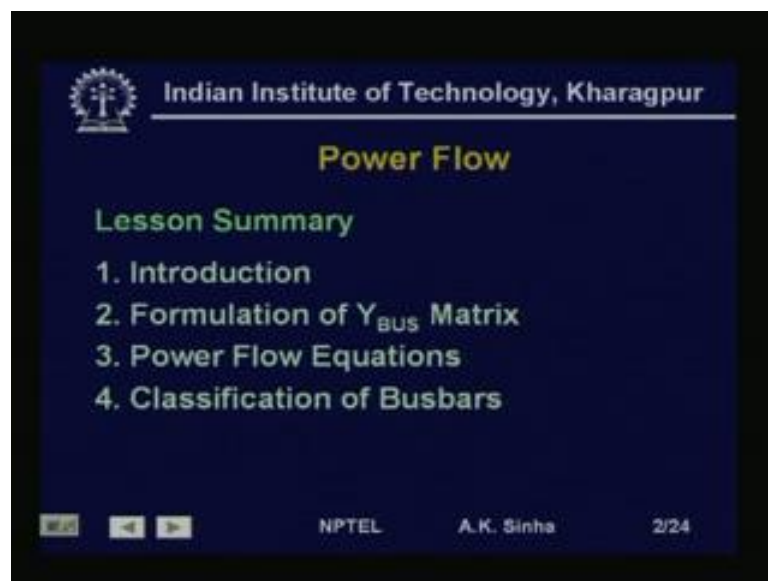


Power System Analysis
Prof. A.K. Sinha
Department of Electrical Engineering
Indian Institute of Technology, Kharagpur

Lecture - 16
Power Flow – I

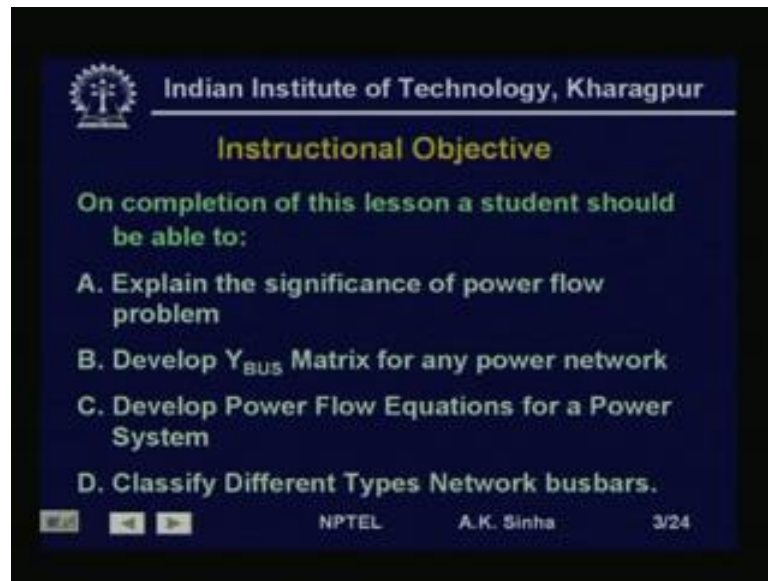
Welcome to lesson 16 on Power System Analysis. In this lesson, we will talk about the Power Flow problem.

(Refer Slide Time: 01:07)



In this lesson, we will start with an introduction to power flow problem. Then, we will talk about how we formulate the Y Bus matrix, then we will derive the power flow equations. And finally, we will be talking about how we classify the busbars into different types for power flow problem.

(Refer Slide Time: 01:40)



Indian Institute of Technology, Kharagpur

Instructional Objective

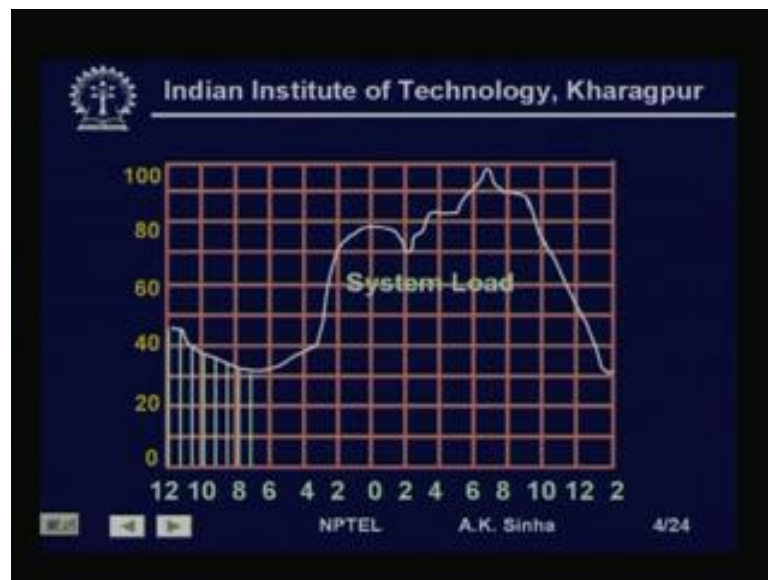
On completion of this lesson a student should be able to:

- A. Explain the significance of power flow problem
- B. Develop Y_{BUS} Matrix for any power network
- C. Develop Power Flow Equations for a Power System
- D. Classify Different Types Network busbars.

NPTEL A.K. Sinha 3/24

Well, on completion of this lesson, you should be able to explain the significance of power flow problem. Develop a Y Bus matrix for any power network, develop power flow equations for a power system. And classify different types of busbars in a power network.

(Refer Slide Time: 02:02)



Well, the power flow problem is one of the most fundamental problem in power systems. Till now we had discussed about the model, for various components in a power system. And now we are starting with the analysis, and power flow analysis is the most

fundamental analysis in power systems. Well, what we mean by this power flow analysis, which is also known as load flow analysis.

Well, in a power system what we have is, as you can see from this diagram, the load on the system keeps on varying all the time. That is the load on the system is not constant, it keeps on changing all the time as we had discussed earlier. And we have a number of generators, which supply this load. Also, this load which we have shown here as a system load is not concentrated at one place.

But, it is dispersed at various locations, that is at various substations we have loads connected to the system. And these substations are connected to the generators, by means of a transmission and distribution network. Now, this power flow analysis, the basic idea here is to find out the voltages at different busbars. That is the sub-stations that we have are the load points which we call; and the flow of power on this lines. So, that is the basic idea.

Now, as we see from this diagram, the load keeps on changing and this load is an aggregation of loads at various substations or busbars. Therefore, we also know that, the load at different busbars are also changing. What we really do is, we consider the loads at any given instant, that is what we do as shown from these lines. We consider the load at this particular instant to be this much, at this particular instant it is this much. And these instance that we choose are of very short duration.

