Power System Analysis
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Lecture - 19
Power Flow – IV

Welcome to lesson 19 on Power System Analysis. In this lesson, we will continue with Power Flow analysis.

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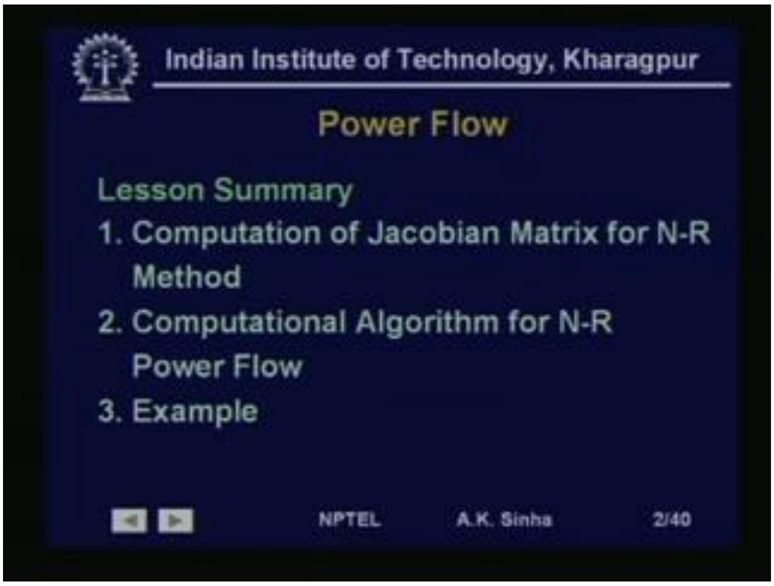

In this lesson, we will talk about the computation of Jacobian matrix for Newton Raphson method. And we will talk about the computational algorithm for Newton Raphson power flow. And we will work out one example for a small power system, how we, do a Newton Raphson load flow or power flow.

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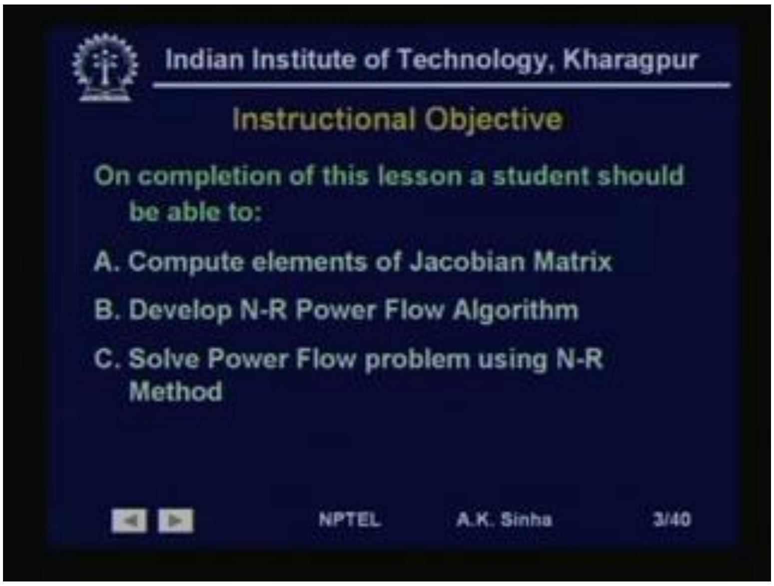

Well, once you complete this lesson, you will be able to compute the elements of the Jacobian matrix. You will be able to develop the Newton Raphson power flow algorithm and solve power flow problems using Newton Raphson method. Before, we go into the computation of Jacobian matrix elements; we will recapitulate some of the equations. For the, that we developed for the Newton Raphson load flow or power flow.

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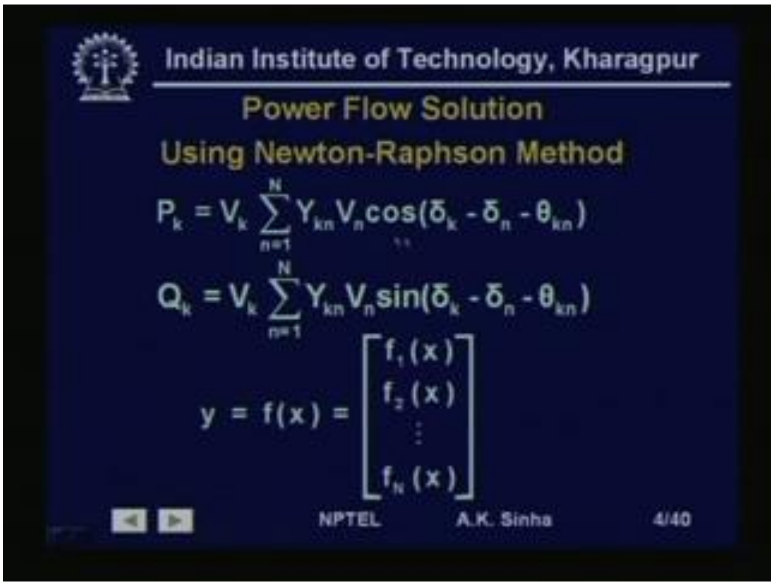

Well, as we had seen in the last lesson, the power flow equations. In terms of real and reactive power injections, at any bus k is given by these relationships. So, which is a function of voltage magnitude at all the buses and the voltage angle at all the buses, as well as the admittance, magnitude and angle for the transmission network. Same, is true for the reactive power, the equations can be return as shown here.

We can write this, as a function of x as y is equal to f x, where for each bus, we will be writing these two equations. So, if there are number of buses, then we will write all these equations as f 1 x up to f n x, total number of equations required.

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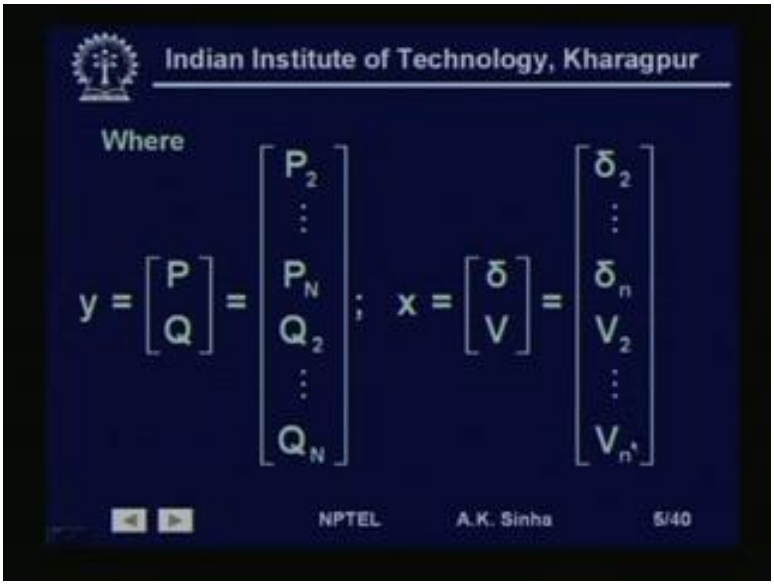

Where, y is basically a vector of real power injections P and reactive power injections Q. Here, we have assumed that bus 1 is a slack bus for our purpose. So, for slack bus, since we know the voltage magnitude and angle, we do not have any real power or reactive power specified. So, we do not have any P 1 and Q 1 in this vector of y. So, we have P 2 to P n and Q 2 to Q n, the unknown vector or the state vector as given by the voltage phase angle delta.

And we again, this delta S for all the buses, except the slack bus for which we already know the angle, as we said earlier, we since the slack bus, can be taken as a reference. So, delta 1 is normally taken as 0 degrees and. So, we have from delta 2 to delta n, the voltage angles and V 2 to V n, the voltage magnitude for all the buses, except the slack bus.

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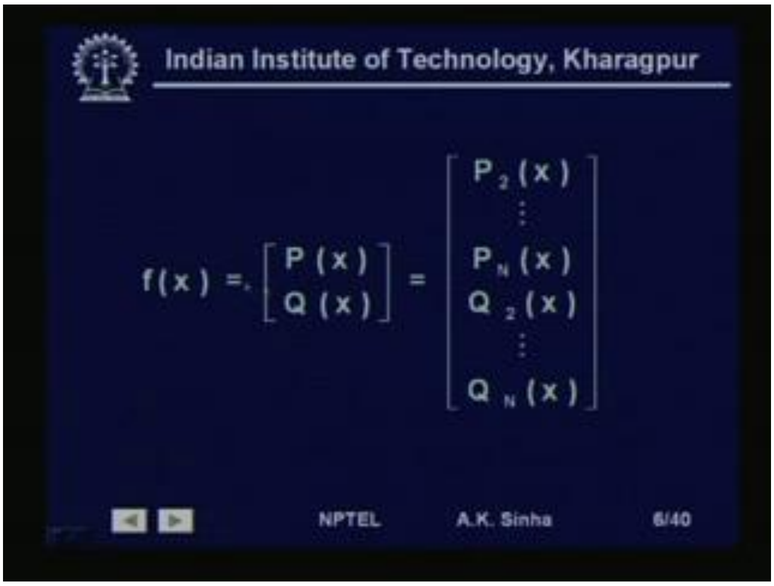

This can be written as f x is equal to P x, Q x in terms of P 2, which is the function of x up to P n and Q 2 which is the function of x, Q which is the function of x, from Q 2 to Q n. This we have already seen. So, I am not going in details of this.

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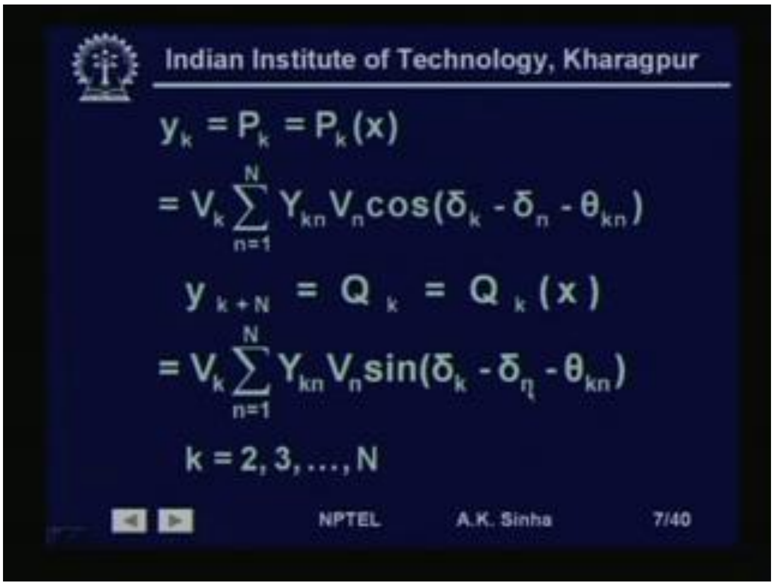

So, for any particular bus, we can write the equation y k is equal to P k, which is P k a function of x. In this term and we can arrange all the reactive power injections from k plus N to k plus. This will be equal to 2 N basically. So, k is equal 2 to N will be the first equations, first N minus 1 equations and N plus 2 to 2 N will be the other N minus 1 equations, for the reactive power. So, we have arranged our equations like this.

