Power System Analysis
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Lecture - 23
Review of Power System Component Models

Welcome to lesson 23 on Power System Analysis. In this lesson, we will review the topics that we had discussed earlier on Power System Component Modeling.

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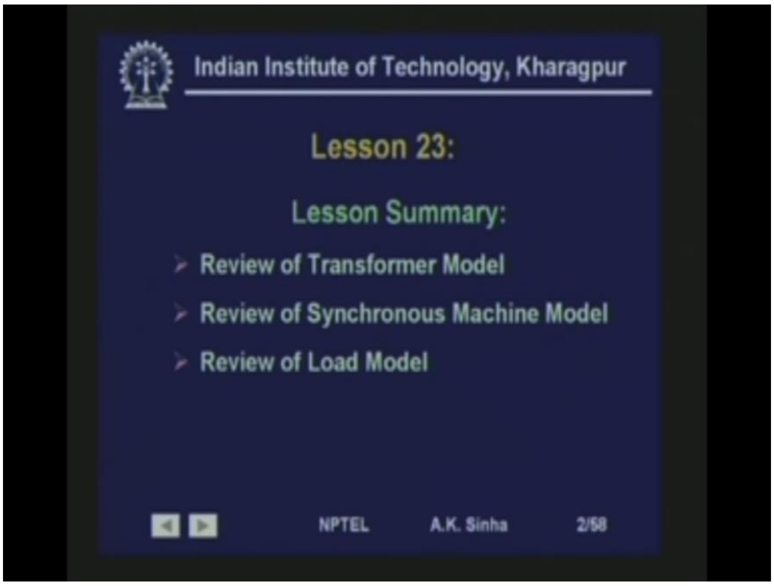

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Lesson 23:

Lesson Summary:

- Review of Transformer Model
- Review of Synchronous Machine Model
- Review of Load Model

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We will start first with the transformer model. Then, we will talk about the synchronous machine model. And finally, we will review the load models. So, since this lesson is basically a review. We will go through the whole thing at a faster phase, because we have already gone through this details of this earlier.

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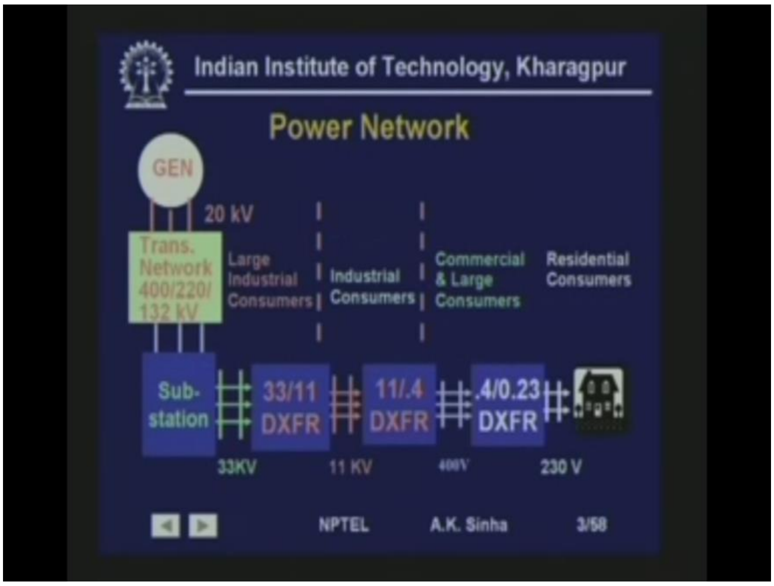

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Power Network

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So, we will start with the transformer model, as we have said earlier. We have in a system different voltage levels. The generator generates at an optimum voltage level, which is somewhere around 11 to 25 KV. So, generator voltage can be anywhere in between that. For transmission, since we are transmitting large amount of power over large distances. We need to transmit it over very high voltages.

And therefore, we need transformers to step up this voltage from the level of 11 to 25 KV, to a much high voltage of the order of 132 KV or 220 KV or 400 KV, depending on the amount of power to be transmitted. And the distance over which it has to be transmitted. So, we need a transformer at this point, where we connect the generator to the transmission system.

These transformers are called generator transformers. And many times these transformers are coupled with each unit of the generator. So, they are also many times called as unit transformers. So, each generator has it is own transformer. So, they form 1 unit of generator and transformer. After transmission, when we have bought the power to load centers, there we have to distribute it to various loads.

And that is done at lower voltage levels which we call the sub transmission voltage levels. So, these voltage levels can be 66 KV, 33 KB and so on. So, we need again a transformer to step down the voltage from the transmission level to sub-transmission level. And so we use these transformers, step down transformers here, at the substation to step down the voltage to may be 33 KV. After that again it is step down to lower voltages for distribution to smaller consumers.

It can be brought down to 11 KV and then, further brought down from 11 KV to 400 volt or 415 volt to be precise. And there this 415 volt three phase power is transmitted and then, you can get it at your home. Here at 230 volt single phase, when you get connected to a line and a neutral of a 415 volt three phase system. So, this is how the power network works. And as we see at each stage we need the transformers.

So, we have to model these transformers mathematically, when we are trying to analyze the transmission system. So, we will now go into the modeling of the transformers. So, before we start with the mathematical model. We must understand that, there are two kinds of transformers that we use. This depends on the basic construction principles.

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Shell Type Transformer

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One type is called the shell type transformer, where the core as you see here, this is made up of iron stampings. And the windings, there are two windings, the primary winding or this winding where current i 1 is flowing, And there is the secondary winding, where current i 2 is flowing. These two windings are put on the central limb. That is the winding is surrounded by the core, this kind of a transformer is called a shell type transformer.

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Core Type Transformer

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We can have a simpler arrangement like this, where the windings are on the outer limbs. There only two limbs of the transformer and the windings are on the outer limbs. And this type of a transformer where the winding is surrounding the core or includes is called the core type of transform. Basically as far as the working of the transformer is concerned, they for our purpose we will find them the working to be very similar.

There are certain differences, but we will not go into those aspects in this course. So, what we find is the transformers consist of an iron core, for providing the path to the magnetic flux. And we have two windings which are linked by the same flux. So, there is a mutual coupling between the two windings by this flux. And because of this, mutual flux which is time varying we get the voltage.

If we energize one side and we get some current flowing into this. Then a flux is set up like this. And since, this flux is time varying and it is cutting this winding also. Therefore, you are going to get a voltage induced here. Now, if you connect a load here you will get a current flowing through this. Now, when the current starts flowing through this, then a magnetic field is set up by this current, which is going to oppose this.

Flux here, and because of this what happens is the flux will in the core will reduce, try to reduce. If the flux here reduces the voltage here cannot be maintained, since this voltage is an applied voltage which is a fixed voltage. So, what happens more current will flow through this to maintain this voltage and thereby maintaining this flux.

So, whenever we connect a load here. And the current starts flowing, we get a reflected current to start flowing from this side also to balance the flux, which is trying to oppose the flux in the core. So, basically the operation of the transformer, you will find is of that of a constant flux. Constant means it is a time varying flux. But, the constant in magnitude as far as, it is the phasor is concerned. And that is this flux is maintained at the same level, in this core whatever may be the load on this side.

So, if you increase the load, this will try to increase the current here. And that will try to reduce the flux here. But, more current will flow here to try and maintain this flux in the core. Now, this is how the transformer works. And before we start doing the mathematical model for this transformer, we will get into the concept of ideal transformer. That is we are considering a transformer, which is ideal in the sense that, it is job is to transform the voltage levels and the currents on the two sides.

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THE IDEAL TRANSFORMER

Transformer Core has Infinite Permiability

- Zero mmf to create magnetic flux in core
- All flux is confined in the core

Transformer Winding has Zero Resistance

- Zero ohmic power loss in winding
- Zero voltage drop in winding

Transformer has Zero Core Loss

Now, this transformer we assume has infinite core permeability. That is the permeability of the core is infinite, what does it really mean? It means that, there is no mmf required to create the magnetic flux in the core. That is there is no current, which is exciting current required to create the flux. All flux is confined in the core. That is it again says that, the core since it has infinite permeability all the flux will try to reside inside the core.

It will not go out where the permeability is lower and so the reluctance to the flux path is going to be high. Another assumption or another property of the ideal transformer is, that the transformer winding has zero resistance. It is simply means, there is no power loss which takes place in this transformer. And there is also no voltage drop, which takes place in this transformer.

Another assumption that we make for an ideal transformer is that, there is no core loss. That is there is no hysteresis or eddy current losses, which takes place in the core. Of course, we know these assumptions are not really valid a physical transformer will have a finite permeability, it has a winding resistance. And it also has the core losses associated with it. But, the concept of an ideal transformer provides a very elegant concept for modeling the transformer.

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$p_1 = v_1 i_1$; $p_2 = v_2 i_2$

$v_1 i_1 = v_2 i_2$; $V_1 I_1^* = V_2 I_2^*$

$S_1 = S_2$

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So, what we find for a ideal transformer, the input power is equal to the output power. What we have as the input power, the instantaneous value of the input power, that is p 1 is equal to v 1 into i 1. Now, these small case letters are meant for instantaneous values. So, the instantaneous power at the input end is v i into i i. And similarly, p 2 is equal to v 2 into i 2.

Now, since we have assumed for this transformer there is no loss. So, p 1 is equal to p 2 or v 1 i 1 is equal to v 2 i 2. If we are talking in terms of the phases, instead of instantaneous values of currents and voltages. Then we can write that v i conjugate, V 1 I 1 conjugate is the complex power, input power is equal to V 2 I 2 conjugate. That is the complex output power. That is we say S 1 is equal to S 2, where S 1 is the complex input power which is equal to p 1 plus j q 1.

And s 2 is equal to p 2 plus j q 2, where p 1 is the input real power, q 1 is the reactive power from the input side. And p 2 is the output real power and q 2 is the output reactive power.

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Physical Transformer

- Finite Permeability
- Core Loss
- Winding Loss
- Leakage Flux

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Now, in case of a physical transformer, what we have a finite permeability. There are core losses, there is loss in winding. Because, winding has a resistance and there is going to be voltage drop. Because, again the core does not have a infinite permeability. So, some flux will pass through air path.

That is if you see ((Refer Time: 14:01)) this transformer, here we are seeing that all the flux is going through the core. But, we will find that some flux may be following this path on this side, like this. Here it will be like this and similarly here it will be like this. That is some of the flux is not going to link the other winding. This kind of flux we call as the leakage flux. So, there is going to be some leakage flux, because of which the flux which is created by this.

All the flux that is created by the current flowing here, does not link this. And therefore, the voltage generated here is not going to be exactly in the same proportion. That is there is going to be some loss in the voltage. And this is occurring, because of the leakage flux. So, we in a physical transformer all these things happen. That is permeability is finite core loss is there, winding loss is there and leakage flux is there. Now, how do we incorporate this into the ideal transformer model.

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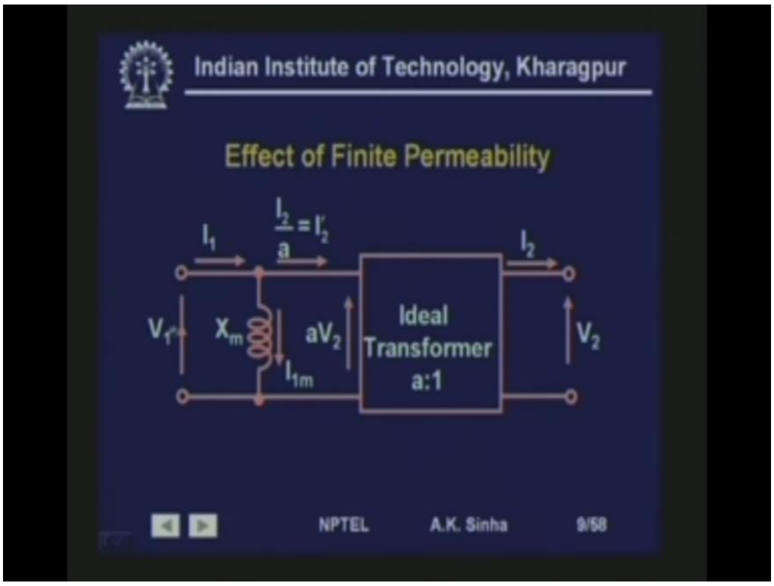

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Effect of Finite Permeability

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So, we have an ideal transformer model, here which basically means. That it is doing only the current and voltage transformation. There no losses no current required, and all that. Whatever is the concept that we have talked about for ideal transformer. So, we want to now take out, that infinite permeability assumption. That is we are saying that the transformer has finite permeability, what does it mean? It means that some current will be required to set up the flux, in the transformer core.

This can be modeled as an inductance, which is in parallel with the ideal transformer. That is an inductance here, will draw a current I m. This current is going to be lagging this voltage by 90 degrees, which is going to be the case for the flux, which is going to be set up as we know. So, we can model the effect of finite permeability, by means of this inductance X m here.