That is if every 3 or 5 minutes, if we are choosing this load values, then we expect the change in load between two successive instances is not match. And for the time between these two instances or two successive instances, we consider that the load remains constant. And therefore, we can consider the network to be working in a steady state or a static operating condition.

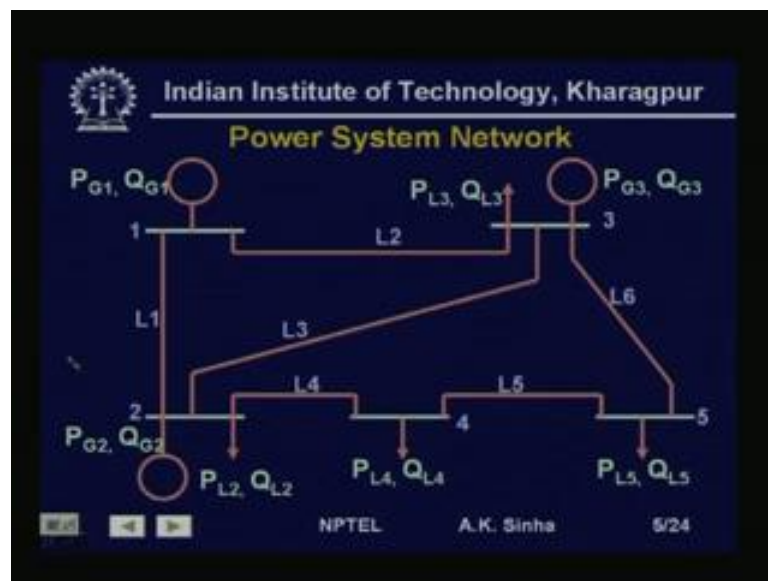
This kind of a classification of power system operating condition is called a quasi static operating condition. That is we are considering the power system loads, and the power flowing on the transmission lines to be constant for a given instant. And to remain constant for a short duration of time, that is the successive time instances at which we are considering.

So, for this small period, we consider the system to be operating on a constant load condition. And therefore, we can consider the system operation as a static operation/

Though the system as such will be changing its operating point from one instant to another instant. So, that is why we call this operating condition as a quasi static operating condition.

So, in load flow analysis what we do is, we try to find out the voltages, the power flows and the power injections at various busbars, for any given operating condition. Therefore, load flow is basically a static network analysis problem for a power network. Now, let us start with a power network.

(Refer Slide Time: 07:07)



When you see this diagram, it is showing a very small power network, where we have considered 5 busbars. These busbars can be considered in an actual system as substations. So, we have a substation which we are calling here as busbars in power system terminology, we call them as buses. So, this bus 1 is having a generator connected to it which is supplying say real power P_{G1} and reactive power Q_{G1} .

Similarly, we have another substation or a bus 2, where we have a generator connected which is supplying power P_{G2} and reactive power Q_{G2} to this bus, bus 1 and 2 are connected by a line which we are calling line L 1. Similarly, we have another substation or a bus 3 which has got a generator connected which is supplying real power P_{G3} and reactive power Q_{G3} . This also has a load of real power P_{L3} and reactive power Q_{L3} it is also connected to bus 1 by a line L 2.

Similarly, we have other busbars, busbar 4 and busbar 5, here what we find is busbar 4 is only having loads P_{L4} and Q_{L4} that is the real power load. P_{L4} and reactive power

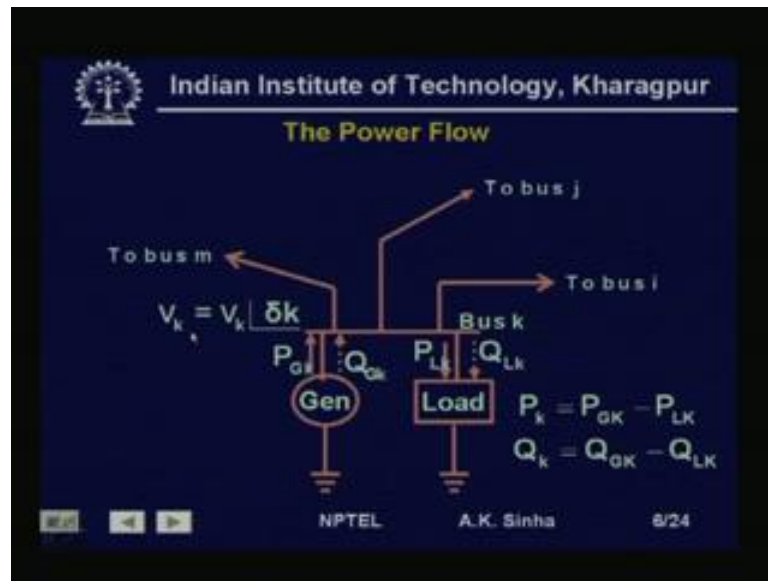
load $Q_L 4$, bus 5 similarly has only load real power $P_L 5$ and $Q_L 5$. Therefore, we see that at some of the busbars or the buses, we have generators connected. And these busbars we generally call as generating busbars, whereas some busbars have only loads connected to them, these we call them as load busbars.

We also have seen in this diagram that some of the generating busbars, like busbar 2 and busbar 3 have also loads connected to it. In fact, most of the generating busbars will also have some load connected to it. Because, if you are running a thermal power station, then for your auxiliaries, as well as for the substation supplying the local power for the presidencies, in that power station all these will have to be supplied. And these are the loads which will be connected to that busbar.

So, in general have a generating busbar and a load busbars. And these are connected by means of transmission lines, transformers and other equipment. And thus these lines and transformers, which are connecting these busbars form the power network. And the whole idea of power flow analysis is to find out, the voltages at various busbars and the power flows in various lines.

Because, we know that we must keep the voltage within a specified limit, as well as transmission lines have a capacity. And therefore, we must keep the power flowing on the line within its capacity. Otherwise, the line will get overloaded and that may lead to other problems like protection system, sensing overload and tripping the line etcetera. So, the idea for power flow analysis is to given these loads and generation, to find out the voltages at various busbars and the power flows in various lines. So, that we can assess the operating condition of the system, and see that none of the variables are beyond their operating range.