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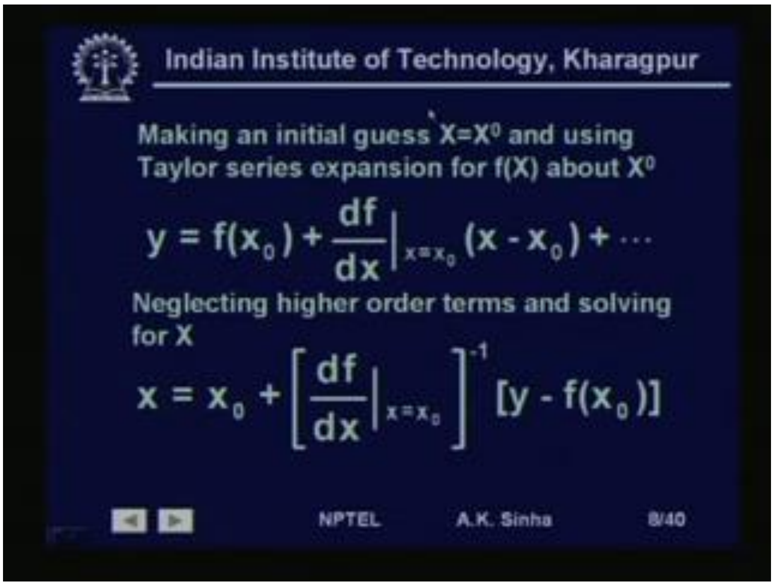

Now, making an initial guess of x at x 0 and using Taylor series expansion for f x about x 0. We can write y is equal to f x 0 plus d f by d x, at x is equal to x 0 into x minus x 0 plus higher order terms. Since, we are assuming that our guess is somewhat near to x. So, the higher order terms are going to be much smaller and they can be neglected. So, neglecting higher order terms and solving for x. We can write this equation as x is equal to x 0 plus d f by d x at x is equal to x 0 inverse into y minus f x 0.

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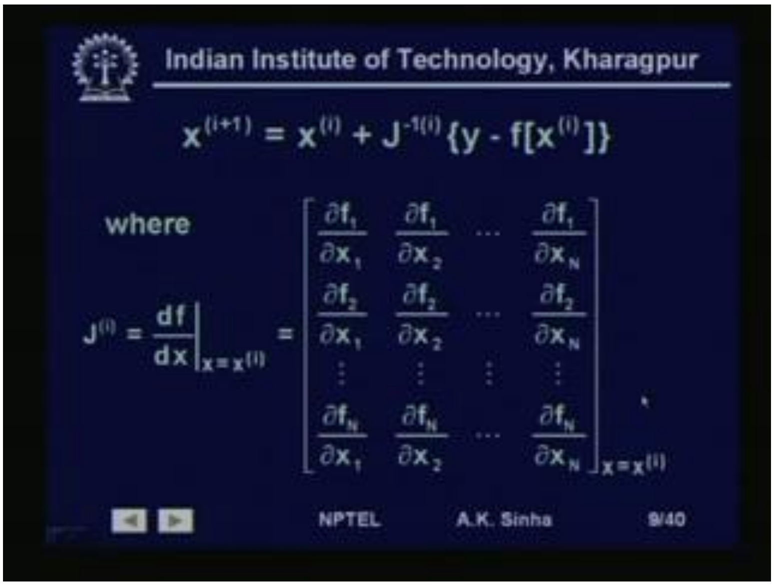

In and in a iterative term, the Newton Raphson power flow equations can be written in terms of x. At iteration i plus 1 is equal to x at iteration i plus J inverse at iteration i into Y minus f x calculated at the x values at ith iteration. So, with this as an iterative form is the basic Newton Raphson algorithm. That we have for power flow solution, where J is basically the Jacobian matrix d f by d x at x is equal to x i.

That is what, we are doing is each equation, we are finding out the partial derivative of the equation with respect to x 1, x 2 up to x n. And for each of these N equations, we have these partial derivatives. So, we will get a Jacobian matrix in this form, which is the first order partial derivative of f with respect to x at x is equal to x i. That is the value of x at the ith iteration.

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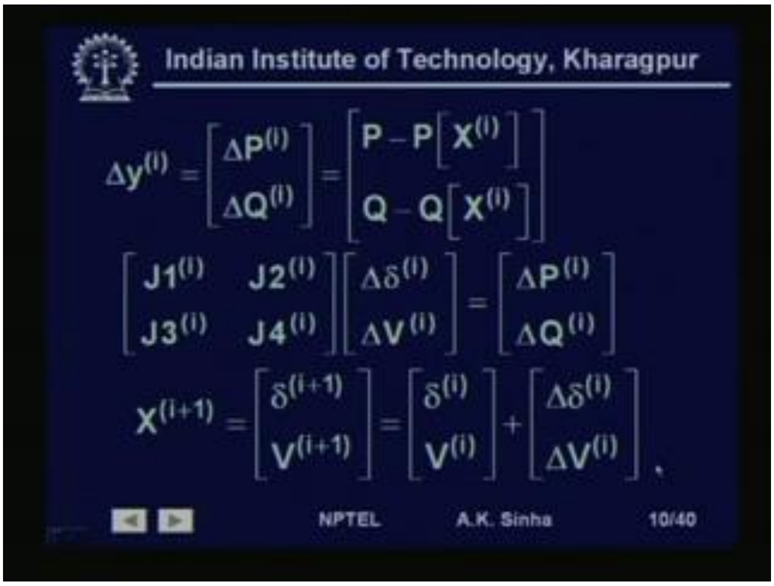

For power flow term, we can write delta y i is equal to delta P i and delta Q i. That is the change in real power and change in reactive power at ith iteration. This will be nothing but equal to the P value, which is specified at the buses minus P value, which is calculated with the state variable x are V and delta calculated at ith iteration. Same thing for Q, Q is specified value at that bus minus Q calculated with x at ith iteration.

These can we written as this Jacobian matrix J 1, J 2, J 3, J 4. That is what, we have done is, we have divided the Jacobian matrix into four parts. J 1 is the part which relates delta P with delta. Delta J 2, will be is the part, which relates delta P with delta V. And J 3, relates the part with delta Q, where delta and J 4 relates the part delta Q with delta V. So, we have this Jacobian multiplied by this delta, delta into delta V is equal to del p i, del Q i. So, this is the equation, now this value we know, using the iteration the value of the state variable at ith iteration.

So, what we need to do is, we need to solve for, the changes in the voltage phase angle and the voltage magnitude at ith iteration. Once, we calculate this, as we see this equation, this is a constant matrix, with the values computed for x at i. So, this constant matrix can be written as the matrix A and this is X. So, A X is equal to Y is the form and we have seen, how we can solve it using the gauss elimination method.

So, once, we solve for this X. That is delta, delta and delta V at ith iteration. Then, we can find out the value of X, which is again same as delta, delta and delta V at i plus 1th iteration. As equal to the value of delta and del and V at ith iteration plus the value of delta, delta at ith iteration and delta V at ith iteration.

So, this is, what we do. So, once, we have got this updated value, then again we calculate the mismatches delta P and delta Q, at i plus 1th iteration. So, again the Jacobian matrix at i plus 1th iteration and solve for this delta, delta and delta V at i plus 2 iteration and so on.

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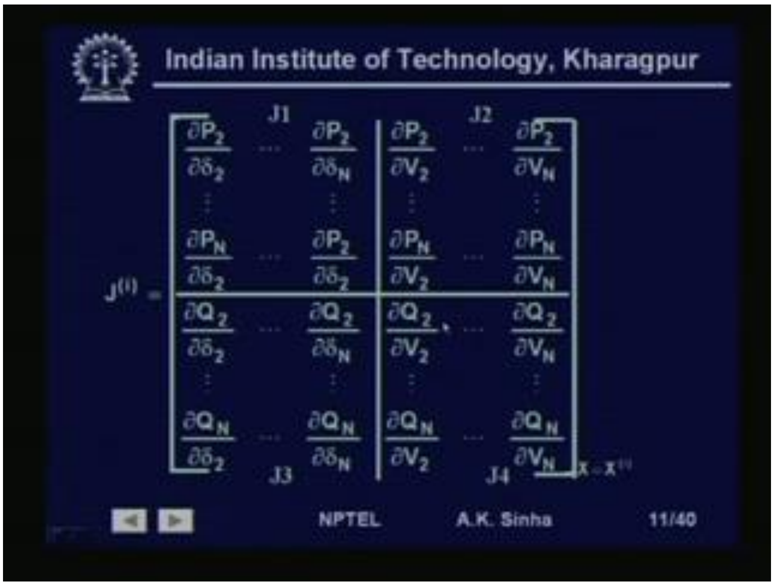

Now, as I said earlier, the Jacobian matrix, we have divided into four parts. J 1, J 2, J 3, J 4, where J 1 is del P by del delta, sub matrix J 2 is del P by del V sub matrix. And J 3 is del Q by del delta sub matrix and J 4 is del Q by del V sub matrix. So, this matrix as we are seeing is N minus 1 into N minus 1 for J 1 and similarly for J 2, J 3 and J 4. All these matrixes are N minus 1 to N minus 1, total size of the Jacobian is 2 into N minus 1.

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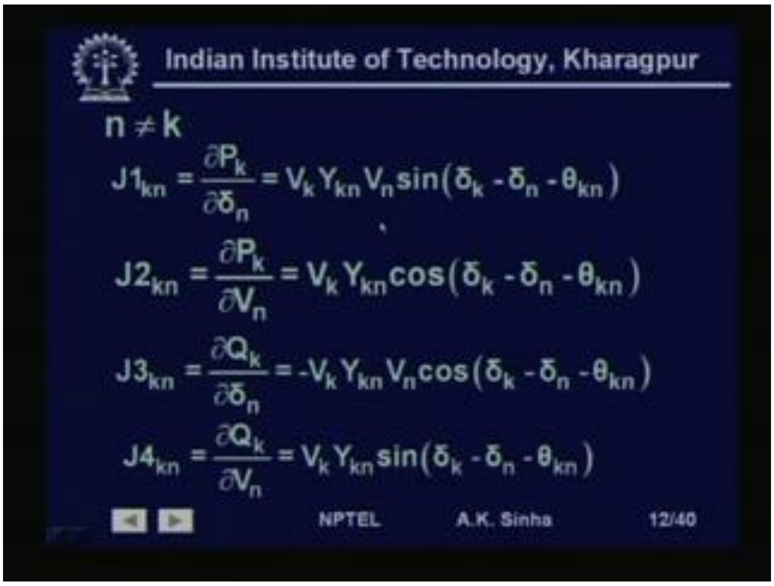

Now, how do, we compute the elements of the Jacobian matrix. If we see, sorry, the elements of Jacobian matrix for J 1 is del P by del, delta. Now, we can calculate the values for N naught equal to K; that means, for the off diagonal terms. So, we can see, how we calculate the off diagonal terms J 1 k n is equal to del P k by del delta N. Now, we know the equation for P k. So, differentiating it with respect to delta n, we can write this as V k, Y k and V n sin delta k minus delta n minus sin theta, sorry, minus theta k n.

Similarly, for J 2 the off diagonal terms can be computed as J 2 k n is equal to del P k by del V n. And this will be equal to V k into Y k n into cos delta k minus delta n minus theta k. J 3 off diagonal terms will be del Q k by del delta n this will be minus V k, Y k n V n cos delta k minus delta n minus theta k n. And J 4 off diagonal terms will be del Q k by del V n, this will be V k, Y k n sin delta k minus delta n minus theta k n. So, if we see, these are the off diagonal terms. That we have for the four sub matrixes.

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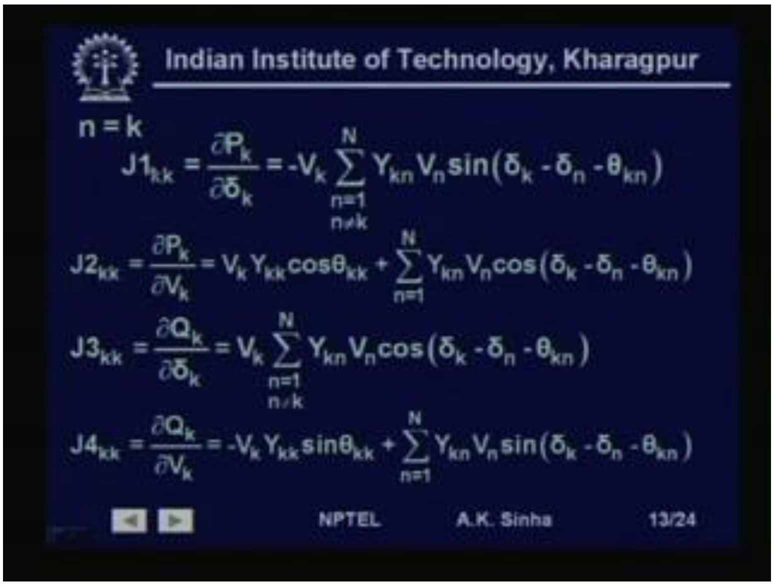

For the diagonal terms, we have again for J 1 the diagonal term for the kth bus can be written as del P k by del delta k, which comes out to be. If we write down the equation for P k and differentiated with respect to delta k, it will come out to be minus V k summation n is equal to 1 to n, where n is naught equal to k. That is for all elements, except that n is equal to k, Y k n, V n sin delta k minus delta n minus theta k n. So, this is in term for J 1 diagonal terms for J 1 sub matrix.