So, here what we have V 1 is the voltage I 1 is the current. Here we have a current, a part of the current is going through this. So, this current is somewhat less, this current is I 2 dash, where I 2 is the current here, which is equal to I 2 by a. And I 2 by a plus I 1 m which is the current in flowing in this inductance, to take care of the finite permeability. If we add these two phases, then we will get the phase of current here.

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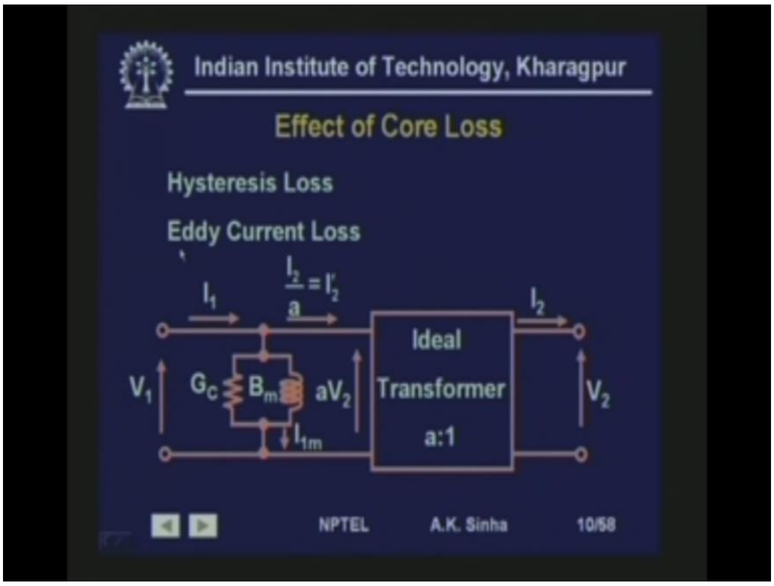

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Effect of Core Loss

Hysteresis Loss

Eddy Current Loss

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Similarly, if we want to take care of the core losses, which consists of hysteresis and eddy current losses. We can modulate by a simply using a resistance or a conductance in parallel with this inductance. Here, since these two are forming parallel branches, therefore it is much easier to work with the susceptance and conductance. Therefore, we are writing this instead of r c, the resistance we are writing this as conductance to take care of the core losses, this as B m, the susceptance to take care of the current for finite permeability. So, the core losses and finite permeability can be taken care of by this kind of the circuits. Where this current I 1 m is the total current which is flowing through this parallel path, which denotes the core losses. And the finite permeability part of the system. Now, why have you put this as a parallel connection, the reason behind this is very simple, as soon as we excite the transformer. There is going to be a current, which will flow through the transformer to set up the flux. And as soon as this time varying flux is set up in the core. We are going to have a hysteresis and eddy current loss in the core. And this is not dependent on the load as soon as we excite the transformer, this is going to happen. So, this is going to be there at even at no load. So, this is dependent on the voltage that we use to excite this transformer. So, or energize this transformer, therefore we have put this as the parallel branch. So, this parallel branch across this ideal transformer takes care of these two parts.

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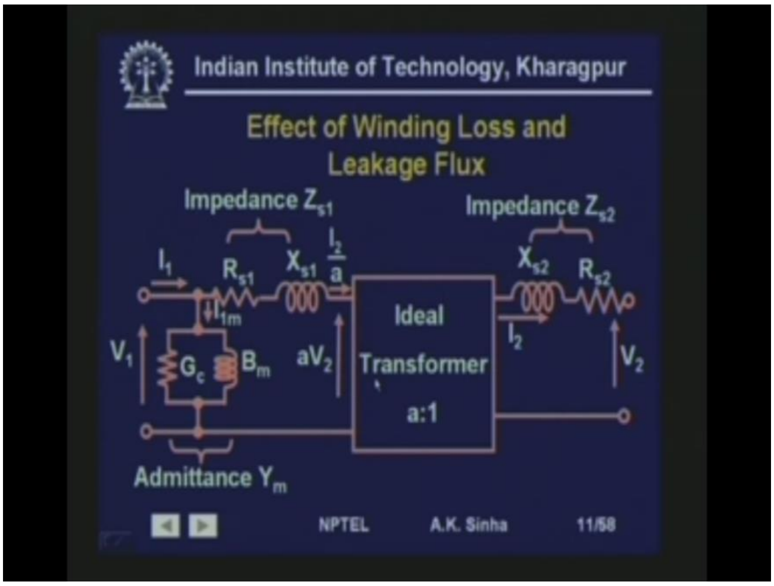

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Effect of Winding Loss and Leakage Flux

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Now, the resistance and the leakage flux parts can be taken care of by means of resistance of the winding here, and the reactance to take care of the leakage flux part. So, this gives the impedance of the winding, where R s 1 is the resistance of the side one, that is the primary side. And X s 1 is the leakage flux part, which is represented by a leakage reactance X s 1.

So, this is what we get, similarly on the other side also we have a winding. So, it has resistance and leakage reactance, so that is given by R s 2 and X s 2. So, this is a model for the transformer. Now, as we know just like voltage and currents are transformed. We have seen, we can transform the impedances from one side to the other side.

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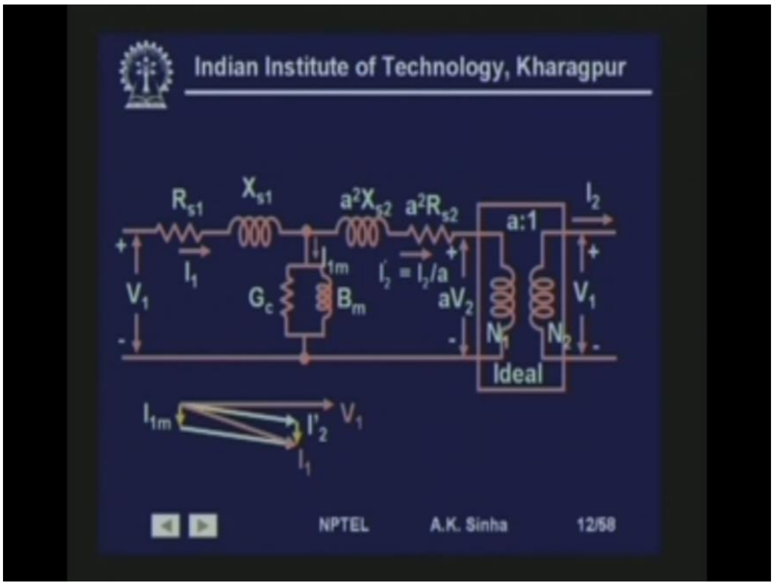

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And therefore, if we do that, then this side impedance which is here can be transformed and put on this side, then we get a like this. Now, once we have a model like this, what we are finding? We have resistance and leakage reactance. We have the core which is having core losses, and the excitation current for building up the flux. Then, we have the winding resistance and the leakage reactance for the secondary winding.

So, this is the model that we have now here. Again we can make an assumption that the current in this is basically reactive in the sense, that this is going to be almost 90 degrees with respect to the voltage. Because, this is much smaller this is much larger. So, this is going to be like this. And since, if we have loaded this transformer the power factor of load will be around 0.8, 0.9. So, the current here is going to be almost in phase with the voltage with a small angle. Here, the phases sum if we take is going to be almost similar to this. That is this I 1 is going to be almost equal to I 2 dash. And therefore, the error in assuming I 1 is equal to I 2 dash is not going to be very large.

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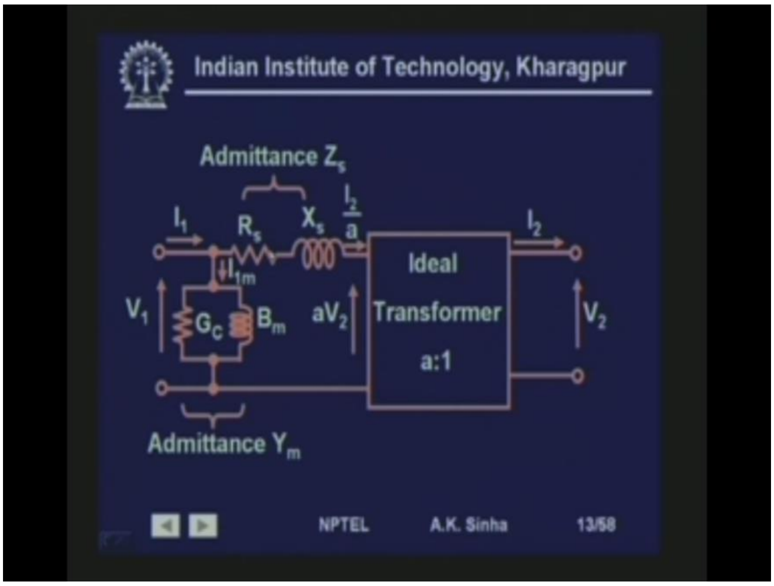

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And if we make that assumption, then we can combine this like this. So, we have shifted this, on that is if you see here, we have shifted this here ((Refer Time: 22:23)). And then, we have combined these two resistance, and reactances which come in series. And therefore, we have got finally a simplified model like this for the transformer. And this is the mathematical model or the circuit model for a practical transformer.

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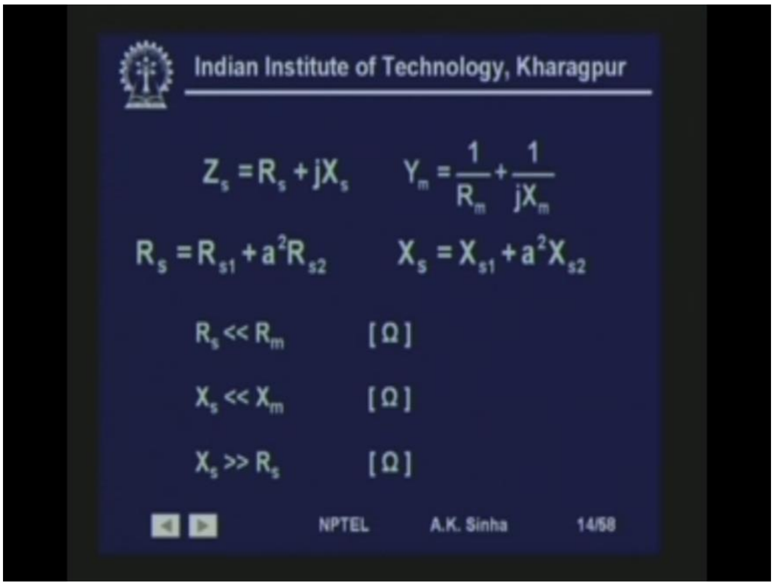

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$$Z_s = R_s + jX_s \qquad Y_m = \frac{1}{R_m} + \frac{1}{jX_m}$$

$$R_s = R_{s1} + a^2 R_{s2} \qquad X_s = X_{s1} + a^2 X_{s2}$$

$R_s << R_m$ [Ω]

$X_s << X_m$ [Ω]

$X_s >> R_s$ [Ω]

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Now, in this case as we see the series impedance Z s ((Refer Time: 22:53)) is going to be like this R s plus j X s. And the shunt admittance is G c minus j B m. So, we have the admittance part as G c which is 1 by R m plus 1 by j X m. That is basically 1 by G c, this is G c minus j B m. Now, here R s is equal to R s 1 plus a square R s 2, that is the transform resistance from the secondary side. To the primary side and X s is equal to X s 1 plus a square X s 2.

That is the transform reactance taking care of the leakage flux of the secondary winding. Now, in practical transformers the R s is very very small compared to R m. X s is very very small compared to X m and X s is much larger than R s. In many cases what we do is, ((Refer Time: 24:00)) many cases this current is generally very small. And also this current is at 90 degrees almost in quadrature with this current. So, this is many times neglected. And the final model of the transformer can be used as just a series impedance, in series with the ideal transformer.

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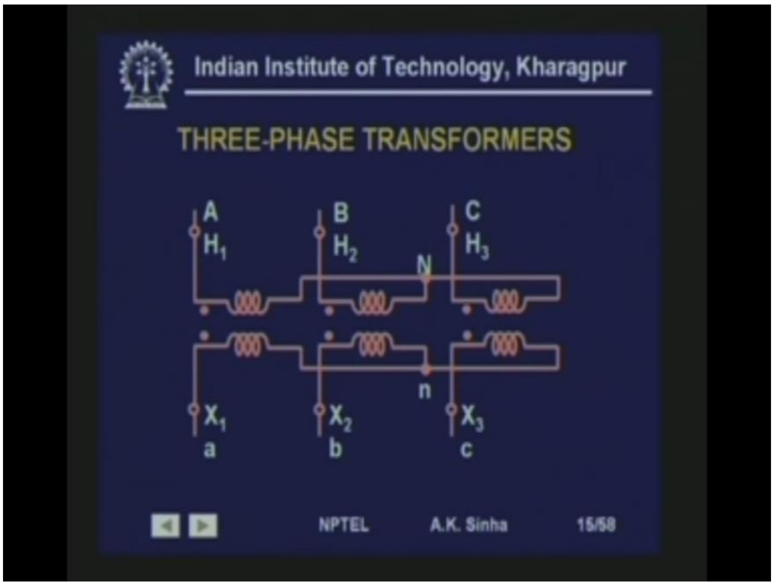
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THREE-PHASE TRANSFORMERS

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Now, most of the transformers since we are using three phase systems. Who are three phase transformers? Three phase transformers can be built as on a single core as a three phase transformer. Or we can build three phase transformers or by using a bank of three single phase transformers. These transformers are the three phase winding of these transformers both on primary. And the secondary side can be connected either as a star or as a delta.

