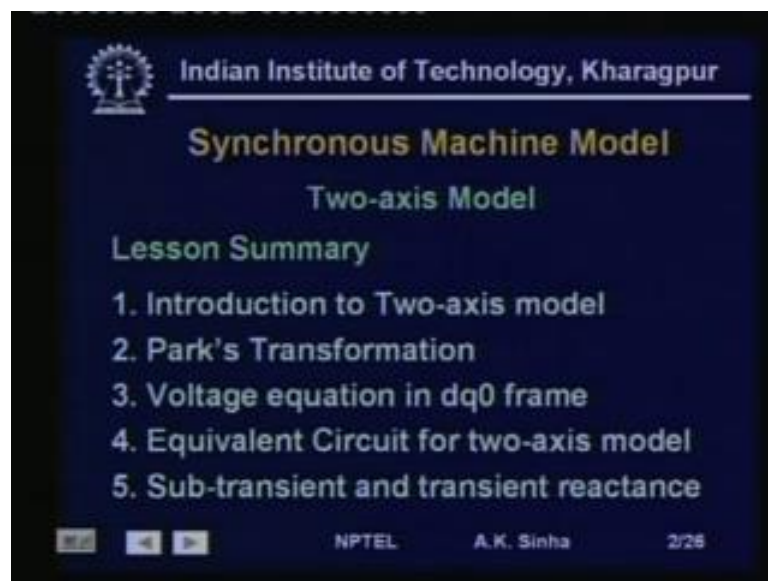


Power System Analysis
Prof. A.K. Sinha
Department of Electrical Engineering
Indian Institute of Technology, Kharagpur

Lecture - 14
Synchronous Machine Model (contd.)

Welcome to lesson 14 on Power System Analysis. In this lesson, we will be continuing on Synchronous Machine modeling, which we started in lesson 13.

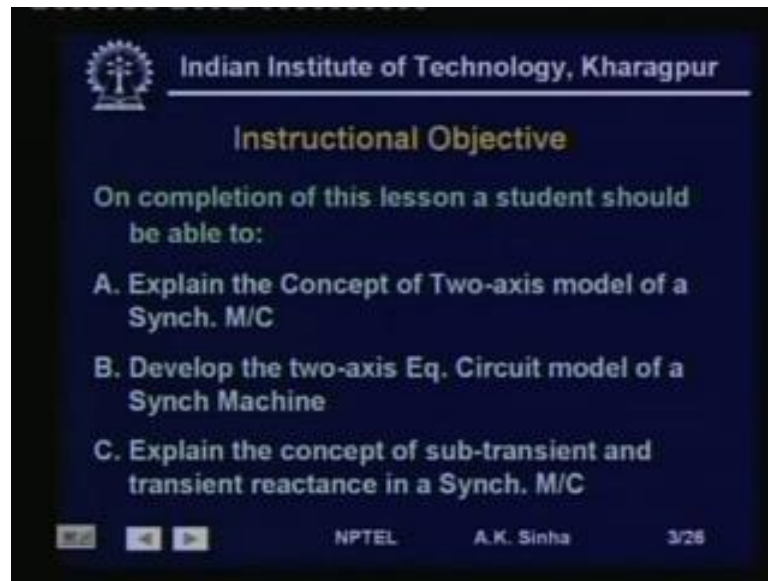
(Refer Slide Time: 01:08)



Well, in this lesson we will discuss about the two-axis model, as we talked in lesson 13. We modeled round rotor machine, we said that the other type of machine which is the salient pole machine. That modeling we will be taking up in this lesson and this will require a two-axis model. Because, the magnetic circuit is not uniform for a salient pole machine as we will see.

So, this we will start with introduction to two-axis model. Then we will go in to Park's transformation, then we will take up the voltage equation in $d q 0$ frame of reference. And we will build the equivalent circuit for the two-axis model. And then we will discuss about sub-transient and transient reactance of a synchronous machine.

(Refer Slide Time: 02:23)



Indian Institute of Technology, Kharagpur

Instructional Objective

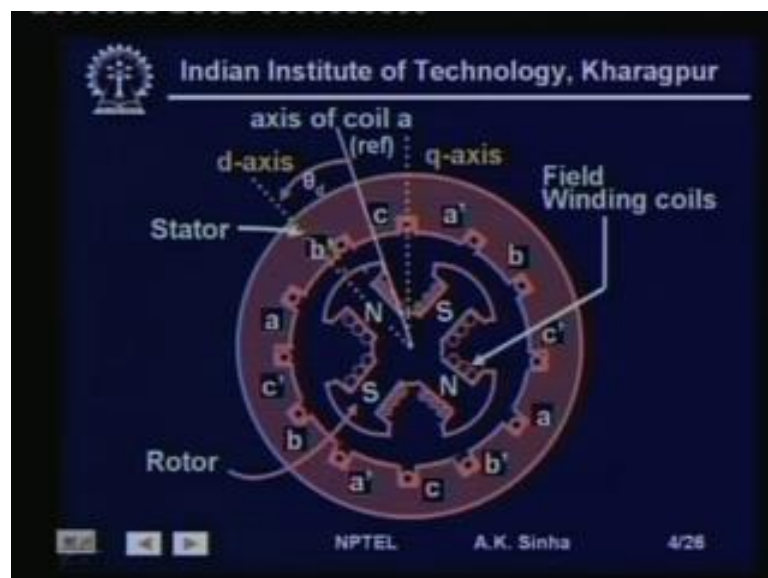
On completion of this lesson a student should be able to:

- A. Explain the Concept of Two-axis model of a Synch. M/C
- B. Develop the two-axis Eq. Circuit model of a Synch Machine
- C. Explain the concept of sub-transient and transient reactance in a Synch. M/C

NPTEL A.K. Sinha 3/26

Well, on completion of this lesson, you should be able to explain the concept of two-axis model of a synchronous machine. Develop the two-axis equivalent circuit model of a synchronous machine and explain the concept of sub-transient and transient reactance in a synchronous machine. Well to start with as we said, in case of a round rotor machine, the rotor is made up of cylindrical, solid cylindrical iron. And the air gap is uniform all around, whereas in case of machines, which are having salient poles and we also said in the last class. That the hydro turbines, which run at much slower speeds require much larger number of poles. And for these machines, we use normally a salient pole construction.

(Refer Slide Time: 03:32)



In this figure here, salient pole machine with four protruding poles are shown in this machine. As we see, we have a North pole followed by a South pole, then again a North pole and a South pole. Now, since there are four poles, we also have the coils according to that. We have coils a dash a for phase a, then we have b dash b for phase b which is lagging this phase a by 120 degree. Then we have c dash c which is lagging this by 120 degree.

Now, this 120 degree is basically the electrical degrees as we told in lesson 13. Whenever we are talking about the angular displacements, we are always referring to the electrical angles. And as we said earlier the angular displacement for electrical degrees is will be equal to p by 2 times the mechanical degrees. Therefore, in this case here since there are number of poles are 4.