Similarly, the diagonal term for J 2 sub matrix will be del P k by del V k. That is given by V k, Y k k cos theta k k plus summation n is equal to 1 to n, Y k n, V n cos delta k minus delta n minus theta k n. Similarly, for J 3 the diagonal terms will be given by del Q k by del, delta k this is equal to V k, summation n is equal to 1 to capital N, where N is not equal to k, Y k n, V n cos delta k minus delta n minus theta k n.

And the diagonal terms for J 4 sub matrix is del Q k by del V k, which is given by minus V k, Y k k, sin theta k k plus summation n is equal to 1 to N, Y k n, V n sin delta k minus delta n minus theta k. So, these are basically computed using the equation for P and Q at bus k and taking the partial derivative with respect to delta and V. That is the magnitude and the phase angle of voltage at different buses. So, once we have computed the Jacobian matrix, knowing the value of V k and delta k at the previous iterations. We have the Jacobian as a constant matrix.

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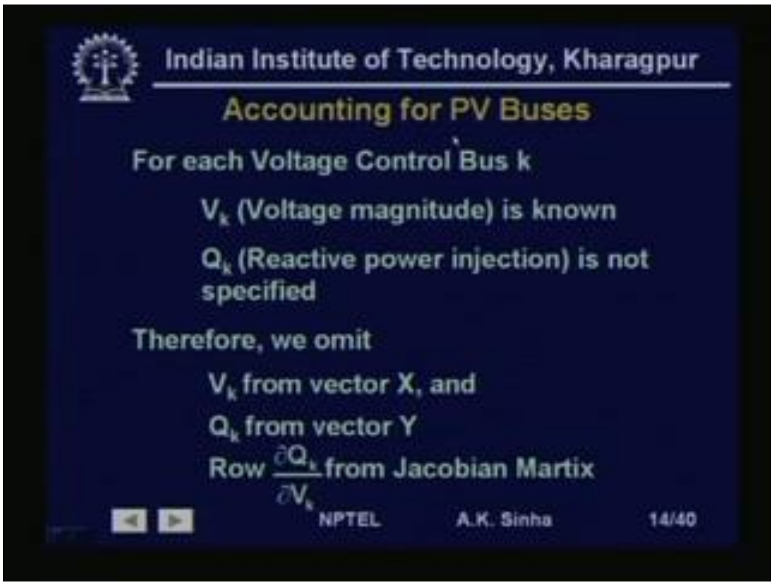

Now, till now what we have done is, we have considered only the P Q buses. Now, we had already said earlier, that most of the generating buses, that we have, are generally having a voltage control. Because, all generators have x excitation system and excitation control system can maintain the terminal voltage at the generator buses. So, most of the generator buses, as well as the buses, where we have reactive power sources available or control reactive power sources available.

For all those buses, we can maintain the terminal voltage and therefore, we call these, for these buses, the voltage magnitude is known and the reactive power is basically being controlled. And that is the unknown parameter for these buses. The buses as we said these generator buses or reactive power control buses are called the voltage control buses or PV buses.

For each of the voltage control bus k, we have V k voltage magnitude is known. Q k reactive power injection is not specified. Because, it is not known, this keeps changing depending on for maintaining the terminal voltage. So, therefore, we omit V k from vector X. That is the state vector; because V k is already known and Q k from vector Y, because Q k is not specified. So, we cannot write any question for this.

Therefore, the row del Q k by del V k, from the Jacobian matrix can be eliminated. That is the del Q k by del V k and del Q k, this row can be eliminated. That is, we do not need to write any equation reactive power injection equation for the voltage control buses.

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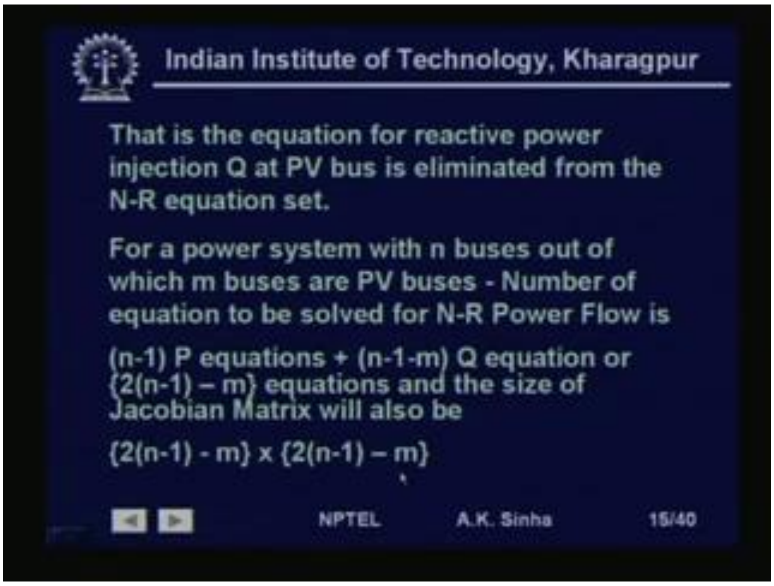

That is the equation for reactive power injection Q at P V bus is eliminated from the Newton Raphson equation set. Therefore, we find that for a power system with n buses, n buses out of which m buses are P V buses. What is the number of equations to be solved for Newton Raphson power flow? Well, as we have seen we need to write n minus 1 P equations. Because, for n minus 1 bus, except for the slack bus, the real power injection at all other buses are specified.

And we need to write n minus 1 minus m Q equations, because we have n minus 1 buses for which we can write the Q equations. But, for m of these n minus 1 bus, the Q values are not specified, rather the voltage magnitude is known. So, we have n minus 1 minus m Q equations only. Also, if we see the number of unknowns, that we have, then we have, n minus 1 voltage phase angle, which are unknown. And n minus 1 minus m voltage magnitudes, which are unknown.

Because, we do not have the value of the voltage magnitude for P Q buses, where Q is specified, but the voltage magnitude needs to be found out. And for P V buses, the voltage magnitude is already known. So, we do not have to find out them. So, the total number of state variables or the unknowns, which we need to find is n minus 1 plus n minus 1 minus m. That is, we have 2 N minus 1 minus m equations for our system and we have 2 into n minus 1 minus m, set of unknowns.

And if we setup this equation, then we will find that the Jacobian. That we have is also having a size, which will be 2 into n minus 1 minus m into 2 into n minus 1 minus m. That is, it is a square matrix, where we have 2 into n minus 1 minus m equations and 2 into n minus 1 minus m, unknowns to be solved for. And as we have seen earlier, we can solve this set of linear equations by using gauss elimination technique.

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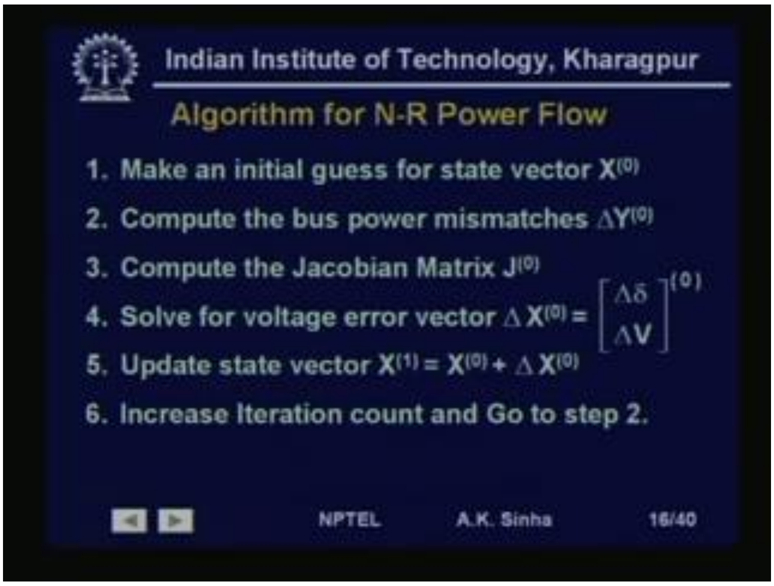

So, what is the algorithm for Newton Raphson power flow? What we have to do is, first we will make an initial guess for the state vector X 0, as we have said earlier the easiest guess, that we have is, a flat voltage start. So, for all the buses, except the slack bus and P V buses, we choose the voltage magnitude, as 1 per unit. And for all the buses, except the slack bus, we have the voltage angle initial guess as 0 degrees.

Of course, for slack bus, the voltage angle is already chosen as 0. Because, it is a reference bus and for the slack bus and all the P V buses, we have already the voltage magnitude, which are specified, so we use those voltage magnitudes. So, using these, we have made the initial guess for the state vector. Now, compute the bus power mismatches delta Y 0. Once, we know the value, we have made the guess of V and delta substituting the value of V and delta into the equation for real power and reactive power.

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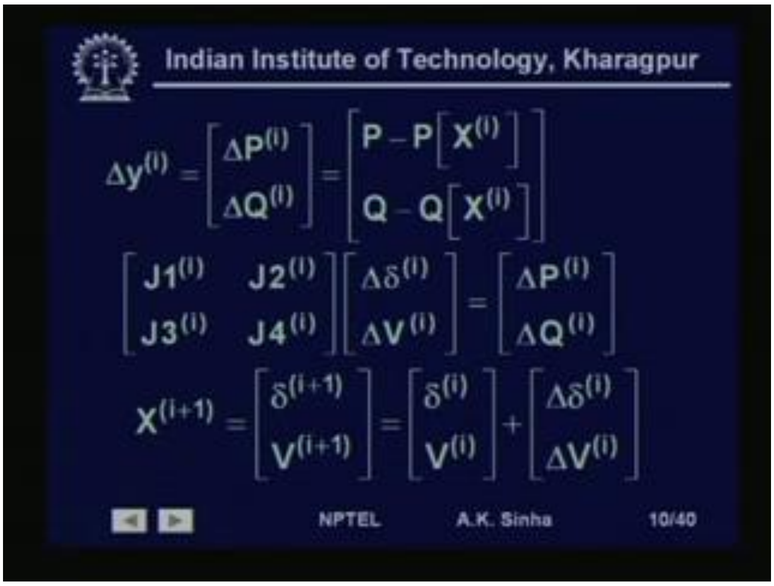

We can find out the mismatch, using this relationship. P specified minus P calculated with the value of X, known at the initial guess. Q specified minus Q calculated with X at the specified value. So, using this, we can find out this delta P and delta Q, which is basically our delta Y. So, compute the bus power mismatches delta Y. Next step is, compute the Jacobian matrix J 0. That is with the initial guess state vector X.

That knows all the voltage magnitude and angles with the guess initial values known. We compute the value for the elements of the Jacobian matrix at the initial operating point. Now, solve for voltage error vector delta X which is delta, delta and delta V, from the initial guess. That is the error vector, based on the initial guess. We can find out using, as we have seen earlier, this relationship using this relationship, we solve for this.

Now, this is already known at the initial value. This is known at the initial value, this is what we need to compute at the initial state. So, once we have computed this error vector, then we update the state vector, as the as value of state vector at first iteration is equal to the value of state vector at initial guess. Plus the voltage error vector or the error vector error in state vector. So, we add this, we get the value at the first iteration.

Now, we increase the iteration count and again go back to compute the bus power mismatches, delta Y 1, now knowing X 1. Similarly, compute the Jacobian matrix J 1, solve for error vector delta X 1 as delta, delta and delta V at for the first iteration. Then, update the state vector, you will get X 2 is equal to X 1 plus delta X 1 and so on. We will keep repeating this iteration, till we find that delta Y has become very small.

