Power System Analysis
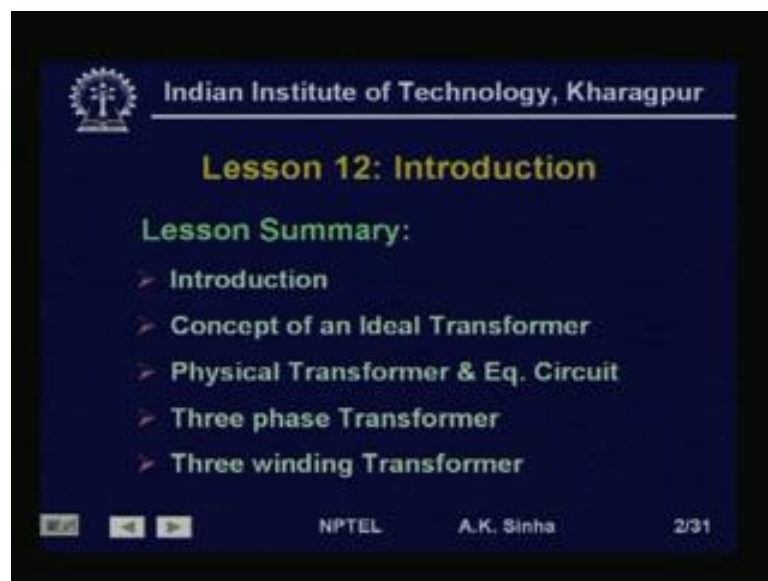**Prof. A.K. Sinha**
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Lecture - 12
Transformer Model

Welcome to lesson 12 on Power System Analysis. And in this lesson we will discuss Modeling of Transformers.

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Well, this lesson we will start with introduction of transformers then concept of an ideal transformer. Then we will go into physical transformers and their equivalent circuit. After that we will talk about 3 phase transformers and 3 winding transformers.

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Well, the main objective of this lesson is to explain the concept of an ideal transformer. That is once you are through this lesson then you should be able to explain the concept of ideal transformer, develop the equation or the equivalent circuits models for single phase transformers. Then 3 phase transformers and 3 winding transformers. Well we all know that transformers are an integral part of any electrical network, as you can see from this diagram.

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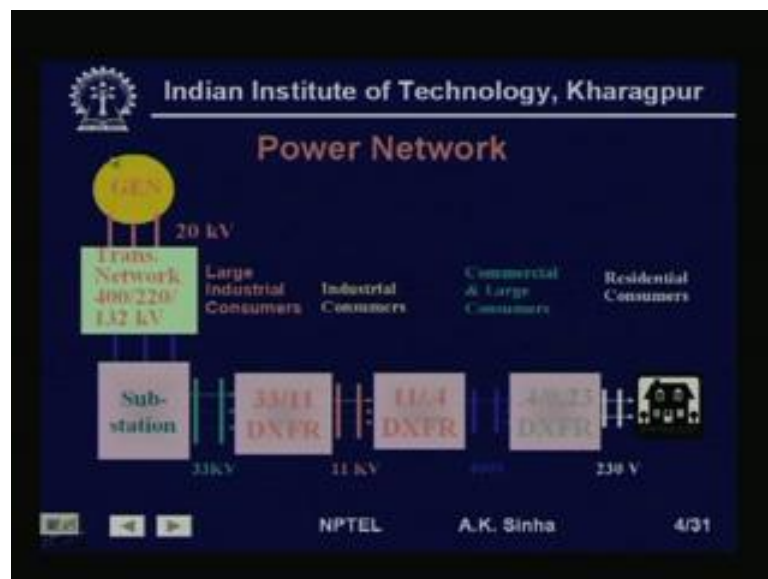
That we have generation from generators, which is normally in the voltage range of 11 to 30 k V and then we connected to a transmission network, which may be working at

levels of 400 to 20 or 132 k V. And then again this goes into sub stations from where we distribute power at lower voltages may be at 33 k V 11 k V or 3 phase 400 volts. So, at all these at different levels, we see we need different voltage levels. And for changing these voltage levels we require these transformers.

Therefore transformers are an integral part of any power system network and we have very large number of transformers in any electrical network. Therefore it is necessary for us for doing a proper analysis to make a good model of the transformer in our analysis. Now, what are transformers? Transformers are basically a static electrical machine, which has a magnetic core on which windings are placed. Now, this magnetic core provides the path for the fluxes as you can see in this diagram.

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We have a magnetic core, a rectangular magnetic core which has two windows cut in it and we have placed the windings on the central limb of this core. Now, this whenever we energize this winding with an alternating voltage, we are going to setup an alternating current in these windings, which will produce the flux, which is an alternating flux and this core provides a path for the alternating flux.

Therefore, the core has to be of a magnetic material. Normally, we use steel and specially the steel which we call cold roll blade oriented silicon steel, because it provides lesser losses. Now, this type of a transformer which we have shown here where we have put the windings on the central limb, and we have the flux parts from both sides, this type of a transformer we call a shell type transformer.
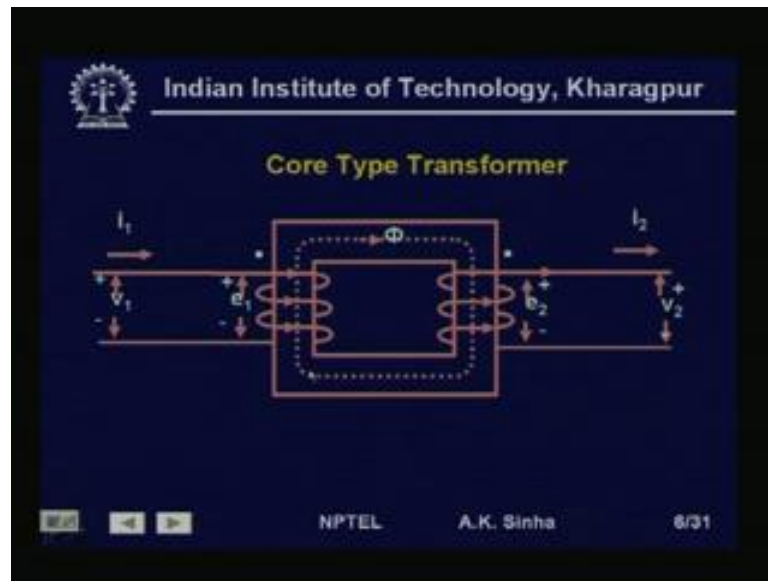
Another type of transformer, which we call core type transformer will have the windings on the two extreme limbs and the flux part is provided by this core as shown here. Now, how does this transformer work, before we model the transformer, we must know how it is working. And therefore, we will see how it is going to work. Now, as if we see this transformer we have two windings placed on this, on the two limbs of this transformer.

Now, once we energize one of the winding say this winding one, with a voltage v 1 what happens is a current will start flowing in this winding. This current will also be an alternating current and this will setup an alternating flux in the core. Now, this flux since it is alternating or time varying it is going to cut this winding also. So, number of turns of this winding and this is going to therefore, produce a voltage.

As per Lenz's law, the voltage which is induced by this flux cutting of this flux lines will be opposing this voltage v 1. So, we will get a voltage even here which will be very nearly equal to v 1. And the flux which is setup here is also going to cut the number of turns of the winding on this other limb. And therefore, it is going to setup a voltage e 2 on this winding also that is across this winding also, and therefore a voltage v 2 at the two terminals of this winding.

This is what how the transformers operates, but in order to get into details of this. What we will do is we will first try to understand the operation of this transformer assuming it to be an ideal machine. And later we will see how we can model the physical transformer where which is not really an ideal one.