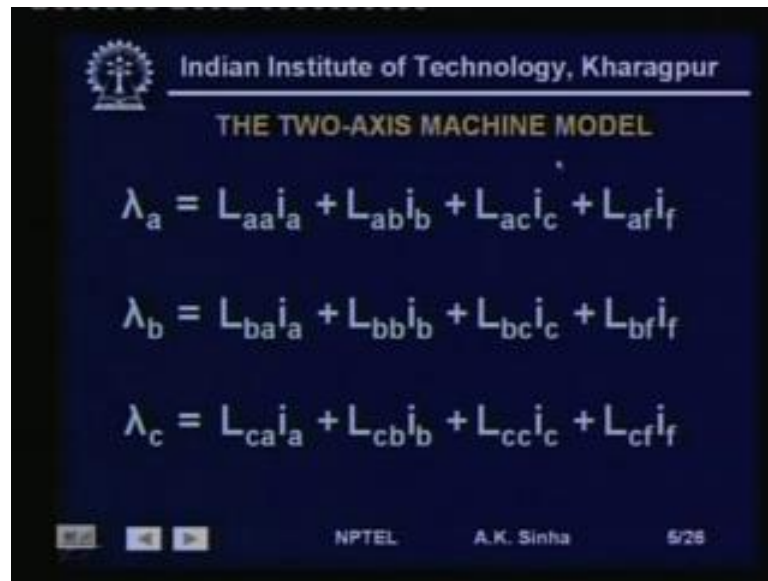
So, the relationship between electrical degrees and mechanical degrees will be that, each mechanical degree is equal to twice the number of electrical degrees. So, each mechanical degree will be equal to twice the number of electrical degrees. So, we have here a 60 degree displacement between a dash and b dash mechanically, whereas electrically this displacement is 120 degrees.

Now, in this machine as we see the magnetic circuit, if you look at the air gap between rotor and stator around the poles. These protogent poles the air gap is much smaller, as compare to the inter polar region, between in the inter polar region the air gap is much larger. That means, the magnetic circuit is not uniform and therefore, we need to consider this difference by considering the reluctance in the two parts separately or distinctly.

That is we say that the axis of the pole is the direct axis, so we call it d-axis the North pole is directing in this way. So, this we call as the d-axis and the q-axis will be in the inter polar region and it will be lagging the d-axis by 90 electrical degrees. In this case we have chosen the axis of coil a as our reference, and we are measuring all the angles with respect to this reference.

So, θ_d here is the displacement of d-axis with respect to the reference at any given time t . So, this is about the physical aspect of this salient pole synchronous machine. Now, for this machine, we will write the flux linkage equations and develop the circuit model.

(Refer Slide Time: 07:36)



Indian Institute of Technology, Kharagpur
THE TWO-AXIS MACHINE MODEL

$$\lambda_a = L_{aa}i_a + L_{ab}i_b + L_{ac}i_c + L_{af}i_f$$
$$\lambda_b = L_{ba}i_a + L_{bb}i_b + L_{bc}i_c + L_{bf}i_f$$
$$\lambda_c = L_{ca}i_a + L_{cb}i_b + L_{cc}i_c + L_{cf}i_f$$

NPTEL A.K. Sinha 5/26

So, if we see this machine we have the flux linkage in phase a given by $L_{aa}i_a$ plus $L_{ab}i_b$ plus $L_{ac}i_c$ plus $L_{af}i_f$, where L_{aa} is the self inductance of the coil a, L_{ab} is the mutual inductance between coil a and b on the stator. L_{ac} is the mutual inductance between coil a and c on the stator and L_{af} is the mutual inductance between coil a and the rotor.

Similarly, for phase b equal, we can write the flux linkage as $L_{ba}i_a$ plus $L_{bb}i_b$ plus $L_{bc}i_c$ plus $L_{bf}i_f$. Again we have L_{ba} , which will be equal to L_{ab} and this is the mutual inductance between coil or phase a. And coil of phase b on the stator L_{bb} is a self inductance of the coil on phase b L_{bc} between coil b or phase b and phase c on the stator L_{bf} between coil b and rotor. Similarly, the flux linkage with coil c can be written as $L_{ca}i_a$ plus $L_{cb}i_b$ plus $L_{cc}i_c$ plus $L_{cf}i_f$.

Now, if we look at these inductances L_{aa} , L_{ab} , L_{ac} , L_{af} all these inductances. We will find that these inductances are going to be varying depending on the position of the rotor. Because, when the position of the rotor is near to the axis of any given coil, then the inductance is going to be much higher at that point. Whereas, when it is away from the axis of that coil or then the inductance is going to be much smaller.

So, all these terms L_{aa} , L_{ab} , L_{bc} all these terms are going to be depending on the position of the rotor that is angle θ_d . Again this angle θ_d is a time varying quantity, because the rotor will be rotating at a synchronous speed. As we have seen earlier for any given frequency, we have a particular synchronous speed. So, the rotor

will be rotating at that speed and therefore, this angle θ_d will keep changing with time.

And which means that, all these inductances are also going to be time varying inductances. This makes this analysis for a salient pole machine somewhat more complex. However for this rotor, if we see as far as the rotor is concerned, the flux will see the same magnetic circuit all the time, through whatever position may maybe. Like the flux seen for this position will be from here it will pass like this, it will go like this, come like this and so on.

Whatever may be the position, the path of this flux will be through the air gap around the poles. So, around the two poles it will pass through that. And this air gap as the rotor keeps rotating still remains same between the rotor tip, rotor poles and the stator. Therefore, the inductance L_{af} , L_{bf} and L_{cf} is going, the inductance of the rotor coil is going to remain constant. That is the inductance of the rotor is going to remain constant, though the inductance L_{af} , L_{bf} and L_{cf} will be varying. Because, the position of the rotor will be the axis of the rotor coil is going to be different from the axis of the particular phase stator coil. And therefore, the L_{af} , L_{bf} , L_{cf} will also be time varying.

(Refer Slide Time: 13:14)

Indian Institute of Technology, Kharagpur

Stator

Self - inductance $(L_s > L_m > 0)$

$$\begin{cases} L_{aa} = L_s + L_m \cos 2\theta_d \\ L_{bb} = L_s + L_m \cos 2(\theta_d - 2\pi/3) \\ L_{cc} = L_s + L_m \cos 2(\theta_d + 2\pi/3) \end{cases}$$

NPTEL A.K. Sinha 6/26

Now, if we find out the value of these self inductances of the stator coil. Then, we will see that the value of self inductance L_{aa} is equal to L_s plus $L_m \cos 2\theta_d$. L_{bb} will be L_s plus $L_m \cos 2\theta_d$ minus 120 degrees, that is $2\pi/3$, L_{cc} will be L_s

plus $L_m \cos 2\theta_d$ plus 120 degree, that is same as minus 240 degrees. That is we are seeing the self inductances of the stator coils are dependent on the rotor position θ_d . And since, θ_d is time varying these inductances are time varying quantities.