Because, once this delta Y or the power mismatches are very small, which means, that the value of the calculated bus power injections are very nearly equal to the specified value, that means, we have got a converge solution. That is the voltage magnitude and the angles, that we have got now is giving me a complete solution for this system. That is substituting these values of voltage magnitude and the angles. We will be able to get the same values as specified for the real power and reactive power injections at the buses.

Now, once we know these values of the voltage magnitude and angle. Then, rest of the values, which are basically the real power and reactive power at the slack bus. And the reactive power injections at the P V buses can be computed by substituting this value of del value of V and delta, which we have found as a final solution.

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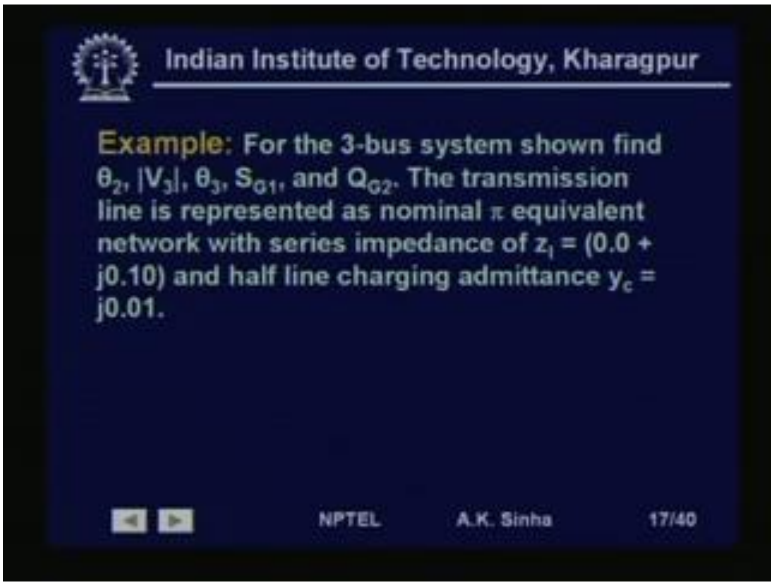

Now, we will take one example for solving this. What we do is, we will take a small three bus system as shown in the figure.

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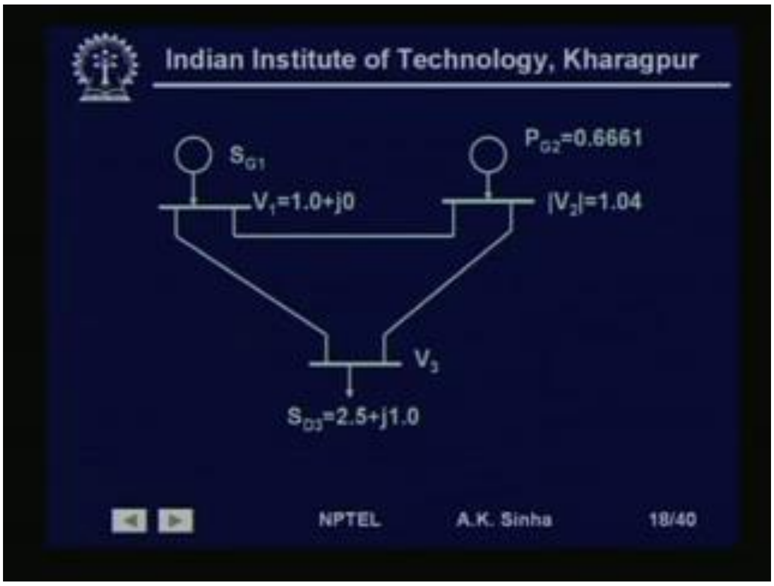

Here, we have a three bus system, this is bus 1, this is bus 2 and this is bus 3. Now, bus 1, we have chosen as a slack bus. So, here the voltage magnitude, as well as angle specified. We have port here voltage magnitude as 1 per unit and angle is 0 degrees. That is, in rectangular coordinate form, we can write it as 1.0 plus J 0. So, voltage magnitude and angle at this is specified. The real and reactive power injection at this bus is unknown.

Bus number 2, as we see here the voltage magnitude is specified as 1.04 and the real power injection into the bus is also specified as 0.661 per unit. Now, here, what is unknown is the voltage angle delta 2 and Q G 2. So, at this bus, we have Q G 2, the reactive power injection into the bus and the voltage phase angle is unknown. This is bus number 3, at this bus; we only have a load, which has a value of 2.5 plus J, 1.0. That is, 2.5 per unit real power and 1.0 per unit reactive power.

So, at this bus, both P and Q injections are known. P injection will be minus 2.5 and Q injection will be minus 1.0. Because, the injections, we are saying will be into the bus as positive. Since, this is coming out of the bus. So, we will have to take the negative of this, which we can say that here S G is 0. So, P G is 0. So, 0 minus 2.5 will give me P I and 0 minus 1.0 will give me Q I at this bus.

So, that again can be written in the same way as minus 2.5 minus J, 1.0 will be the injections into this bus. Now, this is a slack bus, this bus we have the real power specified and the voltage magnitude specified. So, this is a PV bus, this bus both the real and reactive power are specified. So, this is a P Q bus.

So, now if we see the problem, we have for the three bus system shown, find. What we need to find? That is theta 2, here we have written, which is the phase angle of the voltage at bus 2. Magnitude of voltage at bus 3, theta 3, which is the angle of voltage at bus 3 and we also need to find out the real and reactive power injection at the slack bus. And the reactive power injection at the P V bus or bus 2.

Now, the transmission lines, that we have in this system. We have a transmission line between 1 and 2, a transmission line between 1 and 3, another transmission line between 2 and 3. Now, all these transmission lines are represented as nominal pi equivalent network. So, as we had seen, when we derive the equivalent circuit for a transmission line, that we can represent a transmission line by a pi equivalent or a nominal pi equivalent circuit.

For which, we have the series impedance for this system of line given as 0.0 plus J, 0.10. That is resistance is neglected are negligible and the line has a inductive reactive of 1.10 per unit and the half line charging admittance of the line is J, 0.01. That is the half line charging capacitive element at the two legs, two hands of the network or the two buses is equal to J, 0.1 per unit in terms of capacitive, susceptance.

So, this is the system, that we have and now we will see, how we set up the equations and solve using Newton Raphson power flow to get the values for the unknowns. That is theta 2, for this V 3 and theta 3 for this. And of course, after we have, what those values we can compute P G 1, Q G 1 and Q G 2.

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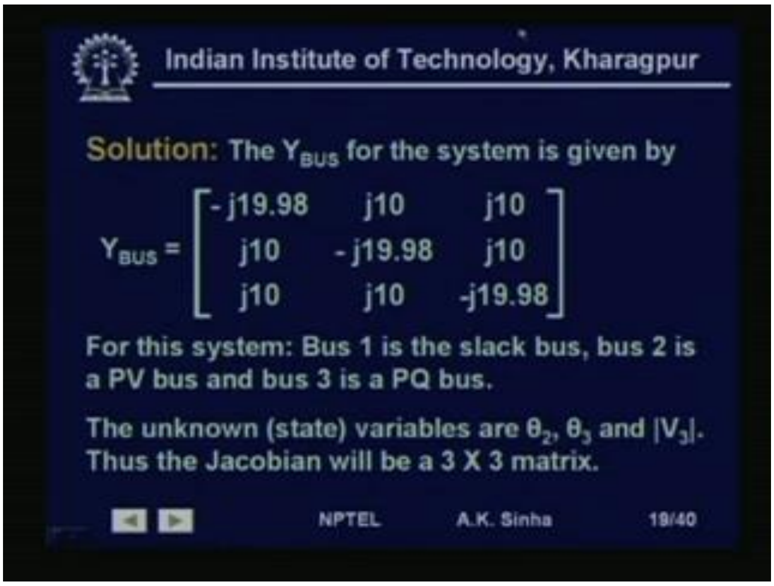

Now, first thing, what we will do is, we will form the Y bus matrix for the system. Now, as we have seen the real power, the series reactance for the line is 0.1. So, in terms of admittance, we will get this as minus 10. So, and the off diagonal terms for the admittance matrix as we have seen is minus Y i j. So, we have got J 10, J 10 as for all the lines are having the same impedance. So, we have got this.

Now, the diagonal terms, as we have seen, will have the sum of all the series admittances connected to the bus, plus the line charging capacitances connected for the lines, which is connected to the bus. So, half line charging at the ends of the line is also to be added. When, we do that, then we see that for this line, we have this bus, we have two lines. One between 1 and 2, another between 1 and 3 and both will have a half line charging of 0.01 per unit.

So, minus 0.02 per unit is going to be added to this. So, we have got minus J 19.98, because the some of the two series admittances for these two lines will be J 20. So, minus J 20 and then we have to subtract from this, because we have the capacitive susceptances for the line charging. So, this comes out to be minus J 19.98. Same will be occurring for bus 2 and bus 3.

Therefore, for this system as I said earlier, bus 1 is slack bus, bus 2 is a P V bus and bus 3 is a P Q bus. The unknown state variables are theta 2, theta 3 and V 3. That is the voltage angle at bus 2, bus 3 and the voltage magnitude at bus 3. Thus the Jacobian matrix for the system will also be a 3 by 3 matrix. What we have is the knowns or the specified values will be P 2, P 3 and we have the specified value of Q 3. So, three known's, three unknowns and we will have. So, three sets of equations and the Jacobian matrix will be a 3 by 3 matrix.

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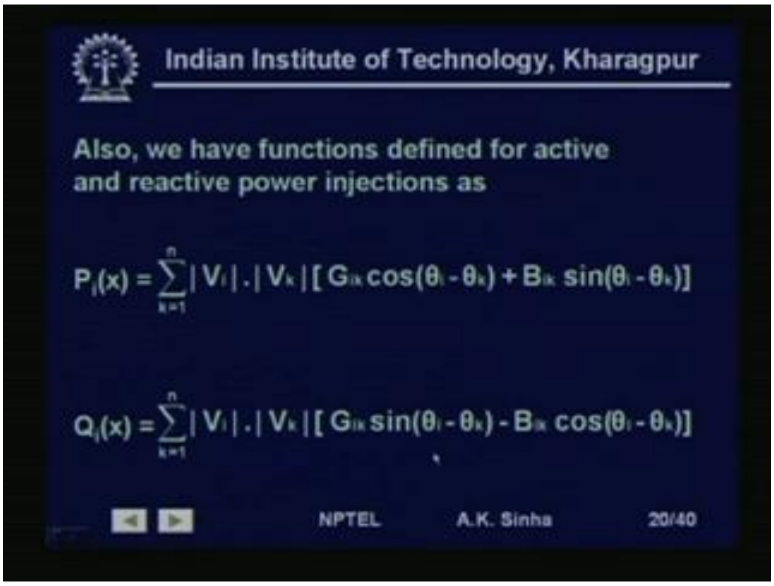

So, we have as us we had written earlier, we are again writing the in real and reactive power injection equations here.

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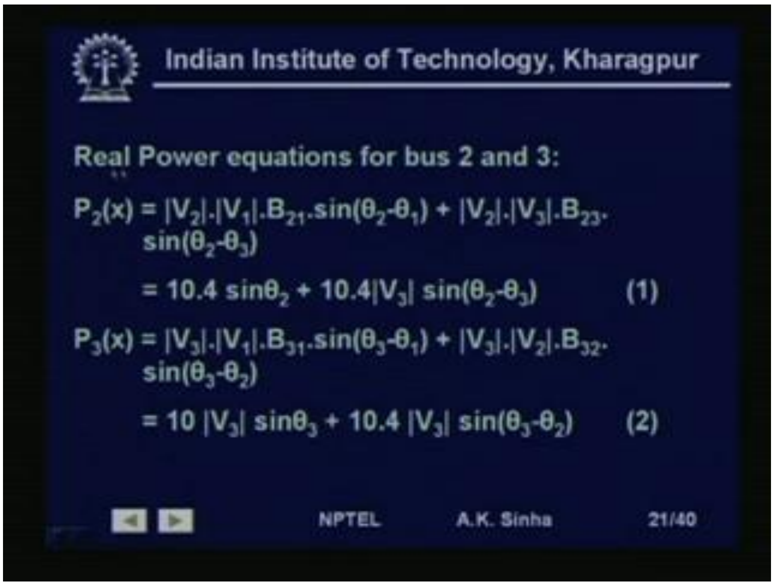

Now, substituting, so real power equations for bus 2 and 3, we can be written in this form. Because, here we have seen that the real part for the Y bus is 0. So, G's are not there. So, only B is present. So, we write the equation like this V 2, V 1, B 21 sin theta 2 minus theta 1 plus V 2, V 3 into V 23 sin theta 2 minus theta 3.