Like here, we have shown this is connected as a star. And this is also connected as a star. So, we have a star star transformer. This is a bank of three single phase transformers, one transformer here, another transformer here and the other one here. And we have connected them as star star transformer, which on a circuit will look like this ((Refer Time: 25:33)).

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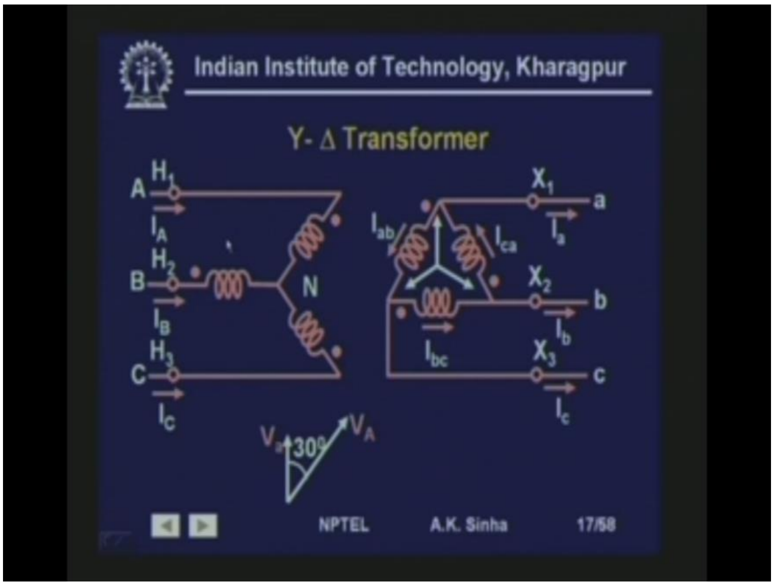

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Y- Δ Transformer

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Similarly, we can have transformers as star and delta connection. Many times we use this type of connection Now, if you look at this circuit, then you find that the phase A. And the phase A here, if you see they have a phasor displacement of 30 degrees, this is what we get. That is when we have a star delta transformer. Then the delta side voltage is leading in this case by 30 degrees.

We can have a inverse also, that is it can lag by 30 degrees as well, if we make connection in that way. So, star delta transformers when we have, we find that the voltage and current on the two sides, have a phasor displacement of 30 degrees.

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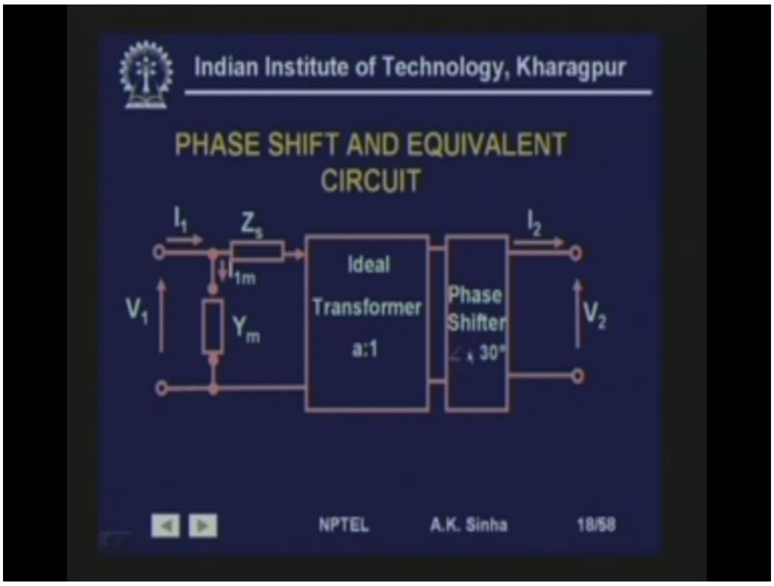

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PHASE SHIFT AND EQUIVALENT CIRCUIT

And we can take care of this in our mathematical model by using the same model that we have. For a transformer that is we have the shunt part taking care of the core losses and the magnetizing current. And the series impedance taking care of the leakage reactance and the resistance of the winding. This is the ideal transformer and we have added a phase shifter of 30 degrees.

Because, if we have a star delta transformer. This will be a plus 30 degrees if the connection is done as shown here. So, if we have this connection and this will be plus 30 degrees. If we have the reverse connection then we will have this as minus 30 degrees. So, this is how we model the three phase transformers.

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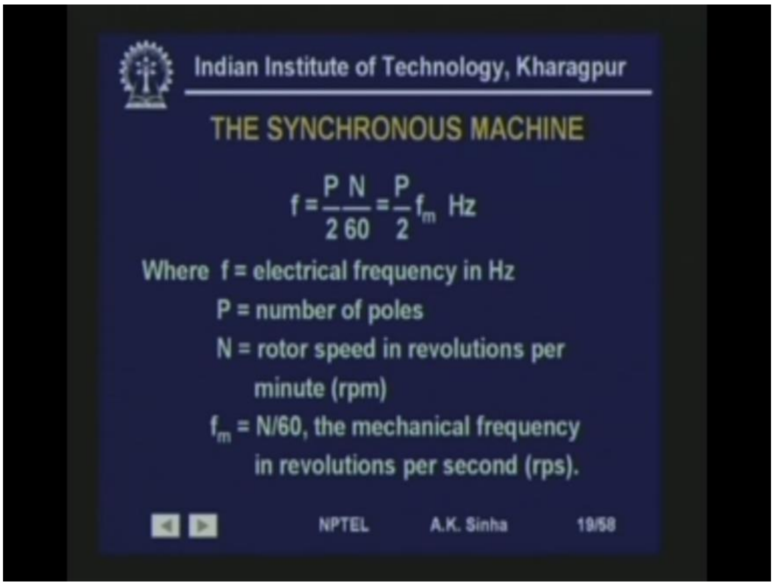

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THE SYNCHRONOUS MACHINE

$$f = \frac{P}{2}\frac{N}{60} = \frac{P}{2} f_m \text{ Hz}$$

Where f = electrical frequency in Hz
P = number of poles
N = rotor speed in revolutions per minute (rpm)
f_m = N/60, the mechanical frequency in revolutions per second (rps).

Next, we will talk about the model for synchronous machine. The synchronous machine have a fixed relationship between the speed and the frequency. And this relationship is given by the frequency f is equal to P by 2 into N by 60, where N is the speed in rpm for the rotor and p is the number of poles of the synchronous machine. This we can write also as P by 2 into P by 2 into f m, where f m is the mechanical frequency.

That is you can see it, in terms of number of rotations per second. Now, this is a very interesting relationship it gives you a relationship between electrical frequency. And the mechanical frequency, that is if we are using a 2 pole machine, then this P is equal to 2. So, f is equal to f m, if we are using a 4 pole machine, then f is equal to for by 2, that is 2 into f m. So, the electrical frequency is going to be twice that of mechanical frequency. Similarly, if we are going to use the 6 pole or 8 pole machines. The electrical frequency is going to be higher than the mechanical frequencies.

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Now, again for synchronous machines also we have. Normally two kinds of machines based on the type of the rotor of the machine. One model that we use as shown, here where we have the rotor which is around cylindrical structure on which the winding is placed. These kinds of machine are called round rotor machines. And these are generally used for large turbo alternators, which are run by steam turbines.

Because, these runs at very high speed are cylindrical rotor structure is much more convenient. As well as stable for the system, because the centrifugal forces at that high speed are going to be very high. And they can be these windings can be braced for those high speed, centrifugal forces on around rotor machine. Whereas, the other kind of machine, we call them as salient pole machine as you see here. The rotors are protruded from the centre they are coming out like this.

So, this machine the poles are salient, they are protruded coming out. And these machines, therefore are called salient pole machines. Now, generally these machines are low speed machines. Now, for hydro generation, where the speed of the hydro turbine is very much dependent on the head. In fact, the optimum speed is based on, what is the height of the water above the turbines.

That is the head and therefore, these machines run at much lower speed. Since, the frequency of the system is same that is we in India we use 50 hertz as the frequency. Therefore, if suppose the speed is only 300 rpm. Then for keeping the frequency 50 how many poles will be required. If we go back to this equation ((Refer Time: 31:52)), we find that the speed is 300 rpm. So, this is 300 by 60, so this is 5 and this is 50, So, here we need how many poles 20, 20 poles will be required for this machine.

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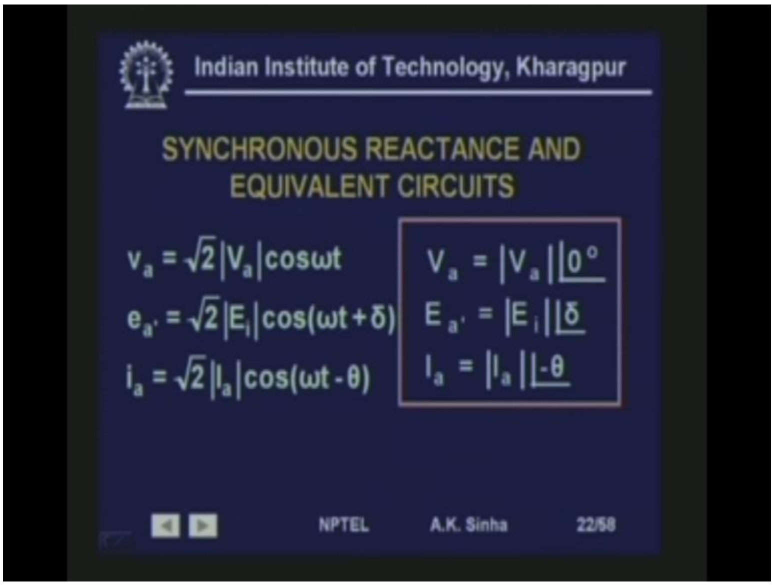

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SYNCHRONOUS REACTANCE AND EQUIVALENT CIRCUITS

$v_a = \sqrt{2}|V_a|\cos\omega t$

$e_{a'} = \sqrt{2}|E_i|\cos(\omega t+\delta)$

$i_a = \sqrt{2}|I_a|\cos(\omega t-\theta)$

$V_a = |V_a|\angle 0^\circ$

$E_{a'} = |E_i|\angle \delta$

$I_a = |I_a|\angle -\theta$

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So, depending on the speed the number of poles change and for low speed machines. We need very large number of poles. And therefore, there we use salient pole fraction. Now, first we will talk about steady state model, for mathematical model for a synchronous machine. Now, in steady state if we see this synchronous machine. The voltage since all the 3 phases are symmetrical, we talk only of one phase. The other two phases, that is phase A.

If we talk about phase B is going to lag, phase A by 120 degrees and phase C is going to lag phase B by 120 degrees. So, voltage and current of phase A and B and C are symmetrical except that, they will be 120 degrees out of phase from each other. So, here if we see, if the voltage generated by the synchronous machine in phase A. Or say the terminal voltage is given by this root 2 times v a cos omega t and E a dash.

That is the internal voltage generated by the machine is E a dash is root 2 into E i cos omega t plus delta. You see what this is and i a is equal to root 2 I a cos omega t minus theta, where theta is the power factor angle or the angle by which the current is lagging the voltage. So, with this in phases if we want to write, we will write V a is equal to V a angle 0 degree. E a dash is equal to E i angle delta and I a is equal to I a angle minus theta, where what is V a the terminal voltage.

