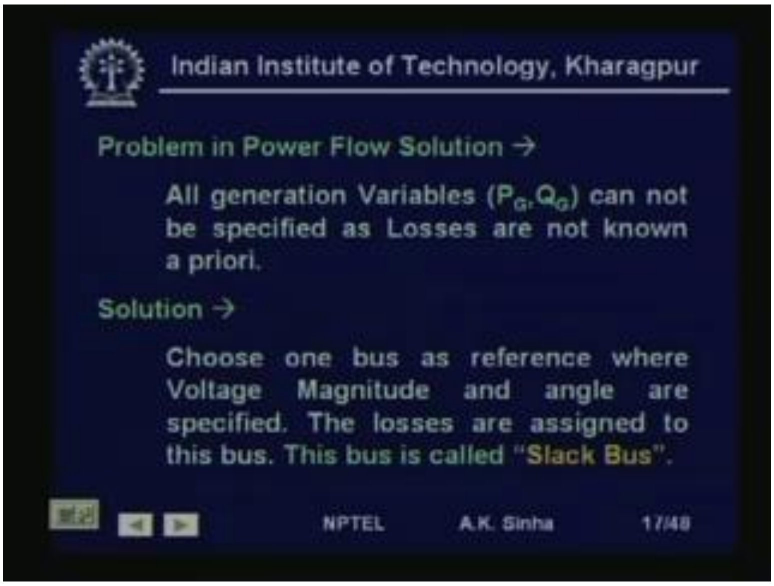

What we do is that, we specify one of the generating bus as a bus, where the power is not specified. Rather the power is allowed to vary. What we do is, we fix the voltage magnitude and the angle at that bus. Normally the angle is fixed as 0, because that is considered as the reference angle, for all the other busbars. And the voltage magnitude is fixed at a particular value.

Now, this is a very valid way of doing things, because a simple reason that generators have automatic voltage regulators, which try to maintain the voltage at it is terminal or at the busbar to which the generator is connected. And therefore, we can assume the voltage at the busbar, generating busbars to be constant. So for this bus which we consider as the reference. The voltage magnitude and phasor angle are specified. And the real and reactive powers are allowed to vary at this bus.

And therefore, what happens at all the other buses are taking other generations. So, whatever losses are coming plus the deficiency and the generation. They are all a sign to this bus. And therefore, this bus is called a slack busbar or a slack bus. So, this is how we try to handle the situation, because we do not know the losses. We cannot specify all the generation. We try to solve it by assigning 1 bus, generating bus as a slack bus.

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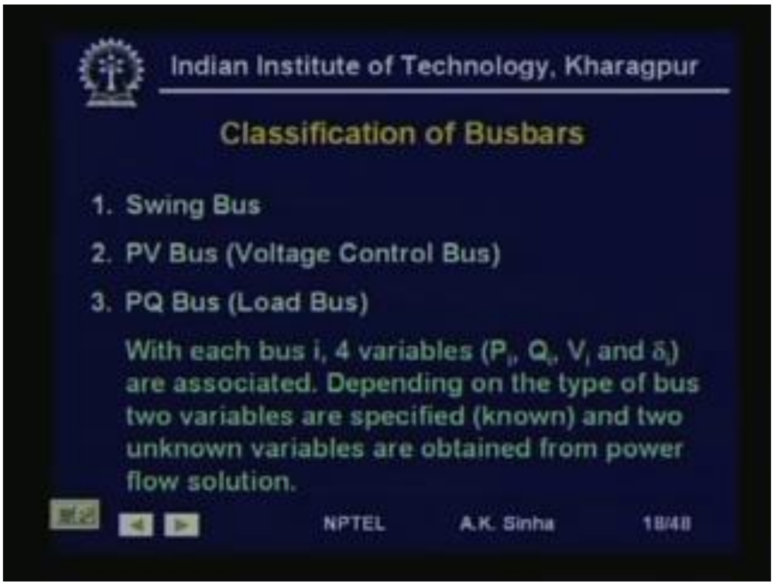

Now, what about other busbars, now some of the busbars where both P and Q are known. Mostly these are the busbars, where the loads are there. So, load buses are substations, which are connected to supply consumers loads. So, there we know the real and reactive power, supplied to the consumers. So, those buses are called P Q busbars or load buses.

And generating buses, as I said earlier have automatic voltage regulators, which can maintain the voltage at it is terminal. Or the busbar to which it is connected, therefore these buses voltage and known. And real power at these buses are also known. What we do not know at these buses are basically the phasor angle of voltage. And the reactive power generated by the generator, because AVR will try to keep changing the excitation. Or vary the reactive power output of the generator to maintain the voltage at that busbar.

So, we see that with each bus I there are four variables associated. What are those four variables, the P, the Q injections. So, at bus i we have P i injection and a real power injection. Q i the reactive power injection, the voltage magnitude V i and the phasor angle delta I at that bus. Now, depending on the type of the bus, two variables are specified or known. And two variables are unknown and are obtained by the power flow solution.

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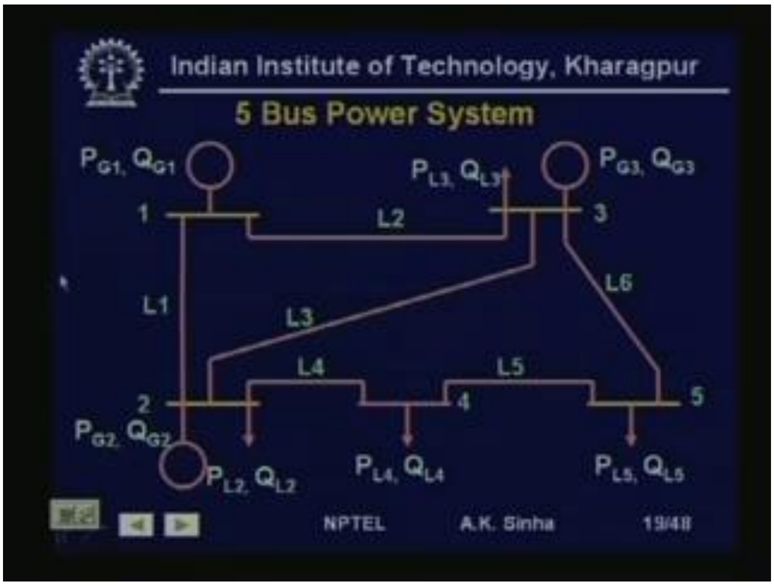

So, again let us take this 5 bus system that we had started with earlier. So, we have this 5 bus system, where we have loads and generation available to us.