Now, substituting the values of B 21, B 23, we get this as 10.4 sin theta 2 plus 10.4, V 3 sin theta 2 minus theta 3. Now, here theta 1 is the slack bus angle, which is 0. So, this is sin theta 2 minus 0. So, we have written this as sin theta 2. Similarly, for P 3, we can write the equation and we get this as V 3, V 1, V 31 sin theta minus theta 1 plus V 3, V 2 sin B 32 sin theta 3 minus theta 2. So, this comes out to be 10, V 3 sin theta 3 plus 10.4, V 3 sin theta 3 minus theta 2.

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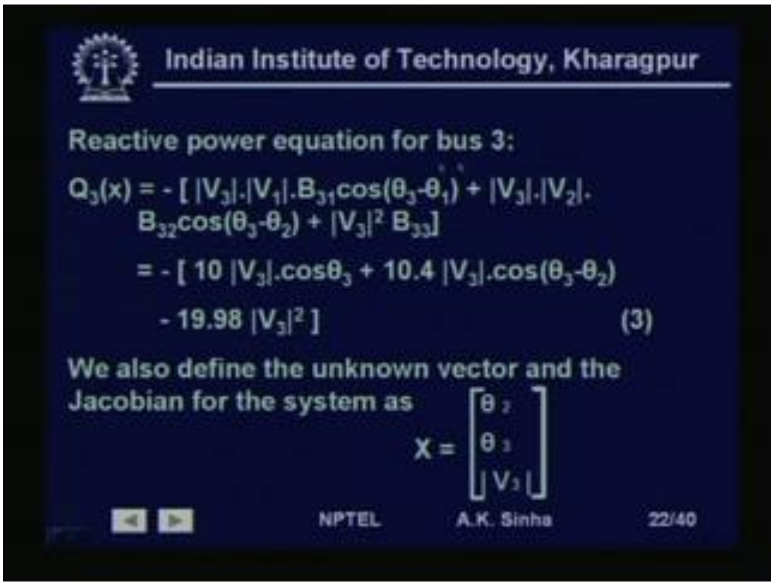

And again the reactive power equation for bus 3 can be written in terms of Q 3 is equal to minus V 3, V 1, V 31 cos theta 3 minus theta 1 plus V 3, V 2, B 32 cos theta 3 minus theta 2 plus V 3 square into B 33. So, substituting again the values of B 31, B 32, and B 33, we will get this as minus 10, V 3 cos theta 3 plus 10.4, V 3 cos theta 3 minus theta 2 minus 19.98 V 3 square. Now, we define the unknown vector and the Jacobian matrix of the system. What is the unknown vector, the state vector, which we need to find is theta 2 theta 3 and V 3 as we had said earlier.

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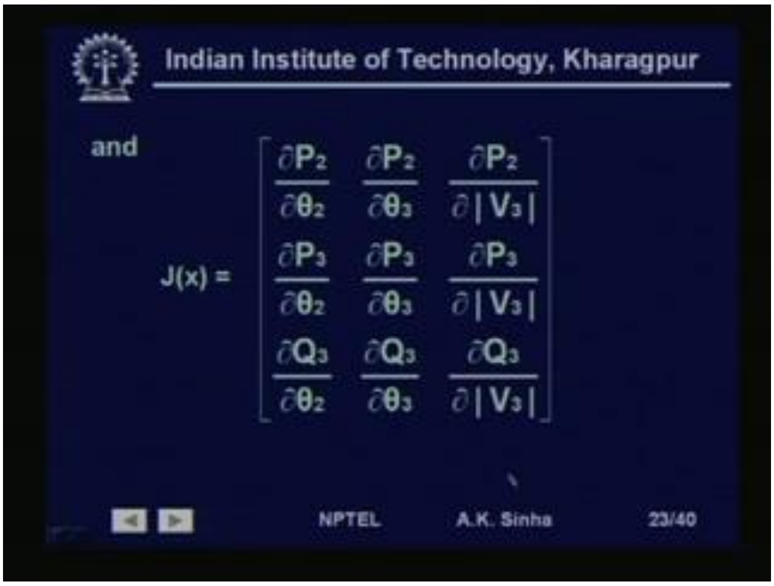

And the Jacobian matrix, that we will have will be del P 2 by del theta 2, del P 2 by del theta 3, del P 2 by del V 3, for the first set of equations for the real power P 2. For real power P 3 equations, the partial derivatives with respect to the state vectors will be del P 3 by del theta 2, del P 3 by del theta 3, del P 3 by del V 3. Similarly, for the reactive power equation at bus 3, the partial derivatives with respect to the state variables will be del Q 3 by del theta 2, del Q 3 by del theta 3, del Q 3 by del V 3. So, this 3 by 3, Jacobian matrix will be there.

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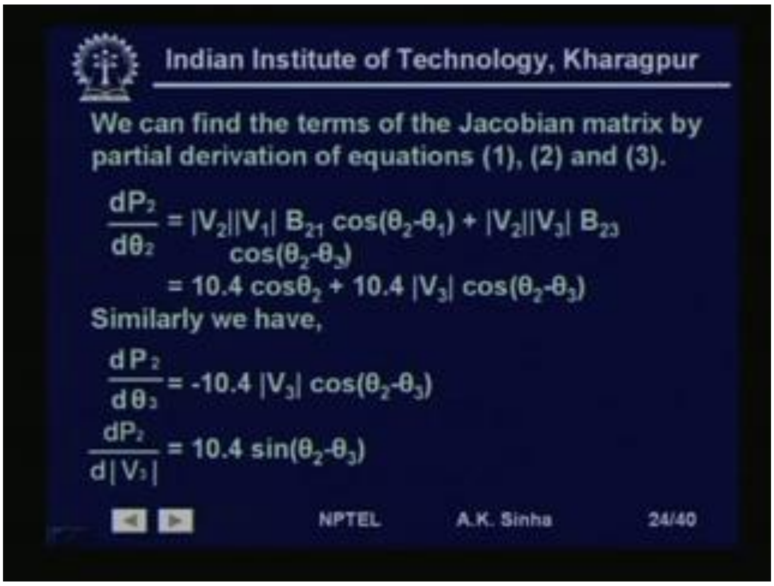

We can find the terms of the Jacobian matrix by partial derivation of the equation 1, 2 and 3. That is what the equations that we had written here.

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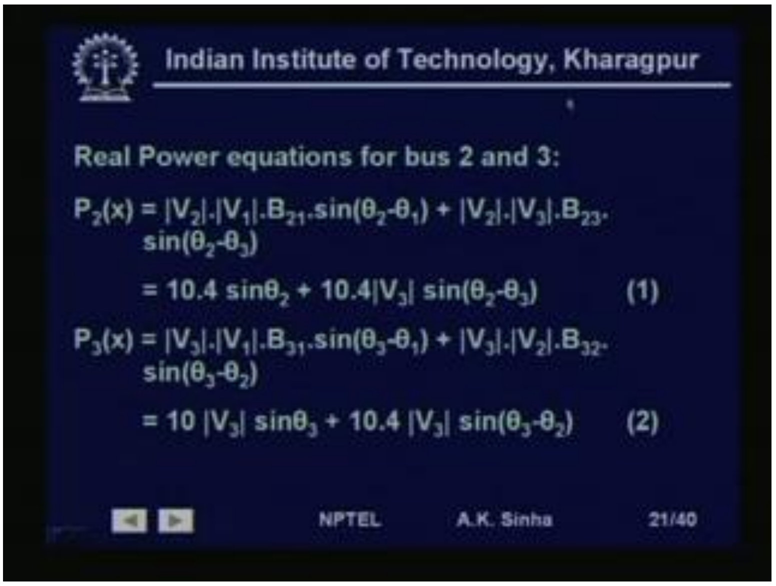

These three equations, 1 2 and 3, so by finding out the partial derivatives for these three equations, we can write the or get the elements of the Jacobian matrix del P 2 by del theta 2. Will come out to be like this, del P 2 by theta 3 will come out to be like this and del V 2 by del V 3, will come out to be like this. That is del P 2 by del theta 2 is 10.4 cos theta 2 plus 10.4, V 3 cos theta 2 minus theta 3. Del P 2 by del theta 3 will be minus 10.4, V 3 cos theta 2 minus theta 3 and del P 2 by del V 3 is 10.4 sin theta 2 minus theta 3.

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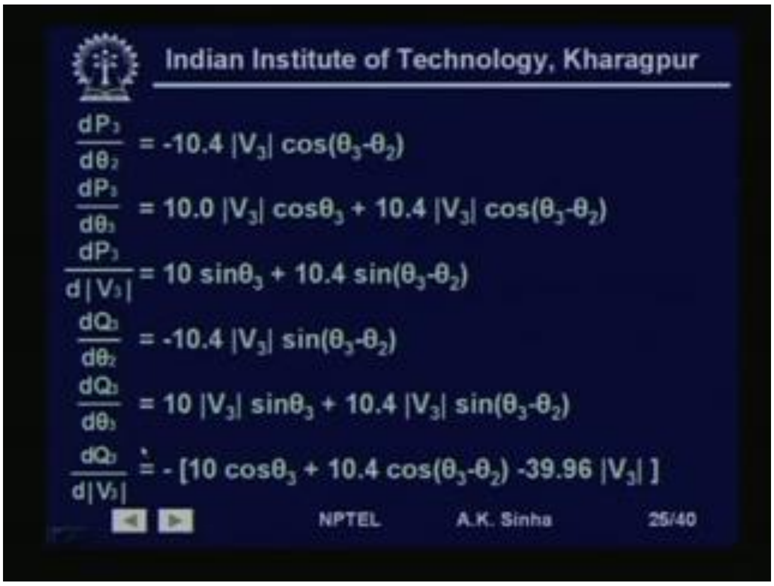

Similarly, for P 3 we will have del P 3 by del theta 2 as this del P 3 by del theta 3 and so on, for all these terms, we can compute.

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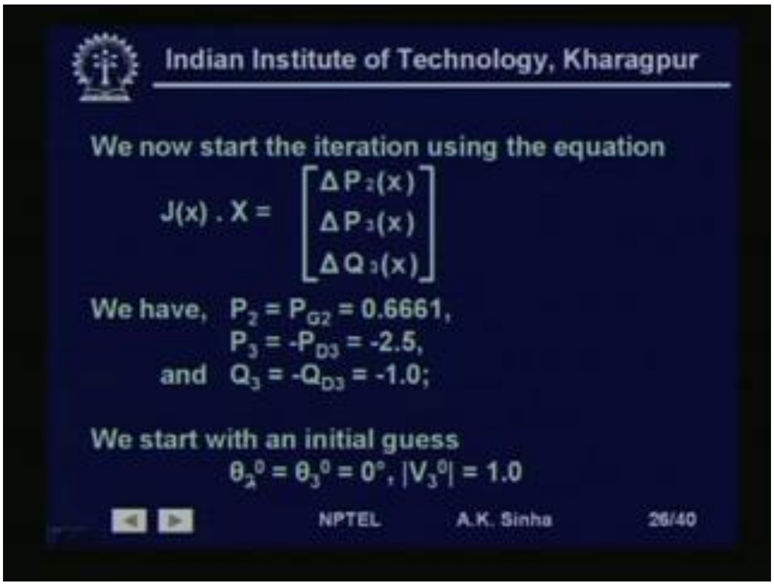

Now, we can start the iteration using the equations J, X J, which is a function of x into X is equal to Y, which is delta P 2, delta P 3 and delta Q 3. Now, we have the specified value of P 2, which is P G 2 is 0.661, 6661 per unit. P 3 the injection is minus P D 3, this is equal to minus 2.5 and Q 3 is equal to minus Q D 3, which is equal to minus 1.0. Now, if we said earlier, we start with an initial guess for the unknown voltages magnitude and the angles as theta 2 0 theta 3 0 is equal to 0 degree and V 3 0 is equal to 1.0. That is, we assume the voltages magnitudes for the unknown voltage as 1.0. And the voltage phase angle for the unknown phase angles of voltage as 0 degrees. That is our initial guess.

