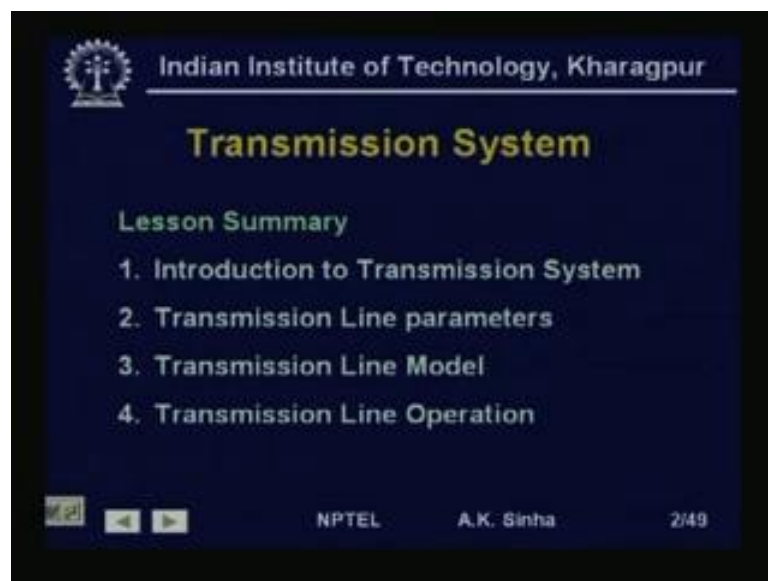


Power System Analysis
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Lecture - 11
Transmission System A Review

Welcome to lesson 11 in Power System Analysis. In this lesson, we will be mainly reviewing, whatever we have learnt on Transmission Lines.

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That is basically we will be again going through the whole set of lessons, that we did. That is from lesson 3 to lesson 10 and we will try to recapitulate, whatever we have learnt in those lessons. In this lesson, we will start with introduction to transmission systems. Then, we will go to transmission line parameters and then we will review; what we have learnt on transmission line modeling, and finally on the steady state operation of the transmission line.

So, we will start with an introduction to the transmission system. If you remember the earlier lessons, we talked about the different types of conductors. That we use for transmission lines. We had said that the overhead transmission lines, which we will be dealing with mostly use, bare conductors. That is conductors, which do not have any insulation on top of that because they will be exposed to atmosphere. And therefore, they can dissipate heat much faster and so they can be worked at higher current levels.

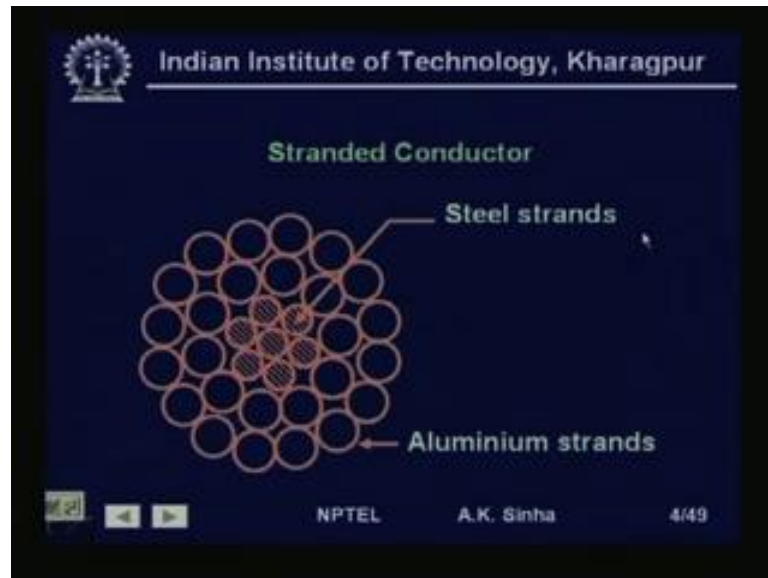
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Now, the type of conductors, that we use are normally copper conductors. Of course, copper being very expensive, now-a-days copper is no longer being used. Though, this has a much higher conductivity as compared to the other materials. That we used now. The conductor, which is now very much in use, in fact, almost 90 percent of the conductors, which I used for over rate transmission systems are ACSR conductors. That is aluminum conductor steel reinforced.

These conductors are made up of small or aluminum conductor filaments, which are twisted together. And since, aluminum does not have very large tensile strength. So, to provide the tensile strength or the mechanical strength to the conductor system, steel wires are used for as reinforcement. So, what we have is the central core is made up of a number of steel wire or steel filaments. And on top of that we have the aluminum filaments, which make the whole conductor.

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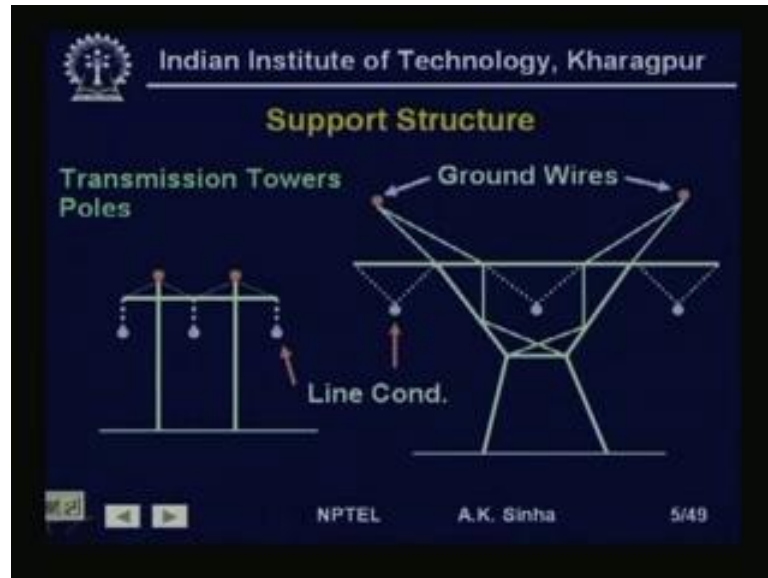
We can see this in this diagram, where we have the either core made up of steel strands. And on top of that we have these filaments of aluminum conductor or the strands made up of these aluminum conductors. So, this is, how ACSR conductor looks, if you take a cross section of the ACSR conductors. The other conductors which are not so much in use, but have been used in some places are aluminum alloy conductors. These conductors again are made up of aluminum alloy. So, that it gives more mechanical strength to the aluminum conductor.

We sometimes, use aluminum clad steel conductors. That is, we have steel conductor core on that aluminum conductor is clad. So, aluminum clad steel conductors are also used, but these are not so much in use. A new type of conductors, which are now being used in extra high voltage lines, sometimes is what we call expanded ACSR. It is the same ACSR or the aluminum conductor steel reinforced conductor. Except that between the steel core and the aluminum, we have some fibrous material, which is placed in between.

This is the filler material, which is introduced there. Just to increase the diameter of the conductor, as we have seen already the inductance of the conductor will reduce, if it is diameter increases. And therefore, this is sometimes used to reduce the inductance of the conductor. So, expanded ACSR conductors are being used in some places. It is very similar to the ACSR, except that here in this position, we will have some filler material,

which will be potent. So, that the overall diameter of this conductor is increased somewhat. Then, comes to the support structures on which these conductors are put. These are basically transmission towers or transmission poles.

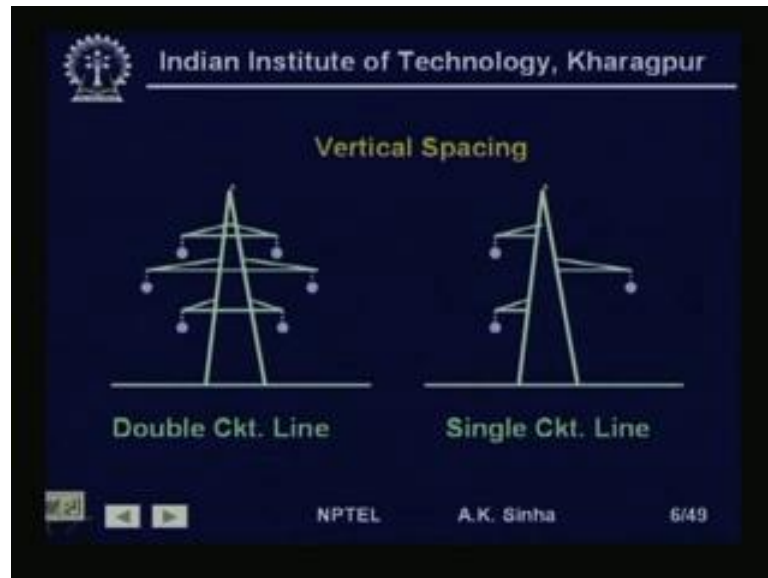
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Now, here we have shown transmission towers. This is for extra high voltage lines, generally what you have is and these three phased conductors are placed like this. It or at a lower voltage, this can be made like this also. Here the configuration of the conductors are in a horizontal plane and three phased conductors are placed like this. Here, at the top, we these red ones, can be seen and these are, what we call the ground wires or the earth wires.