So, first we will get into a concept of ideal transformer. Now, what is an ideal transformer? It is basically a transformer where we make certain assumptions, this we say as the transformer core has infinite permeability. What do we mean by this? We basically mean that zero mmf is required to create magnetic flux in the core. That is it is a negligible current which will setup the magnetic flux in the core or almost a zero current, which is going to setup a magnetic flux in the core, which simply means that the core reluctance is zero, it is permeability is infinite.

Another aspect of this is that infinite permeability is that all the flux will be confined to the core itself, there would not be any flux which will be flowing through the outside air. Because the reluctance offered by the core is zero and the reluctance offered by outside air is going to be much higher as compared to this. Therefore, all the core all the flux will be confined to the core itself.

We make another assumption that transformer winding has zero resistance, this basically means that there is no power loss in the winding. That is I square R loss is taking place in the winding is neglected. We also mean by this that there is going to be no ohmic voltage drop. That is I R drop in the winding is also zero, there is going to be no voltage drop in the winding, because of the current flowing through it.

We make another assumption that transformer core has zero core loss. Now, what does this mean, because we know that if we have a alternating flux setup in any magnetic material. We are going to have losses magnetic losses which will be in terms of

hysteresis loss and eddy current loss for an ideal transformer. We assume that these losses are zero. So, with these assumptions we get into an ideal transformer and to understand the basic operation of a transformer. So, let us see if we see this diagram.

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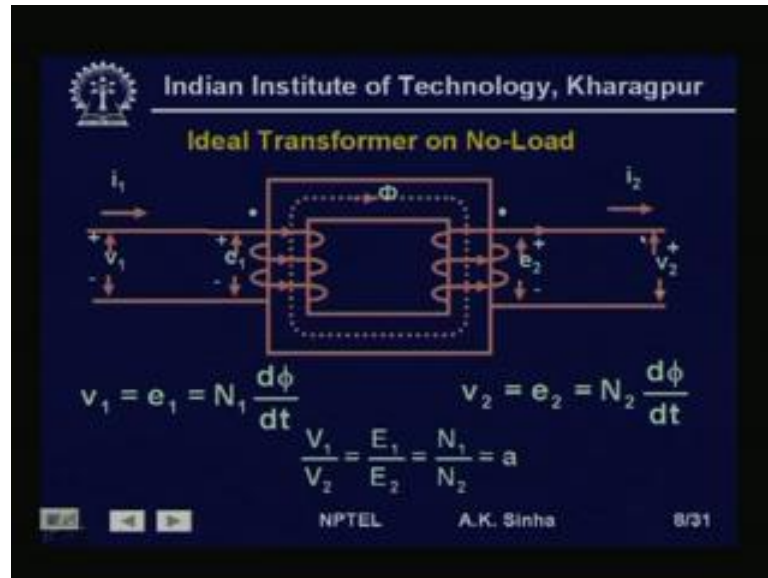
We have a core which we assume to have infinite permeability on, which we have a winding on one limb and a winding on the other limb. Now, these windings are assumed to have zero resistance and we also assume that the core has no losses. Now, generally the two windings are termed as primary winding on which side we put the source. And the other winding where we are going to put the load is called a secondary winding.

Though in a network where power can flow in reverse direction also the concept of primary and secondary winding does not really work. What we need to use in that case is high voltage winding and low voltage winding. So, because you are going to have one side at a high voltage, another side at a low voltage. So, that is the term which is more appropriate to use though for our analysis here. We will term we can term them as this side that is side one as a primary, where we are injecting a source voltage and the other side we can call them as secondary.

There can be this side can be a low voltage side, this side can be a high voltage side. Then we call this transformer as a step up transformer. If this side is at higher voltage than this side then we call it as a low voltage a step down transformer.

So, now we see what how does this transformer work. So, we have connected a alternating voltage source with a voltage V 1 to this primary side or side one winding.

What is going to happen is as we said earlier almost with negligible amount of current flowing through this winding. We will have a flux setup, flux phi setup in the core this flux will be an alternating flux. And this will be cutting the turns of this winding, which will induce a voltage e 1 in this winding.

This e 1 as per Lenz's law will be opposing this voltage V 1. So, the polarities will be as marked here that is this will be positive this will be negative. And so in now what will be the magnitude of this e 1 or what will be this voltage induced voltage e 1. V 1 is the applied voltage which will be well which will be equal to e 1. Because we have assumed there is going to there is no losses or no voltage drop which is taking place. Therefore, V 1 is equal to e 1 and what will be the value of e 1; e 1 will be equal to d lambda by d t that is the flux linkage, the derivative of flux linkage with time, which we can write as N 1 into d phi by d t. So, we have e 1 equal to N 1 into d phi by d t. Now, this flux phi which is a time varying flux is also linking this coil, we have assumed zero permeability. So, all the flux is confined to this core. Therefore, all the flux setup by this finding is also linking this winding. Therefore, we have a voltage induced in across this winding, which will be equal to e 2. And this e 2 will be equal to N 2 into d phi by d t again the rate of change of flux, which is cutting this winding where N 2 is the number of turns of this winding.

So, and since there is no voltage drop, so across the terminals of this winding we will get voltage V 2, which will be equal to e 2. Now, looking at these two relationships, we can write that V 1 by V 2 is equal to E 1 by E 2, which will be equal to N 1 by N 2, which we call equal to a; where a is called the turns ratio, between the primary and the secondary turns of the winding of a transformer.

This is a very important relationships, here we have used capital V 1, V 2 and E 1, E 2 to indicate that if we are using sinusoidal voltage sources then these are our phases. So, the phasor relationship will also be similar that is V 1 by V 2 is equal to N 1 by N 2 is equal to a the turns ratio of the transformer.

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Now, the instantaneous voltage V 1, if we are using a sinusoidal voltage source can be written as equal to root 2 times V 1, where V 1 is the magnitude of the peak value, magnitude of the phasor and sine omega t. And d phi by d t, since we have seen that V 1 is equal to N 1 d phi by d t. Therefore, we can write d phi by d t is equal to V 1 by N 1; where V 1 is given by this relationship. So, we get d phi by d t is equal to root 2 by N 1 into V 1 sine omega t integrating this we will get phi is equal to root 2 by omega N 1 into V 1 cos omega t.

Which we can also write as root 2 by omega N 1 into V 1 sine omega t minus pi by 2. What does this equation tell us? This tells us that the flux, which is produced by this voltage that we have applied is going to lag the voltage by 90 degrees. Because, here we see it is sine omega t minus pi by 2. So, flux is lagging the voltage which is creating this flux in the core by 90 degrees. So, flux lags the voltage by 90 degree also the current which flows in the winding to create this flux is going to be in phase with the flux. Therefore, the current will also be lagging the voltage by 90 degree.

Now, what is the maximum value of the flux in the core, this will be equal to root 2 by omega N 1 into V 1, because when sine omega t minus pi by 2 is equal to 1 that will give me the maximum value of the flux. So, phi max is equal to root 2 by omega N 1 into V 1.

From this we can write V 1 by N 1 which is nothing but voltage per unit or per turn for the winding. So, dividing V 1 by N 1 means each turn is having how much voltage across it or is inducing or how much voltage across it.

So, V 1 by N 1 is equal to omega into phi max by root 2. From here from this equation we can write that V 1 by N 1 is equal to omega goes on this side and root two comes below. So, omega phi max by root 2 this is equal to twice pi f by root 2 into phi max can be written as A into B max where A is the area of the cross section of the core. So, we have omega is equal to twice pi f. So, V 1 by N 1 is equal to twice pi f by root 2 into A into B max, which is equal to 4.44 f A B max, which tells us that the voltage induced per turn on any transformer is proportional to frequency, the area of cross section and the maximum flux density allowed in the core material.

This is a very important relationship for designing a transformer, because this is going to tell what should be the volume of the core material. Now, we will see if we connect a load on the second winding or the secondary winding of the transformer. What is going to happen?