(Refer Slide Time: 14:09)

Indian Institute of Technology, Kharagpur

Mutual-inductance
($M_s > L_m > 0$)

$$L_{ab} = L_{ba} = -M_s - L_m \cos 2(\theta_d + \pi/6)$$

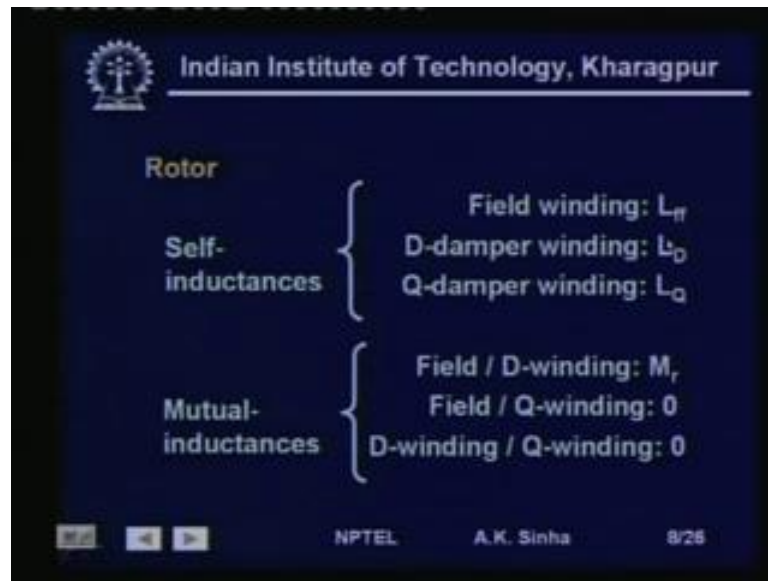
$$L_{bc} = L_{cb} = -M_s - L_m \cos 2(\theta_d - \pi/2)$$

$$L_{ca} = L_{ac} = -M_s - L_m \cos 2(\theta_d + 5\pi/6)$$

NPTEL A.K. Sinha 7/26

Similarly, the mutual inductance between the stator coils L_{ab} is equal to L_{ba} , which is equal to minus M_s minus $L_m \cos 2\theta_d$ plus $\pi/6$, that is 30 degrees. Similarly, L_{bc} will be equal to L_{cb} is equal to minus M_s minus $L_m \cos 2\theta_d$ minus $\pi/2$. And L_{ca} is equal to L_{ac} is equal to minus M_s minus $L_m \cos 2\theta_d$ plus $5\pi/6$. That is these mutual inductances as we said earlier are also going to be time varying. Because, the air gap is not constant and it keeps varying as the rotor keeps rotating around the stator.

(Refer Slide Time: 15:22)



Now, the rotor self inductance as I said L_{ff} is going to be constant, because the rotor is all the time seeing the same magnetic circuit. The flux from the rotor will pass from the pole phases through the air gap into the stator core. And then will come back again from the stator core through the air gap to the rotor poles. So, this remains same whatever may be the position of the rotor, because these two air gaps between the pole, phase and the stator in a cylindrical surface is all the time same.

Now, here we have also introduced some damper windings. Actually most of the synchronous machines have a damper winding, which is placed in the pole phases by invading copper bars in the pole phase and short circuiting them. So, whenever there is any oscillation which takes place or these there will be current voltage setup across these bars. And since, they are short circuited current will flow through them which will oppose the change in motion.

And these damper windings will can also be model. And here, we have put them as D-axis, damper winding. That is the damper winding which is provided by means of putting copper bars on the rotor pole phases. Now, there is another thing which happens, since the rotor is made up of magnetic material. And because of any change in motion, if it speed which takes place there is going to be flux setup in the core material of the rotor.

And this is going to lead to currents in this core, since this core is made up of iron which is also electrically conducting material, which causes eddy current losses in the rotor. And these eddy currents also act as a damper to the change in speed which is causing this

current to flow in it. And this is generally represented by means of a q-axis damper winding. So, we have a d-axis damper winding for which we have an inductances L_D .

Since, the rotor is the flux from the rotor is always having the same magnetic circuit, so L_D and L_Q will also be constant. Now, the mutual inductance between the field and the D winding, that is the direct-axis damper winding and the field is going since the 2 axis are same. There is going to be a mutual inductance between them which we define as M_r .

Since, the 2 circuits are rotating at the same speed they appear to be stationery with respect to each other. And therefore, M_r is going to remain constant. Now, field and Q winding mutual inductance is going to be equal to 0, that is field and Q winding mutual inductance is going to be 0. Because, Q winding inductance is in quadrature with the field winding which is on the d-axis and therefore, the mutual inductance is 0.

Similarly, the D winding and Q winding mutual inductance is also going to be equal to 0, because the D winding is on the direct-axis and the Q winding is on the quadrature axis. And since there is a 90 degree difference between these 2 axis, the mutual inductance is 0 between them. So, what we see here is, we have taken care of the damper winding and the eddy current part, by means of two fictitious winding. That is having an inductance L_D and another having an inductance L_Q .

(Refer Slide Time: 20:35)

Indian Institute of Technology, Kharagpur

Stator-rotor mutual inductances

Armature / field $\left\{ \begin{array}{l} L_{af} = L_{fa} = M_f \cos\theta_d \\ L_{bf} = L_{fb} = M_f \cos(\theta_d - 2\pi/3) \\ L_{cf} = L_{fc} = M_f \cos(\theta_d - 4\pi/3) \end{array} \right.$

Armature / D-winding $\left\{ \begin{array}{l} L_{aD} = L_{Da} = M_D \cos\theta_d \\ L_{bD} = L_{Db} = M_D \cos(\theta_d - 2\pi/3) \\ L_{cD} = L_{Dc} = M_D \cos(\theta_d - 4\pi/3) \end{array} \right.$

NPTEL A.K. Sinha 9/26

Now, we talk about the stator and rotor mutual inductances. As we said since the rotor is rotating the position of the rotor changes with respect to the axis of the coil. So, the

inductance, the mutual inductance between them is also going to change. So, L_{af} and L_{fa} are equal and that is equal to $M_f \cos \theta_d$. This is what we said that, when the d-axis coincides with phase a axis, then the two coils are aligned with each other.

And we are going to have the maximum mutual inductances between them. And that is what we see here, that θ_d in that case will be 0 and therefore, L_{af} is equal to L_{fa} is equal to M_f . Otherwise, at any other angle it is going to be equal to $M_f \cos \theta_d$. Similarly, L_{bf} is equal to L_{fb} is equal to $M_f \cos(\theta_d - 120^\circ)$ and L_{cf} is equal to L_{fc} is equal to $M_f \cos(\theta_d - 240^\circ)$. That is since the coil of phase b and c are displaced or lag, the axis of phase a by 120 and 240 degree respectively.