These wires are directly connected to the tower and through the tower footing resistances, they are grounded. So, these are mainly there to protect the phase conductors from direct lightning stroke. Because, these provide much low resistance path to the lightning current, then what will come for through this phase conductors. And therefore, lightning will rather directly. If it strikes directly, it will strike these ground wires, which are much above the phase conductors and will be grounded. Instead of horizontal configuration of the conductors, we sometimes use vertical spacing also.

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So, for that we have tower structures, which are shown here like this, where, this tower is showing a double circuit line. One circuits here A, B, C the three phase conductors of one circuit and A dash, B dash, C dash the three phase conductors for the other circuits. Both the circuits are running parallelly to each other. And they are placed in the same tower. Again, putting A and A dash like this, B, B dash like this, C and C dash like this, is done mainly, because this helps in reducing the overall inductance of the circuit.

Sometimes, when the load is not very high and the load may build up later. Would, we do is, we use a tower construction, which is very similar to a double circuit tower construction. But, what we do is, we use only three arms like this. And we place our conductors A, B and C like this. And when the load builds up, the other arms will be added and we will get a double circuit, three phase line like this. So, this is about the support structure or the transmission towers.

Next, we come to the electrical parameters of the transmission line, as we know this. The transmission line are consisting of conductors, these conductors will have some amount of resistance. So, the transmission line will have resistance. Since, these conductors will be carrying current; magnetic field will be built up, because of the current flowing in the conductors. And these, magnetic field will interact with the other conductors and which are carrying current. And therefore, we will have some inductance involved.

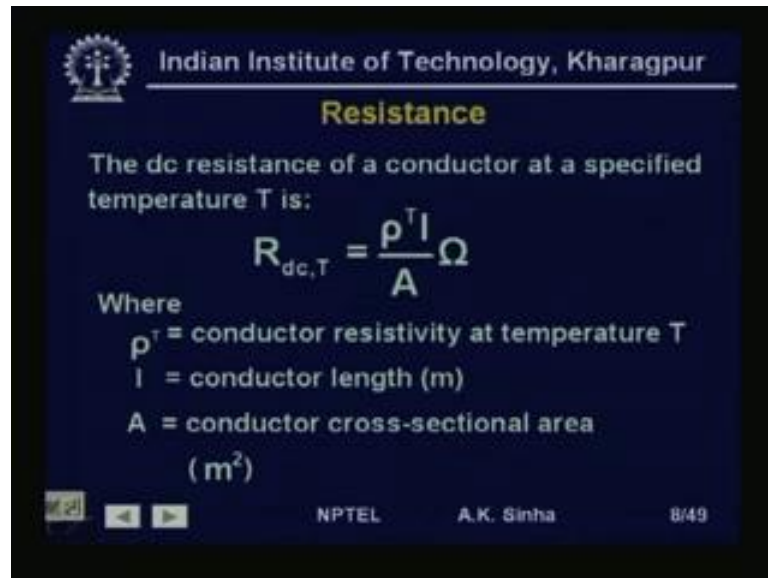
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So, we have inductance on the transmission line. Since, these conductors are the overhead lines are at high voltage. So, there is a voltage difference between the two phase conductors as well as between the phase conductors and the ground. Since, there is a voltage difference and there is insulation in between, which is normally air for overhead conductors. So, we will have some capacitance also involved.

So, transmission line will have three electrical parameters resistance inductance and capacitance. Resistance and inductance will be in series, whereas the capacitance will be between the line and the ground or between the two lines. So, this will be coming as a shunt or a parallel connection. So, we have series impedance of the line given by resistance and inductance and the shunt admittance of the line given by the capacitance, which is coming between the line and the earth or the neutral.

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Resistance

The dc resistance of a conductor at a specified temperature T is:

$$R_{dc,T} = \frac{\rho^T l}{A} \Omega$$

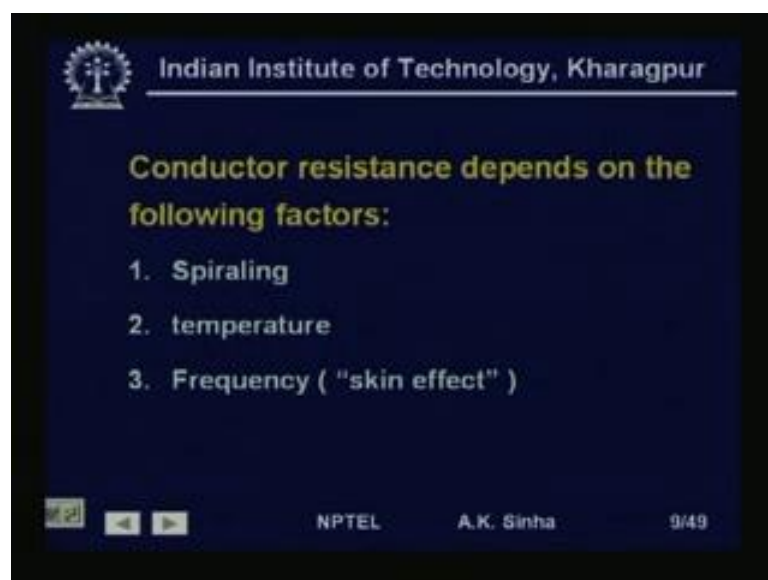
Where

- ρ^T = conductor resistivity at temperature T
- l = conductor length (m)
- A = conductor cross-sectional area (m²)

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Now, for calculating the resistance, we all know resistance. The D c resistance at any temperature R d c at temperature T is given by a rho at that temperature. That is the resistivity of the material of the conductor at that given temperature into l, the length of the line or the length of the conductor and A, the cross sectional area of the conductor. So, it is rho l by A and the unit for resistance is ohms.

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Conductor resistance depends on the following factors:

1. Spiraling
2. temperature
3. Frequency ("skin effect")

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Now, conductor resistance will depend on some other factors as well these factors are since, we are using stranded conductors and these conductors are spiral. So, because of

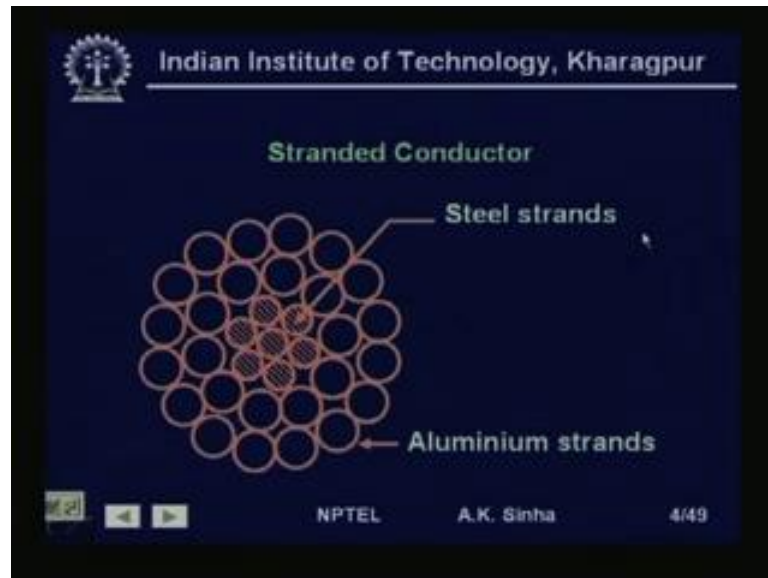
spiraling, what happens is the length, the effective length of the conductor gets reduced. So, the actual length of the each filament will be much longer compared to the effective length of the conductor.

This is, because when we twist the conductors. The conductor each strand of the conductor length, will be larger than the final conductor length, which comes out by making this twisting for all the conductors. So, spiraling, because it increases the length of that is, because it uses a larger length of the conductor, then the actual length of the line. Therefore, the effective resistance of the line will be somewhat larger than what we will calculate by taking the line length. So, this is mainly because this l here, the actual l will be larger than the line length.

Similarly, temperature as we have seen this resistivity of the conductor will increase with the increase in temperature. And therefore, when the conductor is carrying more current, the heat dissipation is going to be more, $I^2 R$ losses are going to be more, which will be dissipated as heat. And therefore, the conductor temperature will go up and this will increase the resistance of the conductor.

Another important factor is, the frequency. In alternating current, what we have is, the current and voltage is going through a cycle. A number of times, every second, like for 50 hertz, the current and voltage waves, go through 50 cycles per second. Now, because of these changing of the current, the magnetic field also gets changed. And therefore, since this magnetic field is changing and it is cutting a conductor. Therefore, some inductance is getting induced.

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So, what actually happens, because of this frequency is that the lower or the central part of the conductor. If you look at this conductor, what we will see is, the either part of the conductor will be experiencing more flux linkages. And therefore, the inductance of these strands will be more than the inductance of these strands. And therefore, an AC current will see a more reactance or the more impedance for to it is flow.