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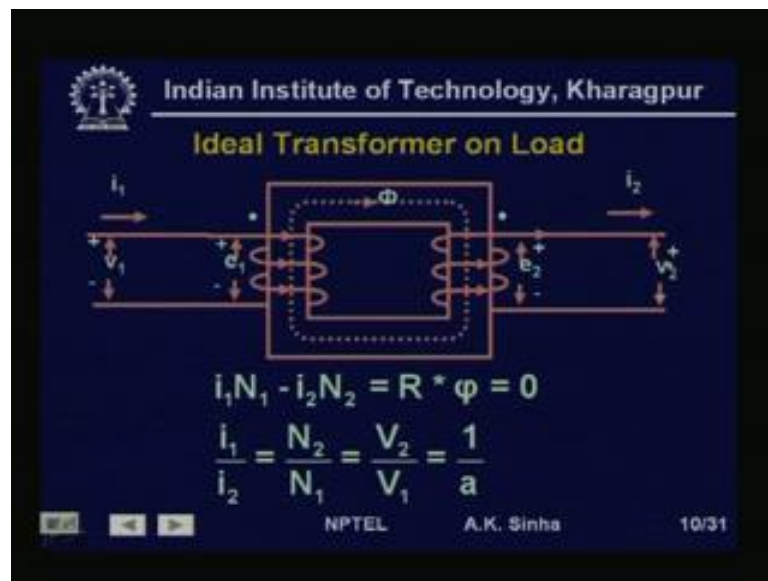
We had seen that if we had applied this alternating voltage here, we had an induced voltage e 1, which was very nearly equal to or equal to V 1 coming across this winding 1 and a voltage e 2 was induced across the winding 2, which was equal to V 2 the terminal voltage across the winding.

Now, if we connect a load here, across these two terminals what will happen? Since there is a voltage V 2 here there is going to be a current i 2 flowing into the load, where i 2 will be equal to V 2 by Z. The Z is the impedance of the node. So, we are going to get this current i 2. Now, what is going to happen because of this current i 2.

This current i 2, when it flows through this winding is going to produce a flux, which will again be an alternating flux which will be as we see from the right hand rule. This flux will be opposing the flux set up by the first winding. That is if we see this thing as the current direction with our fingers and the thumb direction will give us the flux direction 5. So, if we see this direction of the flux due to the current flowing in winding 2 will be opposing the flux setup by the first winding.

Now, as we have seen, the voltage induced here is equal to N 1 into d phi by d t. Now, if the flux produced by this winding is opposing the flux produced by this the resultant flux in the core is going to reduce. If this resultant flux reduces the voltage here is also going to reduce, because it is equal to N into d phi where phi is the resultant flux in the core.

Now, that cannot happen because this e 1 will be equal to V 1, because we have assumed no voltage drop here. So, this voltage must be equal to this V 1, which can come only if we have the same amount of flux prod in the core. For neutralizing the opposing flux produced by the secondary winding, what will happen is. We will have a reflected current i 1 flowing in the first winding which will produce an mmf equal to i 1 N 1 which will be exactly neutralizing the flux produced by the mmf i 2 N 2.

Therefore under conditions of load put on the secondary side we have i 1 N 1 minus i 2 N 2 this will be equal to R into phi where we have assumed reluctance to be zero. So, this is a zero which means i 1 N 1 must be equal to i 2 N 2. That is if a current starts flowing in the secondary winding, there is going to be a current which will flow in primary winding. And which will produce a mmf which will be exactly equal and opposite to the mmf produced by the current flowing in the secondary winding.

So, whenever a current flows in the secondary winding, we have a current also flowing in the primary winding producing an mmf i 1 N 1, which will be exactly equal to the mmf produced by the current flowing in the secondary winding that is i 2 N 2. Therefore from this relationship we get i 1 by i 2 is equal to N 2 by N 1, which is equal to V 2 by V 1 and which is equal to 1 by a where a is the turns ratio of the transformer a is equal to N 1 by N 2.

So, this is how an ideal transformer works, in case when it is loaded. Another important aspect of a transformer is that it does not only transform the voltage or the current levels. And the two sides of the transformer it also provides a transformer of at are

transformation of the impedance level on the two sides of the transformer. Now, we can see this from this diagram.

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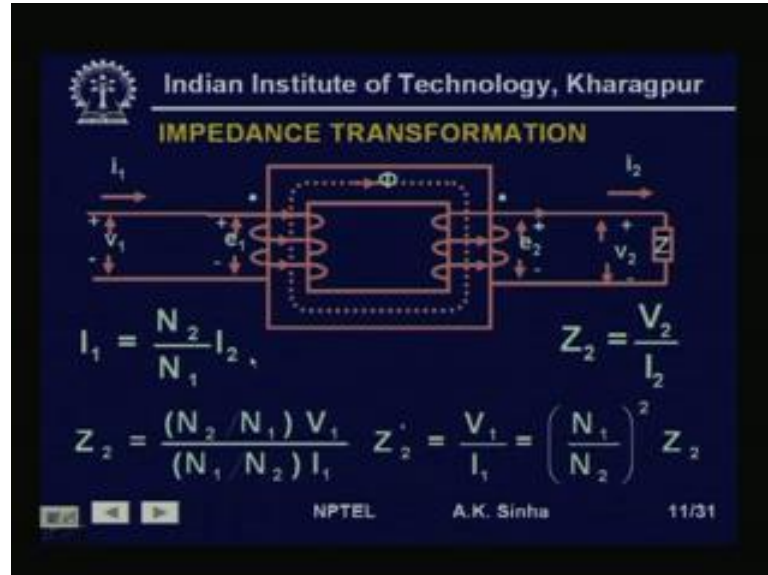
That if we have a voltage V 1 applied across this winding 1 and we have a voltage V 2 across this terminal of the winding 2. And we have put a load Z across this second winding terminals, then we have a current i 2 equal to V 2 by Z flowing in this second winding. Now, when this is flowing in this winding we will have a current i 1 flowing in the primary winding producing i 1 N 1 equal to i 2 N 2. Therefore, we have i 1 is equal to N 2 by N 1 into i 2 and on if we look at this side we have Z 2 is equal to V 2 by i 2.

Now, we can write this as since we know V 2 V 1 by V 2 is equal to N 1 by N 2 or V 2 by V 1 is equal to N 2 by N 1. Therefore, we can write V 2 is equal to N 2 by N 1 into V 1. And for i 2 we can write that i 2 is equal to N 1 by N 2 into i 1. Therefore we can write that Z 2 dash which is the impedance as seen from this side. Impedance seen from this side will be equal to how much V 1 by i 1. So, Z 2 dash is equal to V 1 by i 1. And this will be equal to how much if we see this V 1 by i 1 on this side.

Then this goes in the numerator and this term goes in the denominator. If we do that then this we find that this is equal to N 1 by N 2 square into Z 2. That is if Z 2 is the impedance connected on the secondary side. Then this impedance is seen by the primary side as equal to N 1 by N 2 square into Z 2 that is it is equal to a square Z 2, where a is the turns ratio.

So, if a is more than one, that is it is a step down transformer, then we will find that the impedance a small impedance on this side can be seen as a large impedance on this side. Sometimes this is used a for impedance matching in many circuits. So, impedance transformer transformation is another aspect, which is used in this transformers?

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Now, we have seen that V 1 by V 2 is equal to N 1 by N 2 and i 1 by i 2 is equal to N 2 by N 1. And if we see in these transformers we have assumed no losses in the core as well as no losses in the winding. So, the power on the two side have to be equal, because there is no power which is lost in the windings or in the core.