Now, armature winding and D winding, again we will have mutual inductances, which we can define as L_{aD} . That is between phase a stator phase a winding and the d-axis damper winding L_{aD} is equal to L_{Da} is equal to $M_D \cos \theta_d$, again the same thing, because the rotor is rotating. So, the axis positions will keep changing and because of that, the inductance is a function of $\cos \theta_d$. Similarly, L_{bD} is equal to L_{Db} is equal to $M_D \cos(\theta_d - 120^\circ)$. L_{cD} is equal to L_{Dc} is equal to $M_D \cos(\theta_d - 240^\circ)$.

(Refer Slide Time: 23:11)

Indian Institute of Technology, Kharagpur

$$\text{Armature / Q-winding} \begin{cases} L_{aQ} = L_{Qa} = M_Q \cos \theta_d \\ L_{bQ} = L_{Qb} = M_Q \cos(\theta_d - 2\pi/3) \\ L_{cQ} = L_{Qc} = M_Q \cos(\theta_d - 4\pi/3) \end{cases}$$

NPTEL A.K. Sinha 10/26

And finally, the mutual inductance between armature coils and the Q winding, this again also will be given by the same L_{aQ} is equal to L_{Qa} is equal to $M_Q \cos \theta_d$. L_{bQ} is equal to L_{Qb} is equal to $M_Q \cos(\theta_d - 120^\circ)$ L_{cQ} is equal

to L_{Qc} is equal to M_{Qd} into $\cos(\theta_d - 240^\circ)$. Now, since we have seen that most of these inductances, that we have the mutual and the self inductances are basically dependent on the rotor position.

And therefore, they are varying with the rotor position. And since, the rotor is also a time is changing, rotor position is changing with time as it the rotor rotates at synchronous speed. Therefore, these variables or these inductances are also all time varying inductances. And therefore, in order to analyze the system, it becomes very complex, because we have to deal with all these time varying inductances.

To make this problem somewhat simpler Park, suggested a transformation which we normally call as Park's transformation. This transformation simplifies the problem by making most of these inductances, by transforming them into a $d-q-0$ frame of reference. They inductances, then become constant and the analysis becomes much simpler.

(Refer Slide Time: 25:31)

Indian Institute of Technology, Kharagpur

Called Park's transformation, where

$$P = \sqrt{\frac{2}{3}} \begin{bmatrix} \textcircled{a} & \textcircled{b} & \textcircled{c} \\ \textcircled{a} & \textcircled{b} & \textcircled{c} \\ \textcircled{a} & \textcircled{b} & \textcircled{c} \end{bmatrix} \begin{bmatrix} \cos\theta_d & \cos(\theta_d - 120^\circ) & \cos(\theta_d - 240^\circ) \\ \sin\theta_d & \sin(\theta_d - 120^\circ) & \sin(\theta_d - 240^\circ) \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

NPTEL A.K. Sinha 11/26

Now, the Park's transformation is shown here, this is carried out for the stator coils. The rotor coils, we do not need to carry this transformation, because we find that the rotor circuit is all the time having a same inductance, the same magnetic circuit. Now, this transformation is written as P , in fact this is somewhat a modified transformation, it is not the original Park's transformation.

The advantage of this modified transformation is that, this transformation is orthogonal transformation, which means the inverse of this matrix P is P transpose. Another advantage of this transformation is that, since it is orthogonal transformation, it is power

and variant. That is if we find out the power in a, b, c phase frame of reference. And we find out the power in the Park's transform domain of d q 0 frame of reference, both will be same.

So, this is an advantage of using this modified transformation, which is not true in case we use the original Park's transformation. So, now here this Park's transformation is defined by a 3 by 3 matrix. This P is a 3 by 3 matrix given by $\frac{1}{\sqrt{2}} \begin{bmatrix} \cos \theta & \sin \theta & 0 \\ -\sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$. So, this is the transformation which we use and if we use this transformation, then what we have is that.

(Refer Slide Time: 27:47)

The slide displays the following equations:

$$\begin{bmatrix} i_d \\ i_q \\ i_0 \end{bmatrix} = P \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} \quad \begin{bmatrix} v_d \\ v_q \\ v_0 \end{bmatrix} = P \begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} \quad \begin{bmatrix} \lambda_d \\ \lambda_q \\ \lambda_0 \end{bmatrix} = P \begin{bmatrix} \lambda_a \\ \lambda_b \\ \lambda_c \end{bmatrix}$$

$$\lambda_d = L_d i_d + \sqrt{\frac{3}{2}} M_f i_f$$

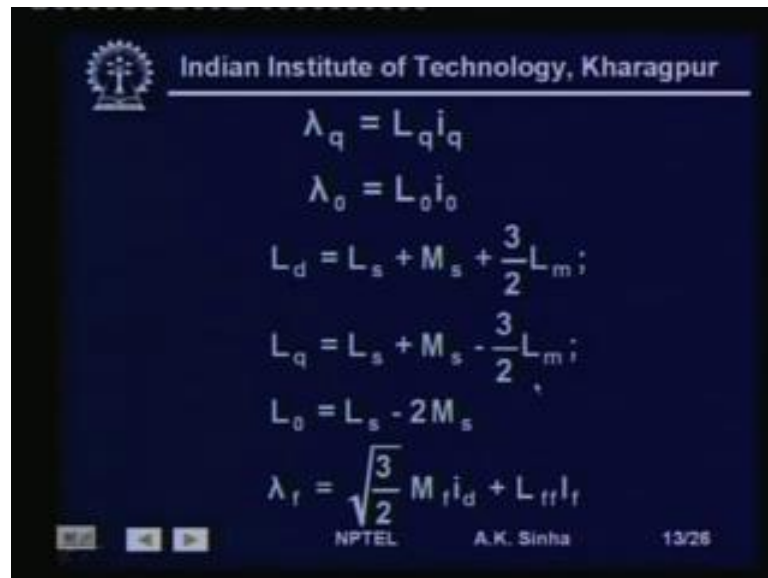
At the bottom of the slide, there is a navigation bar with the text: NPTEL A.K. Sinha 12/26

If we pre multiply this matrix to a, b, c frame currents, that is currents in phase a, phase b, phase c given by i_a, i_b, i_c . If we pre multiplied by the P matrix or the Park's transformation matrix. Then the currents that we will get will be i_d, i_q, i_0 , that is the currents will be transformed into d q 0 frame of reference, where D is the direct-axis and Q is the quadrature-axis and 0 represents the 0 sequence quantities.

Same transformation can be done on a, b, c phase voltages, then we will get the d q 0 frame reference voltages. So, v_d, v_q, v_0 is equal to P into v_a, v_b, v_c . Similarly, the same transformation can also be applied to the flux linkages. So, P into $\lambda_a, \lambda_b, \lambda_c$ will give us $\lambda_d, \lambda_q, \lambda_0$. Now, if we use this transformation P into $\lambda_a, \lambda_b, \lambda_c$. That is we have already seen the relationship for $\lambda_a, \lambda_b, \lambda_c$.

If we substitute these relationship for λ_a , λ_b , λ_c along with the values of the inductances L_a , L_b , L_c etcetera. And multiplied with this matrix P, the Park's transformation matrix, then what we are going to get is the flux linkages in d q 0 frame of reference, where we will get λ_d is equal to $L_d i_d$ plus $\frac{\sqrt{3}}{2} M_f i_f$.