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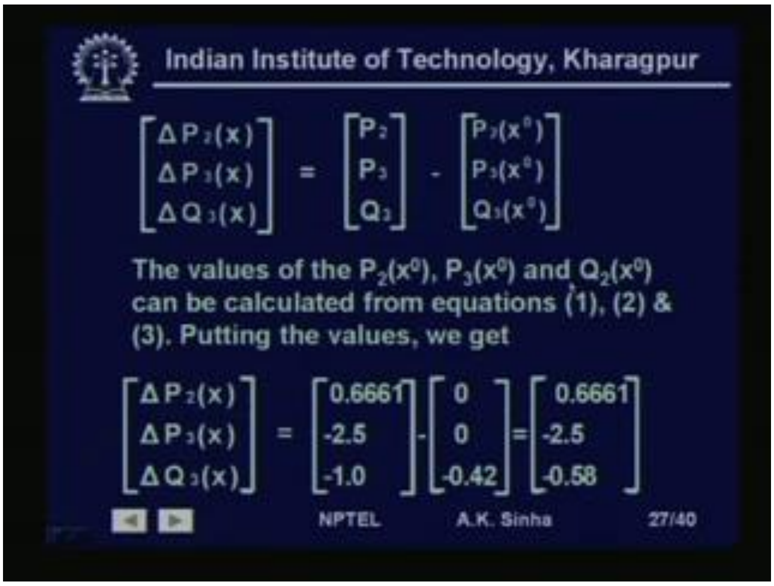

So, we can get the value of the mismatches del P 2 del P 3 and del Q 3 from the specified value of P 2, P 3, Q 3 minus the value of P 2 calculated with the initial guess values of x. So, P 2 calculated P 3 calculated and Q 3 calculated the value of P 2 at x is equal to 0. P 3 at x is equal to 0 and Q 3, this should be Q 3 at x is equal to x 0 can be calculated from equations 1, 2 and 3. Putting the values, we get del P 2, del P 3, del Q 3 as equal to 0.6661 minus this comes out to be 0.

So, this is 0.06661, this is del P 3 is specified value is minus 2.5. The calculated value is 0. So, this comes out to be minus 2.5. Del Q 3 the specified value is minus 1.0. The calculated value comes out to be with the initial guess value comes out to be 0.42. So, we have minus 0.58 as the mismatch del Q 3.

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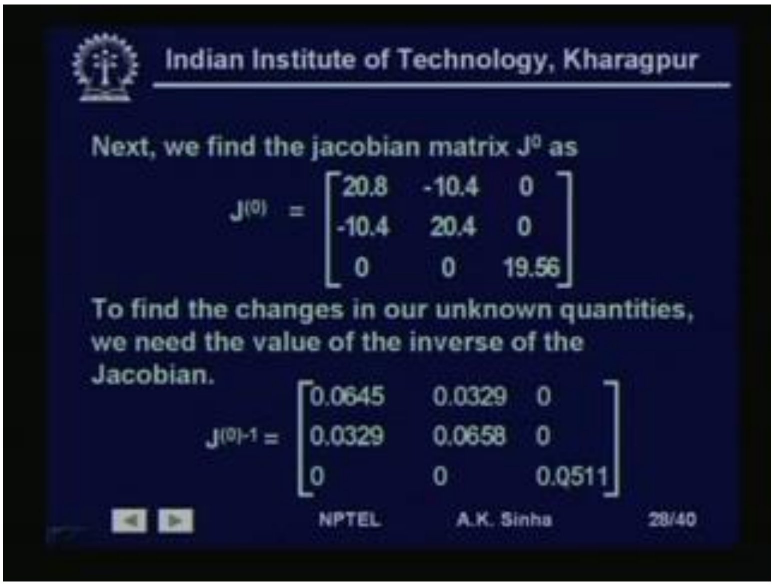

Similarly, we can calculate the Jacobian matrix elements by substituting the value of the state variables with the initial the guess values. So, this comes out to be the 3 by 3 Jacobian matrix, 20.8 minus 10.4, 0 minus 10.4, 20.4, 0, 0,0 19.56. So, this is 3 by 3 Jacobian matrix, that we have. So, what we need to do is, we can solve the equation now, that del P del Q is equal to J del X or that is del delta del V.

So, for that either we can take a inverse or we can solve it using gauss elimination. Since, this is a 3 by 3 matrix and also, this is a block diagonal matrix. That is this 2 by 2 and this 1 by 1. So, we can take the inverse of this 2 by 2 and take an inverse of this. That is 1 by this. So, if we do that, then we get this inverse Jacobian matrix like this.

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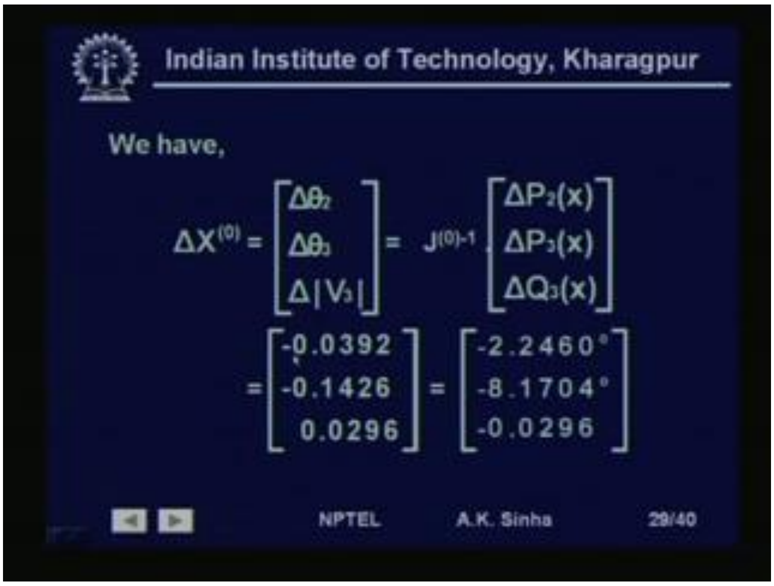

So, using that, we can calculate the change in the state vector. So, initial error of the state vector, will be given by J, 0 inverses in to del P 2, that is del Y. So, if we substitute that then we solving it, we will get the value of delta X, 0 as 0 minus 0.0392 minus 0.1246. So, this is delta theta 2, this is delta theta 3 and this is delta V 3. So, these values, you mind it, the angle values, when we calculate, using this will be in radians.

So, we can convert them degrees, which we will get like this. This theta 2 comes out to be 2 minus 2.246 degrees, delta theta 3 comes out to be minus eight point one seven 0 8.1704 degrees. And delta V 3 comes out to be 0.0296 per unit.

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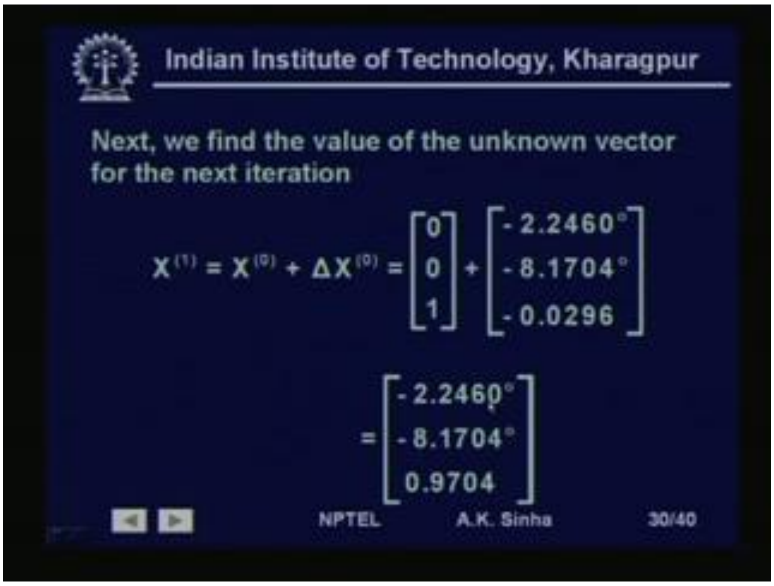

So, using this error values at the first iteration, we get the updated value of the state variables at the first iteration as X 1 is equal to X 0 plus delta X 0. So, the initial guess value of X 0 was 0 degree 0 degrees and 1 per unit. So, we add to this the delta X, which is the value, when we substitute. So, we get the value of the state variable, after first iteration as minus theta 2 is equal minus 2.246 degrees. Theta 3 is minus 8.1704 degrees and V 3 as 0 0.9704 per unit.

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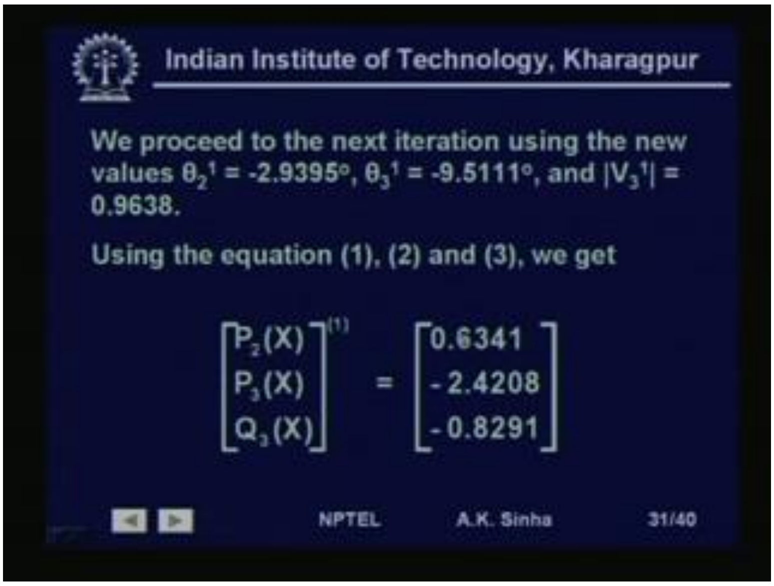

So, once, we have these values now. So, using we proceed to the next iteration using the new values. So, when we substitute this value, again we will calculate P 2, P 3, Q 3 at the first iteration, which we get like this.

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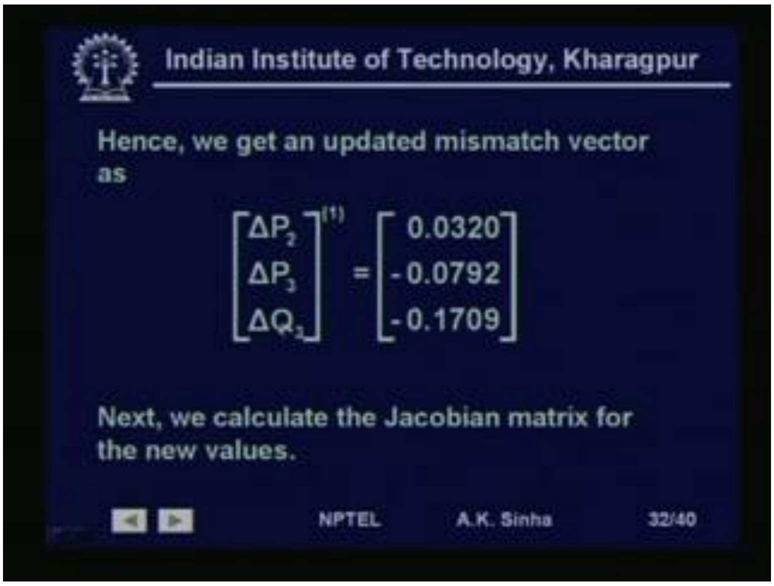

So, we can calculate delta P 2 delta P 3 and delta Q 3, which will be again calculated value specified value minus the calculated value, where the calculated value is computed using the updated or the first iteration value of the state variables. So, when we do that, we get these values like this. Now, we calculate the Jacobian matrix with the new values of X.