And therefore, less and less current will flow through the core and more and more current will flow through the outer edge of the conductor or the strands at the outer edge of the conductor. Because, here the flux linkages will be lower and the inductance seen will be less. And therefore, what we find is, more current will flow through this part and less current will flow through the central part.

This is what happens and that is since disappears as if the current is trying to flow through the outer skin of the conductor. Therefore, this effect is called skin effect. So, because of the frequency, we have skin effect. Higher the frequency, more and more current will be flowing through the outer edge of the conductor. That is, more current will be flowing through the skin and is less the frequency, the current distribution over the cross sectional area was going to be more uniform.

So, higher frequencies will make the current distribution, over the cross sectional area; Non uniform, with more currents flowing, through the edge and less flowing, through the central part of the core.

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Resistivity of conductor metals varies linearly over normal operating temperatures according to

$$\rho^{T_2} = \rho^{T_1} \left(\frac{T_2 + T}{T_1 + T} \right)$$

The ac resistance or effective resistance of a conductor is

$$R_{ac} = \frac{P_{loss}}{(I)^2} \Omega$$

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Well, as we have seen the resistance depends on resistivity and resistivity depends on the temperature. Resistivity of conductor metal varies linearly over normal operating temperatures, according to relationship, ρ^{T_2} is equal to ρ^{T_1} into $\frac{T_2 + T}{T_1 + T}$, where, this T is a temperature constant, which has different values for different materials. So, if you know the resistivity at some temperature T_1 . We can find out the resistivity at another temperature T_2 .

As we have seen, the resistance of a conductor depends on different factors, such as spiraling the temperature and skin effect. Therefore, to that the actual or the effective resistance of a conductor, under which is working under alternating current system. We do this, by experimentally finding out of the resistance, by finding out the loss in the conductor. And therefore, finding out resistance ac loss divided by I^2 ; because we know $I^2 R$ is the loss, mistakes place in the conductor.

So, using experiment, where we flow a certain amount of current, through the conductor and find out the loss in the conductor. We can calculate the A c resistance of the conductor. However, if we calculate the A c resistance, D c resistance of the conductor for at 50 hertz or so most of the time, due to the spiraling skin effect and other effects, if we take them into consolation. The A c resistance will be about 4 to 5 percent higher than in the D c resistance.

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Inductance

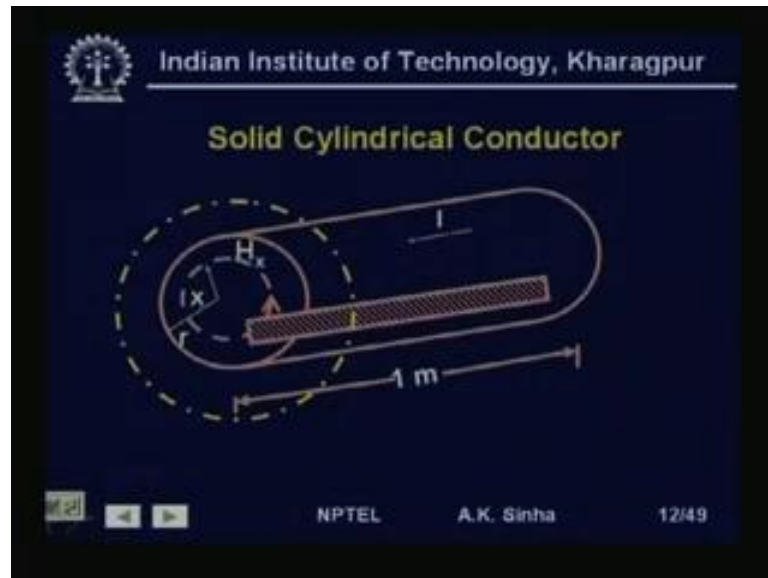
1. Magnetic field intensity H , from Ampere's law
2. Magnetic flux density B ($B = \mu H$)
3. Flux linkages
4. Inductance from flux linkages per ampere
($L = \lambda/I$)

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Next, we will talk about the inductance, we had seen, that we can calculate the inductance of over head conductor using the following steps. First, we calculate the magnetic field intensity H , using Ampere's law. Then, we calculate magnetic flux density B , B is equal to μH . Once, we have calculated H , knowing μ in case of non magnetic conductors, like aluminum or copper μ is same as μ_0 . That is, $4\pi \times 10^{-7}$ to the power minus 7.

So, using that we can calculate the flux density and from flux density, we can calculate the flux and the flux linkages. And then we can calculate inductance from the flux linkages, by L is equal to λ by I , flux linkages per unit current.

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Now, if we take up a solid conductor. Here, what we find is, when a current is flowing through this conductor, it is going to setup magnetic field, inside the conductor also. So, here in this diagram, at a distance X from the center, we have a magnetic field with intensity H_x . And this field is going to link, only this part of the current, which is inside, not the current outside.

Whereas, this current will also setup magnetic field, outside the conductor and this will be linking all the current in the flowing in the conductor. Because, it will be enclosing the full conductor. And therefore, we can find out the total inductance by finding out the flux linkages, which are internal and the flux linkages, which are external.

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$$\lambda_p = 2 \times 10^{-7} I \left(\ln e^{1/4} + \ln \frac{D}{r} \right)$$
$$= 2 \times 10^{-7} I \ln \frac{D}{e^{-1/4} r}$$
$$= 2 \times 10^{-7} I \ln \frac{D}{r'} \quad \text{Wb-t/m}$$
$$r' = e^{-1/4} r = 0.7788 r$$
$$L_p = \frac{\lambda_p}{I} = 2 \times 10^{-7} \ln \left(\frac{D}{r'} \right) \quad \text{H/m}$$

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And this, we can see that at any point P, at a distance D, from the center of the conductor. The total flux linkage will be given by $2 \times 10^{-7} I$, multiplied by $\ln e^{1/4} + \ln \frac{D}{r}$. This $\ln e^{1/4}$ term is coming out, because of the internal flux linkages. And we see, this is independent of the radius of the conductor.

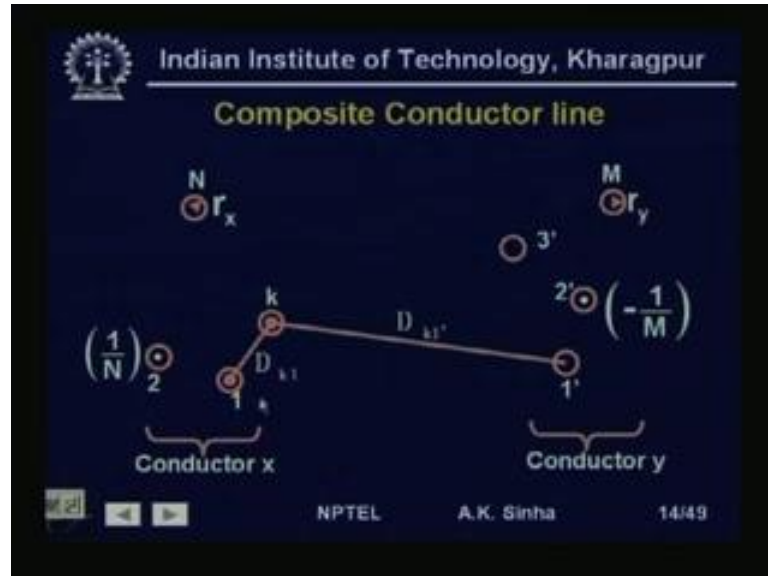
Whereas, this term is coming out, because of the external flux linkages, the point P is at a distance D from the conductor and r is the radius of the conductor. So, if we combine these terms, then we can write this as $2 \times 10^{-7} I \ln \frac{D}{r'}$, where r' is equal to $e^{-1/4} r$ or equal to $0.7788 r$, which tells us, that due to internal flux linkages the effective radius of the conductor gets reduced, somewhat. That is from r, it becomes about $0.7788 r$ or 78 percent of the actual radius of the conductor.

So, $e^{-1/4} r$, which we can write as $0.7788 r$, where, r' is equal to $e^{-1/4} r$ or equal to $0.7788 r$, which tells us, that due to internal flux linkages the effective radius of the conductor gets reduced, somewhat. That is from r, it becomes about $0.7788 r$ or 78 percent of the actual radius of the conductor.

So, once, we know the flux linkage. We can calculate the inductance of the conductor for flux linking up to point P. So, that is λ_p by I. So, this is equal to $2 \times 10^{-7} \ln \left(\frac{D}{r'} \right)$ Henry's per meter. Now, we have seen, that normally

the conductor, that we use is not a single solid conductor, but is made up of a large number of strands of conductor.

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So, here we are trying to depict, that condition as a general case, where, a conductor X is made up of N number strands and each strand is having a radius r_x . This we have said that that is we have made an assumption that all strands are of the same radius. This is normally true and each conductor or each strand of this conductor will be carrying $\frac{1}{N}$ of the current. Because, they are having, equal cross sectional area of the same material.