Now, the power instantaneous power p 1 on the primary side will be equal to v 1 i 1 instantaneous power p 2 on the secondary side will be equal to v 2 into i 2. Since these two power must be equal as there is no losses. So, v 1 i 1 must be equal to v 2 i 2, if you write in terms of Phasors for a sinusoidal voltage sources. Then we have V 1 i 1 conjugate is equal to V 2 i 2 conjugate where V 1 i 1 conjugate is e is called the complex power p plus j 2. And V 2 i 2 conjugate is the complex power on the secondary side. So, we have the complex powers on the two sides equal to each other.

So, this is what we have learned about the ideal transformer operation. Of course the transformers that we use in our system are certainly not ideal transformers. They are made up of components, which are physically available and these components do not have these ideal characteristics. Therefore, we will try to model the transformers a

physical transformer by trying to introduce these physical characteristics of these transformers or transformer material.

Now, first we will take that we know that the iron core that we use does not have infinite permeability, but it has finite permeability. What does it mean? It means that the to set up any flux in the core, we require certain amount of current. That is i 1 N 1 will be required or an mmf will be required to setup a flux in the core because the core has finite amount of reluctance. That is mmf is equal to reluctance into flux and since reluctance is not zero. Therefore, to setup the flux we need a finite amount of i 1 N 1. That is finite amount of current is required to flow in the winding to setup the flux.

Then we will also we also know that whenever there is a alternating flux in the core, we are going to have losses in the core in terms of hysteresis and eddy current losses, we will take care of those also. No winding is having material which has no resistance that is all winding material mostly we use copper and for that has certain amount of resistance. Therefore, there is going to be losses in the winding, because of this resistance also there is going to be a voltage drop in the winding.

Then we will also take care of since we have seen that the core material is not infinite. Therefore, we the whole flux phi may not be confined to this core itself, there may be some amount of core, which will a certain amount of core may be flowing across this path. That is it will not link both the windings and there that kind of flux we call as the leakage flux. Therefore we will take care of the leakage flux also. So, we will build the model for a physical transformer.
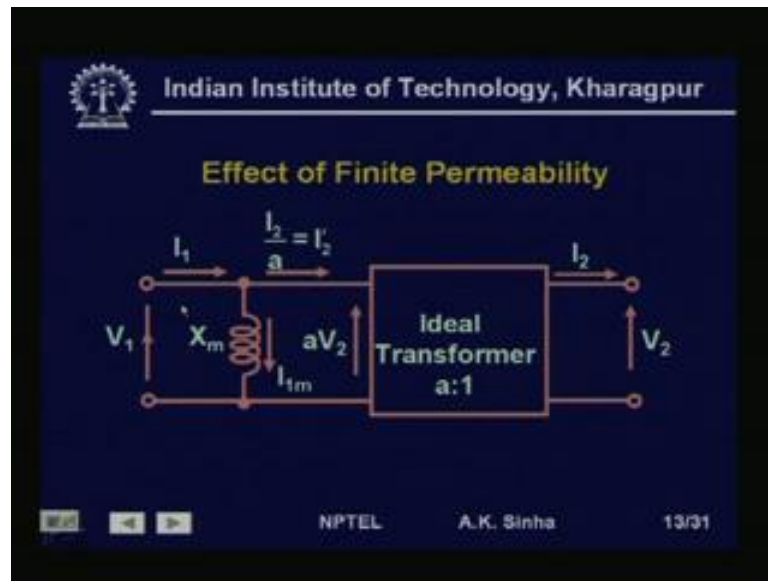
Now, first we will take the effect of finite permeability. What does it say? It says that since the permeability is finite which means we need a certain amount of mmf to produce the flux. Now, we have seen that the current which produces the flux is going to be at 90 degrees to the voltage which is applied, because the flux lags the voltage by 90 degree. And therefore, the current which is producing this flux, since it will be in phase with the flux will also lag the voltage by 90 degrees.

Now, how do we represent this current which is required to produce a flux. We need this current to lag the applied voltage by 90 degree and we know the electrical parameter, which provides this kind of a relationship is an inductance. Therefore, we provide an inductor here, which a reactance X m to represent the finite permeability or the magnetizing part of the current to flow in the circuit.

So, if we have a voltage V 1 applied and we have a reactance X m representing this magnetization or the finite permeability. Then we will have a current I 1 m flowing through this. And this I 1 m will be lagging the voltage by 90 degrees and this is what we want for to represent the magnetizing part of the transformer.

So, I 1 m will lag this voltage V 1 and this will be represented by a reactance X m. Now, since this I 1 m flows here. So, the current which will flow here will be I 1 minus I 1 m or I one the current flowing will be equal to I 2 dash plus I 1 m, where I 2 dash will be equal to I 2 by a on this side. If I 2 is flowing in the secondary side, then on the primary

side the reflected current will be I 2 by a, which we write as I 2 dash. So, I 1 will be equal to I 1 m plus I 2 dash in this case.

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$$I_{1m} = \frac{\Re\varphi}{N_1} = \frac{\Re}{N_1} \frac{\sqrt{2}\,|V_1|}{\omega N_1} Sin(\omega t - \frac{\pi}{2})$$

$$|I_{1m}| = \frac{\Re\,|V_1|}{\omega N_1^2}$$

$$I_{1m} = \frac{\Re V_1}{j\omega N_1^2} = \frac{V_1}{jX_m}; \quad X_m \triangleq \frac{\omega N_1^2}{\Re} = \frac{1}{B_m}$$

Now, what will be the value of I 1 m. I 1 m will be equal to since we know that the flux is going to be produced by the mmf I 1 m into N 1 and this I 1 m into N 1 is equal to reluctance of the core material into phi. Therefore, we can write I 1 m is equal to R into phi divided by N 1, which is equal to R by N 1 into phi will be as we have seen earlier can be represented by root 2 V 1 by omega N 1 sin omega t minus pi by 2.

Therefore the magnitude value of I 1 m will be equal to R into V 1 by omega N 1 square. And since we know that this current is going to lag the voltage by 90 degree, we can write the phasor as I 1 m is equal to R by V 1 into j omega N 1 square where this one by j is indicating a lag of 90 degrees, which is equal to V 1 by j X m that is what we will see here the I 1 m is equal to V 1 by j X m. So, X m therefore, can be defined as omega N 1 square by reluctance R this we can also write as 1 by B m, where B is the susceptance and X is the reactance.

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$$I_1 = I_{1m} = \frac{V_1}{jX_m} \qquad [A]$$

$$P_1 = 0 \qquad [W]$$

$$Q_1 = \frac{|V_1|^2}{X_m} \qquad [VAr]$$

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Now, when the transformer is not loaded then we have I 1 is equal to I 1 m this is equal to V 1 by j X m. And since there is no current which is flowing in the secondary side and we have assumed no losses in the system P 1 will be equal to 0 and Q 1 which is the reactive power will be equal to V 1 square by X m.