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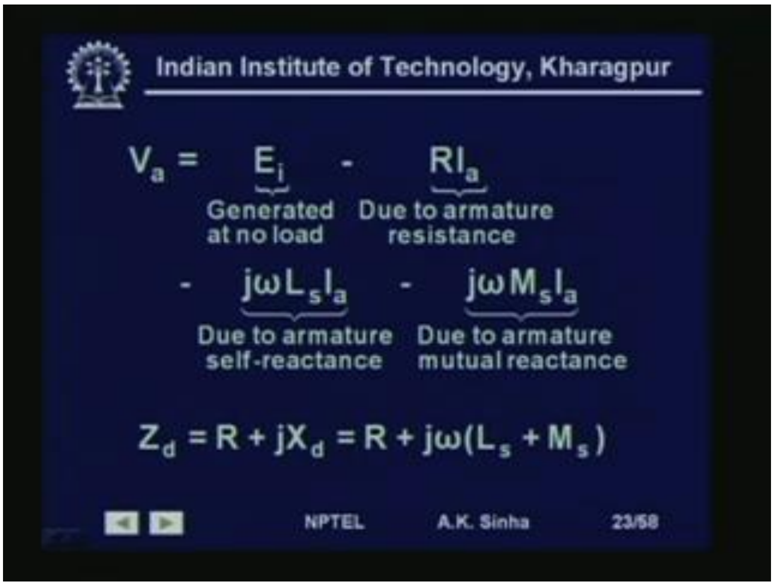

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$$V_a = \underbrace{E_i}_{\text{Generated at no load}} - \underbrace{RI_a}_{\text{Due to armature resistance}} - \underbrace{j\omega L_s I_a}_{\text{Due to armature self-reactance}} - \underbrace{j\omega M_s I_a}_{\text{Due to armature mutual reactance}}$$

$$Z_d = R + jX_d = R + j\omega(L_s + M_s)$$

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Terminal voltage is going to be equal to the generated voltage minus what? The drop, voltage drop due to current flowing through the armature winding resistance. So, due to armature resistance, the voltage drop R I a, minus again the drop due to the self reactance of the rotor winding. Since, the winding well is wound as a coil, so it has a reactance. So, self reactance or self inductance L S. So, it is j omega L s into I a, this is a drop due to armature self reactance.

And since, there is mutual reactances associated. Because, of the currents flowing, in the other phases. With those windings also they have the mutual flux between them. Therefore, there is a mutual inductance M s and for the stator windings. Therefore, we have dropped due to armature mutual reactance as j omega M s into I a. Therefore, we can write the total impedance as Z d is equal to R plus J X t, where which is equal to R plus j omega L s plus M s.

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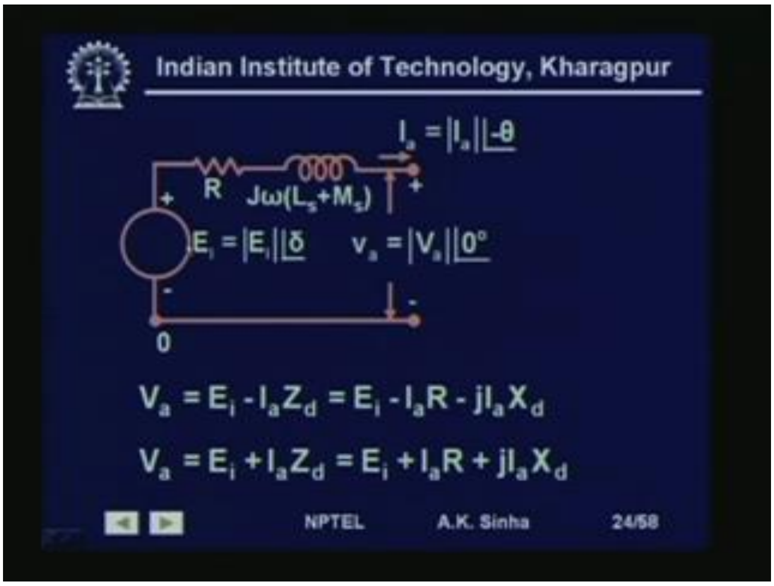

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$$V_a = E_i - I_a Z_d = E_i - I_a R - jI_a X_d$$

$$V_a = E_i + I_a Z_d = E_i + I_a R + jI_a X_d$$

So, this is what we get. And the circuit model for the synchronous machine, can be put like this; where this for the machine acting as a generator. We have V a is equal to E a or E i minus R I into R plus j omega L s plus M s. That is E i minus I a into Z d, which is equal to E i minus I a R minus j X d into I a. If it is acting as a motor, then we are exciting from here.

Therefore, V a is equal to E i plus this, that is E i is equal to V a minus this. So, V a is equal to E i plus I a Z d, that is equal to E i plus I a R plus J I a X d. So, this is a very simple model for the synchronous machine, which is when acting as a generator or acting as a motor. This is a model which is used for steady state operation of the synchronous machine.

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REAL AND REACTIVE POWER CONTROL

$$S = P + jQ = V_t I_a^* = |V_t||I_a|(\cos\theta + j\sin\theta)$$

$$P = |V_t||I_a|\cos\theta \qquad Q = |V_t||I_a|\sin\theta$$

$$I_a = \frac{|E_i|\angle\delta - |V_t|}{jX_d} \quad \text{and} \quad I_a^* = \frac{|E_i|\angle{-\delta} - |V_t|}{-jX_d}$$

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Now, synchronous machine, as we know has excitation control. And therefore, we can control the reactive power also. That is excitation control, can control the generated voltage. And therefore, it can also control the reactive power output of the machine. Now, here we see the complex power S is equal to P plus j Q, which that is the complex if we are e talking about a synchronous generator. This will be the power output from the synchronous generator, will be equal to the terminal voltage into I a conjugate.

We are talking of a single phase. Since, if we are talking of a three phase machines. Then, we will have to multiply the power by 3, or here V t is the phase voltage, not the line voltage, so V t into I a conjugate, which we are writing as V t into I a cos theta plus j sin theta. Since we have taken the conjugate, we are writing this. So, p is equal to V t into I a cos theta, that is the real part.

And Q is equal to V t into I a sin theta, that is the imaginary part that data is coming. So, that is the imaginary part, so the reactive power is this part. Now, what is I a? I a is equal to E angle delta minus V t divided by j X d. That is the reactance and if we take the conjugate of I a. Then it is E i minus delta, the conjugate we are taking minus V t since its angle is taken as 0. So, there is no conjugate for that and minus j X t.

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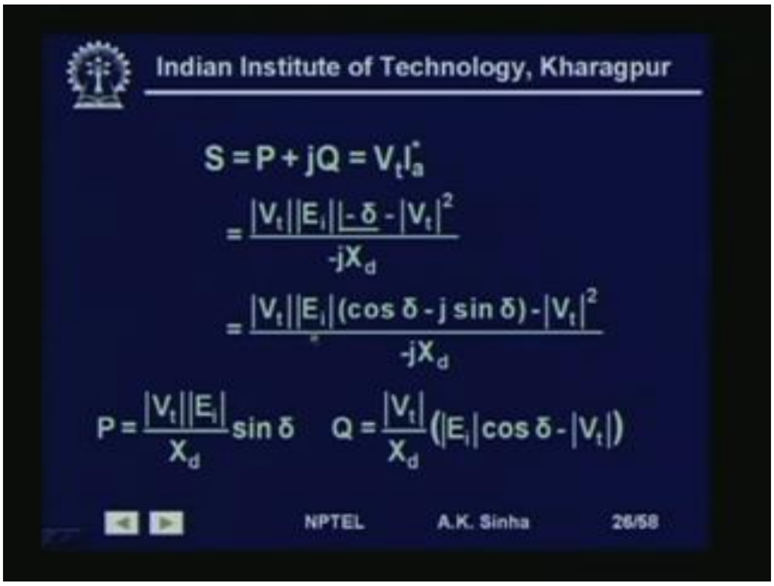

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$$S = P + jQ = V_t I_a^*$$

$$= \frac{|V_t||E_i|\angle -\delta - |V_t|^2}{-jX_d}$$

$$= \frac{|V_t||E_i|(\cos\delta - j\sin\delta) - |V_t|^2}{-jX_d}$$

$$P = \frac{|V_t||E_i|}{X_d}\sin\delta \quad Q = \frac{|V_t|}{X_d}\left(|E_i|\cos\delta - |V_t|\right)$$

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So, now if we substitute that, that is V t into I a conjugate. Then we are going to get this relationship. And if we substitute for all these delta angle here. Then we get finally, P is equal to V t into E i by X d into sin delta. And Q is equal to V t by X d into E i cos delta minus V t. Now, these are very important relationships. It tells us that, the real power output of the synchronous machine depends on the power angle delta.

That is the angle by which the internal voltage or the generated voltage of the machine leads the terminal voltage. If this angle delta is negative. That means, the machine will be acting as a motor, because P will be negative in that case. We also see from this relationship, that Q is dependent on V t or E i. So, what we are finding is that, P is predominantly related with the power angle, and not so much with the voltage.

Of course, there is voltage relationship, but this dependence on sin delta is much more. Because, normally voltage is kept constant, or very near to the rated value. But, Q if we want to change the reactive power output of the machine. Then, we have to change the excitation by changing this E i, we can control the reactive power output of the machine.

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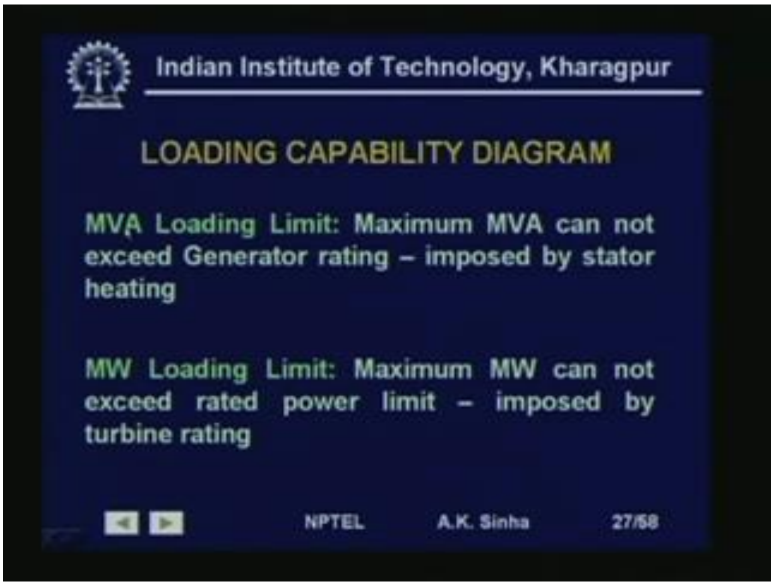

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LOADING CAPABILITY DIAGRAM

MVA Loading Limit: Maximum MVA can not exceed Generator rating – imposed by stator heating

MW Loading Limit: Maximum MW can not exceed rated power limit – imposed by turbine rating

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Now, how much we can load the machine, a synchronous machine can be loaded up to what extent. This we can find out by looking, at what are the limits which come into play. First is MVA loading limit, the maximum MVA cannot exceed the generator rating. Why, that simply depends on the maximum current that can be allowed, because if we allow more current than the rated value, the heating is going to be much more than the cooling which is provided.

And therefore, the temperature will keep rising and the machine may burn out. Therefore, this MVA loading limit is based on the current or the hearting limit, which is permissible by the design. Then, the other one limit is mega watt loading limit. That is the real power loading limit, how much real power we can get out of the machine. The maximum mega watt cannot exceed the rated power limit, why it cannot do that? Because, it is being driven by a turbine, turbine has a rated capacity, you cannot get more output from that. Therefore, that gives us the limit of the real power. That it can take out from the machine.

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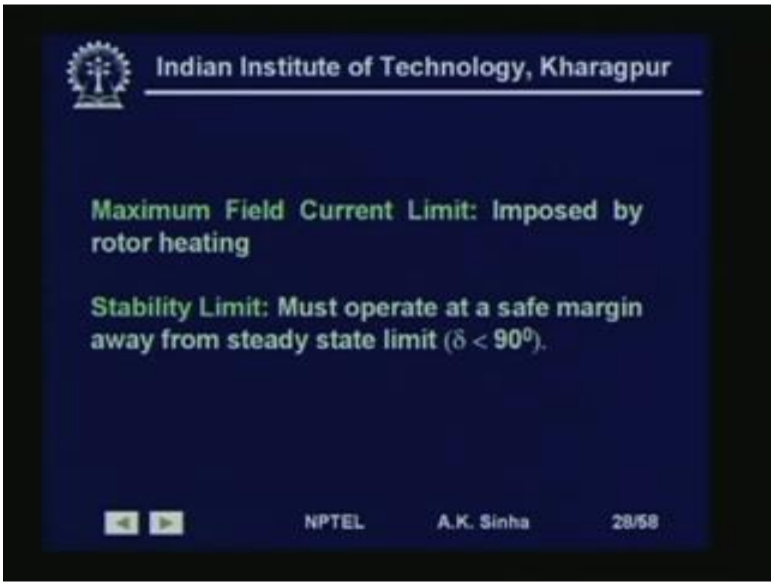

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Maximum Field Current Limit: Imposed by rotor heating

Stability Limit: Must operate at a safe margin away from steady state limit ($\delta < 90^0$).