So, they will be carrying $\frac{1}{N}$ of the current. That is, there are N sub conductors and total current flowing through this is I. So, each sub conductor will be carrying $\frac{I}{N}$ current. The other conductor, which is a return conductor, consists of M number of sub conductors. And each one of them will be carrying $-\frac{I}{M}$ current, because this is a return conductor. So, current I is flowing through this conductor and returning back from the other conductor.

So, conductor Y, has M sub conductors, each carrying $\frac{I}{M}$ current. These are depicted as 1 dash, 2 dash, 3 dash up to M and r_y is the radius of each sub conductor. The distance of any sub conductor to other sub conductor is denoted by like from conductor sub conductor one to sub conductor K N conductor X. It is D_{k1} and from sub

conductor K in X, conductor X to sub conductor 1 dash in conductor set Y, it is D_{K1} dash.

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$$\lambda_x = 2 \times 10^{-7} I \ln \prod_{k=1}^N \frac{\left(\prod_{m=1}^M D_{km} \right)^{1/NM}}{\left(\prod_{m=1}^N D_{km} \right)^{1/N^2}}$$

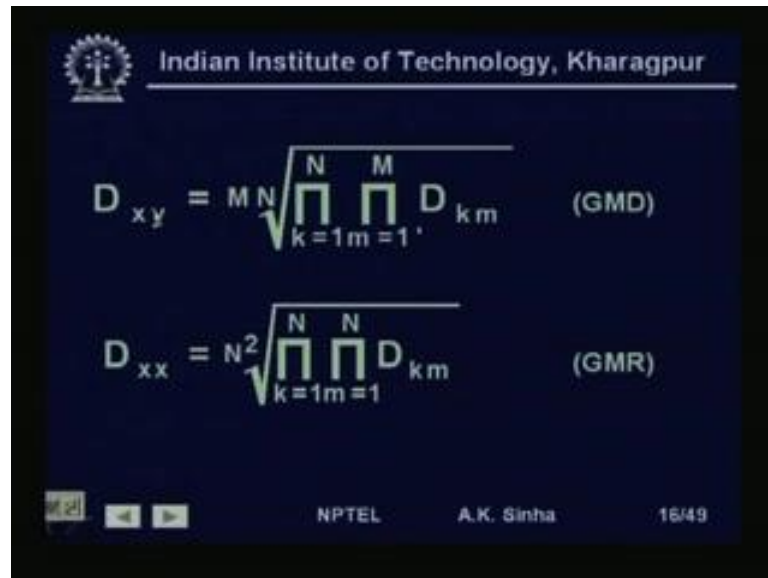
$$L_x = 2 \times 10^{-7} \ln \frac{D_{xy}}{D_{xx}}$$

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So, with these, we can now again find out the flux linkage of the conductors of all the sub conductors in conductor X. That will come out to be $2 \times 10^{-7} I \ln$ multiplication, from K is equal to 1 to N of multiplication m is equal to 1 dash to M of D_{km} . That is distances from conductor sub conductor k to sub conductor m to the power 1 by NM.

And this will be divided by multiplication m is equal to 1 to N into or for D_{km} . That is from conductor k to m and this will be e to the power 1 by N square. This is, what we will get after doing finding out the flux linkages for each sub conductor. And then adding them up for all the sub conductors. Therefore, the inductance of this conductor x will come out to be $2 \times 10^{-7} \ln$, D_{xy} by D_{xx} .

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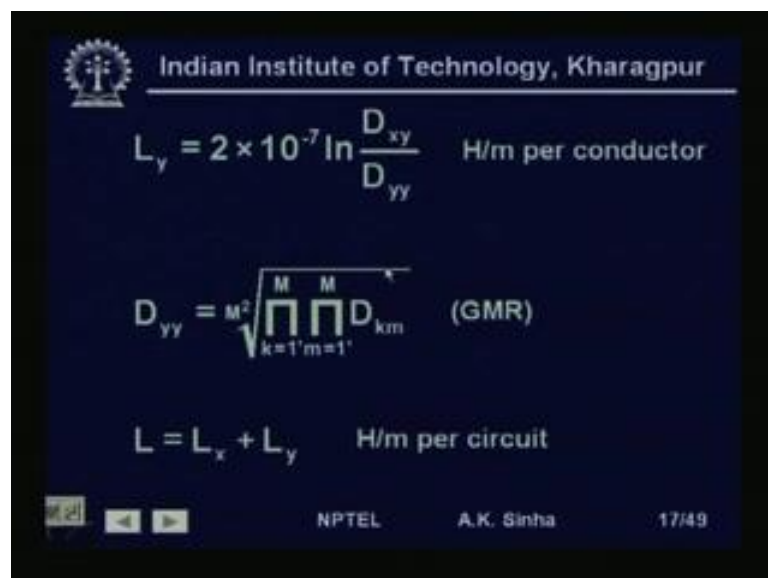
$$D_{xy} = MN \sqrt[N]{\prod_{k=1}^N \prod_{m=1}^M D_{km}} \quad (\text{GMD})$$
$$D_{xx} = N^2 \sqrt[N]{\prod_{k=1}^N \prod_{m=1}^N D_{km}} \quad (\text{GMR})$$

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Where, we are writing D_{xy} as MN th root of product k is equal to 1 to N , product m is equal to 1 dash to M , D_{km} . That is, what we are saying is, that this is the geometric mean distance of each sub conductor, in conductor x , to each sub conductor in conductor Y . For all the sub conductors, in conductor x and D_{xx} is again the geometric mean of the distance of each sub conductor to other sub conductor in conductor x .

So, this is, what we call as GMR or the geometric mean radius, for the conductor x . And GMD is the geometric mean distance, between the two conductors x and y .

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$$L_y = 2 \times 10^{-7} \ln \frac{D_{xy}}{D_{yy}} \quad \text{H/m per conductor}$$
$$D_{yy} = M^2 \sqrt[M]{\prod_{k=1}^M \prod_{m=1}^M D_{km}} \quad (\text{GMR})$$
$$L = L_x + L_y \quad \text{H/m per circuit}$$

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Therefore, similarly we can write for inductance for conductor y as equal to $2 \times 10^{-7} \ln \frac{D_{xy}}{D_{yy}}$ Henry's per meter per conductor, where, D_{yy} will be the geometric mean radius of the sub conductors in conductor y. That is for each sub conductor to all the sub conductors in conductor y, we will find out the geometric mean distance. And that is, what we call as the GMR or the self GMD for that conductor. And the total inductance will be some of the inductance for conductor x and conductor y, because both these are in series.

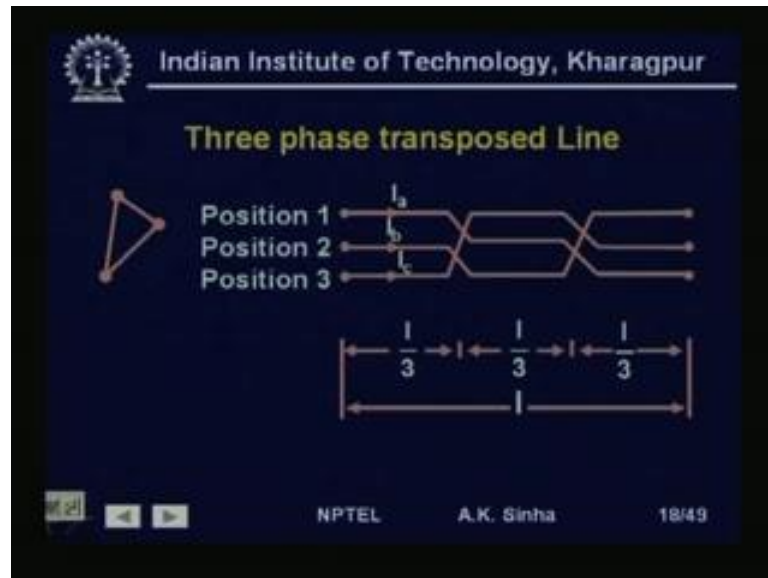
Now, for a three phase system, what we have seen is that if we put the three conductors in a symmetrical spacing. That is on the vertices of an equilateral triangle and then all the three conductors will have same flux linkages, because of the symmetry of their position. But, this is not feasible, practically when we are putting the conductors on transmission towers. Because, this will make our wire of way much larger and transmission tower design will also be more expensive.

So, we have seen that we either put them in some kind of a vertical configuration or may be horizontal configuration or somewhat a scud consideration as such. In that case, since the three conductors will not have the same symmetrical physical position. So, the flux linkages of the three conductors will not be same, which means, that the inductance of the three conductors will not be same.

In that case, what happens is, the transmission system will be an unbalance system. We will have different values of inductances and so reactance for each phase. And therefore, with the same current flowing in them, we are going to get different voltage drops or at the other end of the transmission line. The voltage is phase will not be symmetric is symmetrical and we will have an unbalanced system.