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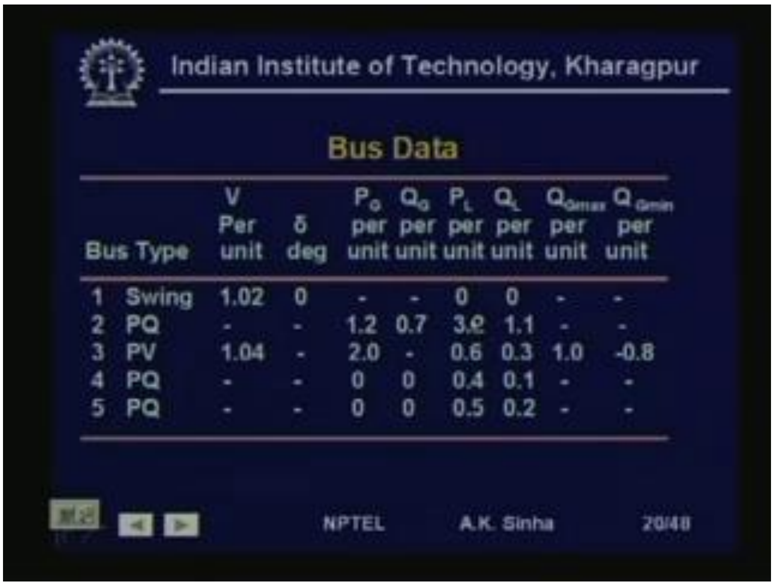

Bus	Type	V Per unit	δ deg	P_G per unit	Q_G per unit	P_L per unit	Q_L per unit	Q_{Gmax} per unit	Q_{Gmin} per unit
1	Swing	1.02	0	-	-	0	0	-	-
2	PQ	-	-	1.2	0.7	3.2	1.1	-	-
3	PV	1.04	-	2.0	-	0.6	0.3	1.0	-0.8
4	PQ	-	-	0	0	0.4	0.1	-	-
5	PQ	-	-	0	0	0.5	0.2	-	-

Now, the data which will be supplied for this bus, will be of this type. That is we have the bus numbers. The type of the busbar will also be given that which bus is considered as a swing bus, which buses the voltages are constant, or maintained. And which buses both P and Q are available. So, here let us say for this 5 bus example, bus 1 is a swing bus. Bus 2 is a P Q bus, bus 3 is a P V bus, bus 4 and 5 are also P Q buses.

And so what we have for P V buses, and the swing bus the voltages are maintained. So, we are given the specified voltage values, magnitude values. Delta angle for the swing bus is 0, the other buses are unknown. The real power is provided at the generating buses, the real power generation. Similarly, reactive power generation at the generating buses. Real power loads and reactive power loads are also specified at various buses.

And at the generating busbars, we also have the reactive power limits for the generators specified. That is at P V bus we specify the reactive power limits. Because, it is excitation cannot go beyond a certain limits. So, the control is has to be within the operating constrain of the machine or the equipment. Therefore, the minimum and maximum value of the reactive power is specified at the P V busbars. If the value if find comes out to be greater than, or goes beyond the limit of the specified value. Then, that bus can no longer maintain the voltage. And therefore, we have to convert that P V bus into a P Q bus with the Q value of that bus, fixed at the limit of the busbar or the rating specified.

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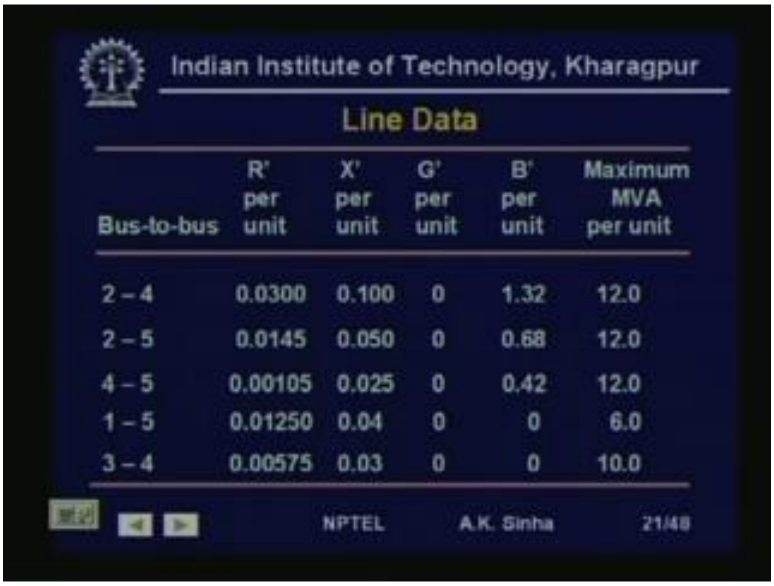

Bus-to-bus	R' per unit	X' per unit	G' per unit	B' per unit	Maximum MVA per unit
2 – 4	0.0300	0.100	0	1.32	12.0
2 – 5	0.0145	0.050	0	0.68	12.0
4 – 5	0.00105	0.025	0	0.42	12.0
1 – 5	0.01250	0.04	0	0	6.0
3 – 4	0.00575	0.03	0	0	10.0

Similarly, we have the data R, X. And the offline surging susceptance for the lines, various lines will be provided along with the ratting of the transmission lines. If you find that the power flow and the transmission line is beyond the rating. We say that the line is overloaded and the operator needs to do something about this.

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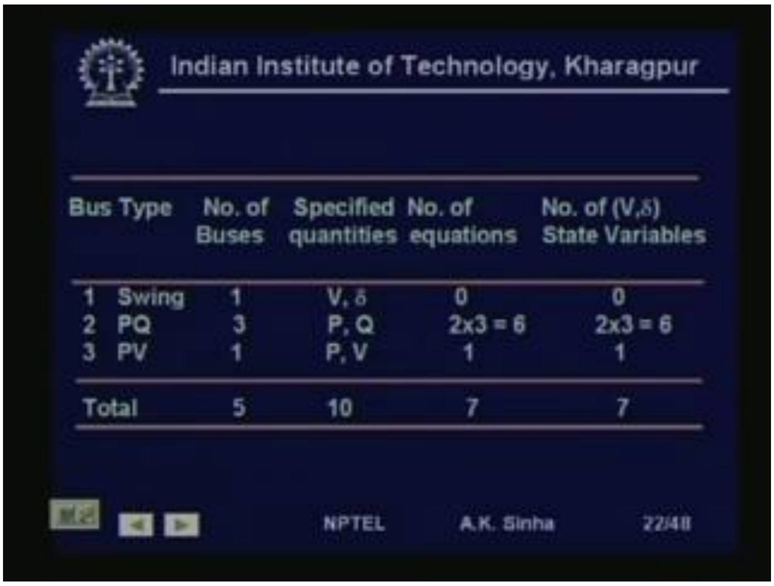

Now, if we look at the power flow problem. Then we find how many equations we need to solve, where the swing bus we have both V and delta specified. So, we do not need to find out V and delta. So, there is no equation, that is required to be solved for the swing bus. For P Q bus in this case, we have 3 P Q buses. And what are the specified values P and Q at these 3 buses are specified. And we have how many equations, we have 6 equations here, and how many unknowns, which we have V and delta for all these 3 buses to be found. So, we have 6 unknowns or 6 state variables which need to be found out. For this system we have 1 P V bus, if we look at the system here, we have 1 PV bus, bus 3. And so we have how many equations, we have at this specified quantities are P and V. So, since V magnitude is known. So, what is the other state variable that we need to find is the delta for this bus. So, the number of equations is only for P, there is no equation for Q, because Q is a variable and V is what is specified.

So, here we need to write only one equation, that is for the real power at that bus. And the number of state variable that, we need to find out is 1, that is delta angle at that bus So, if we see for the system we have 7 number of equations, and 7 number of unknowns which we need to find and this we 2 by means of various solution techniques.