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Maximum field current limit. Again we cannot excite the machine, beyond a certain value. Because, the rotor current is going to exceed and rotor heating is going to go on. Therefore, the maximum current that can be allowed in the rotor limits, the reactive power supply also. Stability limit, this is basically a dynamic aspect. That is generally we cannot allow this machine to work beyond a delta angle of 90 degrees.

If you see here, if we have the maximum power that we can get out from the machine is when delta is 90 degrees. In fact, for practical purposes, we cannot work this machine more than 15 to 20 or maximum 30 degrees. This is imposed, because if we have certain outages or faults on the system. The machines need to absorb this sudden large changes or jerks on the system. And the transient stability of the system may be threatened, if we work at larger delta angle. That is if we go very near to delta is equal to 90 degrees. Then, even small changes may bring oscillations, which can make system become unstable.

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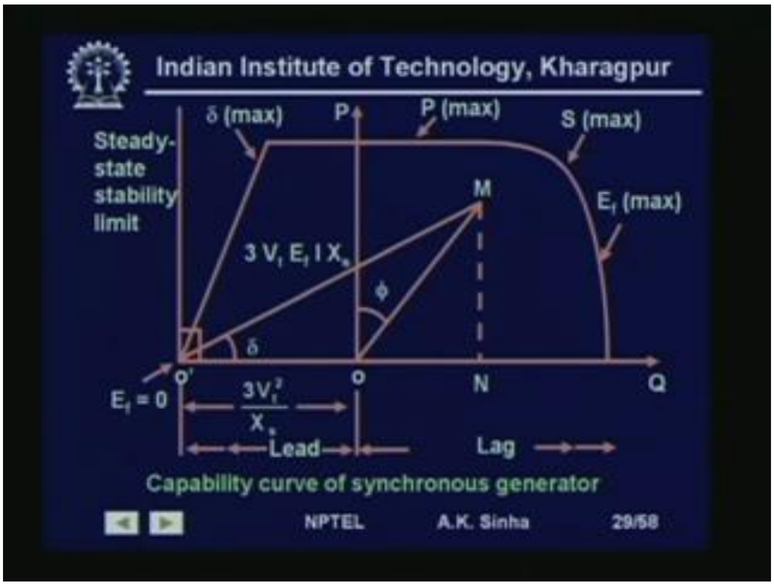

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Capability curve of synchronous generator

So, this is the capability curve. That we get for the generator, this is the delta max that we fix maybe 30 degrees, 35 degrees whatever it is. This is the curve for the P max, this is the curve for the MVA limit. And this is for the field current limits. So, it has to work within this region where this part is P, that is this is P on the Y axis and Q on the X axis. So, this is a very important curve. And we have to see that our operating point at times, is within this region. If it goes beyond this region, then we have violated the operating constraints of the machine.

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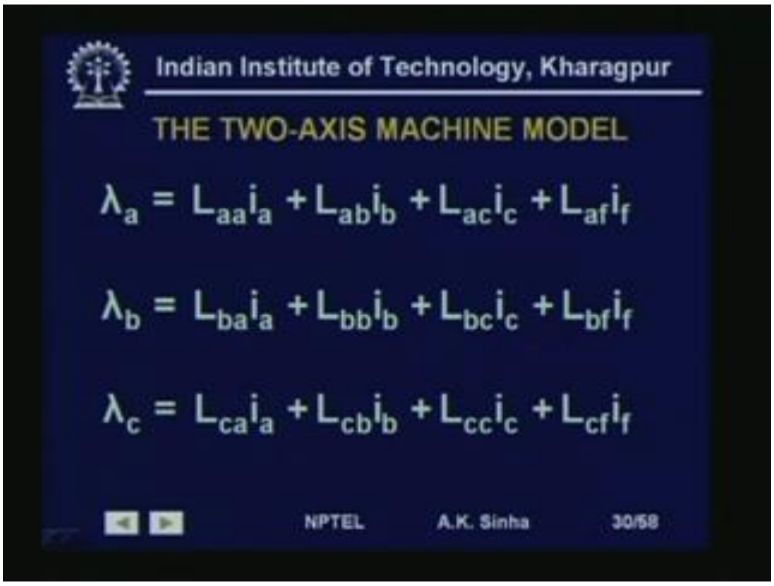

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THE TWO-AXIS MACHINE MODEL

$$\lambda_a = L_{aa} i_a + L_{ab} i_b + L_{ac} i_c + L_{af} i_f$$

$$\lambda_b = L_{ba} i_a + L_{bb} i_b + L_{bc} i_c + L_{bf} i_f$$

$$\lambda_c = L_{ca} i_a + L_{cb} i_b + L_{cc} i_c + L_{cf} i_f$$

Now, this all this that we talked about steady state model. All this is even for dynamics is to some extent valid for round rotor machines. But, in case of salient pole machines, if you see here ((Refer Time: 46:13)). The air gap in the pole region and between the inter pole region is very much different. And therefore, the reluctance of the paths are different. And therefore, the inductances are going to be very different.

And therefore, we cannot use a uniform model for this. We have to in this case use two access models. That is one axis, that is the direct axis along the pole phase. And another axis which is in quadrature with it along the inter polar region. So, this is if you see an angle of 90 degree electrical. The angle here and here between North and South is 180 degree electrical. So, this is 90 degree electrical.

So, we use a two axis model for the case of salient pole machines. I will not go into the details, because we have already done this earlier. So, we can write down the equations, for this machine, for the flux linkage of phase A, phase B, and phase C. In terms of the self and mutual inductances and the currents flowing in the different windings. That is the stator windings A, B and C and the field winding this current I f.

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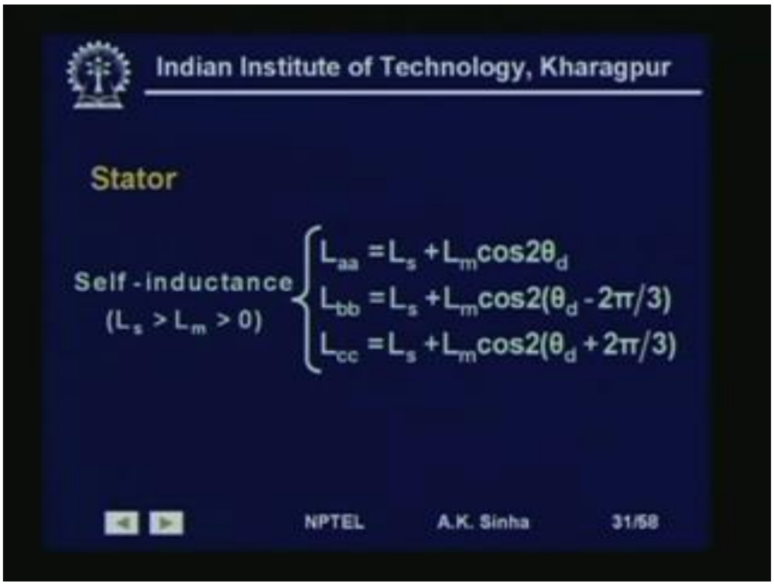

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Stator

Self - inductance ($L_s > L_m > 0$)

$$\begin{cases} L_{aa} = L_s + L_m \cos 2\theta_d \\ L_{bb} = L_s + L_m \cos 2(\theta_d - 2\pi/3) \\ L_{cc} = L_s + L_m \cos 2(\theta_d + 2\pi/3) \end{cases}$$

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Now, one of the major problems as we have seen, because the flux paths are not same, in the inter polar region. What we have, is all these inductances are dependent on the angle theta. And since, they are dependent on the angle theta, this angle theta keeps changing with the rotation. And therefore, all these inductances are time varying inductances.

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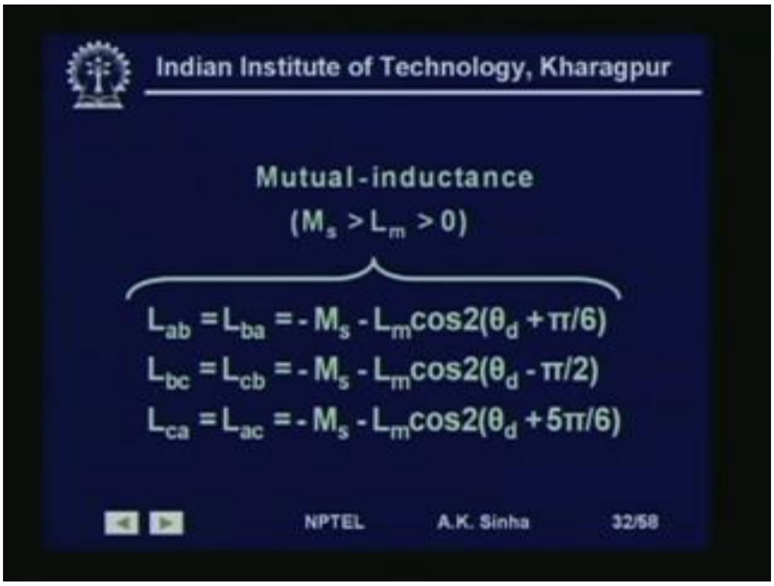

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Mutual-inductance

$(M_s > L_m > 0)$

$$L_{ab} = L_{ba} = -M_s - L_m\cos 2(\theta_d + \pi/6)$$
$$L_{bc} = L_{cb} = -M_s - L_m\cos 2(\theta_d - \pi/2)$$
$$L_{ca} = L_{ac} = -M_s - L_m\cos 2(\theta_d + 5\pi/6)$$

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So, if we see these values and I had already told you earlier, when we talked about the synchronous machine model. So, all the mutual inductances self inductance.

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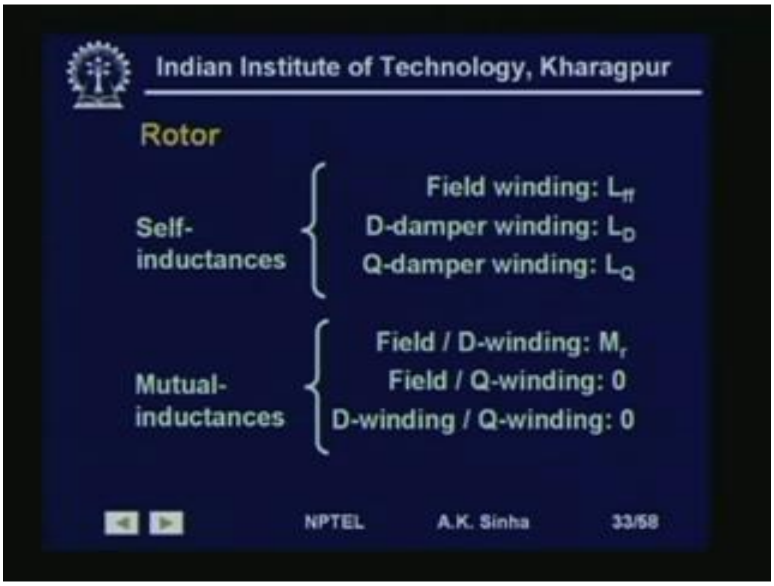

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Rotor

Self-inductances
- Field winding: L_{ff}
- D-damper winding: L_D
- Q-damper winding: L_Q

Mutual-inductances
- Field / D-winding: M_r
- Field / Q-winding: 0
- D-winding / Q-winding: 0

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Rotor inductances.

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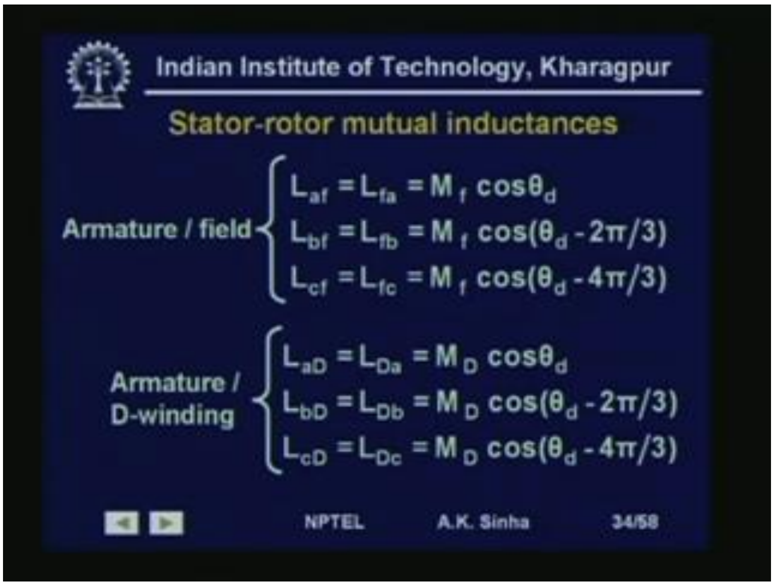

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Stator-rotor mutual inductances

Armature / field
$$\begin{cases} L_{af} = L_{fa} = M_f \cos\theta_d \\ L_{bf} = L_{fb} = M_f \cos(\theta_d - 2\pi/3) \\ L_{cf} = L_{fc} = M_f \cos(\theta_d - 4\pi/3) \end{cases}$$

Armature / D-winding
$$\begin{cases} L_{aD} = L_{Da} = M_D \cos\theta_d \\ L_{bD} = L_{Db} = M_D \cos(\theta_d - 2\pi/3) \\ L_{cD} = L_{Dc} = M_D \cos(\theta_d - 4\pi/3) \end{cases}$$

Rotor mutual inductances, stator rotor mutual inductances. All these we find that most of these inductances are time varying inductances. Mainly because the flux path is not same in the polar. And inter polar region. And therefore, analysis of this kind of machine becomes very, very complex.