(Refer Slide Time: 11:54)

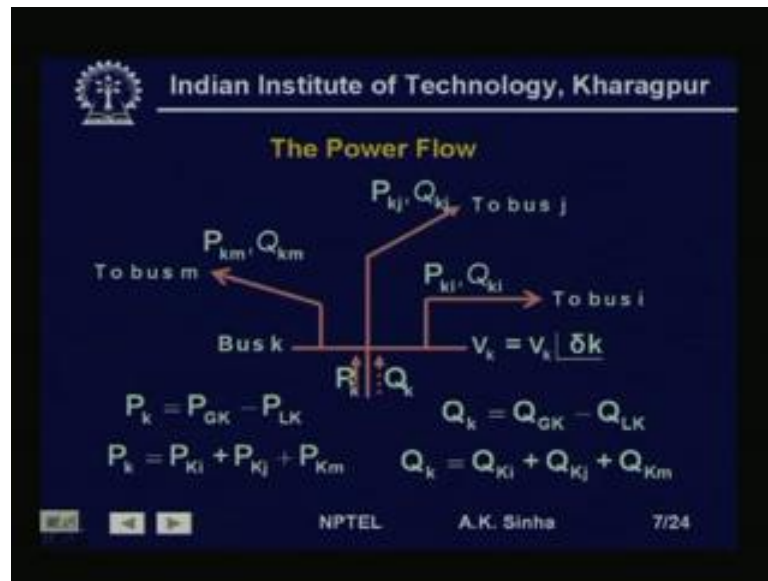


Now, if we see any busbar in detail, like if we say this busbar 2 which we have put here. This busbar, if we want to see it in detail. Then we can see this as a generator is connected to this busbar which is injecting a power real power P_{GK} ; and reactive power Q_{GK} into this busbar. And it has load which is drawing real power P_{LK} from the busbar and reactive power Q_{LK} from the busbar.

This busbar is connected to other busbars, like bus i bus j, this through this line to bus j and through this line to bus m. The voltage at this busbar is V_k , where V_k is equal to the magnitude V_k and angle δ_k for this busbar. Now, one thing which we see from here, that this generator is injecting real or reactive power into the busbar, whereas this load is drawing real and reactive power from this busbar.

What we can always do is, we can take the algebraic sum of these two. That is we can subtract this load from the generation, and that is the real load P_{LK} from the generation P_{GK} . Then, we can say that there is a real power injection into the bus, which is equal to P_{GK} minus P_{LK} . Similarly, we have a reactive power injection Q_k , which is equal to Q_{GK} minus Q_{LK} .

(Refer Slide Time: 13:44)



So, this can be shown as in this diagram, that is we have now instead of writing generation and loads at the busbar. We are now talking about injection into the busbar. So, if we have say load bus, then there is no generation at that bus. So, what is going to be the injection, the injection at that bus is going to be, say for that bus P_k if we look at that. Then, we have no generation that is P_{GK} is 0, then P_k is equal to minus P_{LK} and similarly, Q_k will be equal to minus Q_{LK} .

So, loads can be considered as negative injection. That is the loads are drawing power out of the bus, whereas the injections we have considered to be positive, when it is into a bus. So, if we see here we have injections P_k and real power injection P_k and reactive power injection Q_k . And as we have said we are writing P_k is equal to P_{GK} minus P_{LK} and Q_k is equal to Q_{GK} minus Q_{LK} . Now, we have three lines connected to this busbar, one going to bus i, another going bus j and the third one going to bus m.

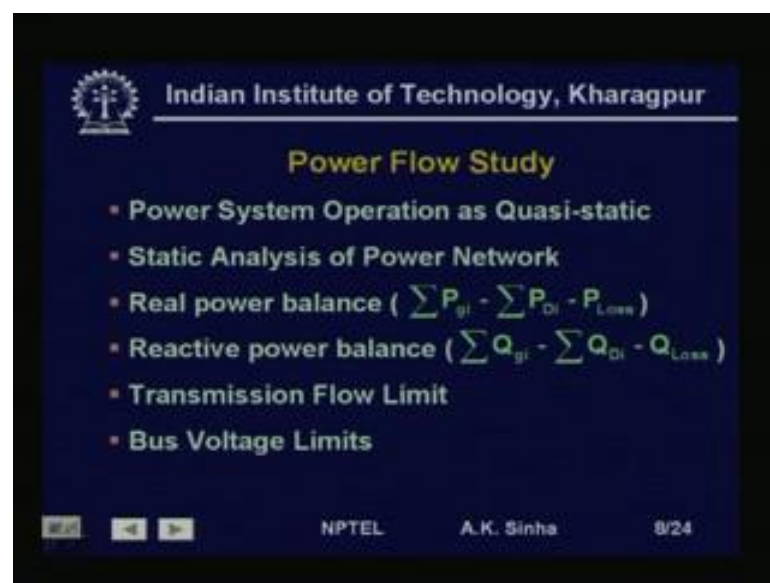
Now, these lines will be carrying power P_{Ki} and Q_{Ki} to bus i, P_{Kj} Q_{Kj} to bus j and P_{Km} Q_{Km} to bus m. Now, some of these powers may be in the reverse direction, that is they may be coming from say bus j to bus k. In that case the value of P_{Kj} and Q_{Kj} will be negative. So, we have here let us say all these are the values of real and reactive power flowing on the line.

Then, we can write that the real power injection into the bus will be equal to the algebraic. Sum of all the real powers flowing away from this bus, through the transmission lines connected to this bus. That is P_k will be equal to P_{Ki} plus P_{Kj} plus

$P_{K m}$. Same thing will hold good for the reactive power. So, reactive power injection Q_K is equal to $Q_{K i}$ plus $Q_{K j}$ plus $Q_{K m}$. That is what we are saying is that, the real and reactive power injection at a bus is nothing but, equal to the algebraic sum of the real and reactive power which is flowing into the bus.

Or negative of the real and reactive power, is equal to the algebraic sum of real and reactive power flowing out of the bus. This follows directly from the Kirchhoff's current law, that is we cannot have any power or current stored here. So, whatever is the power or current flowing into the bus, must go out from the bus through the transmission lines.

(Refer Slide Time: 17:26)



So now, we will start with power flow study, what we really are trying to do in this study. As we said we are considering the power system operation as quasi-static. Therefore, power flow study is basically a static network analysis for a power network. Then, we have real power balance at all the busbars and also for the total system. That is summation of all the generation real power generation, minus summation of all the loads minus the loss in the transmission system is going to be equal to 0.

That is the power which real power, which is injected by the generator has to be equal to the real power drawn by the load, and the real power losses in the transmission line. Similarly, reactive power balance equation also holds good. That is total reactive power generated by the generators or the transmission lines, must be absorbed by the loads. And the losses in the series element of the transmission lines. So, both real and reactive power balance, condition balance equations hold good.

We do this power flow study to find out the power flow on various transmission lines. So, as to check that all the flows are within the limits or the carrying capacity of the transmission line. We also check for the voltages at all the busbars or the substations and to see that the voltage at all points are within a specified limit. Normally this limit will be plus minus 5 percent or 10 percent depending on whether we are talking about the high voltage system. Or we are taking at the consumer level or the low voltage region.