(Refer Slide Time: 30:15)



Indian Institute of Technology, Kharagpur

$$\lambda_q = L_q i_q$$

$$\lambda_0 = L_0 i_0$$

$$L_d = L_s + M_s + \frac{3}{2} L_m$$

$$L_q = L_s + M_s - \frac{3}{2} L_m$$

$$L_0 = L_s - 2 M_s$$

$$\lambda_f = \sqrt{\frac{3}{2}} M_f i_d + L_{ff} i_f$$

NPTEL A.K. Sinha 13/26

And we will get λ_q is equal to $L_q i_q$ and λ_0 is equal to $L_0 i_0$, where, we have this L_d as the direct-axis self inductance. That is what we are saying is that, this is the self inductance of fictitious coil which is representing the a, b, c frame coils or the actual a, b, c phase coils in the direct axis. M_f is the mutual inductances between the field winding, that is the rotor and the direct-axis fictitious coil.

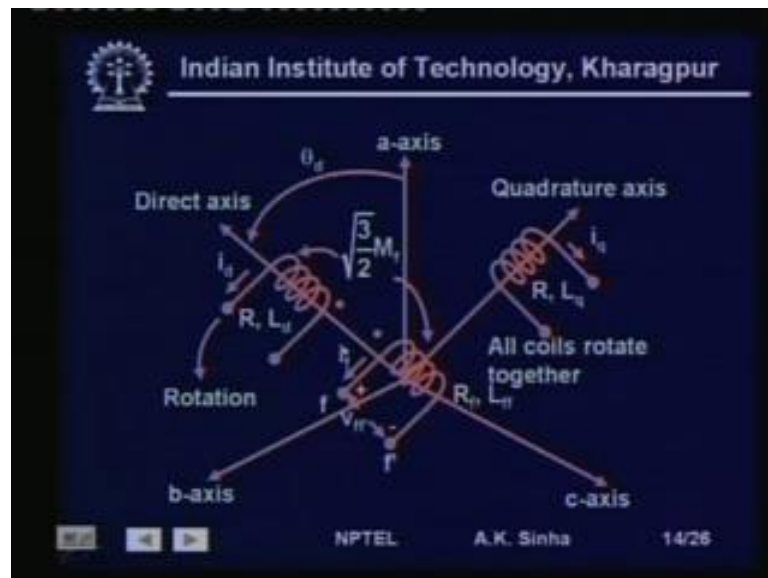
Similarly, L_q represents the self inductance of the coil, in the quadrature axis. So, what we have is we have transform the a, b, c the three phase coils into three separate coils which are along the direct axis. One is along the direct axis, another is along the quadrature-axis and the third one representing the 0 sequence. So, λ_0 is equal to $L_0 i_0$. Now, if we do all this multiplication and find out the values of L_d , L_q , L_0 , then we will find out L_d is equal to L_s plus M_s plus $\frac{3}{2} L_m$.

L_q is equal to L_s plus M_s minus $\frac{3}{2} L_m$ and L_0 is equal to L_s minus $2 M_s$. Now, what are finding here, what is the thing that we see in these inductances. We are finding these inductances are no longer function of theta. And therefore, they are not dependent

on the rotor position and so they are not time varying inductances anymore. This has greatly simplified our analysis.

We can again write λ_f , this is equal to $\sqrt{3/2} M_f$ into i_d plus L_f into i_f . This is the value of λ_f that we had calculated in our round rotor machine model also. Since, the air gap for the field winding is going to remain constant, therefore this relationship is holding good here also.

(Refer Slide Time: 33:33)



Thus what we see is that the a, b, c phase coils are now replaced by three coils. One which is a direct-axis coil, another is at 90 degrees to it, that is at the quadrature axis. And third coil which is basically as representing the 0 sequence quantities. Since, we will be mostly interested in balanced operation 0 sequence quantities will be 0. So, we do not show that winding here at all.

Now, these windings that is L_d and L_q , that is winding direct-axis winding and the quadrature-axis winding of the coil. These are fictitious coil and they are assumed to be rotating at synchronous speed, that is at the same speed as the rotor is rotating. And therefore, they appear to be stationary with respect to the rotor. And this is the reason why, the inductances remain constant in the d q 0 frame of reference.

So, here as we have seen, there is a mutual inductances between the d-axis coil and the field or the rotor as equal to $\sqrt{3/2} M_f$. Each of these direct-axis coil has a resistance R and an inductance L_d . Similarly, the quadrature-axis coil has a resistance R

and a inductance $R L q$. And of course, the field winding or the rotor has a resistance $R f$ and a self inductance $L f f$.

This rotor coil is excited by a voltage $V f f$, $V f f$ dash. So, this is what happens when we have transformed the a, b, c stator coils into a d q 0 frame of reference. In the where we get two coils, one in direct axis, one in quadrature axis. And of course, the third one representing the zero sequence, which we do not consider when we are working on a balanced three phase system. And these two coils appeared to be rotating at synchronous speed and therefore, are stationary with respect to the rotor. So, this is the advantage that we get from Park's transformation.

(Refer Slide Time: 36:47)

Indian Institute of Technology, Kharagpur

VOLTAGE EQUATIONS: SALIENT-POLE MACHINE

$$v_a = -R i_a - \frac{d\lambda_a}{dt};$$

$$v_b = -R i_b - \frac{d\lambda_b}{dt};$$

$$v_c = -R i_c - \frac{d\lambda_c}{dt}$$

NPTEL A.K. Sinha 15/26

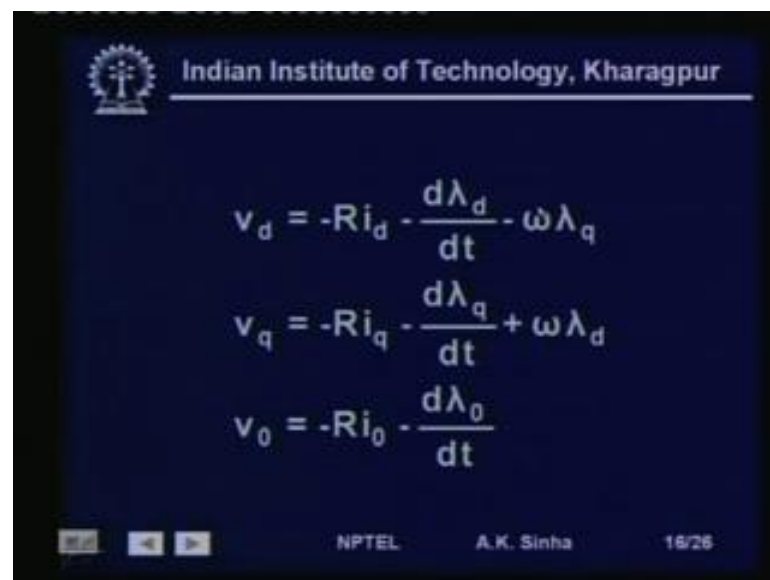
Now, let us write the voltage equation for the synchronous machine. Now, we have the voltage v_a equal to minus R into i_a minus $d \lambda_a$ by $d t$. That is the voltage across the phase a coil is going to be equal to the voltage drop, which is R into i_a and $d \lambda_a$ by $d t$. That is the rate of change of flux linkage with coil a. Now, these negative signs are coming, because we have considered a generator mode of operation.