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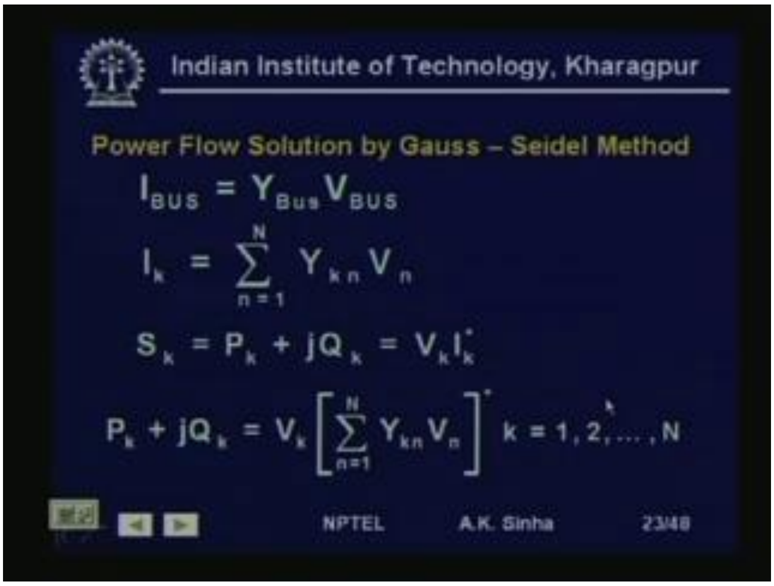

So, one of the techniques which is the simplest one is what we call the Gauss Seidel method. As we have seen earlier the power flow equations are non-linear. And therefore, we need iterative techniques for solution. Gauss Seidel is one of the those techniques, it is one of the simplest techniques for doing that. So, let us start with the very basics, we have the relationship I bus is equal to Y bus into V bus.

And we can write for any bus k, I k is summation n is equal to 1 to N, where capital N is a number of buses. Y k n into V n, Y k n are the k n th element of the Y bus and V n is the voltage at bus N. Now, if you write the complex power injection, S k at that bus this is equal to P k plus j Q k, which is equal to V k into I k conjugate. And we can substitute for I k here, then we have V k plus j Q k is equal to V k into I k conjugate. This is what we need to write for all the busbars k is equal to 1 to capital N.

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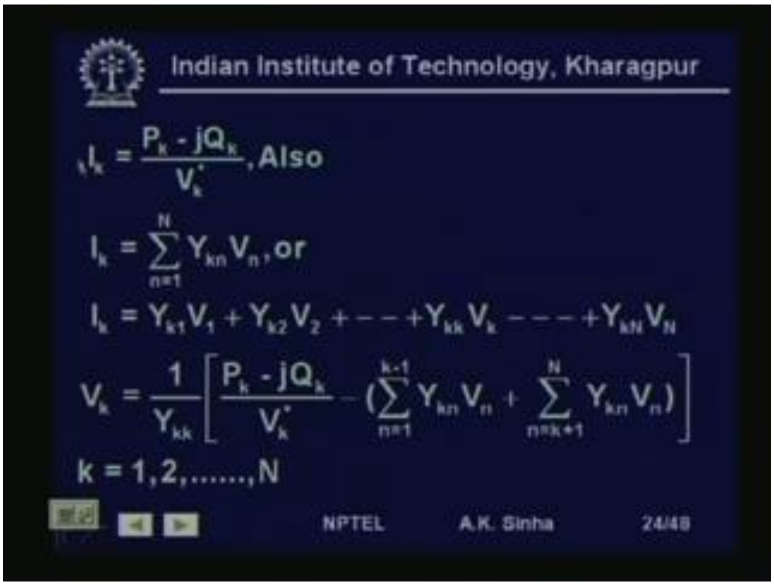

Now, I k is what, the current injection at bus I is equal to P k minus j Q k by V k conjugate. That is instead of writing, here S k is equal to plus j Q k, if you write P k minus j Q k. Then, we can write this as conjugate, V k conjugate into I k. And therefore, I k will be equal to P k minus j Q k by V k conjugate. So, this is what we have written here, I k is equal to P k minus j Q k by V k conjugate. And therefore, and we know I k is equal to this.

So, we can write I k is equal to Y k 1 into V 1 plus Y k 2 into V to N and so on up to Nth. Now, here what we can do is, we can except for this elements Y k k into V k rest all other elements if you take on this side. Then instead of writing this I k, we write this P k minus j Q k by V k conjugate P k minus j Q k by V k conjugate minus summation Y k n into V n. Y k n into V n from N is equal to 1 to k minus 1, that is up to this term.

So, negative all these terms have been taken on this side. Plus Y k n into V n, from n is equal to k plus 1 to N, that is from here to here. So, leaving this term rest all the terms have been taken on this side. So, this comes out to be this expression within the bracket. And now, we divide this by 1 by Y k k. Then we have got V k is equal to 1 by Y k k into this term. And this is the expression for all the bus voltages, we can write this. Now, here all these quantities Y k n V n, all these quantities are complex quantities except for P k and Q k. So, and V k is also a complex quantity.

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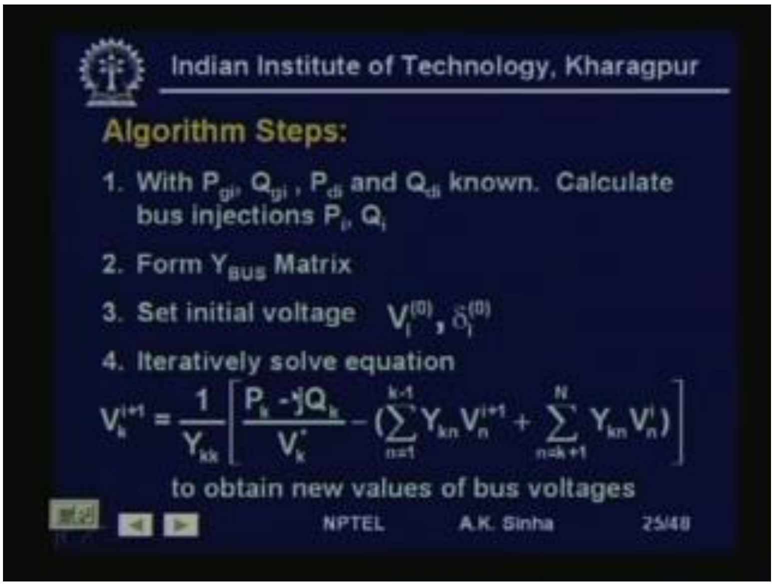

So, if we have this expression, now we can put it into an iterative solution procedure. The algorithm steps are with P g i, Q g i, P d i, Q d i known, calculate the bus injections P i, Q i. Form the Y bus matrix, set the initial voltages V i 0 and delta I 0. As I said earlier, we can choose for P V buses voltages specified, for all other buses very simple starting solution is, that the voltage magnitude is 1 per unit.

And since, the voltage phase angle is generally not very large. So, the starting value we choose as 0 degree. So, the initial starting voltage is normally taken as, 1 per unit in magnitude and delta as 0. So, one angle 0 is the starting value for voltages, at all the buses. Except for P V buses where, the specified value is given. Of course, for slack bus V is specified and delta is anyway 0, because that is the reference.

Now, what we do is, we use the iterative solution here. That is for I plus 1 th iteration, for voltage at k th bus. We can write this expression like this. And here, the only thing that we are seeing is at, we are using the most updated values. That is if we are doing at for bus 5, then up to bus 4, we have the updated value for I plus 1th iteration. So, we are going to use that, so this is what speeds up the solution somewhat.

So, this is how we work for the Gauss Seidel method to obtain the value of the voltages. We keep on this iteration technique going till, we find that the change in the voltage. In between the two different iterations is very small, that is below our threshold value.