In order to avoid this kind of a situation, what we do is, we transpose the transmission line. That is, what we do is we make all the three phase conductors to go through all the three positions, as shown in this figure.

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The conductor of phase A, goes through this position. For one-third length of the line, goes through the position of B. For the other one-third of the line, that is second one-third portion of the line and goes through the position of conductor C, for the last one-third portion of the line. Similarly, the conductor of phase B goes through this position for one-third. Goes through position of C, for the next one-third and goes through the position of A for the next one- third and so on.

Thereby, each conductor or each phase conductor goes through all the three positions. And therefore, the average flux linkage of each phase conductor will be same and there by the inductance for all the three phases are going to be same. This way, we try to make a symmetry and balance for the three phase conductor's inductance.

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$$L_a = \frac{\lambda_a}{I_a} = 2 \times 10^{-7} \ln \frac{\sqrt[3]{D_{12} D_{23} D_{31}}}{D_s} \text{ H/m per phase}$$
$$L_a = 2 \times 10^{-7} \ln \frac{D_{eq}}{D_s} \text{ H/m}$$

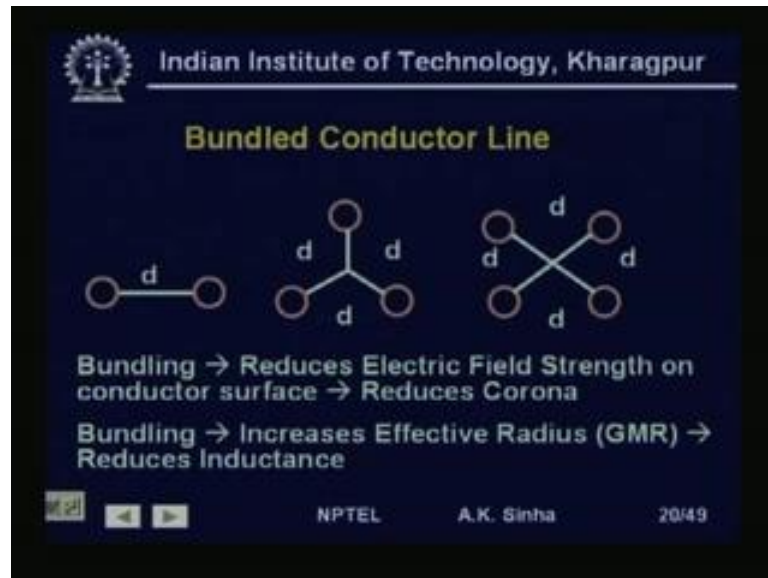
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So, here, if we use this transpose system, then we have inductance of any phase. Here, we are writing for phase A, inductance L_a is equal to λ_a by I_a , which is equal to $2 \times 10^{-7} \ln \frac{\sqrt[3]{D_{12} D_{23} D_{31}}}{D_s}$. This is happening because; we are having all the three positions for each conductor. So, we have got this cube root of D_{12} , D_{23} and D_{31} , divided by D_s .

Where, D_s is the GMR of the conductor and this cube root of D_{12} , D_{23} and D_{31} is called the equivalent distance D_{eq} . And therefore, we can write L_a is equal to $2 \times 10^{-7} \ln \frac{D_{eq}}{D_s}$, where, this D_{eq} or the equivalent distance between the phase conductors is like saying that this is the equivalent equilateral distance between the three phase conductors.

That is, if we would have made equilateral distance for the phase conductors, for the three phase conductors, then we would have had this distance between them.

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Now, if we see the relationship of inductance. We find that the self GMD or the GMR, for the conductor is coming in the denominator. And therefore, if we can increase this value, then we can reduce the inductance and therefore, the reactance of the transmission line. We can reduce the reactance also by reducing $D q$. That is the distance between the phase conductors.

But, this is not feasible, because we have to maintain a certain minimum distance between that the conductors. Because, of the high voltage which is there between the two conductors. We also have you take into account, that there will be wind blowing and because of which the conductors will be swinging with respect to each other. And therefore, the minimum distance between the conductors must be maintain. Otherwise, there can be a flash over between the conductors, if the insulation between them breaks down.

So, for increasing the effective radius or the self GMD or GMR, whatever we call it, we many times use bundle conductor. Another advantage of using bundle conductor is that it reduces the electric field strength on the conductor surface. That is, now what we have is, if we would have used one conductor with the same cross sectional area. If we divide that into two conductors, then the surface area of these two conductors together is going to be large. And therefore, larger and therefore, electric field strength on the conductor surface will get reduced for the same potential.

Now, these conductors, which are connected by means of spaces, which are conducting material, so this is called configuration for bundle conductor. Here, we have two conductors together. That is conductor one here and conductor another here, with the distance D between them. Here, we are showing the configuration with three conductors.

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$$D_s = \sqrt[4]{(r' \times d)^2} = \sqrt[2]{r'd}$$

$$D_s = \sqrt[9]{(r' \times d \times d)^3} = \sqrt[3]{r'd^2}$$

$$D_s = \sqrt[16]{(r' \times d \times d \times d \sqrt{2})^4} = 1.091 \sqrt[4]{r'd^3}$$

$$L_a = 2 \times 10^{-7} \ln \frac{D_{eq}}{D_s} \text{ H/m}$$

The slide also includes three diagrams illustrating conductor configurations: two conductors at distance d, three conductors in a triangle with side d, and four conductors in a square with side d.

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Where, three conductors 1, 2, 3 with a spacing D between each one them and this one shows four conductor. In fact, the higher the voltage, we used large number of sub conductors in a bundle. This reduces the electric field strength, considerably and therefore, it reduces the corona discharge. And corona losses and the associated radio interference and audible noise with it.

As we have also seen, by using the bundle conductors, what we have done is, we have also increase the effective GMR of the conductor of the conductor as such. If we are using two conductors, then the self distance or effective GMR will be given by 4th root of r dash into d, whole square. Because, it is own distance with this conductor will be r dash, it is distance with the other conductor is d. then for this conductor, it is own distance is r dash and the distance of this conductor with this is going to be d.

So, we have r dash into r dash into d into d. That is and we take the 4th root of that. So, we have got the 4th root of r dash into d square, which comes out to be square root of r dash d. Now, this d will be normally around 40 centimeter to 45 centimeter or about 10

times the radius of the sub conductor. And therefore, this square root of $r \text{ dash } d$ is going to be much larger, then $r \text{ dash}$.

Same thing, we can see for this three conductor bundle or for four conductor bundle. We find that for three conductor bundle, it is cube root of $r \text{ dash } d$ square and for four conductor bundle, it is 1.091 into 4th root of $r \text{ dash } d$ cube. And for bundle conductor, we replace the $r \text{ dash}$ by this D_s . That is the surf distance or the GMR of the conductor as calculated here.

Now, once, we have calculated the inductance and the resistance, we have found out the series impedance of the line, per unit length. Now, since we know that the conductors are at high voltage. There is a voltage between the two phase conductors and there is a voltage between the conductor and the ground.

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TRANSMISSION LINE CAPACITANCE CALCULATIONS

Gauss's Law → Electric Field Strength (E)
 Voltage between conductors
 Capacitance → (C = q / V)

$$C_{an} = \frac{q_a}{V_{an}} = \frac{2\pi\epsilon}{\ln(D/r)} \quad \text{F/m line-to-neutral}$$

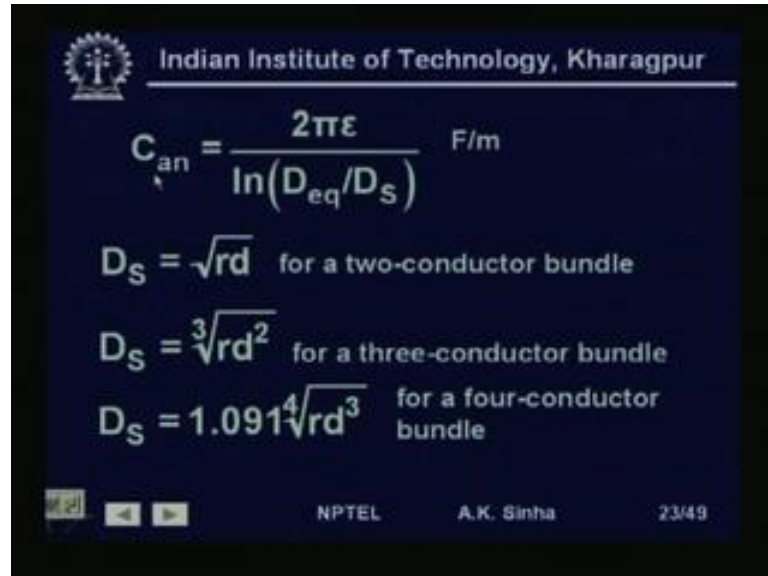
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We have a capacitance involved and we can calculate that the capacitance again by using the Gauss's law. So, using Gauss's law, we find out the electric field strength and then from that we can find out the voltage between the conductors. And then we can find out the capacitance C is equal to q by V , where q is the charge on the conductor. So, we can calculate the capacitance.