Where the current i is leaving the terminals of the machine, that is it is going out from the terminals of the machine. That is why we have this negative sign coming here. Similarly, for phase b we can write v_b is equal to minus $R i_b$ minus $d \lambda_b$ by $d t$ and v_c is equal to minus $R i_c$ minus $d \lambda_c$ by $d t$. So, for the three phase coils we can write these equations. Now, as we see here we have the terms, here time derivative of λ_a λ_b and λ_c .

And as we have seen these lambdas are basically related with inductance which are again a time varying quantities. Therefore, this makes the solution of these equations much more complex, or the analysis of the machine quite complex. So, when we use the Park's transformation we get constant inductances, then these derivatives become much simpler to evaluate.

So, if we use the Park's transformation by multiplying these a, b, c quantities. Whether, they are current voltage or flux linkage by the Park's transformation matrix. And using the inverse transformation, which is equal to the transpose of the Park's transformation matrix.

(Refer Slide Time: 39:36)



Indian Institute of Technology, Kharagpur

$$v_d = -R i_d - \frac{d\lambda_d}{dt} - \omega \lambda_q$$

$$v_q = -R i_q - \frac{d\lambda_q}{dt} + \omega \lambda_d$$

$$v_0 = -R i_0 - \frac{d\lambda_0}{dt}$$

NPTEL A.K. Sinha 16/26

Then, we will get the in d q 0 frame of reference of, as voltages v_d equal to minus R into i_d minus $d\lambda_d$ by dt minus $\omega \lambda_q$, where ω is the synchronous speed of the machine, that is it is $d\theta$ by dt . Similarly, we will get v_q is equal to minus $R i_q$ minus $d\lambda_q$ by dt plus $\omega \lambda_d$; and v_0 is equal to minus $R i_0$ minus $d\lambda_0$ by dt .

Now, from these equations what we find is, now we have to find out time derivatives of λ_d , λ_q , etcetera. Since, these involve constant inductances this is much simpler to evaluate. Another feature which is very important in these equations, is that we are finding these terms minus $\omega \lambda_q$ and plus $\omega \lambda_d$ in... So, minus $\omega \lambda_q$ is coming in v_d , that is the voltage in direct-axis is also dependent on the flux linkage of the q-axis coil.

This is a very interesting phenomena. And this is what we call as the speed voltages. That is the voltage which is coming, because of the rotation of the rotor. In fact, this is what is providing this electromechanical power conversion. That is the d-axis voltage depends on the q-axis flux linkage and the speed, similarly the q-axis voltage depends on d-axis flux linkage and the speed. And this is what is providing as what we call as the speed voltage. And which is basically giving us the relationship for power, which is generated or transformed from mechanical to electrical that these are the terms which provide that path.

(Refer Slide Time: 42:16)

Indian Institute of Technology, Kharagpur

$$\lambda_d = L_d i_d + k M_f i_f$$

$$\lambda_f = k M_f i_d + L_{ff} i_f$$

$$v_d = -R i_d - \frac{d\lambda_d}{dt} - \omega \lambda_q$$

$$v_{ff} = R_f i_f + \frac{d\lambda_f}{dt}$$

$$\lambda_q = L_q i_q$$

$$v_q = -R i_q - \frac{d\lambda_q}{dt} + \omega \lambda_d$$

NPTEL A.K. Sinha 17/26

So, again we can write this λ_d is equal to $L_d i_d$ into $k M_f i_f$ λ_f is equal to $k M_f i_d$ plus $L_{ff} i_f$. This is the flux linkage of the coils in the direct axis. This what we had seen earlier λ_d is equal to $L_d i_d$ plus $\frac{\sqrt{3}}{2} M_f$. If we are writing this $\frac{\sqrt{3}}{2}$ as k , therefore we have here λ_d is equal to $L_d i_d$ plus $k M_f i_f$. Similarly, λ_f is equal to $k M_f i_d$ plus $L_{ff} i_f$.

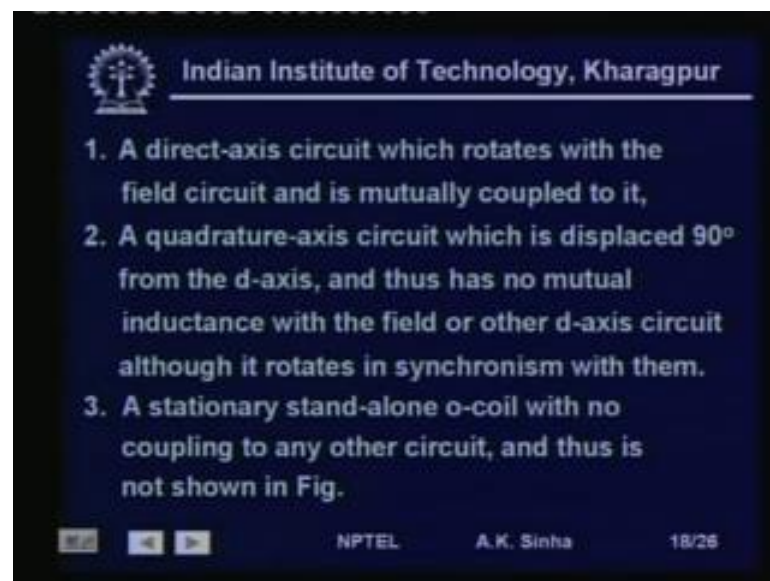
These are the flux linkage equations for the coils along the direct axis, v_d is the voltage of the direct across the direct-axis coil. So, this is equal to minus $R i_d$ minus $\frac{d\lambda_d}{dt}$ minus $\omega \lambda_q$. And v_{ff} is the voltage of the voltage applied across the rotor winding, this is equal to $R_f i_f$ plus $\frac{d\lambda_f}{dt}$. So, these four equations are giving us the for flux linkages, voltages and currents on for the direct-axis coils.

And these two equations λ_q is equal to $L_q i_q$ and v_q is equal to minus $R i_q$ minus $\frac{d\lambda_q}{dt}$ plus $\omega \lambda_d$ are providing us the equations, for the coil

on the quadrature axis. Now, here if we see these equations show that, there is a coupling between the direct-axis coil and the field, or the rotor coil. Whereas, there is no coupling for the quadrature-axis coil λ_q is equal to L_q into i_q , it is dependent only on the current in that coil.