(Refer Slide Time: 19:58)

Indian Institute of Technology, Kharagpur

Static Analysis of Power Network

Mathematical Model of the Network

Transmission line - nominal π model

Bus power injections -

$$S_k = V_k I_k^* = P_k + jQ_k$$

$$P_k = P_{GK} - P_{LK}$$

$$Q_k = Q_{GK} - Q_{LK}$$

NPTEL A.K. Sinha 9/24

Now, how do we do this power flow analysis, which is basically a static analysis or the power network. What do we do here, first thing that we need to do is create a mathematical model for the network. For this purpose as we had already seen, when we develop the model for the transmission lines, all the transmission lines are represented by their nominal pi model.

In fact, the very short lines can also be represented by just a series impedance, whereas long lines can be represented by an equivalent pi model that we had developed. So, a nominal pi or an equivalent pi can be used for medium line and long lines. Whereas a simple series impedance can be used for a short line, we have also seen how we can model the transformer.

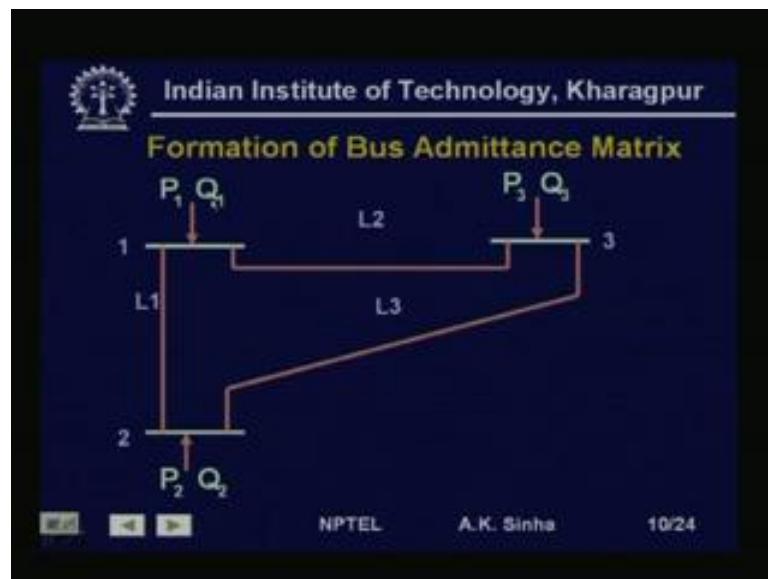
Transformers are generally modeled by their leakage impedance only, we do not consider the magnetizing and the core loss part. Because, that is much smaller percentage of the current which is flowing, or the power which is going into this shunt part or the

magnetizing part, but also because the current in the two elements are almost 90 degree out of phase. So, the effect of this magnetizing part is not much.

Now, we considered bus power injections, rather than generation and loads separately. So, what we do is, as we saw just now that we find out the injections at each busbar, which we have seen is equal to the generation minus load at that busbar. So, we can write the injection at any bus k as S_k , this is the complex power injection which is $P_k + jQ_k$.

And we know this $P_k + jQ_k$ are the complex power injection is equal to V_k into I_k conjugate; where V_k is the voltage phasor at that bus. And I_k is the current injection phasor at that bus. So, using this that is S_k is equal to V_k into I_k , we will formulate the equations for power flow. So, once we know this P_k and Q_k and we know this V_k , then we can always find out I_k . That is the current injection into the bus, where P_k as we had already seen is the real power injection $P_G - P_L$ Q_k is the reactive power injection $Q_G - Q_L$.

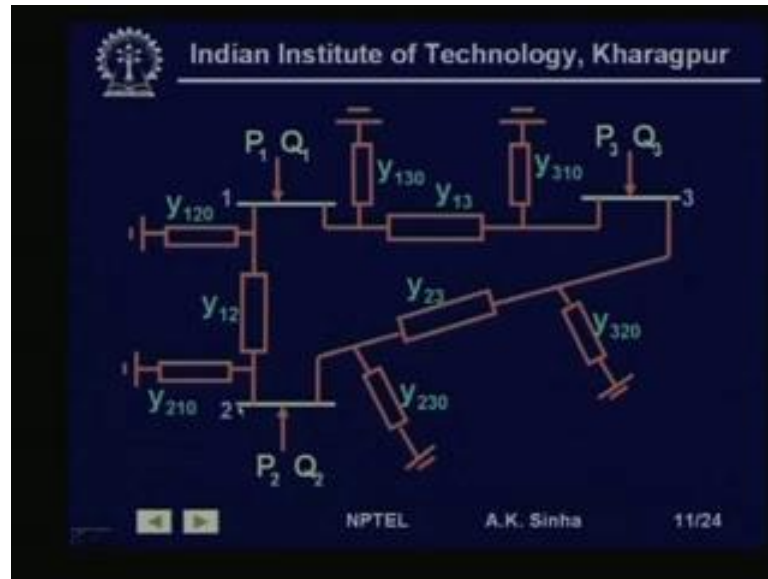
(Refer Slide Time: 23:17)



Now, let us formulate the model for this power flow analysis. In formulating the mathematical model, we need to formulate first what we call as bus admittance matrix, we will see what this actually is. Now, I have taken here a small power system with three busbars. The busbar 1 is having a real power injection P_1 and a reactive power injection Q_1 , whereas busbar 2 has a real power injection P_2 , reactive power injection Q_2 .

And busbar 3 has real power injection P_3 reactive power injection Q_3 . These buses are connected to each other through lines, like bus 1 is connected to 2 through line L 1, 1 is connected to 3 by line L 2 and bus 2 is connected to 3 by line L 3. Now, if you model these lines L 1, L 2, L 3 by their pi equivalent model, then the circuit will look something like this.

(Refer Slide Time: 24:31)



As shown here, where for a line L 1 which connecting bus 1 to bus 2, y_{12} is representing the series admittance of the line y_{120} is representing the shunt admittance of the line, which is basically the line charging capacitance of the line and at the end towards bus 1. Similarly, y_{210} is the shunt admittance of the line at the bus end 2. So, this pi equivalent model is representing line L 1 which is connecting bus 1 and 2.

Similarly, we have the pi equivalent model for line two, L 2 which is connecting bus 1 and 3. And the series admittance is given by y_{13} , the shunt admittance part on the bus 1 side is y_{130} and on bus 3 side is y_{310} and so on. For line L 3 we have the series admittance y_{23} and shunt admittance towards bus 2 is y_{230} and towards bus 3 is y_{320} . Now, if we take these power injections and convert them into current injections, then having this circuit with us we can now using Kirchhoff's law, we can write the network equation for this system as shown now.