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Effect of Core Loss

Hysteresis Loss

Eddy Current Loss

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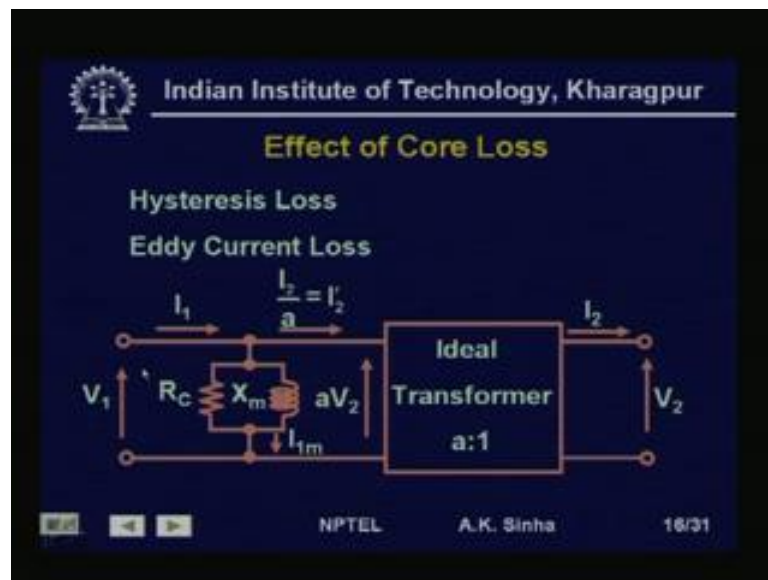
Now, we will take care of the core losses. Now, we say that core losses have two parts one is hysteresis another eddy current. So, we you can take care of both this losses by providing an element of resistance R c. So, if we put a resistance R c in parallel with this X m that will provide us the model for core losses, because here what will happen is V 1

by R c will provide a current flowing through this which is we call as I c. This I c into I c square into R c will be equal to the core losses of the transformer.

Now, why we put this in parallel here; that is or across voltage V 1 is that once we energize this transformer a flux is setup and core losses are going to be present, whether we have any load on the transformer or not. So, even for unloaded transformer since a flux is setup this alternating flux is going to create losses. And therefore, these losses have to be taken care of. So, as soon as you apply a voltage here you will have a current flowing through this and therefore, I square R c losses will be coming also.

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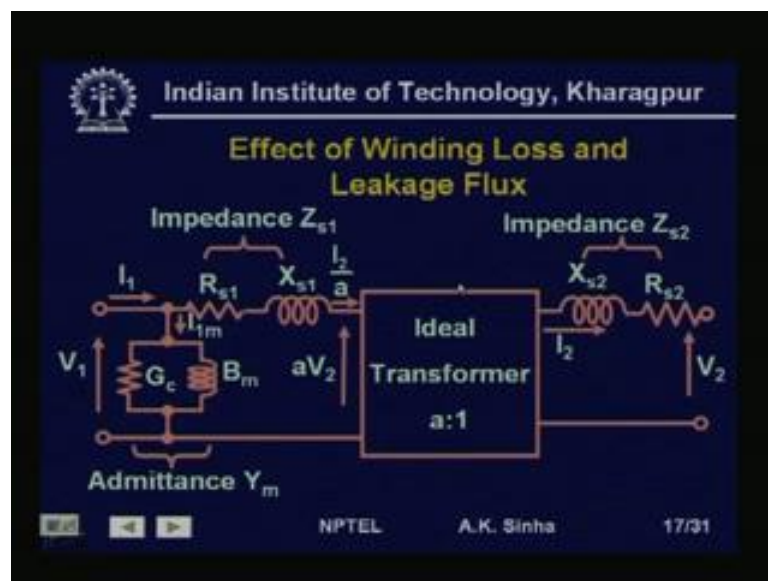
Now, we will take care of the effect of winding loss and leakage flux. Now, for winding losses, since we know that each winding is made up of copper or aluminum which has finite resistance. So, the winding resistance can be used to represent the losses in the winding. Like on the primary side of the winding we have the resistance of the winding as R s 1 this will be put here to represent the losses in the primary's winding. Similarly on the secondary side the winding will have resistance R s 2. And so this R s 2 represents the losses on the secondary side winding.

Now, as we said certain amount of flux may not be linking both the windings, these flux we call as the leakage flux. That is some flux produced by winding 1 may not be linking the second winding because the flux path can be through the air also. It may not be all the flux may not be confined to the core. Therefore, in that case that part of the flux will not be linking the other winding and this flux we call as the leakage flux.

And the leakage flux can be represented by means of a reactance. So, X_{s1} here the reactance is representing the leakage flux of primary winding. Similarly the current flowing in on the secondary side due to the load will also produce a flux. And all this flux may not link the primary side of the winding. That is some will leak through the air path and therefore, this is represented by X_{s2} here, these will be in series with the resistance of the winding, because these are going to be proportional to the current flowing in the winding.

So, we have the complete transformer model here; where we have represented the finite permeability by this B_m the core losses by this G_c. G_c is the conductance which is 1 by R. So, 1 by R_c is this G_c since these two arms are in parallel it will be easier to work with admittances that is conductance and susceptance rather resistance and reactance. Therefore, most of the time we write them in terms of conductance and susceptance conductance, G_c representing the core loss part susceptance B_m representing the magnetization magnetizing current path.

Rs represents the resistance of the primary winding X_{s1} represents the leakage reactance of the primary winding X_{s2}. The leakage reactance of the secondary winding R_{s2} is the resistance of the secondary winding. And we can generally combine this two as the impedance Z_{s1} of the primary winding and impedance Z_{s2} of the secondary winding where Z_{s1} is equal to R_{s1} plus $j X_{s1}$ and Z_{s2} is R_{s2} plus $j_s X_2$.

The current here on the primary side at the terminal will be I_1 the current in this winding will be I_1 minus I_{1m} and which will be equal to I_2 dash I_2 dash. And that will be equal to I_2 by a. So, I_2 by a current will flow in this winding and I_2 current will flow in this winding depending on the load on the secondary side of the transformer.
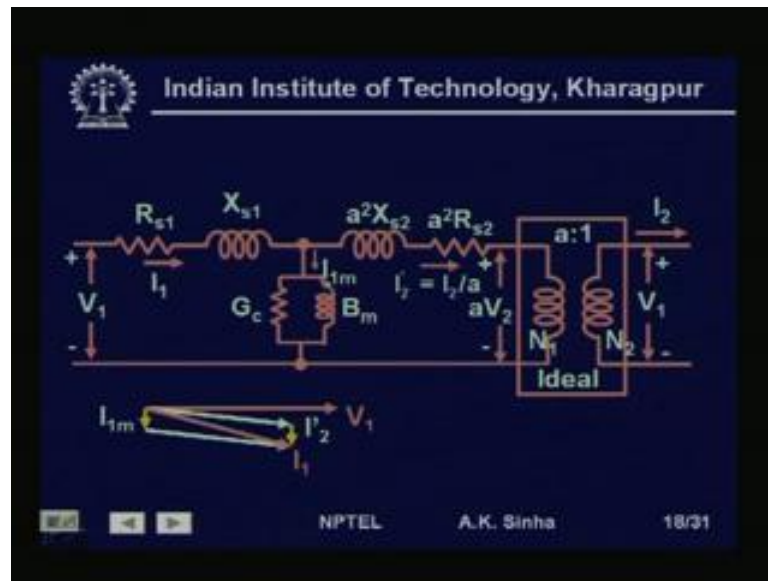
Now, as we have seen earlier we can transform the impedance from one side to the other side. Therefore, what secondary side impedance what we have done is transform them on the primary side or brought them to the primary side or we are looking at all the impedance from the primary side. In that case the secondary side impedances X s 2 becomes a square X s 2 and R s 2 becomes a square Rs two on the primary side.

So, this is now complete circuit of equivalent circuit of a transformer with an ideal transformer placed here. Now, there is another aspect, which we can use to further simplify this circuit. Now, we know this current I 1 m will be lagging the voltage applied by 90 degrees. Therefore, we almost by 90 degrees, because this current the losses in the core is not very large. So, this current being 90 degree this current being in phase with V one the actual current will be lagging the voltage by an angle which will be less than 90 degree, but very near to ninety degree, so we are representing this current like this.

And since this current I 1 is going to depend on the load current the loads will have a power factor very nearly equal to 8.85 0.9 or so. So, this current I 2 dash here which is a reflected current for I 2 will be very nearly in phase with voltage V 1. And therefore, the current I one is going to be the Phasor sum of this I 2 dash and I 1 m is also going to be very much nearly equal to I 2 dash here. Because this is very small the losses or the magnetizing current for a transformer or a unloaded transformer takes only about 3 to 5 percent of the full load current.