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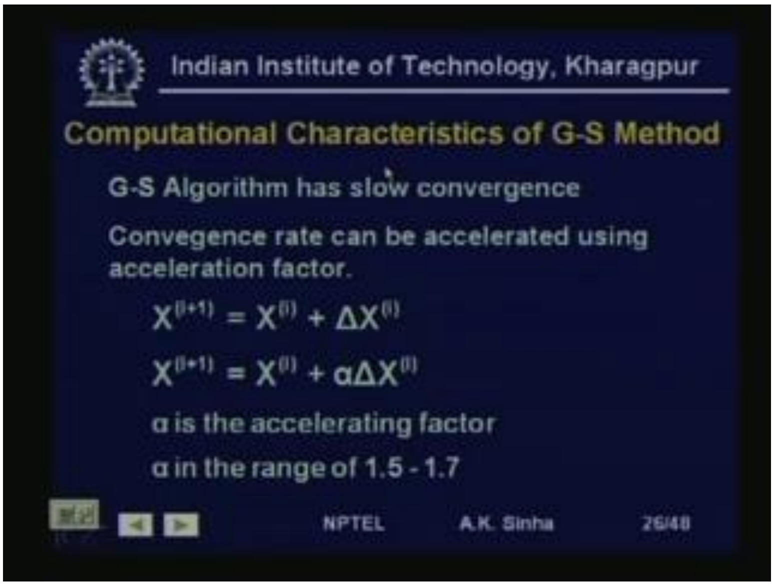

Now, computational characteristics of this method is very simple. But, it has slow convergence. So, for large power systems this method is generally not preferred. Now, people have tried to speed up the convergence characteristics of this by using some acceleration factor. So, instead of using this delta X, addition between the two voltages, we multiplied by alpha also. So, X at I plus 1th iteration is equal to X at i th iteration plus alpha delta X i.

What is this delta X, this is basically X is what our voltage complex voltages. And delta X is nothing but the difference of the voltage between two successive iterations for that particular bus. So, instead of using delta X, X i plus 1 is equal to X i plus delta X. We multiply this delta X by alpha, where alpha is between 1.5 and 1.7. Then, it is found that the convergence improves somewhat. But, still the convergences much slow compared to other methods. And therefore, this Gauss Seidel method is now a days not preferred for large power system, power flow analysis.

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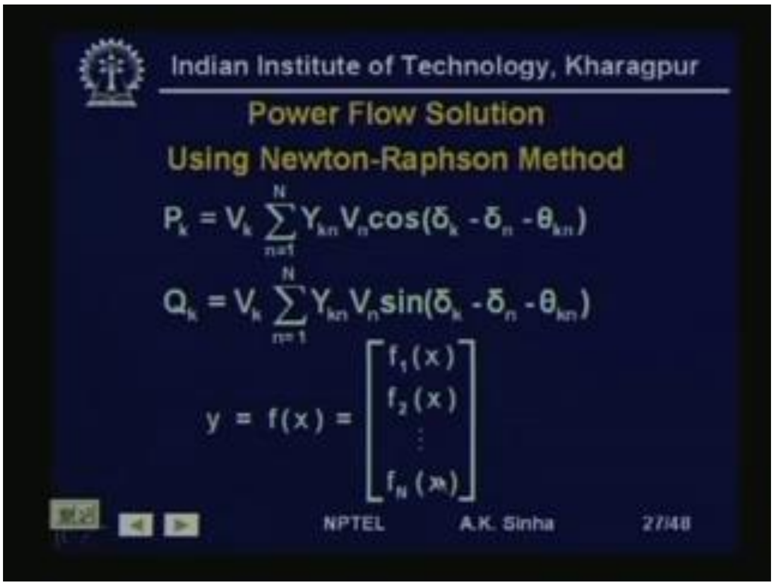

The other methods, which have much faster convergence are methods such as Newton-Raphson method. Now, here again if we go back and to our power flow equations. The power injection V k and Q k, in terms of V and delta and the circuit elements. We can write this expression as we have down earlier. Now, we can write this as for each bus, if you are writing. Then, this we can write as y is equal to f x, where we are writing this f x is f 1, f 2 up to f n, where n is the number of equation, that we have. If we have say n number of nodes, then we will have two n equations, one n number of equations for P and n number of equation for Q.

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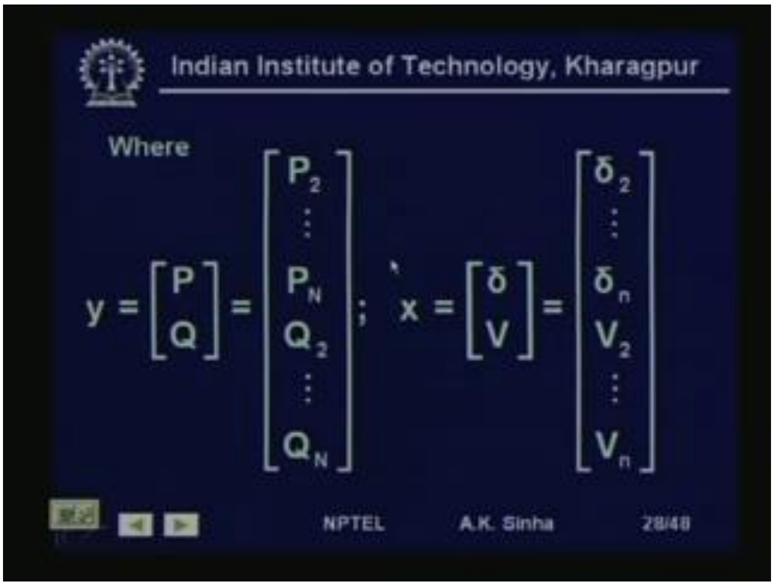

So, here y is equal to P and Q which is P 2 to P n and Q 2 to Q n. Now, here why we are not taking P 1 and Q 1, the reason behind that is, we have assumed that bus 1 is the slack bus. So, that is y we do not also calculate delta 1 and V 1. So, X is our delta V, which is delta 2 to delta n and V 2 to V n.

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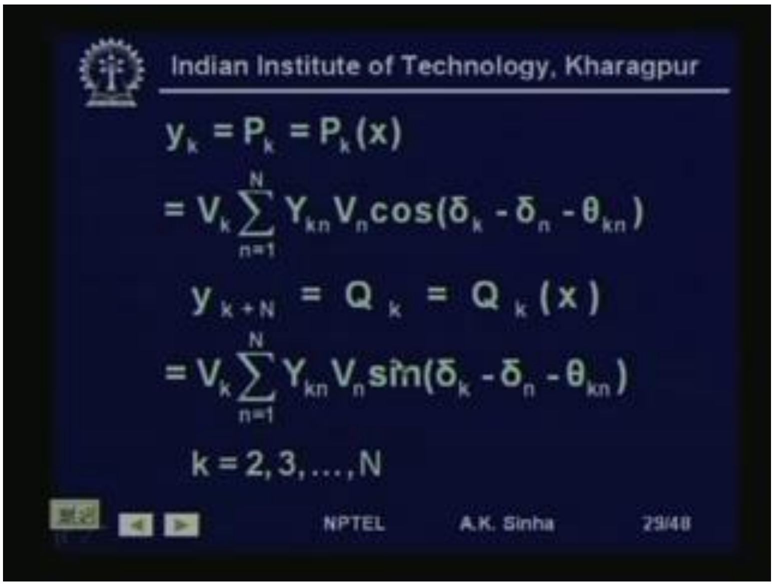

So, again we have y k is equal to P k which is a function of X. And this is written same expression, as we have written y k plus N is Q k. So, what we are doing is arranging all the equations P equations first and then, the Q equations. So, we are writing in this form. So, Q k is like this or y k plus N is the Q k expression like this, k is equal 2 to N. So, this is have we can arrange the equations. And now if we can see this equation are non-linear equations. So, we can always expand them in Taylor series.