(Refer Slide Time: 26:18)

Indian Institute of Technology, Kharagpur

$$I_1 = y_{120} V_1 + y_{12} (V_1 - V_2) + y_{130} V_1 + y_{13} (V_1 - V_3)$$

$$I_2 = y_{210} V_2 + y_{12} (V_2 - V_1) + y_{230} V_2 + y_{23} (V_2 - V_3)$$

$$I_3 = y_{310} V_3 + y_{13} (V_3 - V_1) + y_{320} V_3 + y_{23} (V_3 - V_2)$$

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} (y_{120} + y_{12} + y_{130} + y_{13}) & -y_{12} & -y_{13} \\ -y_{21} & (y_{210} + y_{12} + y_{230} + y_{23}) & -y_{23} \\ -y_{31} & -y_{32} & (y_{310} + y_{13} + y_{320} + y_{23}) \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$$

NPTEL A.K. Sinha 12/24

So, if we do that then we can write using the K c 1, that I 1 the current which is injected here will be equal to the current flowing through this branch, which will be equal to V 1 into y 1 2 0. Current flowing through this branch which is equal to V 1 minus V 2 into y 1 2 2, and current flowing through this branch which is y 1 3 0 into V 1 minus V 3, current flowing through this branch which is y 1 3 0 into V 1.

So, if you write this equation, we have I 1 is equal to y 1 2 0 V 1 plus y 1 2 into V 1 minus V 2 plus y 1 3 0 into V 1 plus y 1 3 into V 1 minus V 3. Same way we can write the equations for the current injection at bus 2, I 2 will be equal to y 2 1 0 into V 2 plus y 1 2 into V 2 minus V 1 plus y 2 3 0 into V 2 plus y 2 3 into V 2 minus V 3. Same way for I 3 that is current injection at bus 3, that will be equal to y 3 1 0 into V 3 plus y 1 3 V 3 minus V 1 plus y 3 2 0 into V 3 plus y 2 3 into V 3 minus V 2.

So, these three equations which are relating the current injections I 1, I 2, I 3 into the network with the voltage at the 3 busbars. That is V 1, V 2 and V 3, can be arranged in a matrix form as I 1, I 2, I 3 is equal to here in this matrix we have y 1 2 0 plus y 1 2 plus y 1 3 0 plus y 1 3 into V 1. So, we have this element minus y 1 2 minus y 1 3. Similarly, here we will have minus y 2 1 into V 1 plus y 2 1 0 plus y 1 2 plus y 2 3 0 plus y 2 3 into V 2 minus y 2 3 into V 3 for I 2.

And similarly for I 3, we have minus y 3 1 into V 1 minus y 3 2 into V 2 plus y 3 1 0 plus y 1 3 plus y 3 2 0 plus y 2 3 into V 3. So, this equation can be arranged in this fashion in a matrix form.

(Refer Slide Time: 29:18)

Indian Institute of Technology, Kharagpur

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} \\ Y_{21} & Y_{22} & Y_{23} \\ Y_{31} & Y_{32} & Y_{33} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$$

$$Y_{11} = y_{120} + y_{12} + y_{130} + y_{13}$$

$$Y_{22} = y_{210} + y_{12} + y_{230} + y_{23}$$

$$Y_{33} = y_{310} + y_{13} + y_{320} + y_{23}$$

NPTEL A.K. Sinha 13/24

And this we can write as I_1, I_2, I_3 is equal to capital Y we are writing here $Y_{11}, Y_{12}, Y_{13}, Y_{21}, Y_{22}, Y_{23}, Y_{31}, Y_{32}$ and Y_{33} . V_1, V_2, V_3 , where Y_{11} the diagonal element is $y_{120} + y_{12} + y_{130} + y_{13}$. And the diagonal element Y_{22} is $y_{210} + y_{12} + y_{230} + y_{23}$. Similarly diagonal element Y_{33} is equal to $y_{310} + y_{13} + y_{320} + y_{23}$. This is what we get from the matrix that we had seen here. So, we are replacing this element by Y_{11} , this element by capital Y_{12} , this element by capital Y_{13} .

(Refer Slide Time: 30:27)

Indian Institute of Technology, Kharagpur

$$Y_{12} = Y_{21} = -y_{12}$$

$$Y_{13} = Y_{31} = -y_{13}$$

$$Y_{23} = Y_{32} = -y_{23}$$

Y_{ii} is called Self-Admittance (Driving Point Admittance)

Y_{ij} is called Transfer-Admittance (Mutual Admittance)

$$I_{BUS} = Y_{BUS} V_{BUS}; V_{BUS} = Z_{BUS} I_{BUS}$$

NPTEL A.K. Sinha 14/24

So, we can see that the other off diagonal elements Y_{12} is equal to Y_{21} and that is equal to minus small y_{12} . That is the series admittance of the line between 1 and 2, it is negative of that. And Y_{13} is equal to Y_{31} is equal to minus y_{13} , Y_{23} is equal to Y_{32} is equal to minus y_{23} . Now, ((Refer Time: 31:03)) this matrix Y , matrix of Y admittance elements is relating the current injection at the various busbars to the voltages had the busbars.

And that is why this matrix, Y matrix is called the bus admittance matrix, it relates the current injections at the busbars to the voltages at the busbars. Now, the elements of this Y bus matrix Y_{ii} , that is the diagonal elements is called the self admittance or the driving point admittance of the network at bus i . And the element Y_{ij} , that is the off diagonal element is called the transfer admittance or the mutual admittance between bus i and j .

And in short we can write this as I_{bus} is equal to Y_{bus} into V_{bus} . We can in the same way write this also as V_{bus} is equal to Z_{bus} into I_{bus} , where Z_{bus} is the inverse of Y_{bus} matrix. And Z_{bus} matrix is called the bus impedance matrix. Now, we normally work with the bus admittance matrix and not with the bus impedance matrix. There are various reasons behind this, one of the reason is it is very easy to form the Y_{bus} matrix by just inspecting the power network.

Whereas, Z_{bus} matrix is much more complex to form. The other aspect which is there is that Y_{bus} matrix is generally sparse, whereas Z_{bus} matrix is a full matrix, we will see these things.

(Refer Slide Time: 33:11)

Indian Institute of Technology, Kharagpur

Characteristics of Y_{BUS} Matrix

- Dimension of Y_{BUS} is $(N \times N) \rightarrow N = \text{Number of Bus}$
- Y_{BUS} is Symmetric Matrix
- Y_{BUS} is a Sparse Matrix (up to 90 to 95 % sparse)
- Diagonal Elements Y_{ii} are Obtained as Algebraic Sum of All Elements Incident to Bus i
- Off-diagonal Elements $Y_{ij} = Y_{ji}$ are Obtained as negative of Admittance Connecting Bus i and j

NPTEL A.K. Sinha 15/24

So, let us see what are the characteristics of this Y bus matrix. The dimension of this Y bus matrix is N into N , where N is the number of busbars in the system. So, if you have a 3 busbar system, as we had seen here, we had a 3 busbar system for this 3 busbar system we will get a 3 by 3 Y bus matrix. If we have a 10 busbar system we will get a 10 by 10 matrix.