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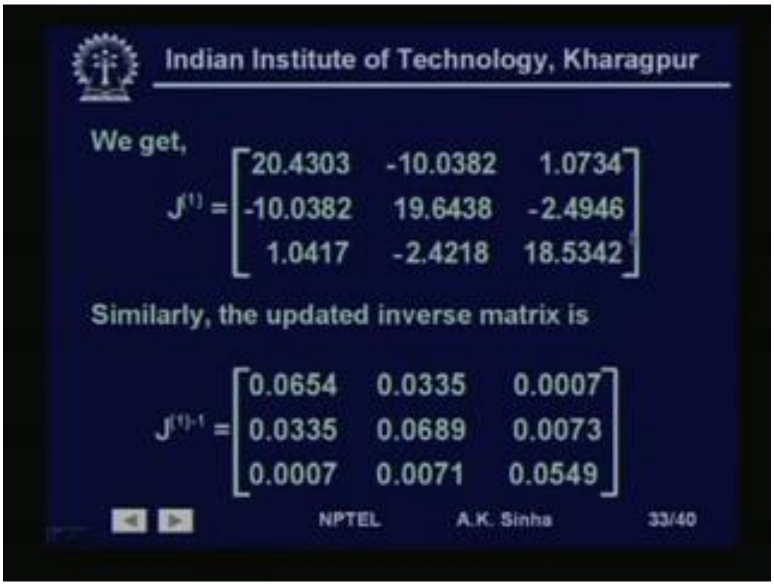
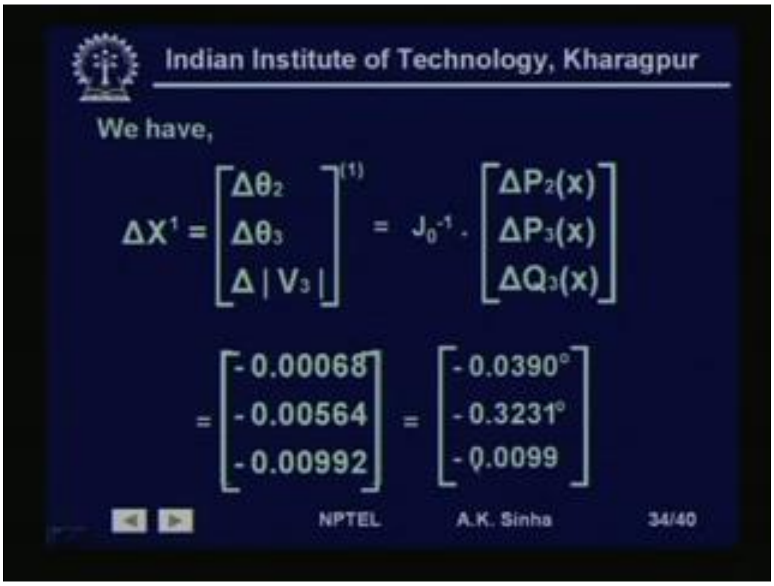


We see that we have a Jacobian matrix, a 3 by 3 Jacobian matrix given like this at the first iteration values of X. Similarly, the updated inverse can be computed like this. (Refer Slide Time: 48:01)



And once, we have this, we can get delta X 1, the error in state variables at first, after first iteration. So, this comes out to be 0.00068. That is delta theta 2. After, first iteration comes out to be this, delta theta 3. After, first iteration comes out to be minus 0.00564. Radians and delta V 3, after first iteration substituting the first iteration values or solving after using the first iteration values, comes out to be minus 0.00992.

When, we convert these two values, which are in radians in to degrees comes out to be this much. That is minus 0.0390 degrees minus 0.3231 degrees and the voltage will be change in voltage will be minus 0.0099 per unit.

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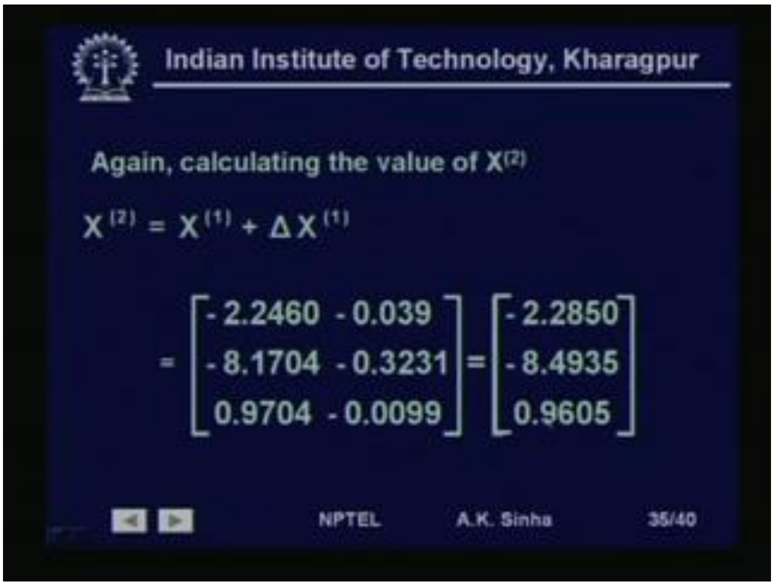

So, again, we can get the updated value at the second iteration as the value at the first iteration plus the change or the error computed after first iteration. So, when we do that, we get the values as minus 2.2850 degrees minus 8.4935 degrees and the voltage magnitude V 3 as 0.9605 per unit.

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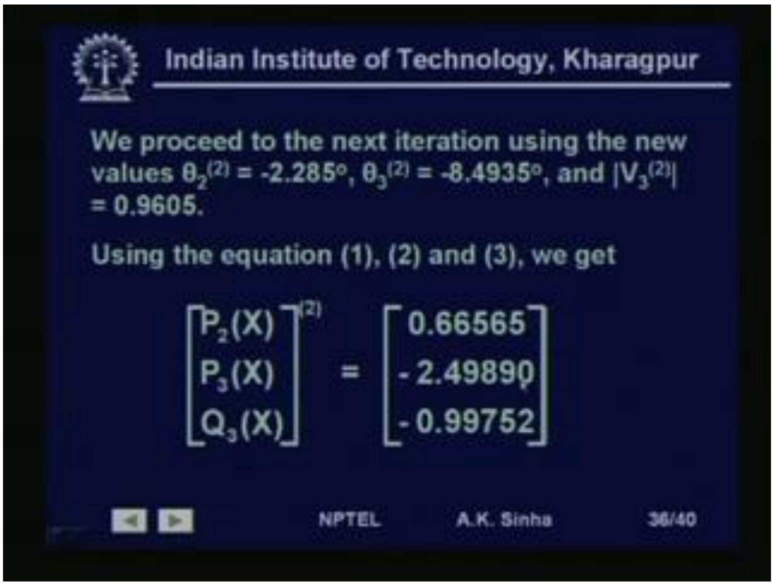

Now, again using these values, we can compute the calculated values for P 2, P 3 and Q 3, which comes out to be this much. That is P 2 with the new values of the voltage magnitude and angle comes out to be 0.66565. And the P 3 comes out to be minus 2.49890 and Q 3 comes out to be minus 0.99752.

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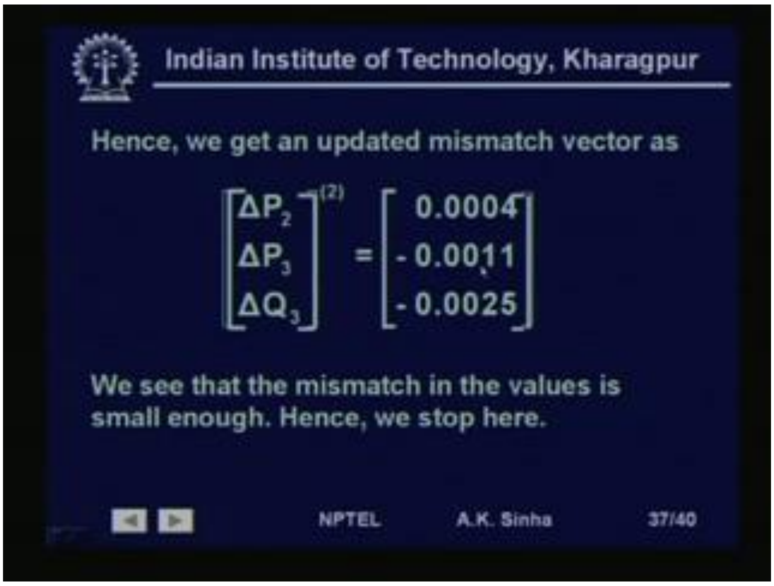

Now, if we use these calculated values, then we can calculate the mismatches del P 2 del P 3 and del Q 3, which will be again P 2 specified minus P 2 calculated, When, we do that this comes to be 0.004 per unit, del P 3 comes out to be minus 0 0.0011 per unit and del Q 3 comes out to be minus 0.0025 per unit. What we are saying is, that the mismatch values have now come to be very small. If we do another iteration, the values will become very, very small. So, what we are saying is, that we can if accept this kind of an accuracy, then we have already obtained our solution, where we have the value of X.

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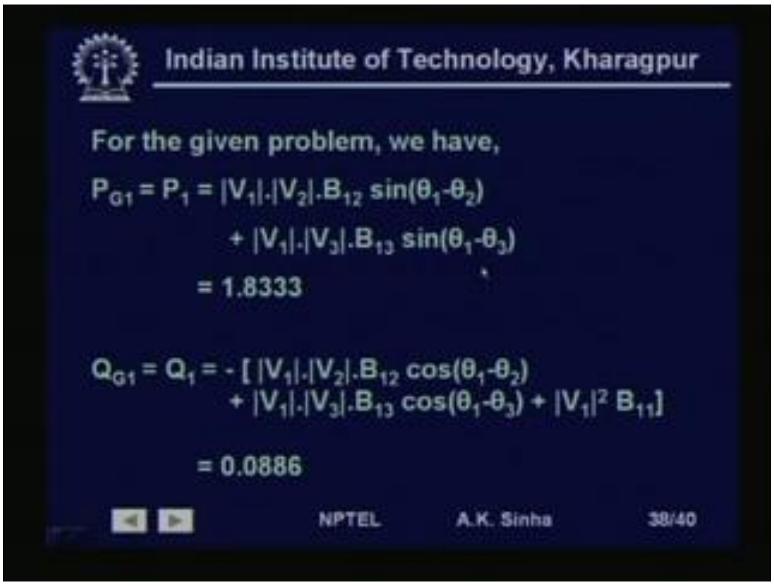

That is theta 2 as minus 2.285 degrees theta 3 as minus 8.4935 degrees and V 3 as 0.9605 per unit. So, this is the solution that we have, that is with these values, we have got quite a good convergence and we have got this convergence in just two iterations. If you do one more iteration, the convergence can be up to four or five decimal places. So, we have got the solution now, so using this value of V and delta for all the buses. Now, since they are known if we substitute in the equation for P G 1 and Q G 1, we get P G 1 as 1.8333 per unit and Q G 1 as 0.0886 per unit. Sorry and Q G 2 as 1.288 per unit.