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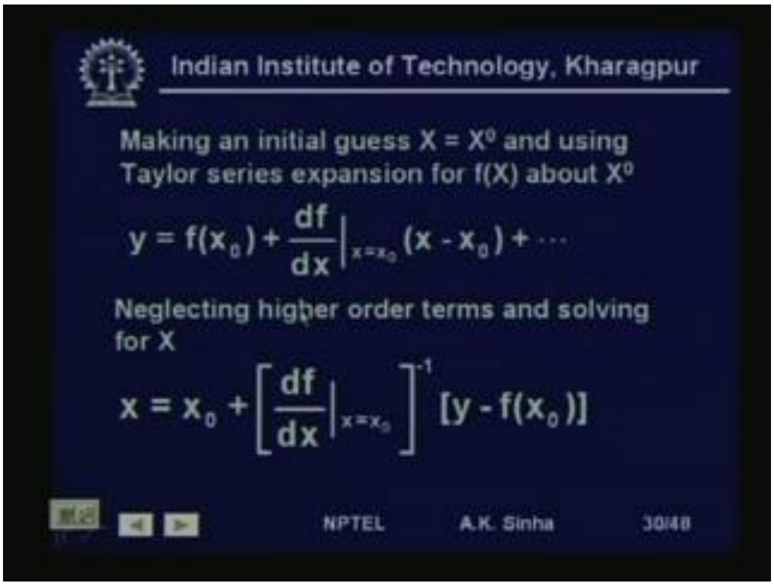

So, if we do that, then we can write that as Taylor series. In terms of y is equal to f x 0 plus d f by d x at x is equal to x 0 into x minus x 0 plus higher order terms, which will be of x minus x 0 square and so on. Now, the since if we say that our initial gas, that is x 0 is very close than the higher order terms, which are x minus x 0 square. And other terms are going to be much smaller. So, we can neglect them. So, neglecting higher order terms and solving for x. We can write this as x is equal to x 0 plus d f by d x at x is equal to x 0 inverse into y minus f x 0. That is from here this expression itself, we are calculating x like this.

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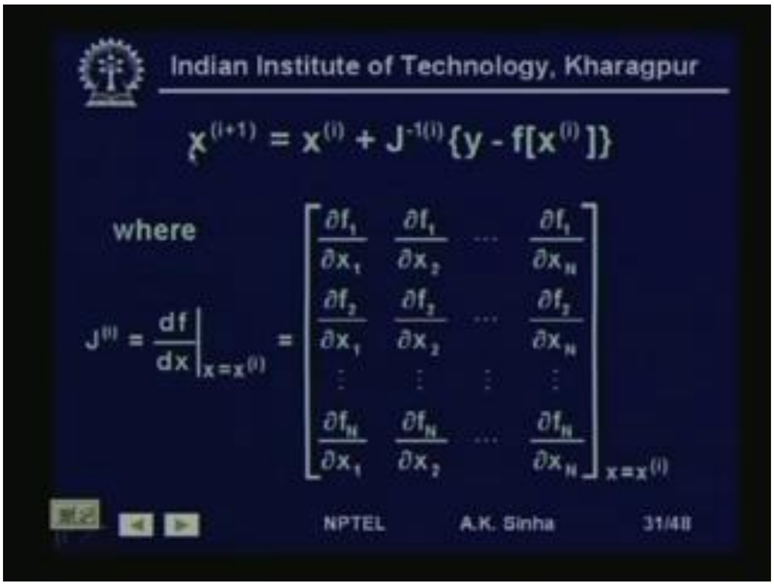

So, once we have this expression like this, in iterative form. We can write that x at I plus 1th iteration is nothing but equal to x at i th iteration. Plus J inverse calculated at i th iteration value, that is at x is equal x at i th iteration into y minus f at calculated for x at i th iteration. And this J is the Jacobian matrix, because we have n number of equations. And this is the partial derivative of all those equations.

So, J i is equal to d f by d x at x is equal to x i is given by this matrix, which is del f 1 by del x 1 and so on. So, this is N by N matrix, if you see for the n number of variables, that we need to solve.

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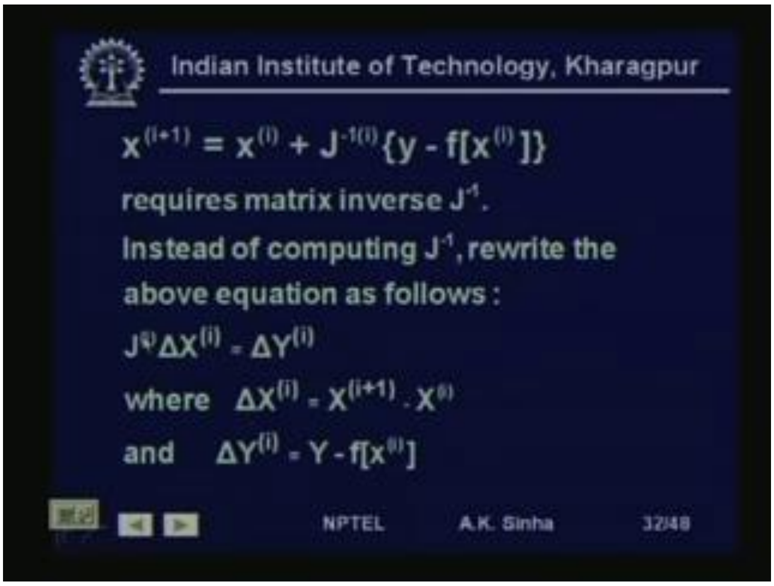

Now, if we look at this expression, what we find as we have to require an inverse of this matrix J. So, we need J inverse, which is for a large system going to be very large. Because, suppose we are taking 1000 bus system, then we have say 2000 equations to be solved. And in that case, the J is going to be almost 2000 by 2000 and taking inverse of such a large matrix, is going to be very time consuming. Therefore, we can arrange this equation in a different form like this. That is J into delta X is equal to delta Y, where this term is delta y and this minus this is delta X. So, J into delta is equal to delta Y, where as I said delta X is this term delta Y is this term.

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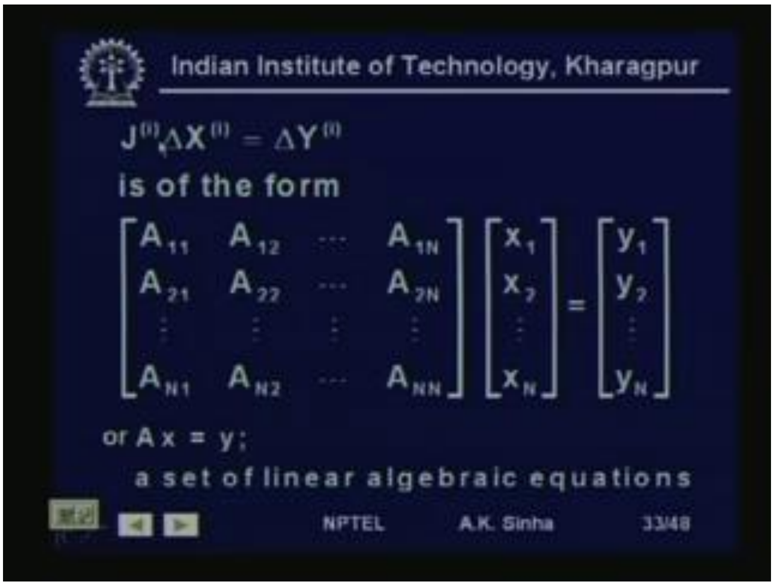

So, if we write this, then this can be put in this form. That is this matrix is a Jacobian matrix J, which we can write as a form A x is equal to y or A x is equal to b whatever you say. So, this is a set of linear equations form. And since this matrix, Jacobian matrix will also be a sparse matrix. Because, it depends on the is elements of the y bus matrix. And therefore, sparse matrix techniques are used to solve this equation.