Normally the power networks are very large, they are more than 1000 busbars, with many 1000 lines connecting various busbars. And therefore, we will be dealing with Y bus matrices which can be as large as 1000 into 1000 or even more. Another characteristics that we see for this Y bus matrix is, that it is a symmetric matrix. That is what we are seeing here is Y_{12} is equal to Y_{21} is equal to minus y_{12} , that is minus small y_{12} .

So, what we are saying Y_{12} is equal to Y_{21} Y_{13} is equal to Y_{31} Y_{23} is equal to Y_{32} , that is we are saying that the matrix is a symmetric matrix. Another feature of this Y bus matrix is that Y bus matrix is a very sparse matrix. That is almost 90 to 95 percent of the elements of the Y bus matrix are zeros and very few elements which will be there are non-zeros. This comes mainly from the property that we have seen here.

Now, if we see here ((Refer Time: 35:32)) in this network, there is no line which is connecting bus 1 to bus 4 and bus 1 to bus 5. So, the Y_{14} and Y_{15} are 0. And therefore, we will have no element, non-zero element for these in the Y bus matrix. Similarly, in a very large network, we do not have all the busbars connected to all other

busbars. So, if we are talking about a 1000 bus system, then in that case each busbar may be connected to some 5 or 7 busbars.

So, what we see is that there will be only 8 entries in that row for that busbar or in the column for that busbar. So, rest of the other elements 992 elements will be 0's, and so will be the case with all the other busbars also. So, we are finding that out of the 1000 elements most of the rows and columns will be having only some 8, 10 elements. And rest the elements will be 0's and which shows that the number of 0's in the Y bus matrix will be very large.

And therefore, these matrices are called sparse matrices and we try to take advantage of the sparsity of this matrix, in our computation process very much. In fact, sparsity is a property which is available or prevalent in all very large systems. So, Y bus is a sparse matrix, diagonal elements of Y_{ii} are obtained as algebraic sum of all elements incident to bus i . That is we had seen that Y_{11} or Y_{12} is nothing but the algebraic sum of all the admittance which is connected to that bus.

So, if you see here Y_{11} will be equal to Y_{120} plus Y_{12} plus Y_{130} plus Y_{13} , all the admittances which are directly connected to this busbar. So, this is how we can calculate or find out the diagonal element of the Y bus. Off-diagonal elements Y_{ij} is equal to Y_{ji} , because the matrix is symmetric, are obtained as negative of the admittance connecting bus i and j .

That is again if we see this Y_{12} is nothing but, negative of this Y_{21} that we have. In fact, these Y is should have been small letters, whereas the Y bus matrix elements we have used capital Y. So, these are some of the characteristics of Y bus matrix. Now, we will go in and see how we can derive the power flow equations. So, again we will go back to our basic equation.

(Refer Slide Time: 39:24)

Indian Institute of Technology, Kharagpur

Power Flow Equations

$$I_{BUS} = Y_{BUS} V_{BUS}$$

$$I_k = \sum_{n=1}^N Y_{kn} V_n$$

$$S_k = P_k + jQ_k = V_k I_k^*$$

$$P_k + jQ_k = V_k \left[\sum_{n=1}^N Y_{kn} V_n \right]^* \quad k = 1, 2, \dots, N$$

NPTEL A.K. Sinha 16/24

I_{BUS} is equal to Y_{BUS} into V_{BUS} . And if we take any particular bus injection, then I_k will be equal to summation Y_{kn} into V_n . That is if we go back and see here, then I_2 is this, if we look at this, this is I_2 is equal to y . Or let me see from here ((Refer Time: 39:59)) I_2 is equal to Y_{21} into V_1 plus Y_{22} into V_2 plus Y_{23} into V_3 , so it is summation of Y , I_k is summation of Y_{kn} into V_n .

So, this is what we have written here. I_k is equal to summation n is equal to 1 to capital N , where capital N is the number of busbars in the system. So, I_k is equal to summation Y_{kn} into V_n for n is equal to 1 to 2, small n is equal to 1 to 2 capital N . Now, we can write the complex power injection into bus k S_k is equal to P_k plus jQ_k and this is equal to V_k into I_k conjugate.

Now, I_k conjugate we can write from here will be conjugate of this, so we can write P_k plus jQ_k is equal to this V_k into this term is for I_k , so I_k conjugate. So, conjugate of this term, where this can be written for all the busbars, that is k can be from 1 to n . So, we can write this kind of a equation, that is the real and reactive power injection at any bus K can be written in this form.

(Refer Slide Time: 41:34)

Indian Institute of Technology, Kharagpur

$$V_k = V_n e^{j\delta_{kn}}$$

$$Y_{kn} = Y_{kn} e^{j\theta_{kn}} \quad k, n, = 1, 2, \dots, N$$

$$P_k + jQ_k = V_k \sum_{n=1}^N Y_{kn} V_n e^{j(\delta_k - \delta_n - \theta_{kn})}$$

$$P_k = V_k \sum_{n=1}^N Y_{kn} V_n \cos(\delta_k - \delta_n - \theta_{kn})$$

$$Q_k = V_k \sum_{n=1}^N Y_{kn} V_n \sin(\delta_k - \delta_n - \theta_{kn})$$

NPTEL A.K. Sinha 17/24

Now, we know V_n or the voltage at bus n is a phasor. And it is equal to V_n the magnitude in into e to the power $j\delta_n$, this should be $j\delta_n$. So, V_n is equal to V_n magnitude into e to the power $j\delta_n$, this k should not be here, $j\delta_n$. And Y_{kn} , the admittance is equal to magnitude Y_{kn} into e to the power $j\theta_{kn}$, where θ_{kn} is the angle of the impedance. We have the impedance written as $R + jx$.

And the admittance will be $1 / (R + jx)$ which we can write as $g - jb$. Therefore, we will have an angle depending on the magnitude of g and b for this admittance. So, we have Y_{kn} is equal to Y_{kn} magnitude into e to the power $j\theta_{kn}$. So, substituting the phasor V_n and for the admittance vector Y_{kn} , we can write $P_k + jQ_k$, the real and reactive power injection at bus k is equal to V_k which is the phasor V_k into summation Y_{kn} into $V_n e^{j\delta_k}$ is the angle for this.