So, this is almost 1 by 30 or 1 by 20 of this therefore, we find this I 1 here will be very nearly equal to I 2 and very much in phase with I 2 dash. Therefore what we can do is we can shift this from here to this side itself, because I 1 and I 2 dash are very nearly equal then our equivalence circuit becomes like this, because we can now combine R s 1 and R s 2 and X s 1 and X s 2.

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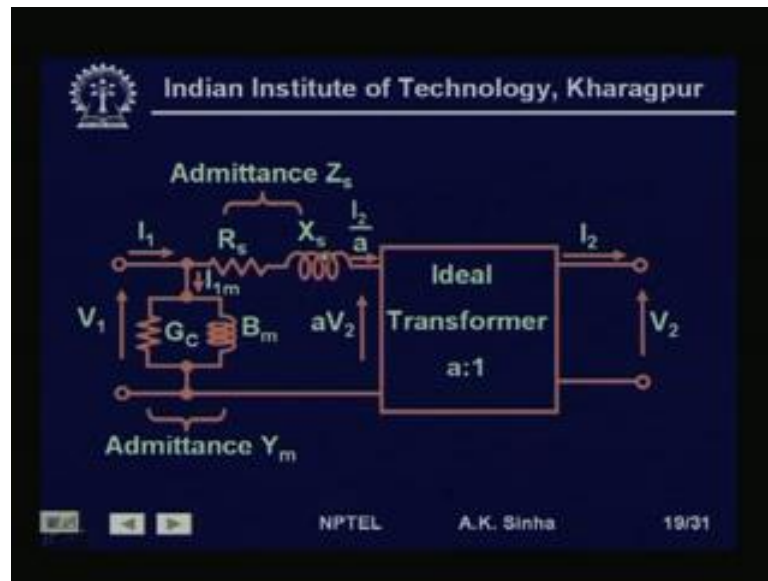
So, we can write them as Rs plus j X s, where R s is equal to R s 1 plus R s 2 and X s is equal to X s 1 plus X s 2. So, this is now the equivalent circuit, which represents the magnetization current, the core losses, the winding losses and the leakage flux. So, this is an equivalent circuit for a physical transformer.

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So, as we said the impedance winding and the leakage part is taken care of by Z s, Z s is equal to R s plus j X s and Y m is the admittance of the parallel path, which is equal to G c or 1 by R m plus 1 by j X m R s is equal to R s 1 plus a square R s 2 and X s is equal to X s 1 plus a square X s 2.

And for any physical transformer the value of R s is very, very small compared to R m, because core losses are very small. So, they are represented by a very large resistance because the current flowing through them is very small it is V 1 by R m and this current is very small. So, R m will be much larger than R s. Similarly, X s is much smaller then X m and also X s is much greater than R s for any physical transformer.
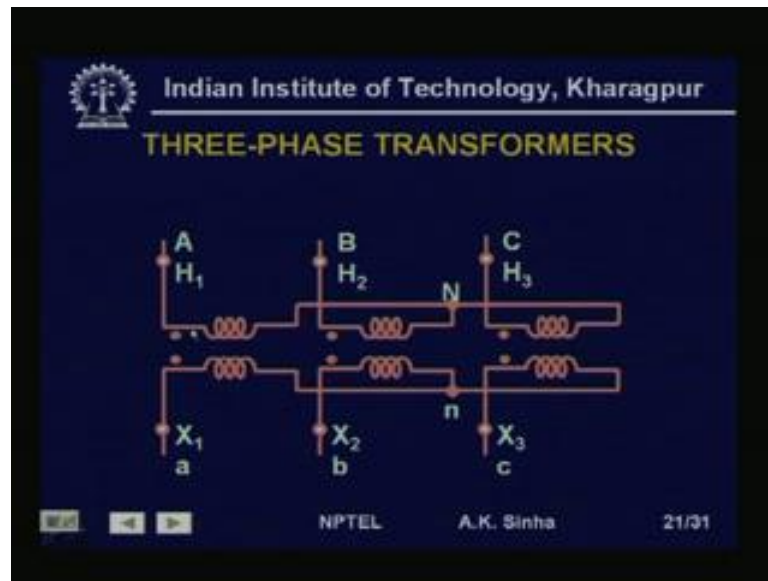
Now, we will go into 3 phase transformers. Now, three phase transformers are basically built up by connecting three single phase transformers or we can have three sets of windings on the same core or a single core with 5 limbs or 3 limbs. So, we can build three phase transformers.

Now, 3 phase transformers have another characteristics, because we have three windings we can connect them in star or in delta. So, we have different combinations of windings, the primary side can be star, secondary side can also be star. The primary side can be delta, secondary side can be star, the primary side can be star, secondary side can be delta and so on. So, we have various combinations which are available. Now, here we have shown how three windings can be connected to create a star-star transformer, which will look like this.
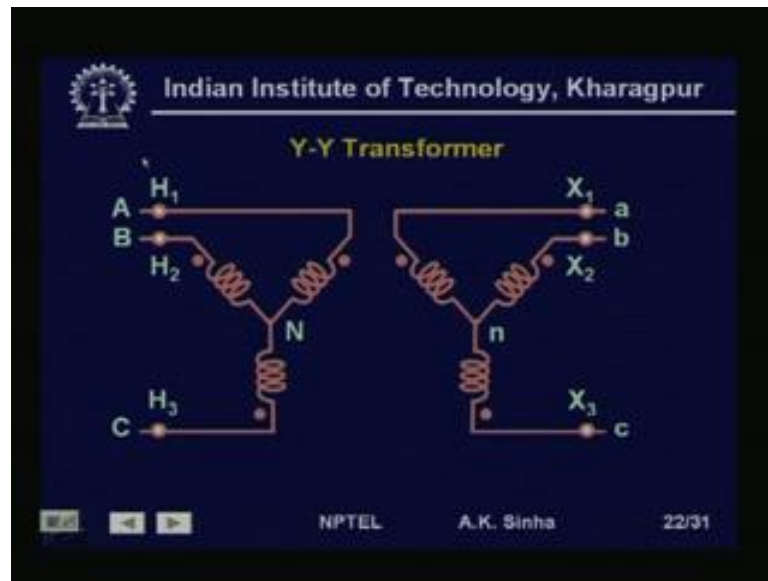
Now, here the high voltage windings we are representing by H. So, H 1 H 2 H 3 represents the three terminals of the high voltage winding to which phase A, B and C are connected. Similarly, X 1, X 2, X 3 represents the three terminals of the low voltage winding to which a, b and c phase are connected. The small case letters indicates low voltage, the bold case letters indicates the high voltage this is a star-star transformer, which can be created like this.