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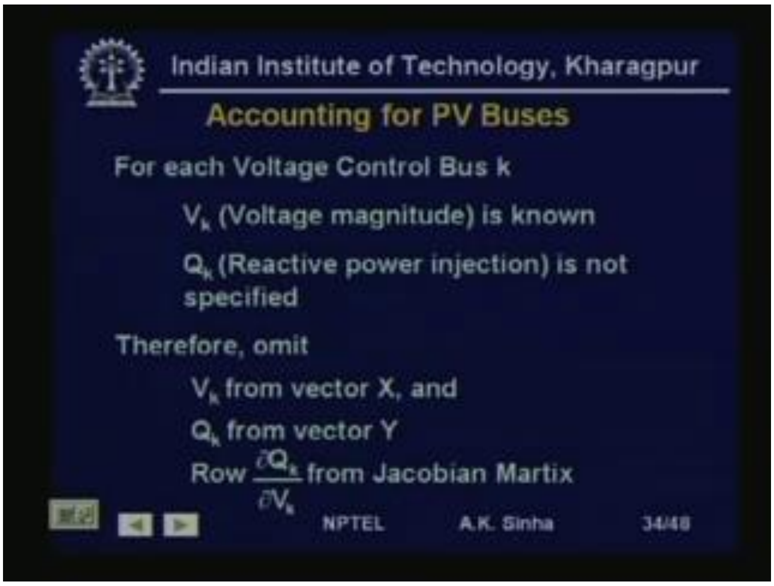

Now, what we do for P V busbars, for P Q busbars we have two equations. For P V busbars, we know the voltage magnitude. So, we do not need to write the equation for the voltage magnitude. So, what we do is, since Q is not known for that those busbars.

So, there is no equation for Q. And therefore, the row of the Jacobian matrix for Q also for that bus is eliminated. And we do not need to calculate V for that particular bus. So, V k from the vector X, Q k from vector Y. And row del Q k by del V k from the Jacobian matrix are eliminated.

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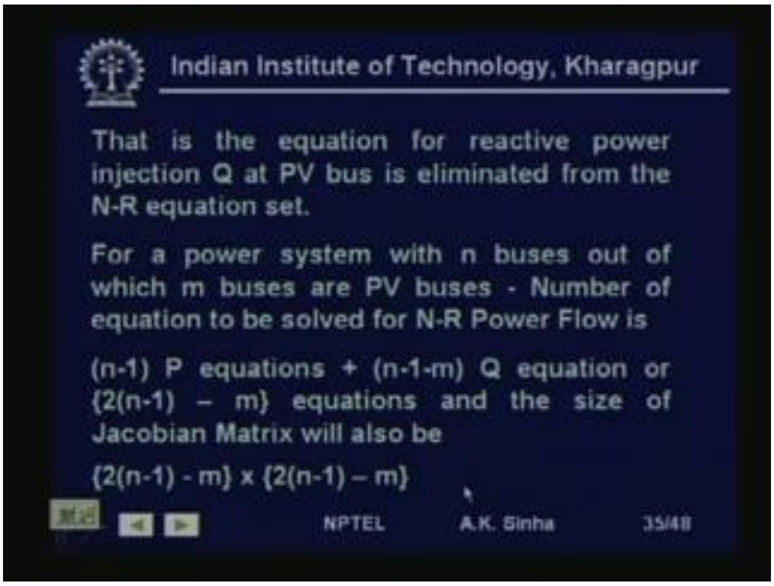

And what we get is that, we get less number of equations. Also the size of the Jacobian becomes much less, because if we have say a power system with say n buses and with m P V buses, then we have n minus 1 P equations. One equation for slack bus is not required and n minus 1 minus m Q equations, because one equation for slack buses is not required and m equations for P V buses are not required. And therefore, we have finally 2 into n minus 1 minus m equations. That we have and also the Jacobian matrix is 2 into n minus 1 minus m into 2 into n minus 1 minus m size of the matrix.

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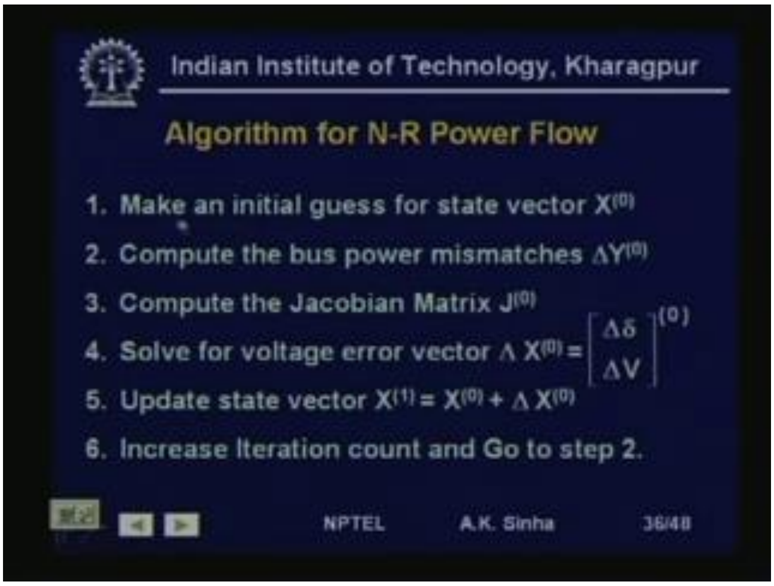

The algorithm is again very simple, make initial guess state vector X 0. That is again flat start we assume all bus voltages magnitude to be 1, where it is not specified. And the phase angle at all buses are assumed to be 0. Compute the bus power mismatches delta Y, compute the Jacobian matrix. Solve for voltage error, vector update, state vector increase the iteration count and again go to bus power of mismatch calculation. So, this is how this Newton Raphson method works.

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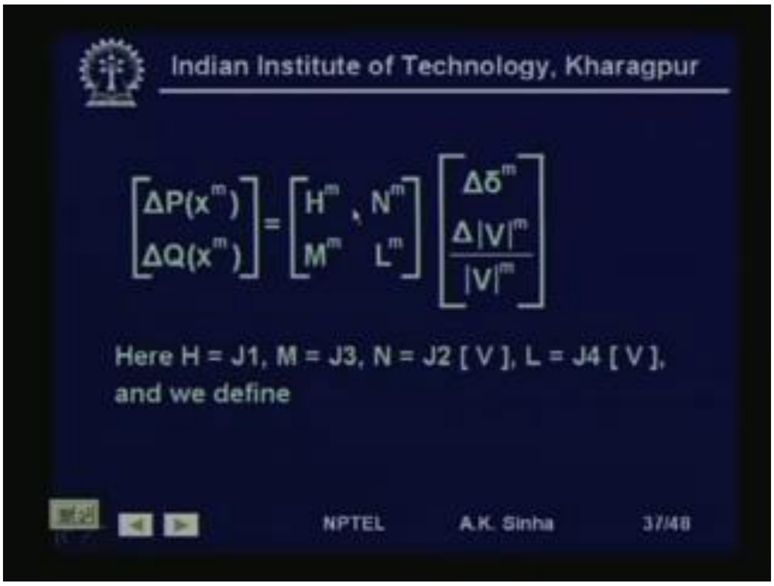

Now, we can try and make some improvement in this. That is the calculation of Jacobian elements can be again reduced. If we multiply the element of this, del P by del V term by

V and del Q by del V term by V. Then, what happens is the all the off-diagonal elements for this matrix. And this matrix, this matrix and this matrix, they will be coming out to be same terms. And therefore, the number of terms that we need to compute for the Jacobian elements gets reduced considerably, then if we are multiplying it by V this to sub-matrices. Then, we are dividing by V this state variable term here for the Newton Raphson algorithm.