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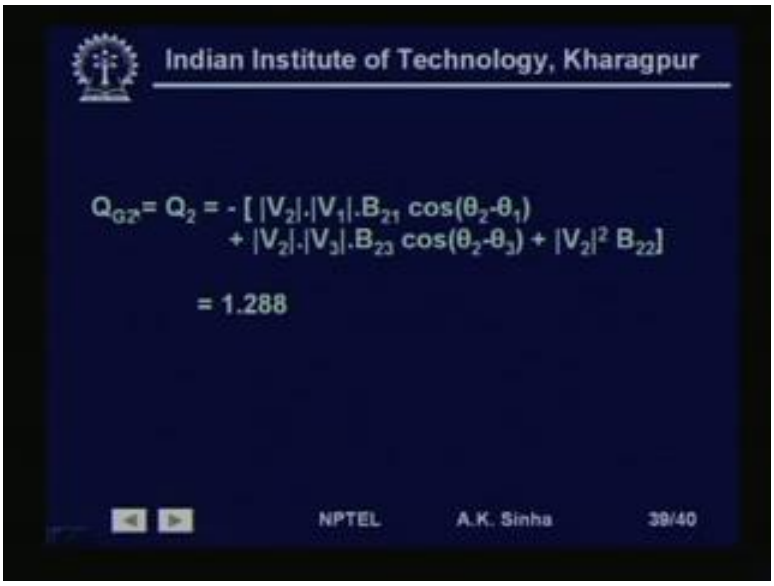

So, this is the complete solution for this network. That is, we know now all the unknowns. Now, knowing these values of voltage magnitude and angle at all the buses, as well as the injections at all the buses. Now, if we want, we can calculate the power flow in each of the lines as well. So, this is all about solving the power flow equations using Newton Raphson method. The advantage of using Newton Raphson method, as we had seen is you need much less number of iterations. In fact, even for very large systems of 1000's buses and so you need only 4, 5 iterations to achieve the convergence up to acceptable levels.

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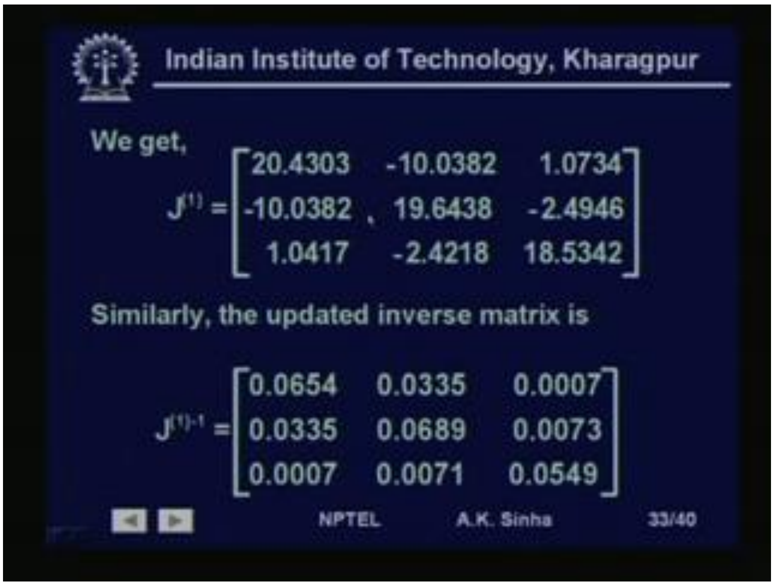

Another accept, which is very important here is that the Jacobian matrix. That we have here, since it is a three bus system and all the buses are connected to all other buses is a full matrix. But, in a general the Jacobian matrix, will be a spars matrix. Just like, the Y bus matrix, which is a spars matrix. Because, we know all the buses are not connected to all other buses in a large system.

In fact, each bus maybe connected to just a few other buses and therefore the Y bus will contain large number of 0's. Similarly, the Jacobian will also contain large number of 0's, these matrices as we said earlier are called spars matrices. And we take advantage of the sparsity of these matrices, if in solution to make the solution much more, faster. So, with this, we have completed the Newton Raphson load flow.

Now, in the next lesson, we will talk about another version of load flow, which is very popular, which we call fast decoupled load flow. So, with this we end today's lesson.

Thank you very much.

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Welcome to lesson 20, on power system analysis. In this lesson, we will continue with the power flow analysis.

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We will start with computations and computational aspects of Newton-Raphson load flow method. Then we will develop fast decoupled power flow algorithm and we will take up an example for solving the fast decoupled power flow problem.

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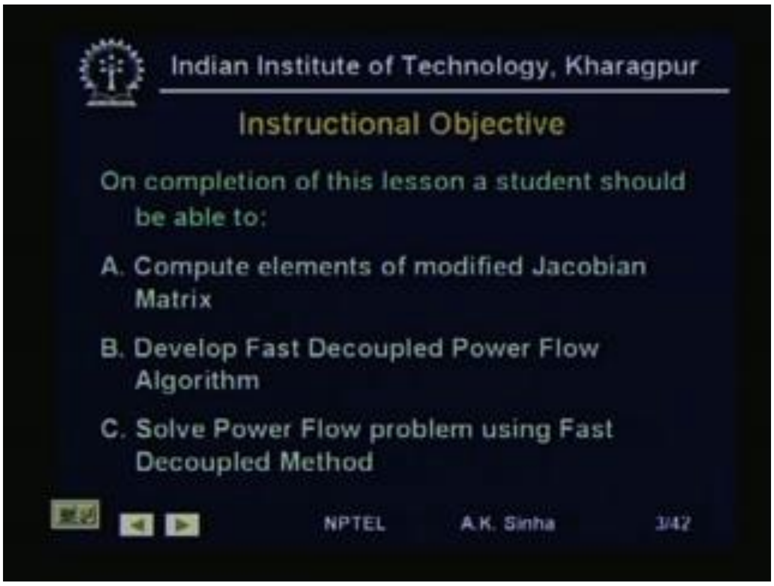

On the completion of this lesson, you should be able to compute elements of the modified Jacobian matrix. Develop, fast decoupled power flow algorithm and solve power flow problem using fast decoupled method.

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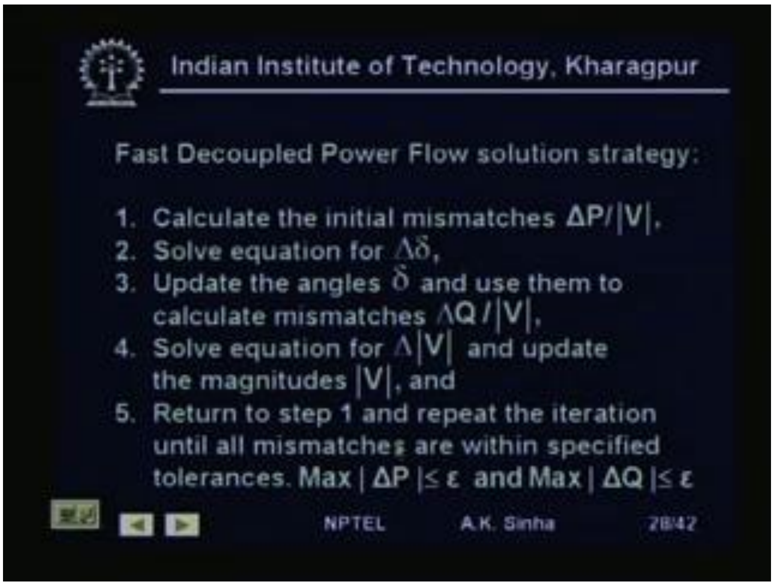

Now, we will take one example, for solving a power flow using fast decoupled method.

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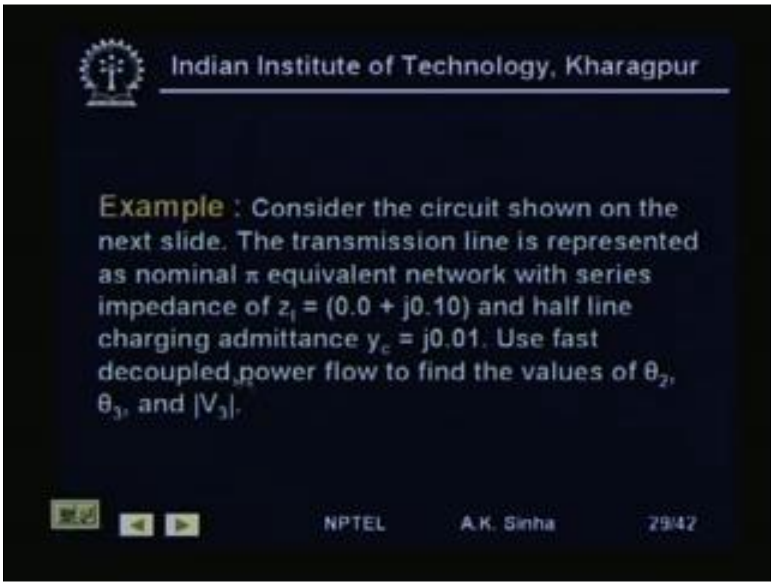

So, consider the circuit shown on the next slide, the transmission line is represented as nominal pi equivalent network, with series impedance Z l is equal to 0.0 plus j, 0.1 and half line charging admittance Y c is equal to j 0.1, 0.01. Using fast decoupled power flow, find the values of theta 2, theta 3 and V 3.

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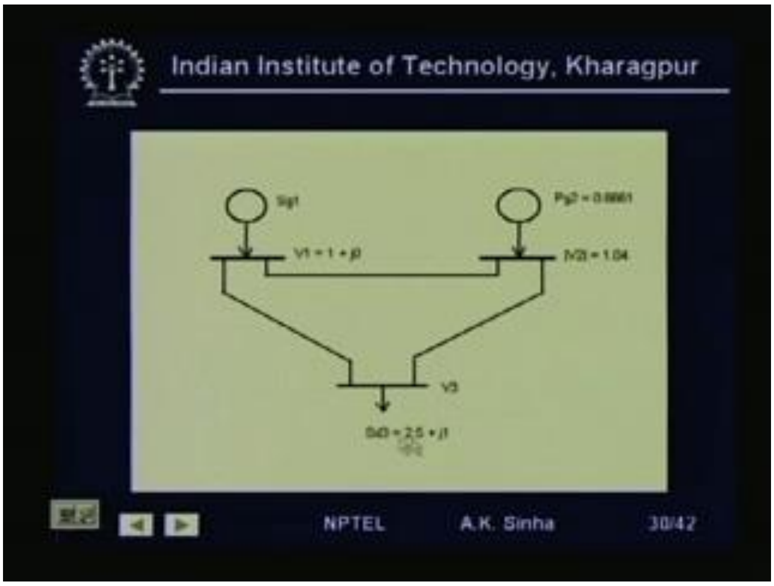

Now, if we see this system, this is a three bus system. This bus 1 is a slack bus, where the voltage is specified as one angle 0. That is 1 plus j 0, bus 2 is a P V bus, where we have the real power injection given as 0.6661 per unit and the voltage magnitude at this bus is given as 1.04 per unit. Bus 3 is a P Q bus, where the load value is given as 2.5 plus j 1.

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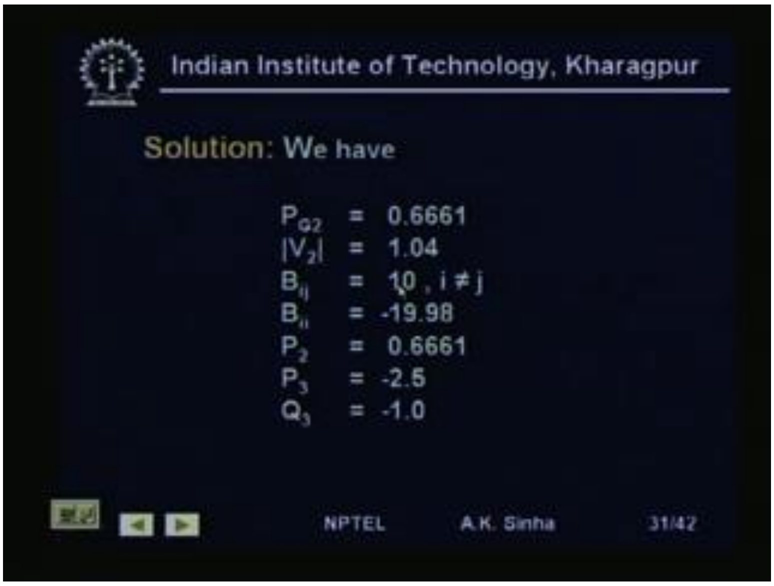

Now, using the transmission line impedance values, we have got B i j is equal to 10 for i naught equal to j that is B 12, B 13, B 23, B 32. All these terms will be equal to 10, because it is equal to 1 by 0.1, 1 by 0.1 j 0.1. So, that becomes minus J 10 and B i i is equal to minus 19.98, because we add all the susceptances connected to the bus, plus this half line charging of the two lines at that bus.

Thank you very much.