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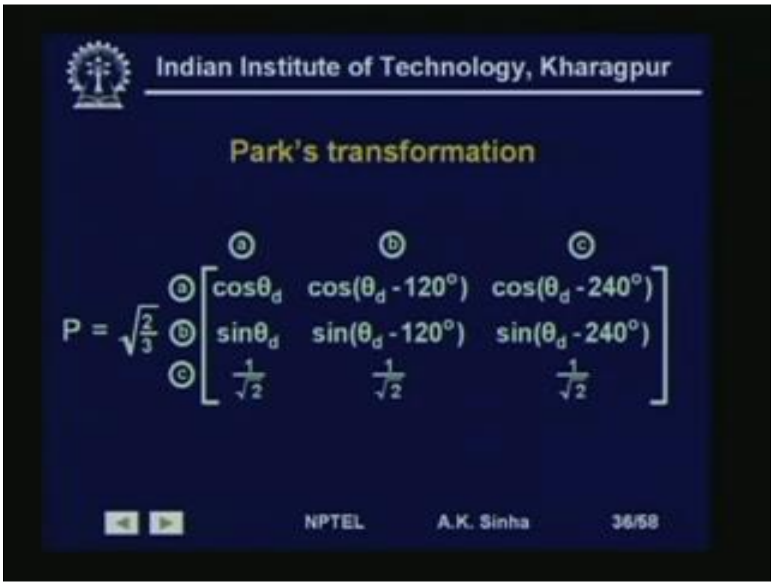

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Park's transformation

$$P = \sqrt{\frac{2}{3}} \begin{array}{c} \\ \textcircled{a} \\ \textcircled{b} \\ \textcircled{c} \end{array} \begin{array}{ccc} \textcircled{a} & \textcircled{b} & \textcircled{c} \\ \cos\theta_d & \cos(\theta_d - 120^\circ) & \cos(\theta_d - 240^\circ) \\ \sin\theta_d & \sin(\theta_d - 120^\circ) & \sin(\theta_d - 240^\circ) \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{array}$$

In order to make this simpler what we do is, we use a transformation called Park's transformation. The idea of this transformation is to transform the three phase stationary. Three phase winding of the stator into two windings, basically or three windings. One which is in phase with the rotor other which is in quadrature with the rotor. And the third one which is along the axis of the rotor, since that has nothing much to do.

So, we generally do not use that part. But, we have a axis which is which we call the direct axis, which is in the phase with the rotor. A quadrature axis which is at 90 degrees to the rotor and the third one a 0 degrees axis. So, D Q 0 frame of reference, we transform the A B C phase reference values into D Q 0 phase D Q 0 frame of reference by using this Park's transformation. And this D Q 0 frame of reference, since it is along with the rotor it rotates at the same speed as that of the rotor.

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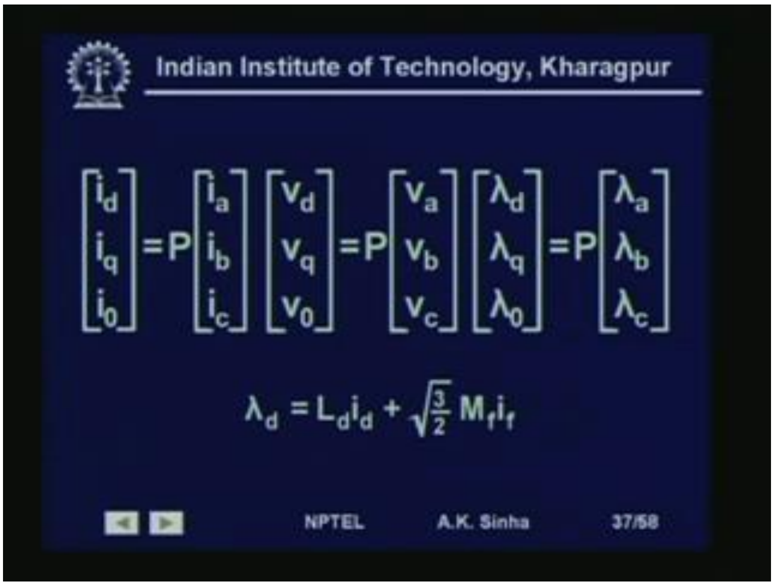

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$$\begin{bmatrix} i_d \\ i_q \\ i_0 \end{bmatrix} = P \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} \quad \begin{bmatrix} v_d \\ v_q \\ v_0 \end{bmatrix} = P \begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} \quad \begin{bmatrix} \lambda_d \\ \lambda_q \\ \lambda_0 \end{bmatrix} = P \begin{bmatrix} \lambda_a \\ \lambda_b \\ \lambda_c \end{bmatrix}$$

$$\lambda_d = L_d i_d + \sqrt{\frac{3}{2}} M_f i_f$$

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So, what we do we this transformation. That is for the current D Q 0 can be got by multiplying this transformation matrix with the phasor values. Same thing for the voltage D Q 0 can be obtained by P multiplied by voltage. And same thing for the flux, we can do that if we use this transformation.

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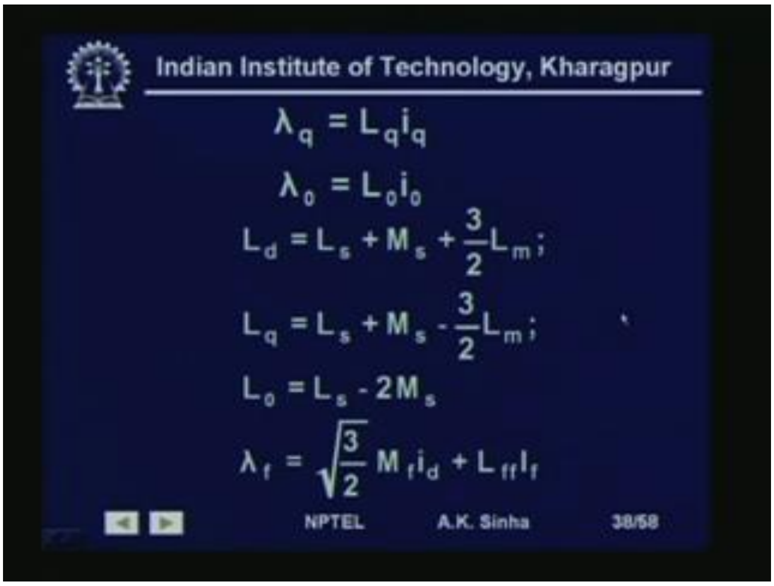

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$$\lambda_q = L_q i_q$$

$$\lambda_0 = L_0 i_0$$

$$L_d = L_s + M_s + \frac{3}{2} L_m;$$

$$L_q = L_s + M_s - \frac{3}{2} L_m;$$

$$L_0 = L_s - 2M_s$$

$$\lambda_f = \sqrt{\frac{3}{2}} M_f i_d + L_{ff} I_f$$

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Then we can write all these equations for inductances, and the flux linkages. And we find that in this case most of the inductances are have become constant, because now the D Q 0 frame is also rotating along with the rotor. So, these time varying inductances are no longer there.

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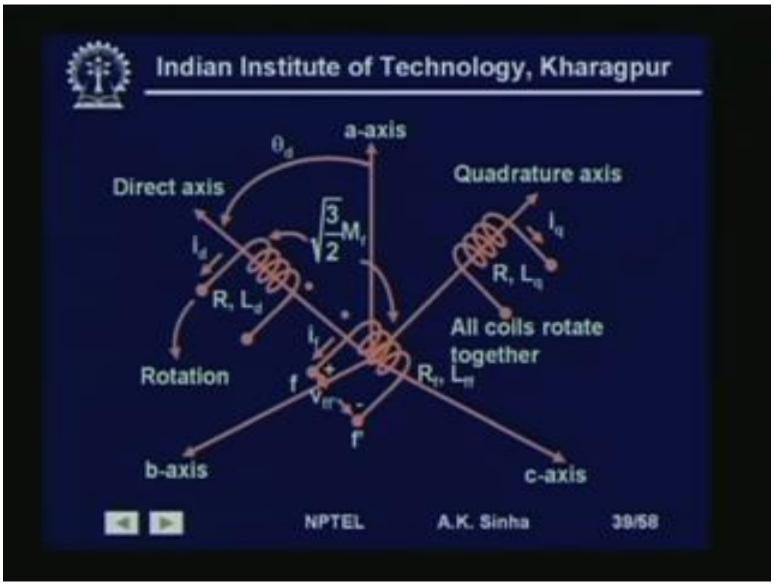

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So, this is about this as I said, this is the rotor the direct axis, and the quadrature axis. And this keeps rotating at the same speed along with the rotor.

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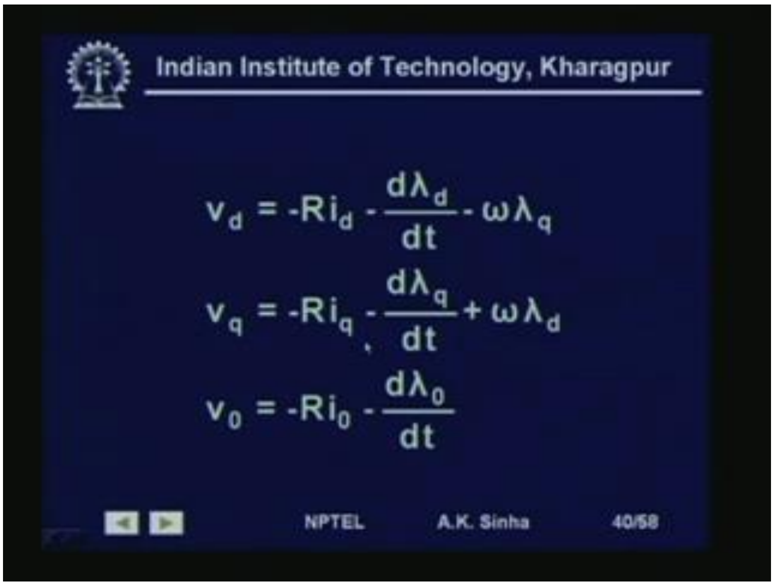

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$$v_d = -Ri_d - \frac{d\lambda_d}{dt} - \omega\lambda_q$$

$$v_q = -Ri_q - \frac{d\lambda_q}{dt} + \omega\lambda_d$$

$$v_0 = -Ri_0 - \frac{d\lambda_0}{dt}$$

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So, we do that, then we will write all the relations for voltage, and the flux linkages. In terms of D Q 0 frame of reference. And we avoid the time varying inductances in this case.

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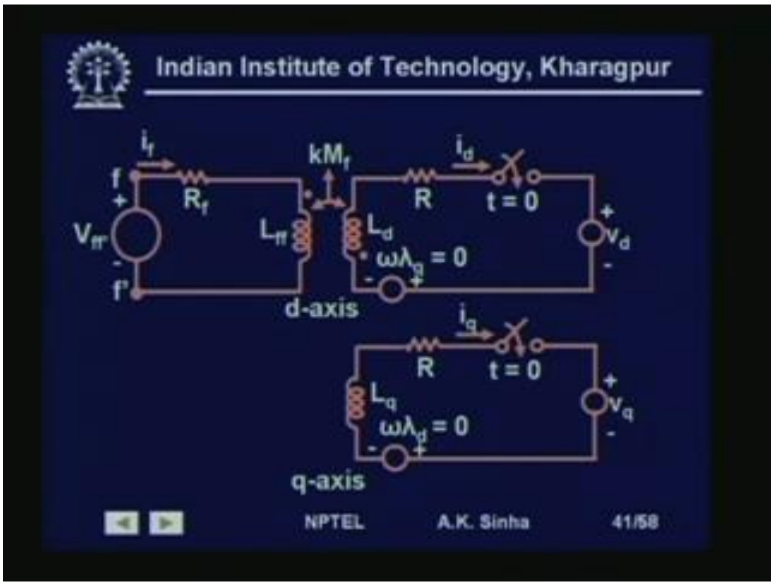

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So, this is what we get if we do this. We have got this direct axis circuit, where we have this is the field circuit, resistance and the inductance of the field. And this has a coupling with the direct axis, because the two are in the same axis. So, here we have k M f as the coupling factor. L d is the direct axis, inductance R is the resistance of the stator winding. And omega lambda Q is the speed voltage generated, because of the Q axis flux, because the voltage is normally generated at 90 degrees to the flux.