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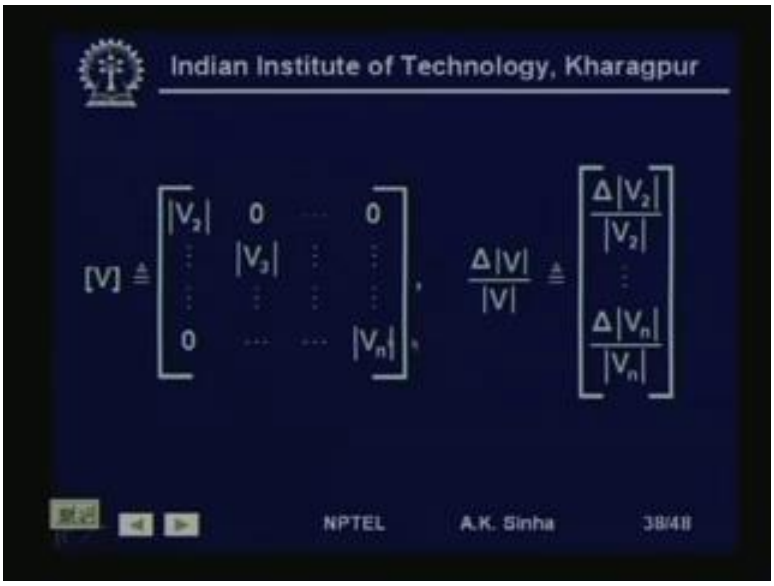

Of course, this is the same thing that we talked about.

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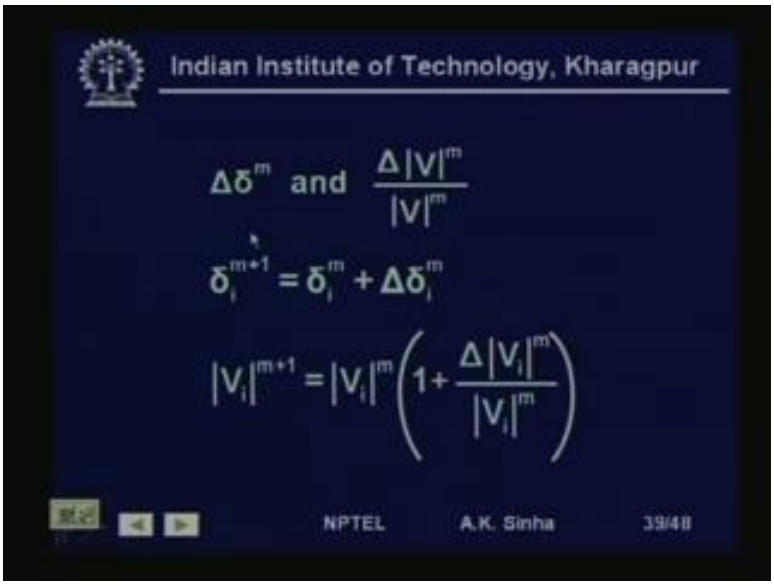

So, instead of updating, what we have is delta m plus 1 iteration is delta I at m th iteration. Plus delta I at m th iteration, and similarly for voltage, because we are dividing the voltage. So, we are getting this term, so update is done accordingly.

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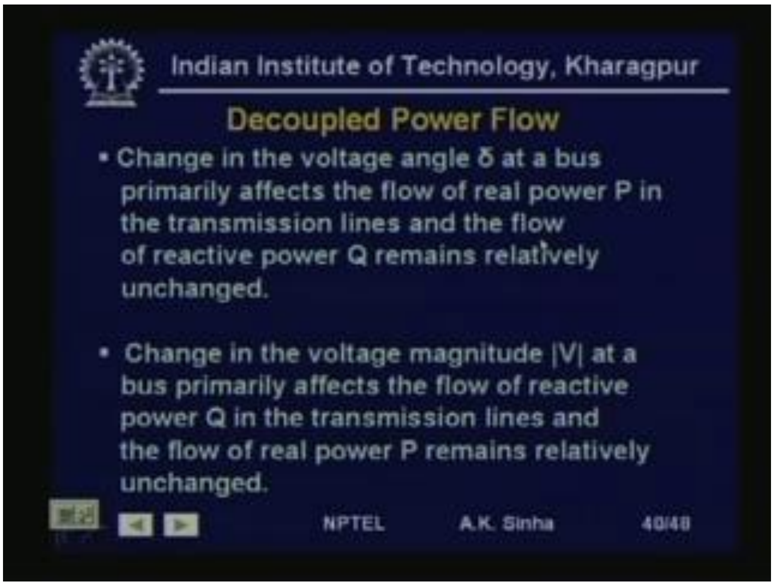

Now, one thing which we get is that, we know that the real power and voltage phase angle are very strongly coupled. And reactive power and voltage magnitude are strongly coupled. Whereas, there is a very weak coupling, between P and V, that is voltage magnitude and real power. And Q and delta, that is reactive power and voltage phase angle. This is also found from these, if we check the elements of these matrices.

Then we will find the elements of these matrices N and M are very small, compared to H and L elements of H and L. And therefore, we can think of decoupling the equations by neglecting those two terms. That is by assuming del P by del V terms to be 0, and del Q by del delta terms to be 0.

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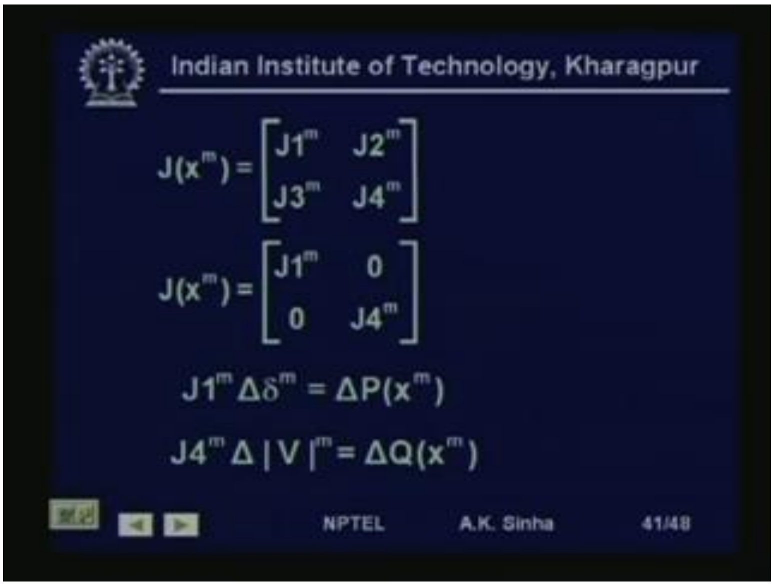

So, this is what we try to do, that is here as it is shown these two sub-matrices are assumed to be 0. If we do that, then we have two separate equations, that we can solve and the size of the Jacobian matrix becomes much smaller. This is what we do in decoupling.