So, the q-axis coil has no coupling, whereas the d-axis coil has a coupling with the rotor, that is the coils, since the rotor coil or the field coil is on the d-axis they have a coupling. So, using these equations, we can now find out the equivalent circuit for the synchronous machine in 2 axis, that is d-axis and the q-axis

(Refer Slide Time: 45:48)

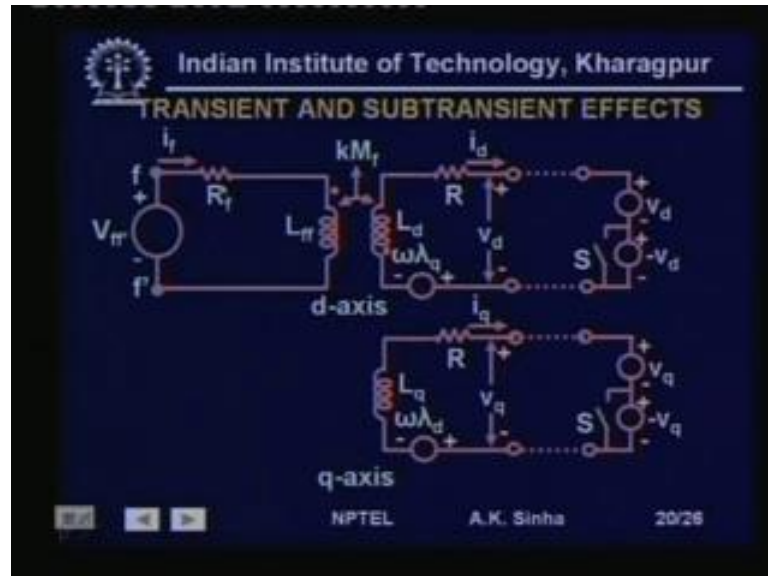


Before we do that, we will like to see what is the importance of these equations. Now, these equations tell us that a direct-axis circuit, that is the Park's transform converts the stator coils into a direct-axis circuit which rotates with the field circuit and is mutually coupled to it. We have a quadrature-axis circuit, which is displaced at 90 degrees from the direct axis. And has no mutual inductance with the field of the other d-axis circuit.

Although it rotates in synchronism with them, that is we have replaced the a, b, c phase coils by two fictitious coil, One on direct-axis and another on quadrature axis, both rotating at synchronous speed. And displaced with one another by 90 degree, the d-axis coil has a mutual coupling with the rotor coil, whereas the q-axis coil does not have any coupling. We also have a stationary stand alone 0 sequence coil with no coupling to any other circuit, and thus is not shown.

That is if we see here ((Refer Time: 47:27)), we have this v_0 is equal to minus $R i_0$ minus $d \lambda_0 / dt$. This has no coupling with the other coils. And since, this will be 0 for a balanced operation we normally do not show this at all.

(Refer Slide Time: 47:52)



In $d-q-0$ frame of reference, the circuit model of the synchronous machine can be as shown here. Now, we have field winding with inductance L_{ff} and the resistance R_f , in which the current i_f flows due to the application of a d c voltage source V_{ff} . Then we have a direct-axis winding which has inductance L_d and a resistance R and a current i_d flows in this. Because, there is a speed voltage $\omega \lambda_q$ with a negative sign and positive sign as shown here, with this i_d flowing when a load is connected to this winding.

Then we get a voltage v_d as shown here. Similarly, in the q-axis winding we have L_q . The inductance of the winding and R the resistance of the winding the speed voltage, due to the flux linkage in the direct-axis is given by $\omega \lambda_d$, as shown here. And with current i_q flowing in this winding, the voltage at the terminal of the q-axis winding is given by v_q . So, this is the circuit model for the synchronous machine.

Now, when the synchronous machine goes through transient condition or sub-transient condition, which is created when a fault occurs on the system, when a short circuit occurs, the machine behavior shows that initial current values are much larger and the current decays. And finally, settles down to a steady state fault current value. Now, this

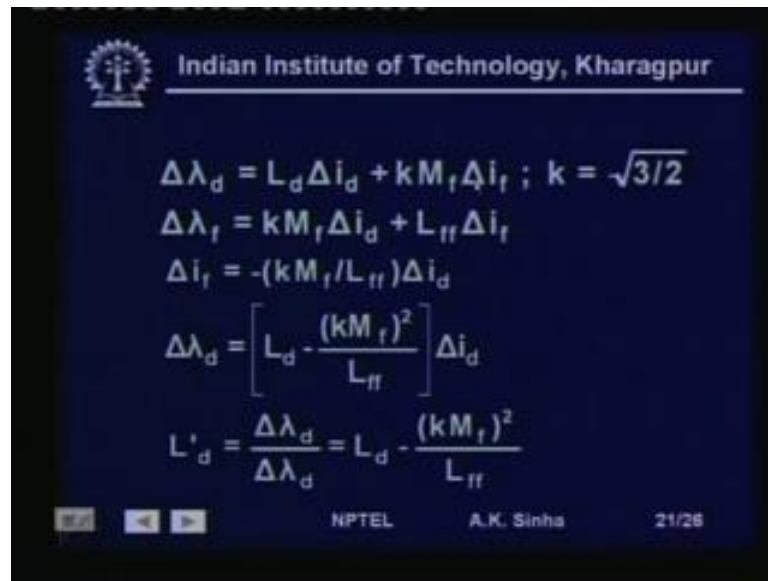
happens mainly, because of the changes which occur in the flux linkages by sudden application of the fault.

And this is represented by the variation of inductance, it as shown by the machine. Now, in order to simulate this short circuit condition, what we can do is we can connect two sources of v_d and minus v_d in series, across the terminals of v_d . That is across the terminals of the direct-axis winding, similarly v_q and minus v_q across the terminals of the q-axis winding. Now, when this switch is open, then we have only this v_d connected here, which shows the normal operating condition.

When we close this switch then we have only this v_d acting, which is same as the normal operating condition. The terminal voltage is v_d , when we open this switch, then these two voltage sources are in series. And the voltage across these terminals becomes 0 which indicates a short circuit condition. So, in order to create the short circuit condition, what we are seeing is that, the changes are produced by applying this minus v_d voltage source across the terminals.

That is we can use the superposition principle, where all we can have the voltages created by each voltage source separately and we can see this condition. Now, if we see what we have done during short circuit is only applying this voltage source, with all other voltage sources shorted. Then, this is showing the changes which occurred when the fault occurs on the system. Same thing for this minus v_q being applied to the system which will show the changes which occur, because of the creation of the fault. Because, when this minus v_d is not included the system condition, is same as the normal operating condition. So, now when we apply this minus v_d source, across this terminals we can write down the equation for the direct-axis system.