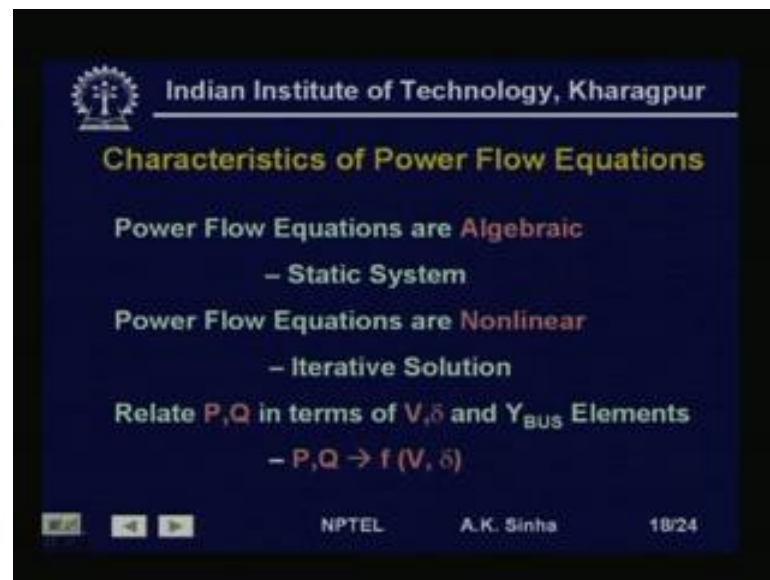
δ_n is the angle for this minus δ_n , because this part was our conjugate. So, conjugate means the magnitude remains same the angle becomes negative. So, minus δ_n in into Y_{kn} was here, so Y_{kn} we are writing with Y_{kn} with an angle minus θ_{kn} . Because, we this term is this whole term is conjugate we have to take, so these angles θ_n and θ_{kn} are negative.

So, this is what we get from here. And if we now separate out the real and reactive part from this equation, then we can write P_k is equal to V_k summation n is equal to 1 to capital N , $Y_{kn} V_n \cos$ of δ_k minus δ_n minus θ_{kn} , here this k is missing in this. And Q_k is equal to V_k summation from small n is equal to 1 to capital N , $Y_{kn} V_n \sin$

$n \sin \delta_k - \delta_n - \theta_{kn}$. So, we have the real and reactive power separated.

And we can write the relationship between the real power and the reactive power, in terms of the voltage magnitude and the angles at different busbars. And the admittance which is connecting the various busbars in the system, that is real and reactive power equations are given like this. These equations are relating the real power and the reactive power, with voltages at various busbars V_n , where n is changing for all the busbars. And Y_{kn} , where Y_{kn} is the admittance which is connecting the bus k to all the other busbars.

(Refer Slide Time: 45:52)



So, let see what is the characteristic of these power flow equations? Some of the important characteristics are that these power flow equations are algebraic equations. That is if we see this equations, these equations are algebraic equations, because there is no derivatives involved here or differentials involved here, so these are just algebraic equations, because we are using a static system condition. So, there is no differential equations involved, it is only an algebraic equation.

Now, power flow equations are also we see they are non-linear. That is we have, if we look here sine and cos terms involved here, also we have multiplication of V terms and so on. Therefore, these are non-linear algebraic equations. Since, these equations are non-linear, the general procedure for solving these equations will be an iterative solution procedure.

So, these power flow equations relate the P Q injections at any busbar. In terms of V and delta the voltage magnitude and angle at various busbars and the Y bus elements. That is P Q are a function of V and delta.

(Refer Slide Time: 47:34)

Indian Institute of Technology, Kharagpur

Power Flow Problem

Characterization of Variables –

- Loads (P_L, Q_L) → Uncontrolled (Disturbance) Variable
- Generation (P_G, Q_G) → control Variable
- Voltage (V, δ) → State Variable

For a Given Operating Condition → Loads and Generations at all buses are known (Specified)
→ Find the Voltage Magnitude and Angle (V, δ) at each bus

NPTEL A.K. Sinha 19/24

Now, we will see exactly what is this power flow problem, and how we try to solve them. First thing that we have to do is, we will have to see what are the different variables which we have... Now, we see that in a power network, we have at certain buses loads. Now, these loads are controlled only by consumers, and the power system has no control on them. Therefore, these variables are called uncontrolled variables or disturbance variables.

Whereas, generation P_G and Q_G are controlled by the power system operator. Depending on the load on the system, he raises or lowers the generation at different generators. As we will see later he also tries to optimize the cost. That is for a given loading condition, he tries to find out what should be generation at various bus generating station. So, that the total cost of operation is minimum, that we call as economic dispatch problem.

So, he adjust this generation at various busbar, source that he gets an optimal performance. And therefore, these variables generation variables P_G, Q_G are our controlled variables. Whereas, the voltage magnitude and angle at all the busbars, which basically relates this controlled variables. As well as disturbance variables in the system is the state variable of the system.

In fact, if we know these values of V and δ at all the busbars in the system. We can find out all the other values and therefore, we called this V δ V and δ . That is voltage magnitude and angle at busbars as the state variables. Now, what is the power system problem, a power flow problem, the power flow problem is for a given operating condition, that is loads and generation at all the buses.

We have to find out the voltage magnitude and angle at each bus. That is given the and disturbance variables and the controlled variables. That is if these are specified find out what is the state variable. That is for any given operating condition, any particular time when we have all the loads and generation known. Then, we can what we need is we need to find out for these generation and load, what is the voltage at various busbars.

And once we know this, then we can calculate all the line flows and all the other variables. Now, what is the problem in formulating or solving this equation, well one of the problem that we face when trying to formulate this power flow.

(Refer Slide Time: 51:16)

Indian Institute of Technology, Kharagpur

Problem in Power Flow →

All generation Variables (P_G, Q_G) can not be specified as Losses are not known a priori.

Solution →

Choose one bus as reference where Voltage Magnitude and angle are specified. The losses are assigned to this bus. This bus is called "Slack Bus".

NPTEL A.K. Sinha 20/24

Problem is that all generation variables P_G and Q_G at all the busbars cannot be specified. Why we cannot specify the generation at all the busbars, the loads are known we can say, that we can give the generation also to satisfy this load. Unfortunately this is not possible, because the losses in the transmission line are not known. They will be known only after we know, what are the flows in the transmission lines? And so we know the current flowing in each line and we can find out the loss taking place in each line.

So, unless we know the voltages at all the busbars, we cannot find out the current flowing in various lines. And therefore, we do not know the losses. So, if we do not know the losses, then we cannot specify all the generation, because the total generation must be equal to the total load plus the transmission losses. So, how do we solve this problem? Well, in power flow problem what we do is we say that, one of the busbars normally a large generating busbar, we do not specify its generation.

We say that, this busbar is going to take or absorb all the transmission line losses. So, we specify the generation at all the other generators or generating busbars except for this one busbar. And since this busbar takes the slack, that is the losses in the system. Therefore we call this busbar as a slack busbar and we have one slack busbar in one system. So, what we do is basically we are choosing a reference bus, that is one bus where we are not specifying the real and reactive power generation.

Now, if we do not specify real and reactive power generation at this particular busbar. Then, we have to specify some other two variables at this busbar. And normally the variables that we specify at this busbar will be the other two, that is the voltage magnitude and its angle. Since, the angle has to be specified we normally will choose an angle equal to 0. And therefore, the voltage angle at all other busbars will be related to this busbar's angle.