The capacitance to ground or capacitance to neutral for any conductor in over head system will be given write q_a by V_{an} , which on calculation comes out to be twice pi

Epsilon, log n by D by r, where, Epsilon is the permittivity of the material. In this case, it is for over head lines, this is the permittivity of air.

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$$C_{an} = \frac{2\pi\epsilon}{\ln(D_{eq}/D_S)} \text{ F/m}$$

$D_S = \sqrt{rd}$ for a two-conductor bundle

$D_S = \sqrt[3]{rd^2}$ for a three-conductor bundle


$D_S = 1.091\sqrt[4]{rd^3}$ for a four-conductor bundle


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So, this C_{an} is given by twice pi Epsilon log n, D_{eq} , D_S for a three phase system, where, D_{eq} is the equivalent equilateral distance for a transpose line. And if we are using bundle conductor, we have to replace this D_S by the effective GMR or the self GMD of the bundle conductor system. Now, since we are putting these conductors on towers and the distance between the phase conductors.

Normally, for a 220 k v line will be of the order of around ten meters. As well as the distance of or the height of the conductor from ground, that is the minimum clearance that is required is also of the same order. Therefore, we need to consider also the effect of earth, when we are calculating the capacitance, because the height of the conductor or the distance from earth of the conductor is of the same order as the distance between the phase conductors.

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Effect of Earth on Transmission Line Capacitance
 Earth surface is considered as an equi-potential Surface
 Method of Images
 Image conductors → below the ground
 → depth equal to the height of the O/H conductor above the ground → opposite charge.


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Now, earth can be seen as an equipotential surface. That is a surface, where potential is equal. It is normally the ground potential or the 0 potential, that we have and to take this into effect, what we have to do is, we use, what we call as a method of images. That is, what we do is, we consider image conductors below the ground at a depth equal to the height of the over head conductor, above the ground. And this image conductor, has opposite charge.

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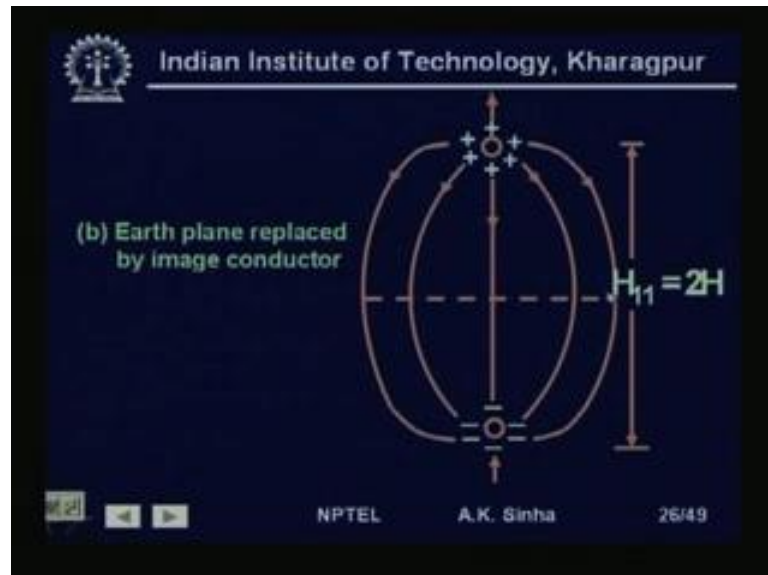
(a) Single conductor and earth plane


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In fact, if we look at the electric field, contour, when we have a conductor placed above the ground, this conductor has a positive potential. Therefore, the field lines will look

like this. Now, the earth being an equipotential surface, these lines will be coming to the earth at 90 degrees are perpendicular to the ground.

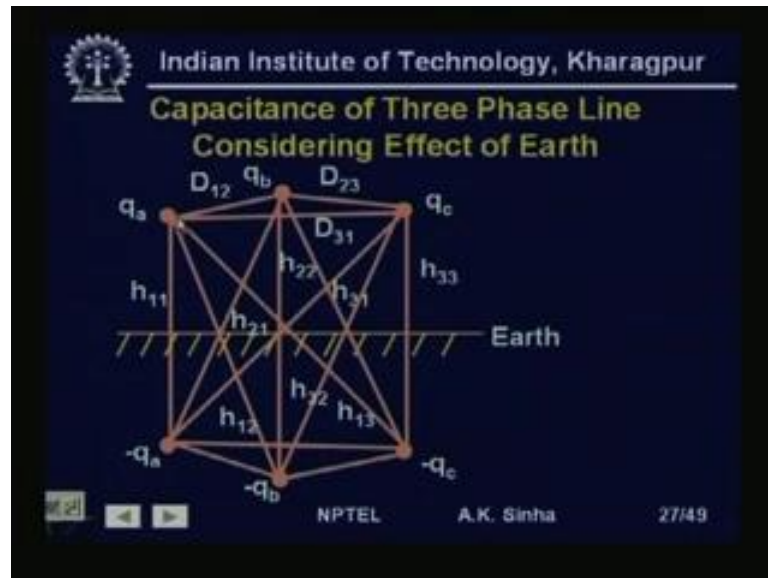
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Now, this situation can be given or can be made by considering an image conductor, with negative charge at a distance, H below the ground. Because, if we put this image conductor here likes this, then the field lines will look like this. Now, we see, this field lines configuration is same as if the ground was there. So, now, even if we remove the ground with this image conductor placed here, the field line configuration does not change or the effect of capacitance will not be changed.

So, for taking this effect of earth into consideration, what we do is, we place an image conductor, below the ground at a height equal to the height of the conductor, about the ground and we put the image conductor, with opposite polarity.

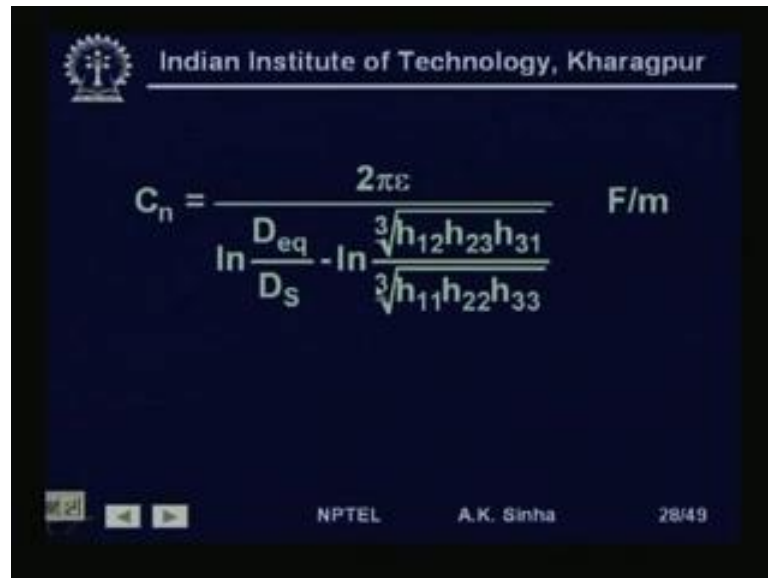
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For a three phase system, we have shown here. We have phase conductor a, phase conductor b and phase conductor c, q_a is the charge on that q_b is the charge on phase b conductor; q_c is the charge on phase c conductor. We have taken this height to be h , then we have taken an image conductor for with a charge minus q_a and the height h_{11} will be equal to twice h .

Similarly, for b we have image conductor here and c we have image conductor here and we have all these distances given for conductor to image conductors, as well as between the conductors. And if we use this configuration, then this earth effect is taken in to account. So, whether this is here or it is not here, it does not matter and with these six conductors now, we need to find out the capacitance.

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$$C_n = \frac{2\pi\epsilon}{\ln \frac{D_{eq}}{D_s} - \ln \frac{\sqrt[3]{h_{12}h_{23}h_{31}}}{\sqrt[3]{h_{11}h_{22}h_{33}}}} \quad \text{F/m}$$

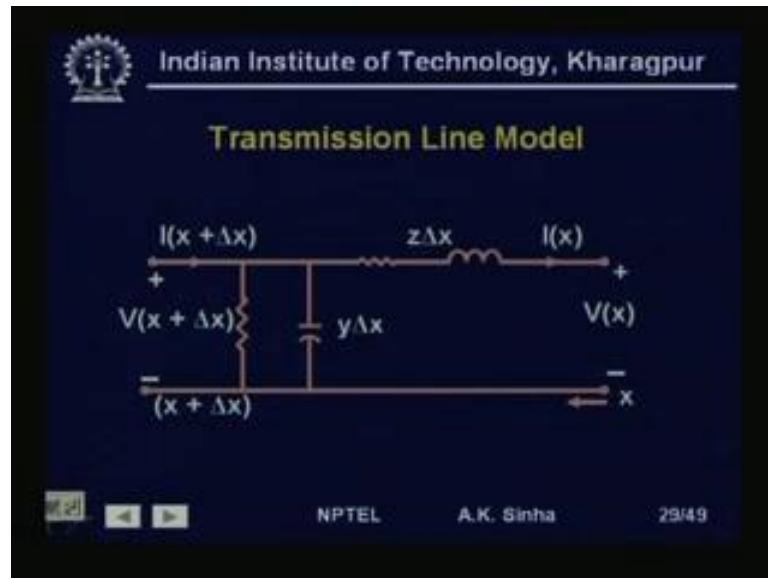
NPTEL A.K. Sinha 28/49

If we do this calculation, then we find the earth the capacitance to neutral for any of the phase conductors will be given by twice pi Epsilon, divided by log n in D e q by D s minus log n cube root of h 12 into h 23 into h 31, divided by cube root of h 11 into h 22 into h 33. Now, what we are saying is, the effect of ground is to introduce this term in the denominator.