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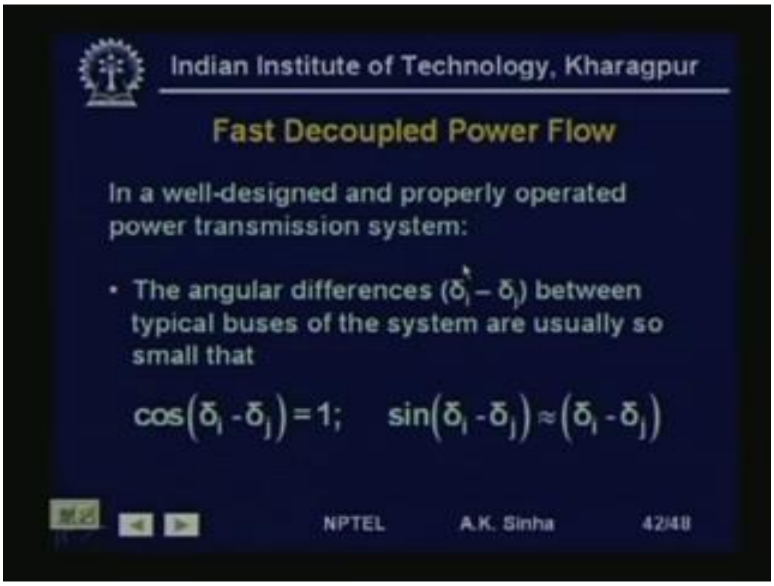

If we make certain other assumptions, then we can make the elements of the Jacobian matrix also constant.

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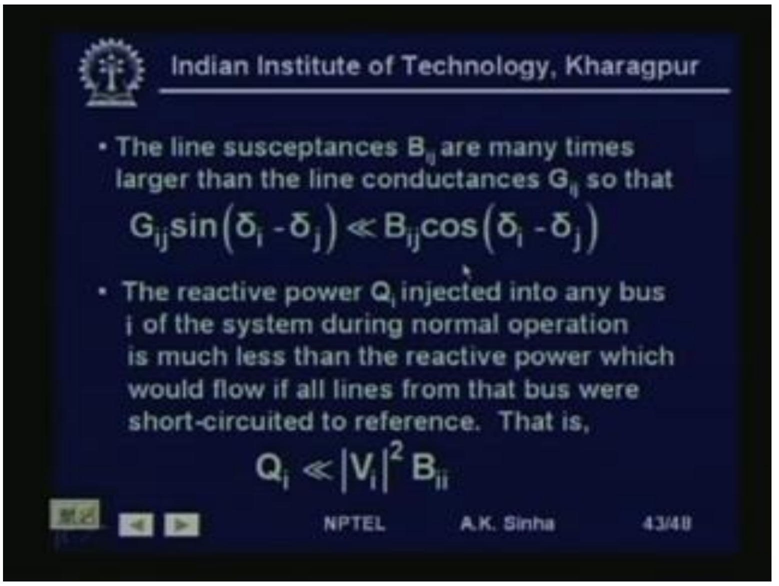

This is what we do in case of fast decoupled load flow algorithm. In fact, in that case the Jacobian matrix terms becomes constant. That is we have terms as V dash and V double dash which are nothing but the susceptance elements of the Y bus matrix. Negative of the susceptance elements or the elements of the Y bus, susceptance elements of the Y bus matrix.

So, these ((Refer Time: 56:44)) two are decoupled equations. That needs to be solved for to calculate delta delta and delta V. This is what we do in fast decoupled load flow, since here these matrices are constant. So, what we need to do is factorize that only once at the beginning. And these factors can be retained for all other iterations. This makes this fast decoupled algorithm very fast. And this method is pretty accurate and therefore, this is one of the most popular algorithm, that we use.

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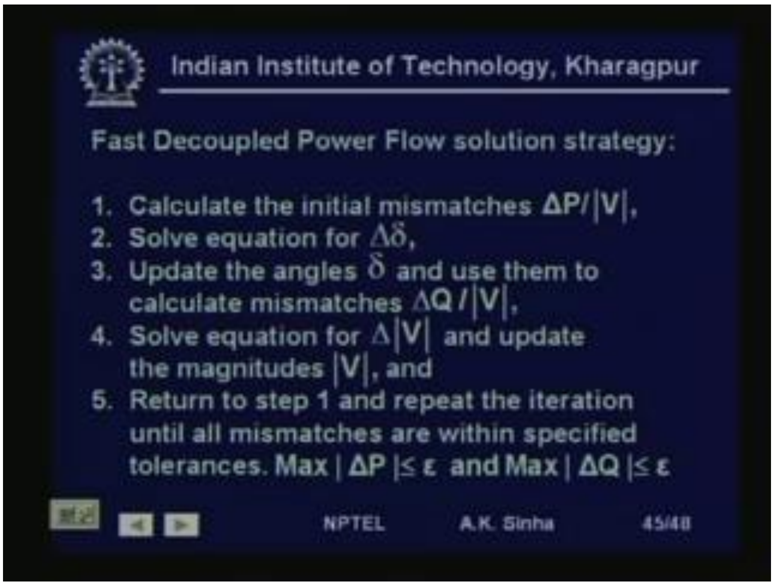

So, again the algorithm for fast decoupled is shown here, which is what we have done earlier.

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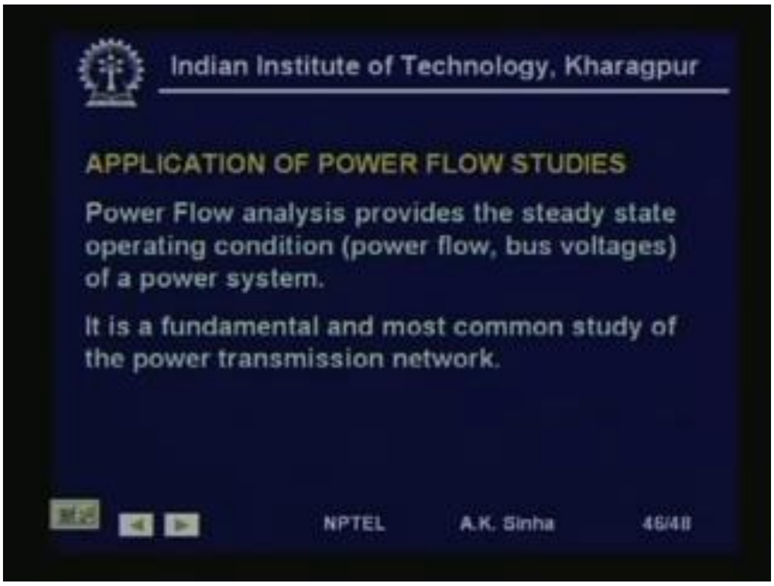

Now, what are the applications of the power flow analysis. Power flow analysis provides the steady state operating condition. That is power flow bus voltages in a power system. And it is a fundamental and most common study of power transmission network.

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It is an integral part of most of the planning and operating studies. That we do in power system. It is also a starting point for many other studies. Such as short circuit studies, transient stability analysis and other studies, that is all.

Thank you very much.