So, this busbar can be seen as a reference busbar, where both voltage magnitude and angle are known or specified. So, we see that we have different kinds of busbars in the system. One we have just now seen we have to specify a slack bus, that is a bus which can take care of the transmission losses.

(Refer Slide Time: 54:32)

Indian Institute of Technology, Kharagpur

Classification of Busbars

Each bus 'k' is Classified into one of the following three bus types :

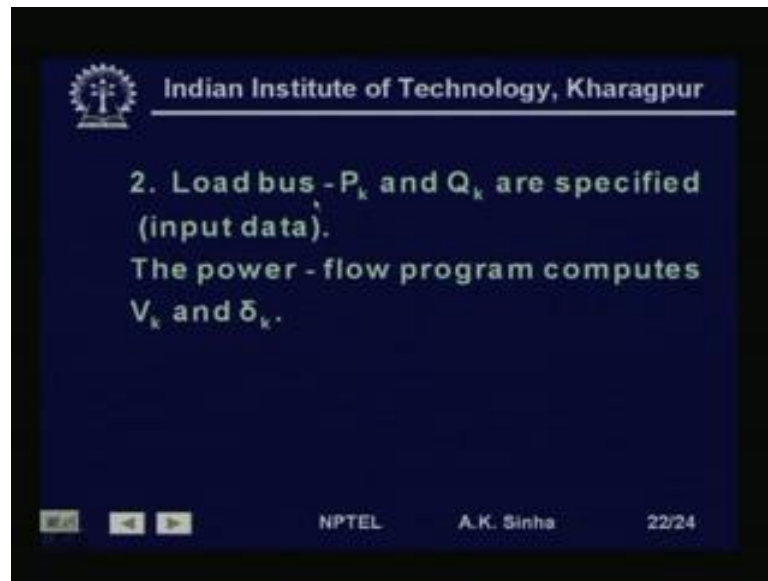
1. Swing bus - There is only one swing bus, which for convenience is numbered bus 1. The swing bus is a reference bus for which $V_1, |\delta_1|$, typically $1.0 \angle 0^\circ$ per unit, is Specified (input data). The power - flow program computes P_1 and Q_1 .

NPTEL A.K. Sinha 21/24

So, we can say that each bus k is classified into one of the following three bus types. One is the swing busbar or the slack busbar. There is only one swing bus, which is for convenience numbered bus 1. We can number it, it can be any bus as such, but we will generally use bus 1 as the swing bus. The swing bus is a reference bus for which V_1 and δ_1 which we typically $1.0 \angle 0$ per unit is specified, that is the input data.

The power flow program computes P_1 and Q_1 for this bus. That is the unknown for this bus will be P_1 and Q_1 , that is injections at this bus, because we have not specified the generation at this bus. So, we need to calculate that finally, after we have calculated the voltage at all the busbars.

(Refer Slide Time: 55:42)



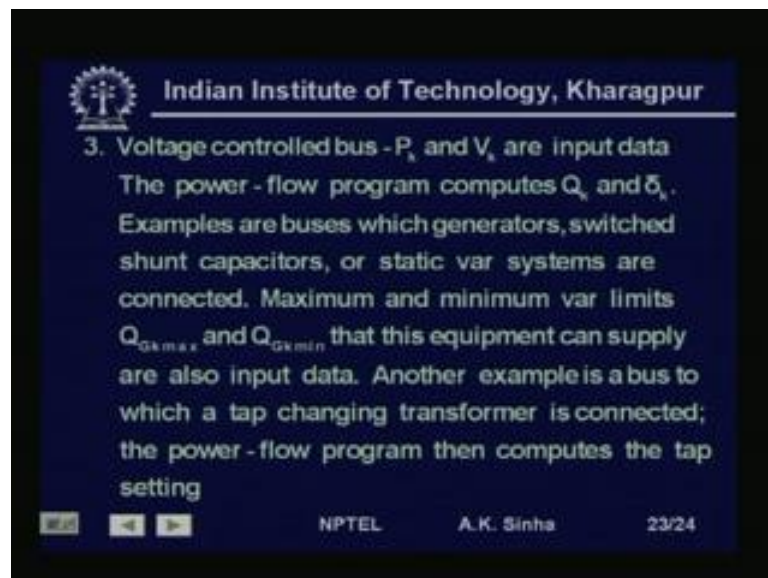
Indian Institute of Technology, Kharagpur

2. Load bus - P_k and Q_k are specified (input data).
The power - flow program computes V_k and δ_k .

NPTEL A.K. Sinha 22/24

Then, other type of busbar is a load busbar, where both real and reactive powers are specified. In at this busbar what are the unknowns, if we know V and δ at this busbar, we have all the variables. So, here P and Q are specified, that is injection real and reactive power injection is specified. And the unknowns at this busbar are V and δ a voltage magnitude and it is phase angle.

(Refer Slide Time: 56:20)



Indian Institute of Technology, Kharagpur

3. Voltage controlled bus - P_k and V_k are input data
The power - flow program computes Q_k and δ_k .
Examples are buses which generators, switched shunt capacitors, or static var systems are connected. Maximum and minimum var limits Q_{Gkmax} and Q_{Gkmin} that this equipment can supply are also input data. Another example is a bus to which a tap changing transformer is connected; the power - flow program then computes the tap setting

NPTEL A.K. Sinha 23/24

We have a third type of busbar, which we called as voltage controlled busbars. These are generally busbars, where we have generation at these busbars P_k and V_k are specified. That is the real power and the voltage magnitude is specified and the power flow

program or the solution of power flow will provide us the reactive power and the voltage phase angle. Now, these busbars as I said are generally the generator busbars, or switched shunt capacitors or static var systems.

So, busbars which have these they are generally P V bus or voltage controlled bus. The main reason for this is at these busbars, since we have reactive power control available. So, we try to keep the voltage controlled or the voltage magnitude at some specified value. And therefore, the voltage magnitude can be specified and there since, these are also generator busbars or may be load busbars, the real power at these busbars are known.

So, once we have fixed this magnitude of voltage, once we calculate the delta angle for this voltage, then we can calculate Q for this voltage. So, Q and delta are the unknowns for this. Sometimes maximum and minimum var limits, that is $P_{Gk\max}$ and $P_{Gk\min}$ for the equipment is also specified. And in case we find after the solution, that the value of the reactive power, that is Q_k is exceeding these limits.

Then, we fix the value of Q_k to these limits and consider these busbars as load busbars. Because, both P and Q are now fixed at these busbars and V and delta needs to be found out. So, with this we come to the end of today's lesson, we will talk about the solution methods for power flow equation in the next lesson.

Thank you very much.