That is the denominator is getting reduced by this amount and therefore, the capacitance is somewhat increase. So, effect of earth is to increase the capacitance somewhat. Now, this term h 12, h 23 and h 31 and h 11 and h 22, h 33; if the height of the conductor is much above the ground. Then, these terms will be almost equal and this effect will be negligible.

Next, we will now, that we have calculated the resistance, the inductance and the capacitance. Next, comes, how to model the transmission line.

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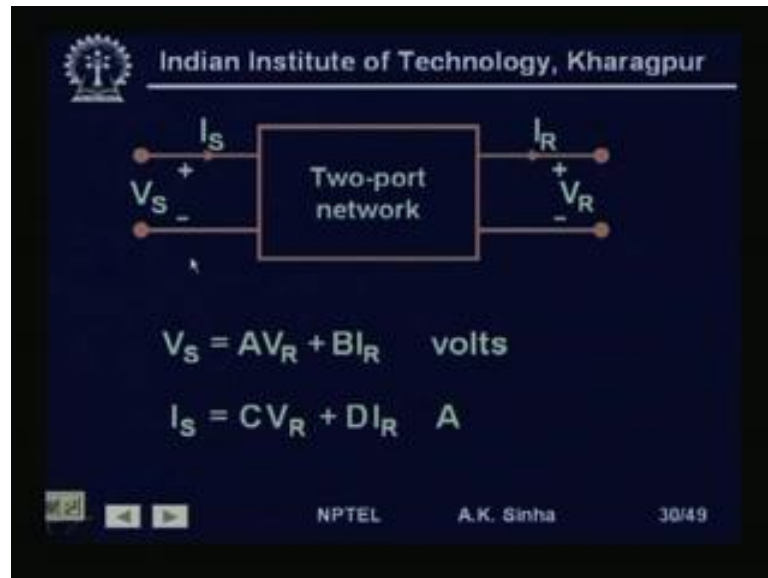
Now, we have seen that we are having resistance per unit length of the line. We have inductance per unit the length of the line and we have a capacitance per unit length of the line. That we have calculated. Here, I have shown also a conductance, which most of the time, we neglect. This is any current, which is flowing, any leakage current, which flows across the insulators of the line, which is normally very, very small and is most of the time, neglected.

So, we have model of the transmission line, where we have resistance and inductance, per unit length of line and a capacitance per unit length of the line. So, we will have basically a distributed parameter model of the transmission line. That is this series induct the resistance and inductances are in series and the capacitances is in the shunt and this will be there for per unit length of the line.

Now, this model, if we use in detail, leads to somewhat complicated mathematical model and complex for calculation. So, most of the time, we try to use simpler equivalent models. For any of these line configurations, this line if we see, here we have voltage and here we have voltage at the other end. We have resistance, inductance and capacitance involved in between.

So, we have one port here of two terminals and one port here of the two terminals. We have voltages at this end, we have voltages at this port, we have current flowing at this port and current into this port.

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So, we can represent this transmission line as a two port network, where we have at the sending end voltage V_S and the current I_S at the receiving end we have current I_R and the voltage V_R . Now, this can be represented as V_S is equal to AV_R plus BI_R volts and I_S is equal to CV_R plus DI_R . That is, we can represent this two port network in terms of A, B, C, D parameters.

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$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

$AD - BC = 1$

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The slide displays the matrix equation for a two-port network. The input variables V_S and I_S are in a column vector on the left, followed by an equals sign, a 2x2 matrix with elements A, B, C, and D, and another column vector on the right containing the output variables V_R and I_R . Below this equation, the condition $AD - BC = 1$ is stated.

We can write this as V_S, I_S is equal to A, B, C, D, V_R, I_R and one of the property of this A, B, C, D parameter is AD minus BC is equal to 1.

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Short Line Model

- Line Length < 80 km
- Generally MV / LV Lines
- Capacitance can be neglected

$Z = z l = (R + j\omega L) l$

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Now, as I said earlier, we do not use the full distributed parameter model for the transmission line, most of the time. In fact, if the line length is less than 80 kilometers, then we can make a much simpler model of the transmission line. Normally, these lines will be low voltage lines and therefore, the effect of the charging current or the capacitance of the line can be neglected.

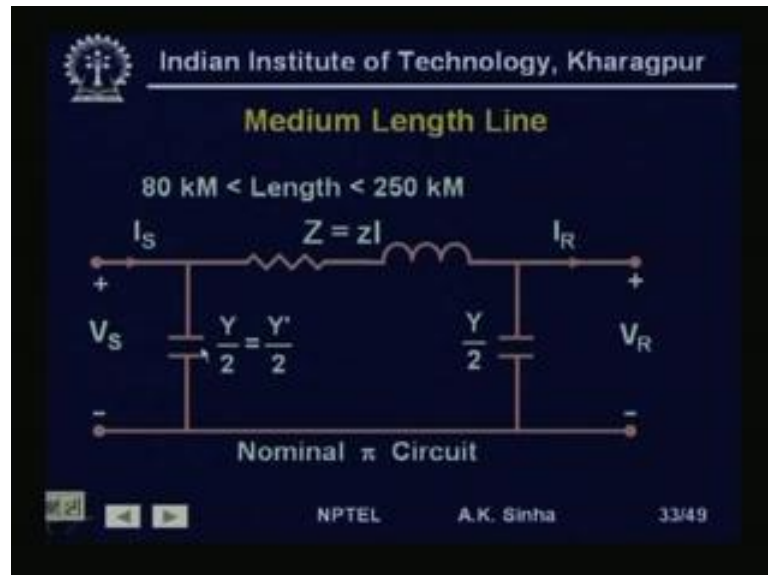
In that case, the line will have only series resistance and inductance. And we can lump of all these resistance and inductances into single resistance and inductance, for the whole length of the line. So, we have a lump parameter model, where we have only series impedance of the line, which is Z will be equal to z into l , small z is the series impedance per unit length of the line, l is the length of the line.

This is equal to R plus j omega L , where R is the resistance per unit length of the line and L is the inductance per unit length of the line multiplied by l . So, this is model, which is a very simple model, where this can be used only for short length lines. That is, lines we which has length less than 80 kilometers. Otherwise, the errors which will creep will be significant.

In case, the line length is somewhat more then 80 kilometers, but less than 250 kilometers. We call these lines as medium length lines, since these lines will generally be high voltage lines, we can no longer afford to neglect the capacitive part of the model.

That is the charging current is going to be significant and it needs to be model and therefore, we have to have the capacitance of the line included in the model.

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What here we do is, normally we use a pi model, where we lump the total series impedance into single series impedance for the whole length of the line. And we lump also all the capacitance of the line, that is if we have calculated the capacitance per unit length multiplied by the total length. Then, we get the total charging capacitance of the line.

We divide this total charging capacitance into two parts and put them the half on one. That is receiving end and half at the other end that is the sending end. So, half of it is put at the sending end and half of it is put at the other end, the receiving end. And this leads to normal pi circuit for the medium length line.