Now, since the equivalent circuit of single phase transformer, and 3 phase transformers will be exactly same as 3 phase transformers are built using 3 single phase transformer or you use with a core with three links on which the three phase windings are placed. So, here again also you have core losses you have finite permeability, you have winding resistance and leakage flux. So, they can be represented in the same way. So, the transformer 3 phase transformers are very similar or almost same as equivalent circuit though. So, except for one difference that we find and that difference is...
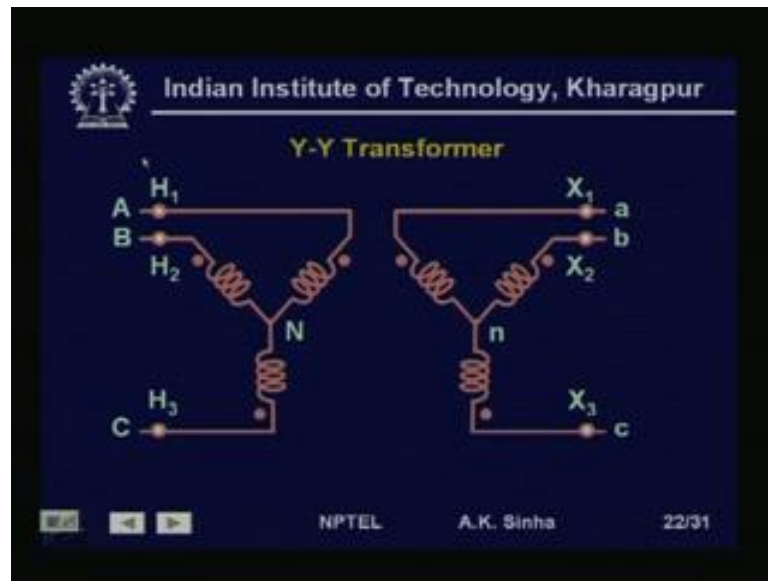
If we have a star delta transformer then if you see the phase voltages of these, for say this transformer the phase voltage A is in this direction, whereas the phase voltage A will be in this direction as indicated here. So, V a is in this direction and V A capital A that is the high voltage side voltage is in this direction as shown here, there is going to be difference of 30 degree in the phase angle, whenever we have a star delta transformer. This needs to be taken care of in the equivalent circuit.
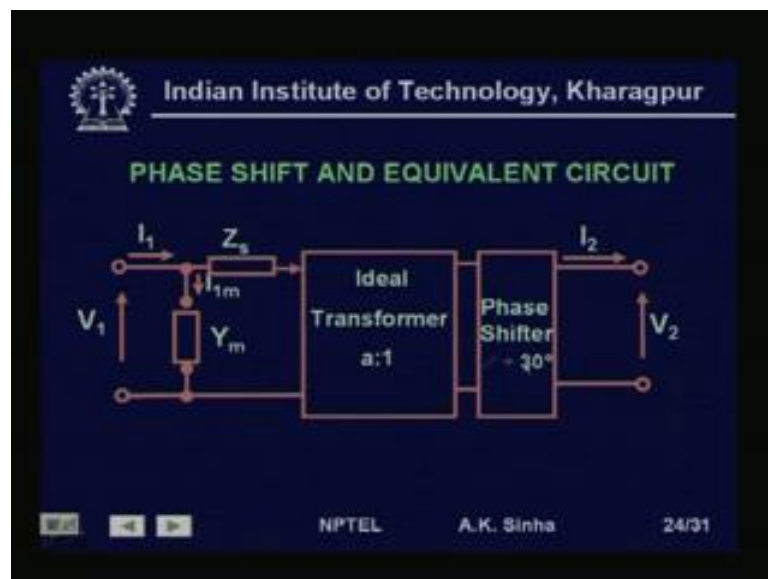
This we do by using the same kind of an equivalent circuit that we have built earlier where we have the admittance Y m in parallel, which is indicating the core losses and the

magnetization current and impedance Zs in series which is representing the resistance and the leakage flux of the windings. The ideal transformer and we have a phase shifting introduced. This phase shifting will be plus or minus 30 degree depending on which side is delta and which side is star.

So, the equivalent circuit of a 3 phase transformer is very similar to that single phase transformer. In fact, for all analysis of 3 phase transformers what we do is generally we work with a single phase or a Phasor values of voltage and current, because we most of the time work with balance circuits. And therefore, each of the three phases will have the same voltages and current except that they will be one 20 degree out of phase from each other.

There is another kind of transformer which we use in power systems which we call 3 winding transformers. Now, instead of the two windings that we have seen a primary and a secondary winding or a high voltage low voltage winding on a normal transformer, these transformers have three windings. This is generally used in certain places where we need two different voltage levels or three different voltage levels.

Say you have a power coming from say in a power station you have the generator voltage at say 20 k V and you are connecting it to the transmission site. So, you have say 400 k V transmission line then the voltage on the other side is 400 k B. And for the auxiliaries of the transform of the generators we need since you have feed, water feed water pumps you have compressors you have. So, many equipment in the power plant which require power supply and this is provided by means of another winding which is at a much lower voltage of say 3.3 k V or 6.6 k V, which is used for supplying the power to the auxiliaries of the power plant.

So, also many places we use devices which are used for reactive power compensation and other things. Now, these devices can also be connected at the much lower voltage because the cost of devices at lower voltage is much lower than that at high voltages. So, many places we need another voltage level there we use this three winding transformers. The third winding is normally called a tertiary winding.

There is another advantage with this three winding transformer. That is if you have unbalanced loading, then if you have a star-delta transformer. Then in that case the on the delta side the zero sequence current can flow, because it will appear as a closed circuit for the 0 sequence current which are basically a third harmonic current.

But if you have a star-star transformer then this zero sequence currents cannot flow in that and this creates a shifting of the neutral. So, many times to provide path for these zero sequence currents in the transformer, we provide this tertiary winding which is normally connected in delta and this provides the path for that.

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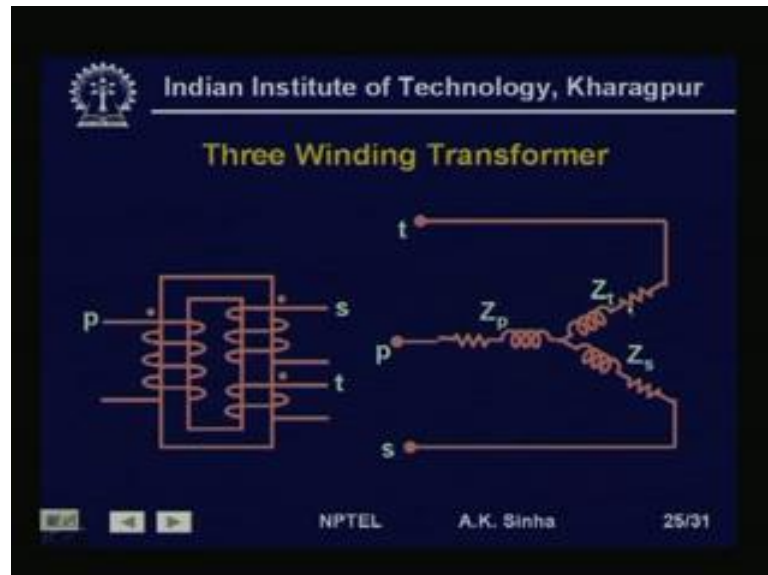
Now, here we are showing these three windings Z_p is the primary, Z_{cs} is the tertiary and Z_s is the secondary and Z_t is the tertiary winding.
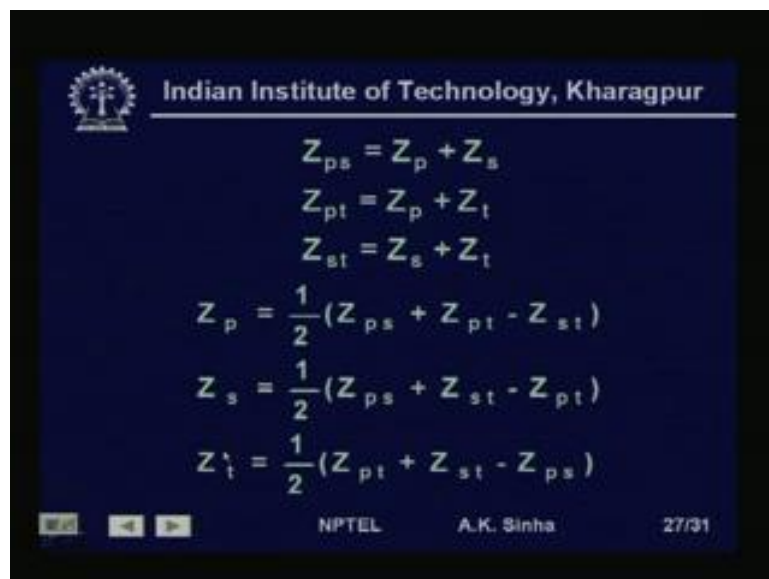
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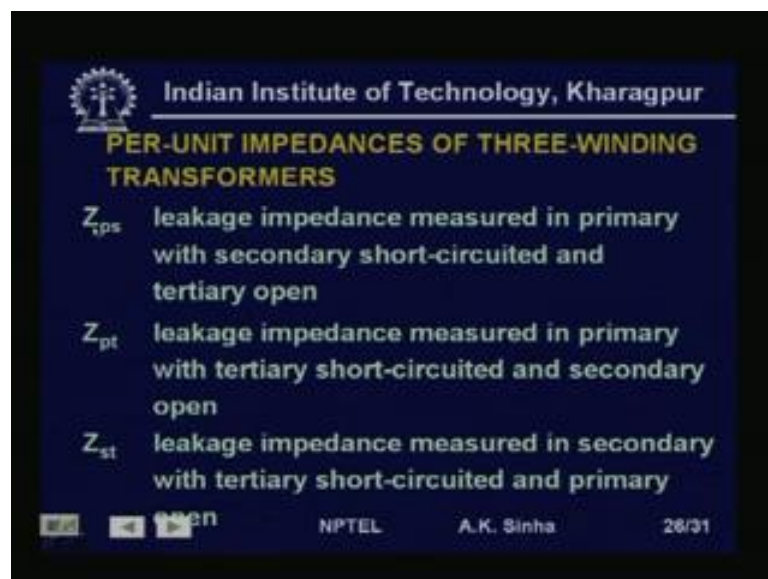
We can now find the values of these windings by conducting test. Say leakage impedance measured in primary with secondary short circuited and tertiary open. Z_{pt}