(Refer Slide Time: 52:14)



Indian Institute of Technology, Kharagpur

$$\Delta \lambda_d = L_d \Delta i_d + k M_f \Delta i_f ; k = \sqrt{3/2}$$
$$\Delta \lambda_f = k M_f \Delta i_d + L_{ff} \Delta i_f$$
$$\Delta i_f = -(k M_f / L_{ff}) \Delta i_d$$
$$\Delta \lambda_d = \left[L_d - \frac{(k M_f)^2}{L_{ff}} \right] \Delta i_d$$
$$L'_d = \frac{\Delta \lambda_d}{\Delta i_d} = L_d - \frac{(k M_f)^2}{L_{ff}}$$

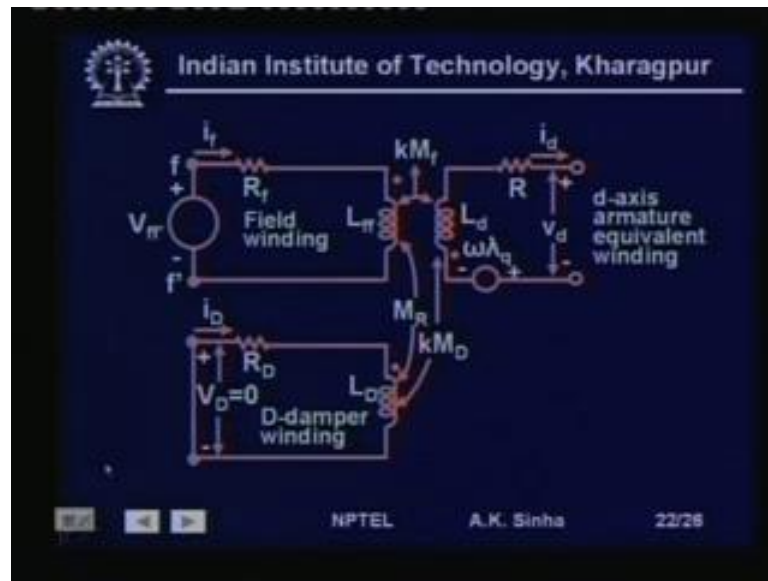
NPTEL A.K. Sinha 21/26

So, we have this changes created by applying of minus v_d across the terminals are shown here as $\Delta \lambda_d$ is equal to $L_d \Delta i_d + k M_f \Delta i_f$, if where k is root 3 by 2. And $\Delta \lambda_f$ is equal to $k M_f \Delta i_d + L_{ff} \Delta i_f$. Now, since here, all the other sources have been shorted when we are applying this source. So, this is shorted which means $\Delta \lambda_f$ is 0, therefore using this $\Delta \lambda_f$ as 0 we can calculate Δi_f .

From this equation which gives minus $k M_f$ by L_{ff} into Δi_d . And substituting for this Δi_f in this expression we get $\Delta \lambda_d$, $\Delta \lambda_d$ is equal to L_d minus $k M_f$ whole squared divided by L_{ff} into Δi_d . Now, this term is basically showing the inductance due to the short circuit which is created in the system. So, $\Delta \lambda_d$ is showing this change in inductance and this inductance we call as the transient inductance of the direct-axis winding.

So, this is L'_d , which is $\Delta \lambda_d / \Delta i_d$, this is equal to this, And since, this term is subtracted from L_d , we find that the transient inductance is smaller than the series state inductance or direct-axis inductance of the winding. Now, if we see the in quadrature axis, since there is no linkage with the field winding or other winding. Application of this is not going to create any changes in the flux linkages. And therefore, there is no transient quadrature-axis inductance or reactance for the machine. Now, the reactance shown under transient condition for the quadrature-axis will be same as that of steady state condition.

(Refer Slide Time: 54:47)



Now, most of the machines synchronous machines have also a damper winding, which are basically copper bars, which are embedded in the pole phases and shorted at the two ends. So, that represents a short circuited winding, because of the eddy currents flowing in this winding, which has the mutual inductances with the field and the direct-axis winding. And also the same the these damper and windings in the quadrature axis, will also show the mutual and the effect with the quadrature axis winding. So, they will create flux changes during the period, when current flows through this winding i_d .

(Refer Slide Time: 55:44)

$$\begin{aligned} \Delta\lambda_d &= L_d\Delta i_d + kM_f\Delta i_f + kM_D\Delta i_D \\ \Delta\lambda_f &= kM_f\Delta i_d + L_{ff}\Delta i_f + M_r\Delta i_D = 0 \\ \Delta\lambda_D &= kM_D\Delta i_d + M_r\Delta i_f + L_D\Delta i_D = 0 \\ \Delta i_f &= -\left[\frac{(kM_f)L_D - (kM_D)M_r}{L_{ff}L_D - M_r^2}\right]\Delta i_d \end{aligned}$$

So, again using the same concept we can write down the changes, which occur in the flux linkages due to the application of short circuit. Now, here again since the damper winding is associated, so we get the damper winding terms associated here. Now, again this term is 0, because this is a shorted winding, the field winding is shorted. Because, we are seeing only the effect of the short circuit, similarly the damper winding is a shorted one. So, these two equations are showing this is equal to 0, from these two equations we can calculate delta i f, like this in terms of delta i d.

(Refer Slide Time: 56:23)

Indian Institute of Technology, Kharagpur

$$\Delta i_D = - \left[\frac{(kM_D)L_{ff} - (kM_r)M_r}{L_{ff}L_D - M_r^2} \right] \Delta i_d$$

$$\frac{\Delta \lambda_d}{\Delta i_d} = L''_d = L_d - k^2 \left(\frac{M_i^2L_D + M_o^2L_{ff} - 2M_iM_oM_r}{L_{ff}L_D - M_r^2} \right)$$

NPTEL A.K. Sinha 24/26

And similarly delta i capital D in like this in this term. Therefore, writing down now the equations in terms of for delta lambda d, in terms of delta i D. We can write to get the inductance which we call the sub transient inductance as L d minus k square into this term. Now, here we see this term is much larger than what we had seen earlier. And therefore, the sub-transient inductance as seen by the direct-axis winding is much smaller.

And this shows that immediately on the application of the fault, this is the inductances, which is seen by the machine, and therefore the current is much larger. Now, this the current in the damper winding quickly decays, because the resistance of the winding is much larger and inductance is smaller. Therefore, the sub transient effect is seen only for one or two cycles.

(Refer Slide Time: 57:31)

Indian Institute of Technology, Kharagpur

$\Delta \lambda_q = L_q \Delta i_q + kM_q \Delta i_Q$
 $\Delta \lambda_Q = kM_q \Delta i_q + L_Q \Delta i_Q = 0$
 $\Delta i_Q = -\left[\frac{kM_q}{L_q} \right] \Delta i_q$
 $\Delta \lambda_q = \left[L_q - \frac{(kM_q)^2}{L_q} \right] \Delta i_q$

NPTEL A.K. Sinha 25/26

Same effect can be seen for the quadrature-axis winding also, where we will get L_q double dash term as shown here. So, we see that, the effect of damper winding is to initially create inductances which are much smaller. And therefore, larger currents which die out quickly and then the transient situation comes which is due to the current flowing in the field winding. That also decays after sometime and finally, we have the steady state current flowing. So, this completes our modeling of synchronous machine. In this we 2 lectures or 2 lessons 13 and 14, we have developed the model for round rotor machine and salient pole synchronous machine also.

Thank you.