So, this relationship is there. So, Q axis flux generates the direct axis voltage. Similarly, we have the circuit for the Q axis, where the voltage generated is speed voltage omega lambda Q L Q is the direct quadrature axis inductance. And R is the winding resistance. And we find that these two circuits are that is the Q axis circuit, is not coupled to any circuit, it is independent.

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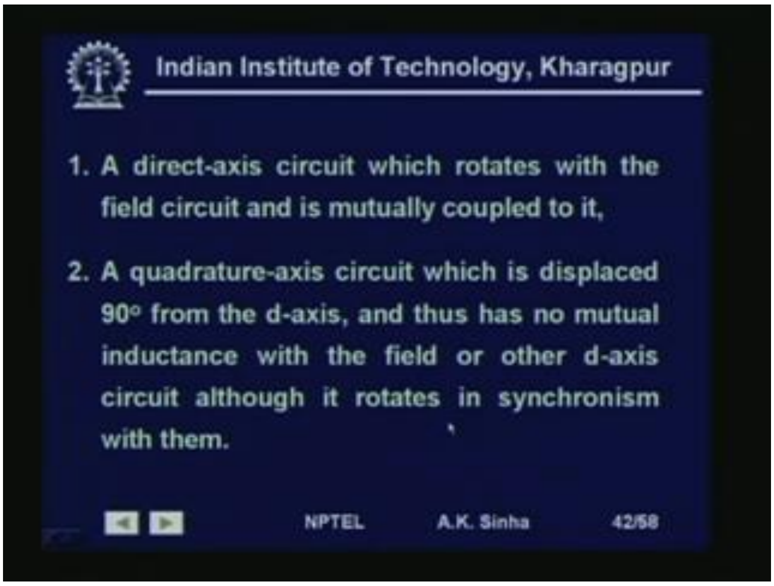

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1. A direct-axis circuit which rotates with the field circuit and is mutually coupled to it,
2. A quadrature-axis circuit which is displaced 90° from the d-axis, and thus has no mutual inductance with the field or other d-axis circuit although it rotates in synchronism with them.

So, the cross transformation provides what, a direct axis circuit which rotates with the field circuit, and is mutually coupled to it. A quadrature axis circuit which is displaced 90 degrees from the d-axis. And thus has no mutual inductance with the field. Or other d-axis circuit although it rotates in synchronism with them.

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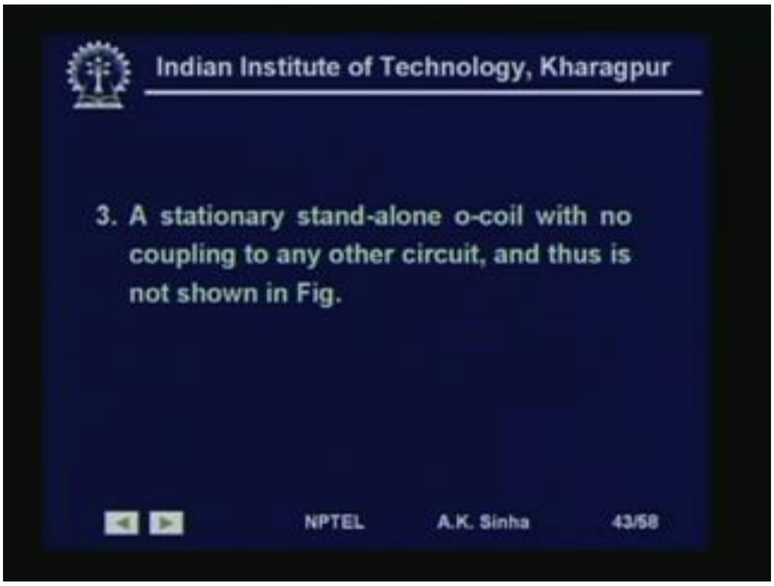

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3. A stationary stand-alone o-coil with no coupling to any other circuit, and thus is not shown in Fig.

And a stationary stand alone o-coil with no coupling to any other circuit, and thus is not shown it is not used as such.

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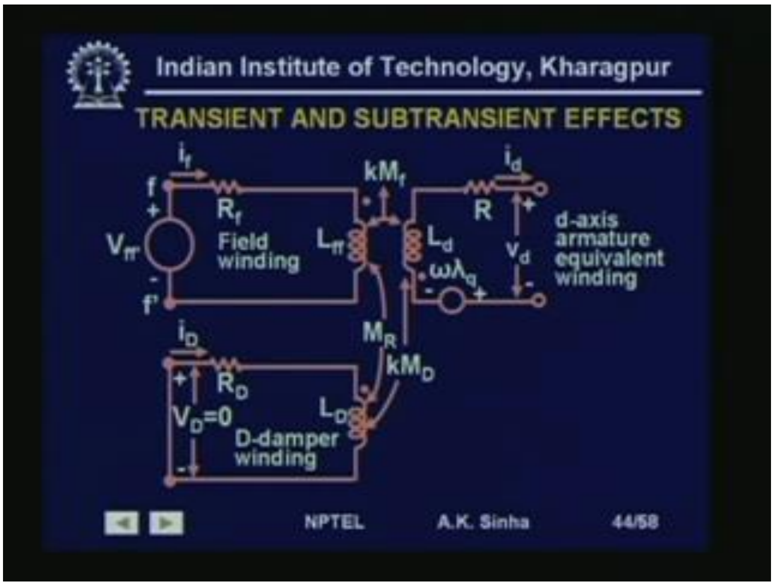

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TRANSIENT AND SUBTRANSIENT EFFECTS

For transient conditions, the same thing happens. Except that, we have the damper winding coming into play. And using this circuit, we can use the transient and sub-transient effect. What happens is that, this damper circuit. Whenever there is oscillation in the machine, this damper circuit comes into play. And since it is time constant is very small, this dies off very quickly.

And therefore, this circuit becomes open. There is no current flowing through this, and these two circuits are only valid, and then this field time constant somewhat larger. So, that again dies out within some time. And then finally, we have only the synchronous impedance of the circuit remaining. And therefore, we define three different reactances or impedances for this circuit.

One is in which all the three circuits are there. Since these three will be in parallel. Therefore, the total inductance is much lower. And this is what is happening in the initial part. And this we call as the sub-transient reactance. When the damper part dies out, then we have only these two inductances which will be in parallel. And therefore, this we call as the transient reactance, or the direct axis transient reactance. And finally, when this also this current dies out, then we have only this circuit which is the steady state or the synchronous reactance of the machine. So, we normally have three different associated with this.

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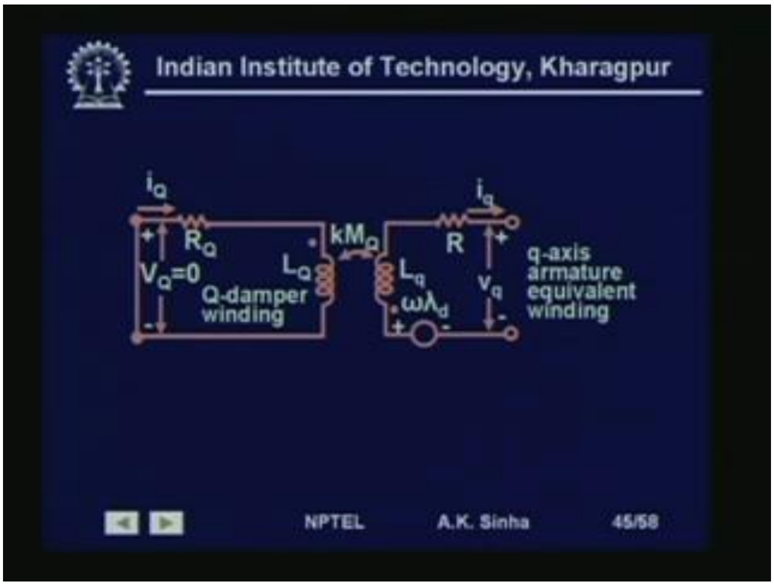
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In quadrature axis we have the damper winding, and we have the Q axis coil. We do not have any field winding coupling. Therefore, in this case we have a sub-transient reactance and the damper winding. And the quadrature axis winding are present, when the damper current dies out, we have only the Q axis current flowing. So, we have a sub-transient and a steady state quadrature axis, reactance, synchronous reactance of the machine.

So, we have x Q and x Q double dash which we use for the sub-transient path, x Q dash is generally not there. X Q dash normally people take same as x Q. Whereas, in case of the direct axis, we have x D double dash which is much smaller than x D dash and which is smaller than x D. The quadrature axis we have two windings, the damper winding and the q axis winding on the stator.

Now, when the fault occurs initial current will flow in the damper winding. As well as in this quadrature axis winding, which are coupled. And since, R will be much larger than L, these current decays very fast. So, initially we see the sub-transient condition. And after these current decays, how we get only the x Q part. So, we have in quadrature axis, the sub-transient reactance. And then, we have the quadrature axis reactance.

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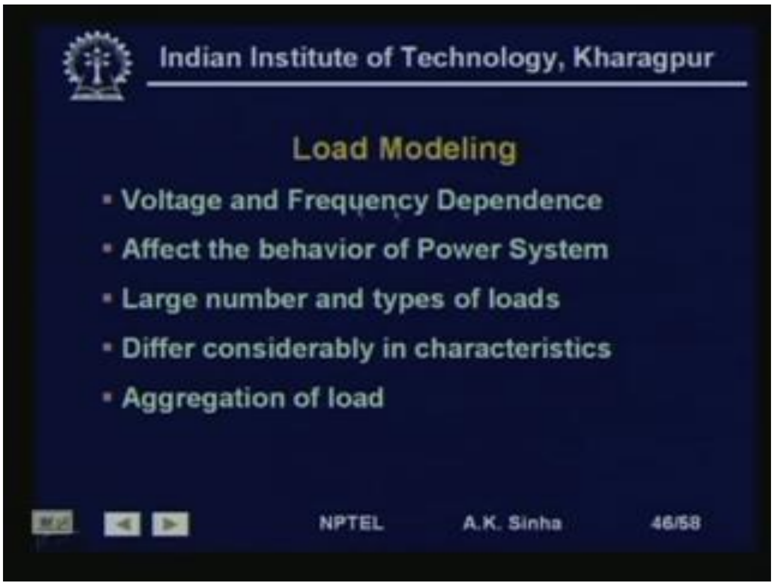

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Load Modeling

- Voltage and Frequency Dependence
- Affect the behavior of Power System
- Large number and types of loads
- Differ considerably in characteristics
- Aggregation of load

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Next we will take up the load modeling. Loads have voltage and frequency dependence. And because of this change in voltage and frequency in a system, the value of the loads change and therefore, they affect the behavior of the system. We have a large number and different types of load, which differ considerably in characteristics. And most of the time, what we do is at any particular bus, we aggregate these different types of load into a single load model.

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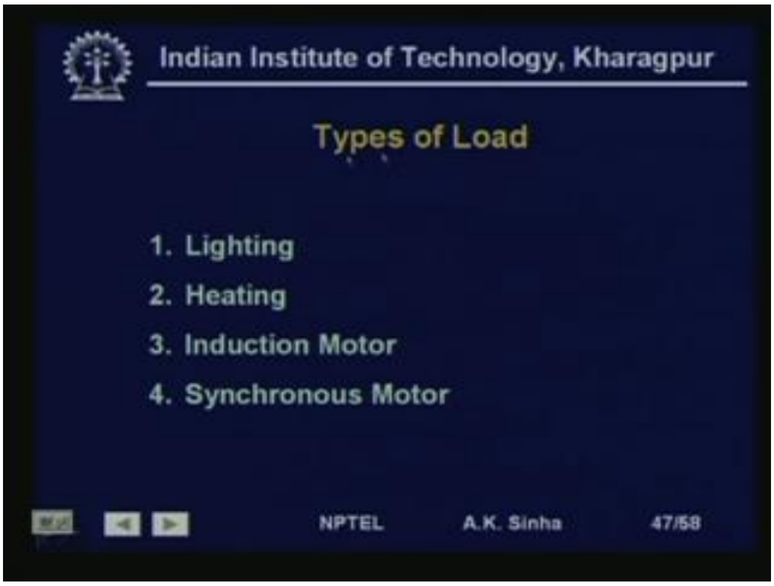

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Types of Load

1. Lighting
2. Heating
3. Induction Motor
4. Synchronous Motor

So, the different type of loads that we have, are lighting loads, heating load, induction motor loads and synchronous motor loads.