we can find out as leakage impedance measured in the primary with tertiary short circuited and secondary open. That is you short circuit the tertiary open the secondary winding.

And provide a voltage source on the primary side whatever voltage divide by current you get will be the leakage impedance measured in primary with tertiary short circuited and secondary open. Similarly, if you leakage impedance measured in secondary with tertiary short circuited and primary open can give us Z s t.

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$$Z_{ps} = Z_p + Z_s$$

$$Z_{pt} = Z_p + Z_t$$

$$Z_{st} = Z_s + Z_t$$

$$Z_p = \frac{1}{2}(Z_{ps} + Z_{pt} - Z_{st})$$

$$Z_s = \frac{1}{2}(Z_{ps} + Z_{st} - Z_{pt})$$

$$Z_t = \frac{1}{2}(Z_{pt} + Z_{st} - Z_{ps})$$

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Now, once we calculate this we can find out the values of Z p Z s and Z t as is. So, with this we have now been able to model the transformers a single phase transformer, 3 phase transformers and also 3 winding transformers.
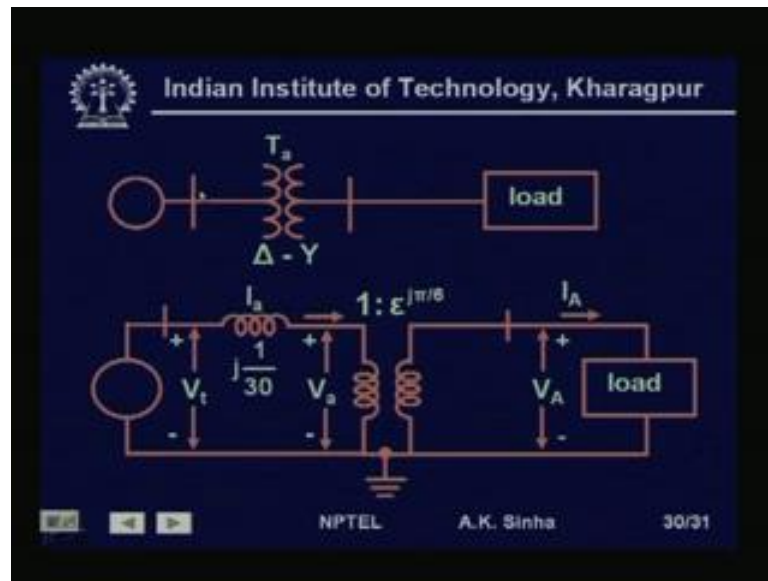
Now, what in a system we have say generator a transformer transmission line. Another transformer a step up transformer here a step down transformer here, then the total circuit will look like this. If we have a star delta transformer then we have the phase shifting introduced here. So, this is the way we get the equivalent circuit for the system.
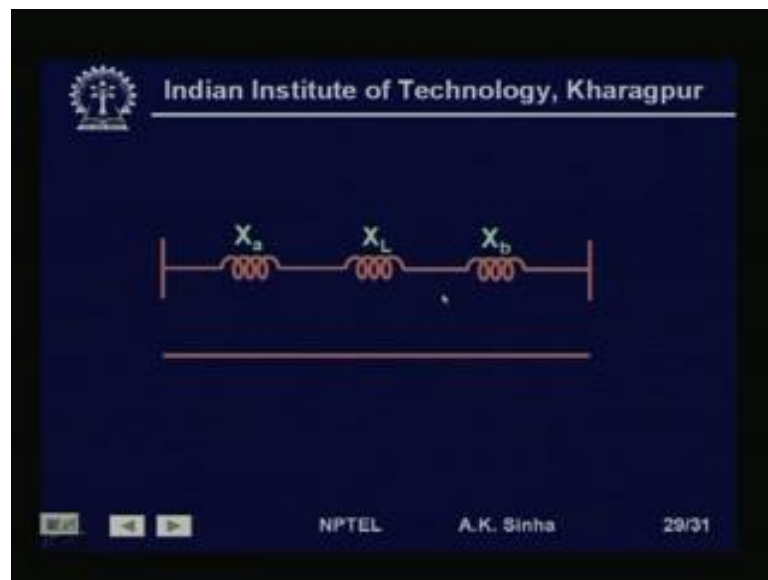
And most of the time the resistance being very small and the core loss part and the magnetization current is very small. So, we represent the transformer by a simple reactance also. This is what is shown here.
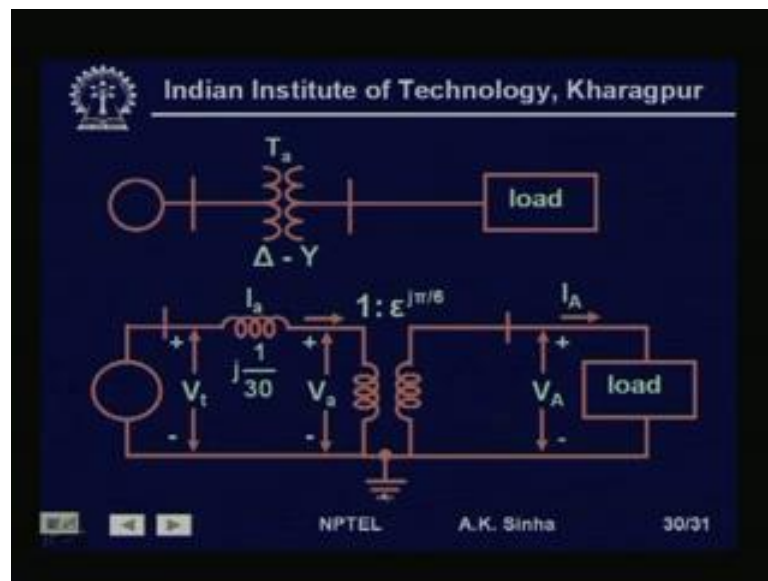
This is again another circuit with a load and a transformer again here we have introduced this phase shifter, because this is a delta star transformer. So, this is all about transformer modeling.

Thank you.