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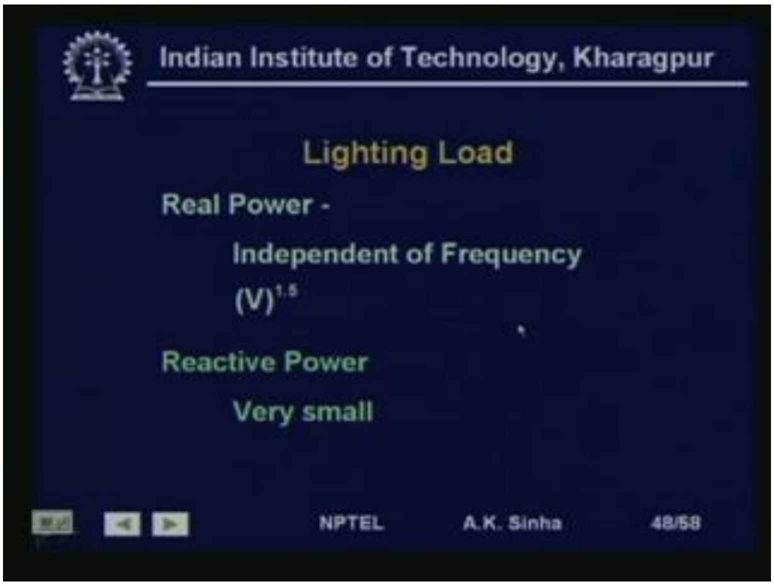

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Lighting Load

Real Power -

Independent of Frequency

$(V)^{1.5}$

Reactive Power

Very small

We have already discussed about the characteristics of these. Like lighting load has a real power independent of frequency and voltage to the power 1.5 reactive power is very small.

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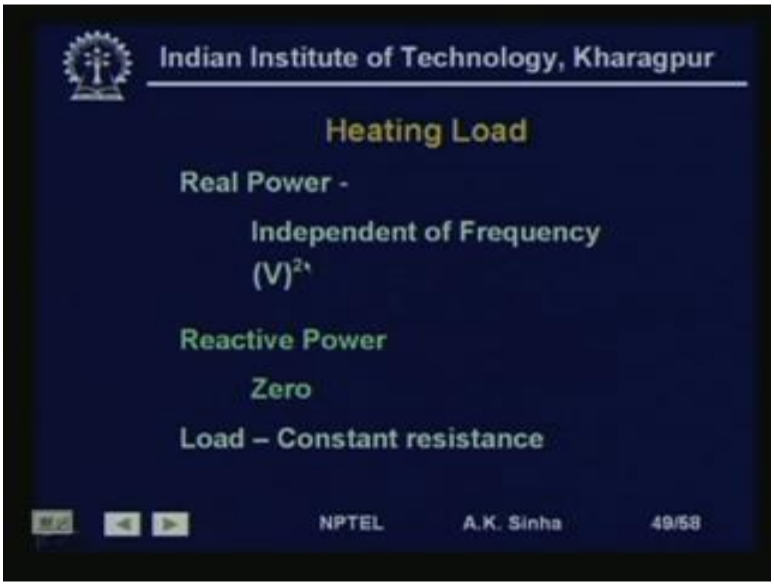

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Heating Load

Real Power -

Independent of Frequency

$(V)^2$

Reactive Power

Zero

Load – Constant resistance

Similarly, for heating loads it is independent of frequency. And real power is dependent on V squared, and it can be represented as a constant resistance.

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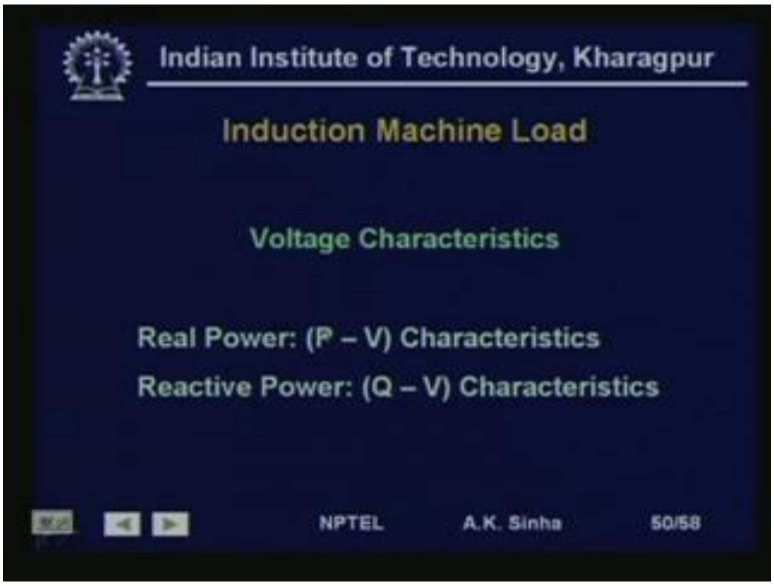

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Induction Machine Load

Voltage Characteristics

Real Power: (P – V) Characteristics

Reactive Power: (Q – V) Characteristics

We have induction machine loads, which have also P V and Q V characteristics. As well as P F and Q F characteristics, we will not go into details of this.

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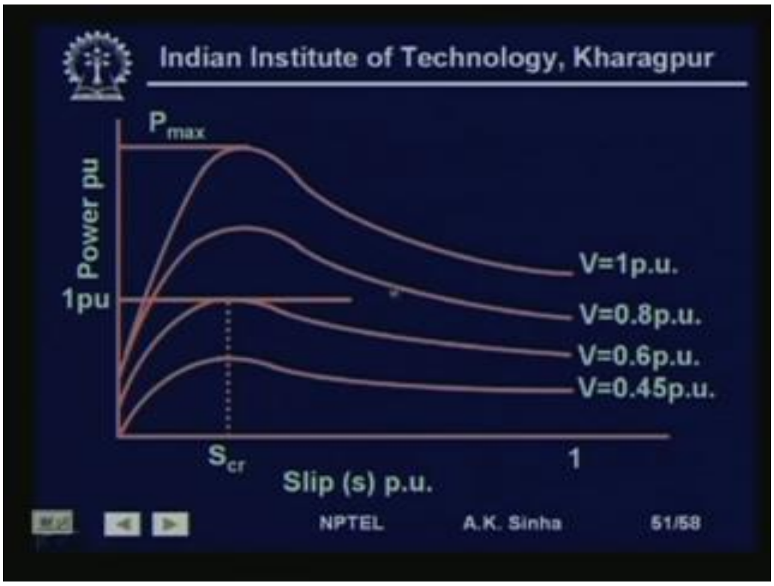
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This is just showing a family of curves.

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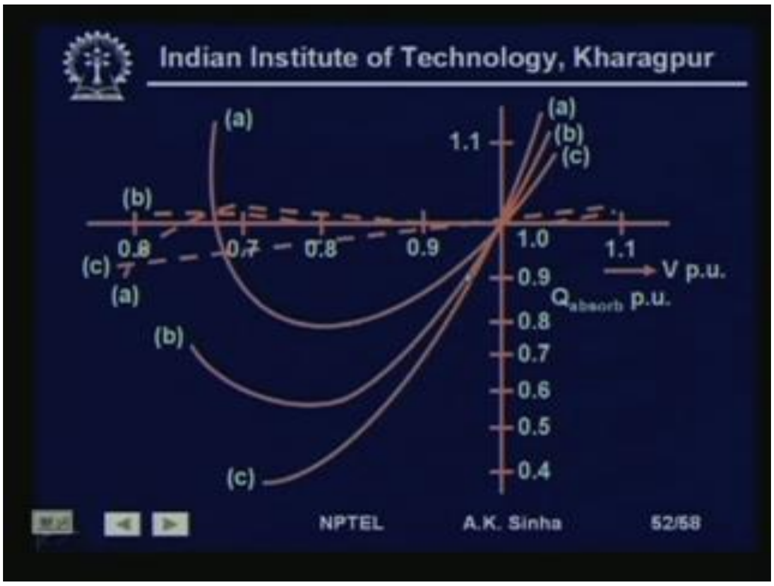
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For variation of voltage for P and Q.

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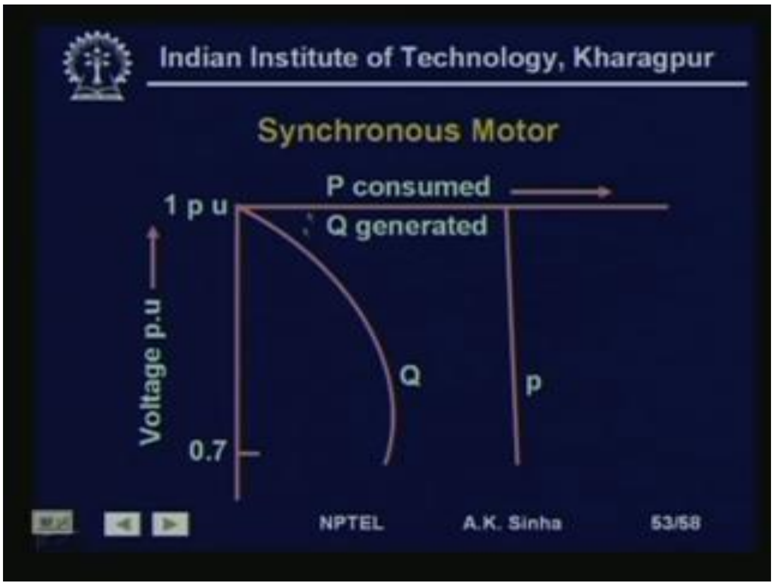

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Synchronous Motor

And similarly for synchronous machine, the Q varies like this, whereas P varies like this.

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Categories of Load

- Residential
- Industrial
- Commercial

And there are other categories in which we can categorize the loads are residential load, industrial and commercial load. This is done for aggregation of the loads.

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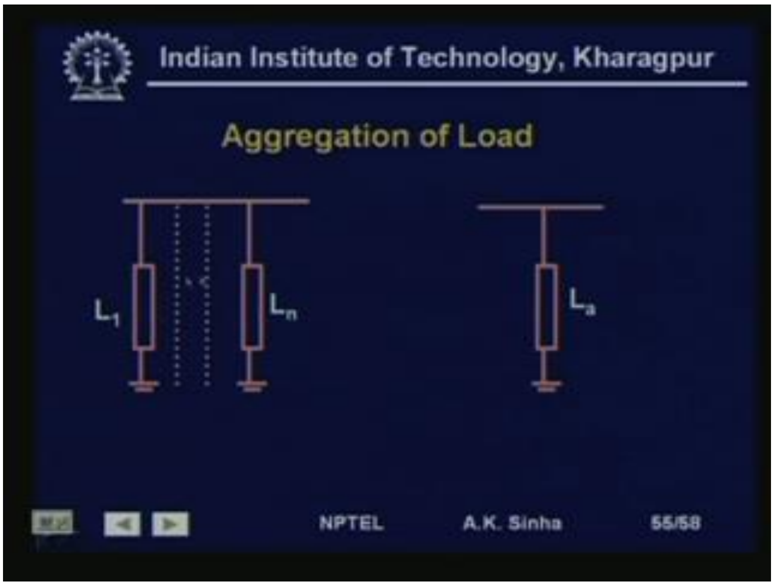

Aggregation of Load

Like we add up various loads into one aggregated load.

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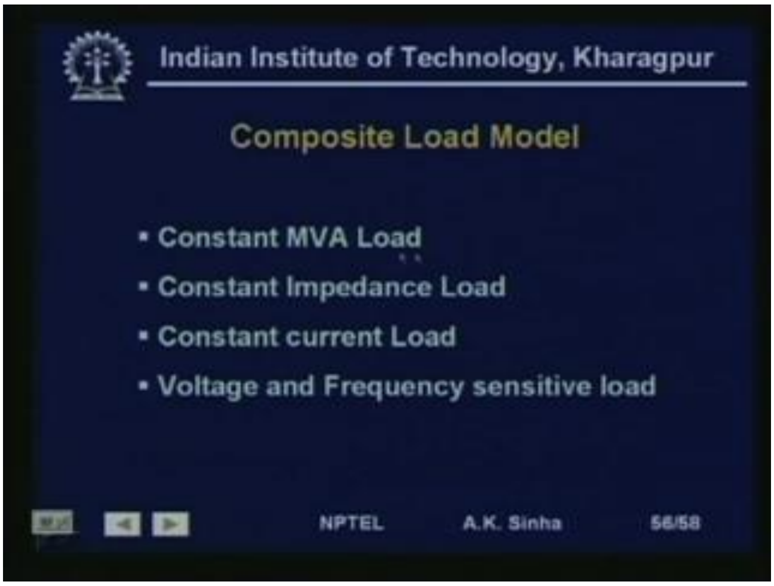

Composite Load Model

- Constant MVA Load
- Constant Impedance Load
- Constant current Load
- Voltage and Frequency sensitive load

So, we can composite loads, can be modeled as constant MVA load. Constant impedance load, constant current type loads and voltage and frequency sensitive loads. All these types of load can be aggregated as a complex model.