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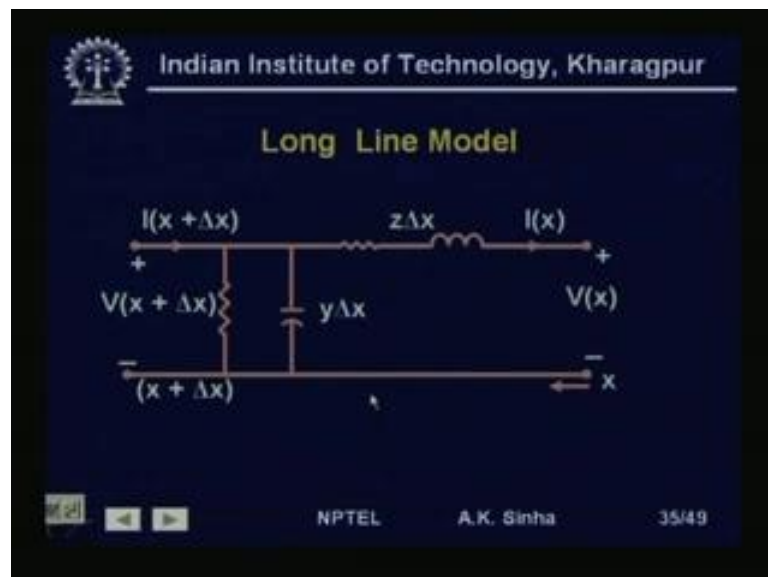
$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} \left(1 + \frac{YZ}{2}\right) & Z \\ Y\left(1 + \frac{YZ}{4}\right) & \left(1 + \frac{YZ}{2}\right) \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

$A = D = 1 + \frac{YZ}{2}$ per unit
 $B = Z \Omega$
 $C = Y\left(1 + \frac{YZ}{4}\right) S$

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For this line, if we do the A, B, C, D parameters, then we have V_S, I_S is equal to $1 + \frac{YZ}{2}$, Z in Y into $1 + \frac{YZ}{4}$, $1 + \frac{YZ}{2}$, V_R, I_R . A is $1 + \frac{YZ}{2}$, B is Z C is Y into $1 + \frac{YZ}{4}$ and D is equal to $1 + \frac{YZ}{2}$. That is A is equal to D and if we do again $AD - BC$ will find that it is equal to 1.

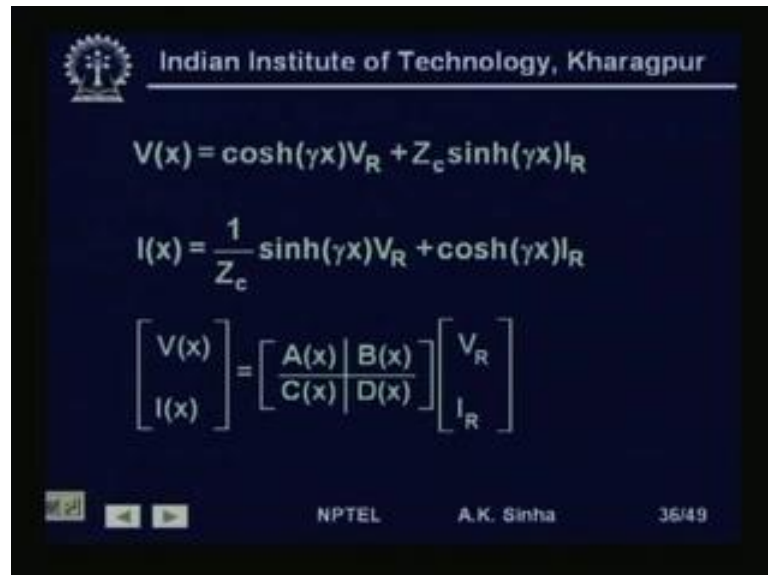
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Now, when we take the long line model, we will have to use distributed parameter models. That is for lines lengths greater than 250 kilometers, we can no longer afford to

lump all the series impedance and the shunt admittances. We will have to use distributed parameter model for the long line.

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$$V(x) = \cosh(\gamma x)V_R + Z_c \sinh(\gamma x)I_R$$

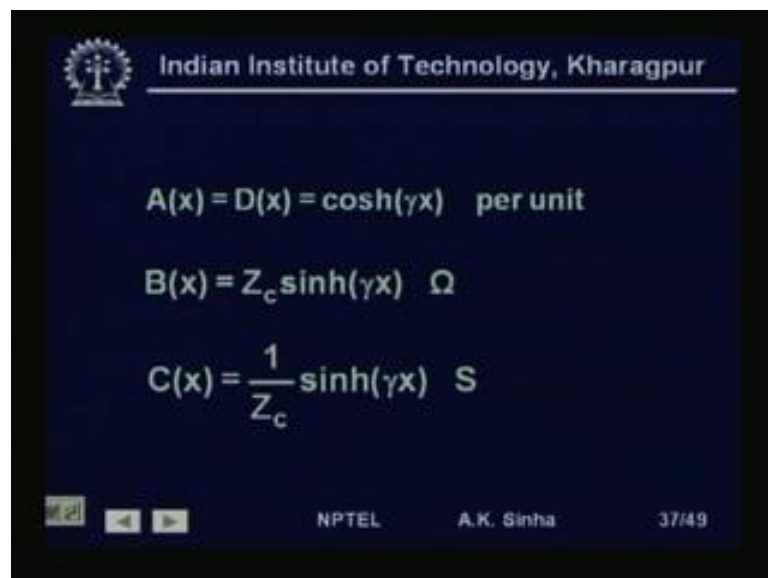
$$I(x) = \frac{1}{Z_c} \sinh(\gamma x)V_R + \cosh(\gamma x)I_R$$

$$\begin{bmatrix} V(x) \\ I(x) \end{bmatrix} = \begin{bmatrix} A(x) & B(x) \\ C(x) & D(x) \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

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Now, when, we do that then if we can again make the A, B, C, D parameters for this line.

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$$A(x) = D(x) = \cosh(\gamma x) \text{ per unit}$$

$$B(x) = Z_c \sinh(\gamma x) \Omega$$

$$C(x) = \frac{1}{Z_c} \sinh(\gamma x) \text{ S}$$

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And here, what we find is the A, B, C, D parameters will be given by at for any distance x. We can calculate this, if we put x is equal to l and then we have for the whole length of the line. So, A is equal to D is equal to cos hyperbola gamma l B is equal to Z c sin hyperbola gamma l and C is equal to 1 by Z c sin hyperbola gamma l.

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II Equivalent Circuit (Long Line)

$$A = D = 1 + \frac{Y'Z'}{2} \quad C = Y' \left(1 + \frac{Y'Z'}{4} \right) S$$

$$B = Z' \Omega$$

The diagram shows an equivalent circuit for a long line. It consists of a series impedance Z' in the middle. On both sides of Z' , there are shunt admittances of $\frac{Y'}{2}$. The input terminals on the left are labeled V_S and I_S , and the output terminals on the right are labeled V_R and I_R .

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Where, Z_c is given by sorry, where Z_c is given by root over Z by Y .

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$$A(x) = D(x) = \cosh(\gamma x) \text{ per unit}$$

$$B(x) = Z_c \sinh(\gamma x) \Omega$$

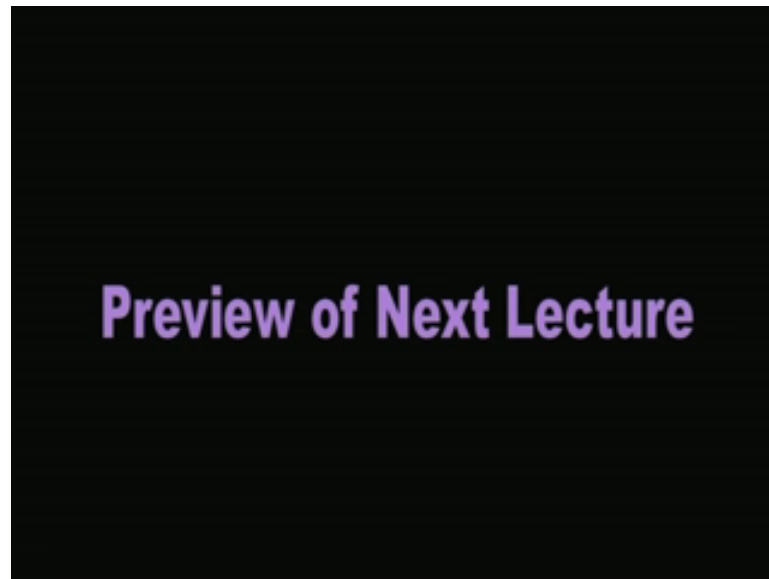
$$C(x) = \frac{1}{Z_c} \sinh(\gamma x) S$$

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Where, Z is the series impedance per unit length of the line and Y is the shunt admittance per unit length of the line and γ is given by root Z into Y . So, by substituting those values we can calculate A , B , C , D parameters. Only thing here we see is, we have hyperbolic function involved in this case and the computation is somewhat cumbersome. So, with this, we stop today and we will continue with this review in the next lesson.

Thank you.

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Welcome to lesson 12, on power system analysis. In this lesson, we will discuss Modeling of Transformers.

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Lesson 12: Introduction

Lesson Summary:

- Introduction
- Concept of an Ideal Transformer
- Physical Transformer & Eq. Circuit
- Three phase Transformer
- Three winding Transformer

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Well, this lesson we will start with introduction of transformers. Then, concept of an ideal transformer, then we will go in to physical transformers and their equivalent circuit. After that we will talk about three phase transformers and three winding transformers.

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Instructional Objective

On completion of this lesson a student should be able to:

- Explain the Concept of an Ideal Transformer
- Develop the Eq. Circuit model of:
 - Single Phase Transformer
 - Three Phase Transformer
 - Three Winding Transformer

NPTEL A.K. Sinha 3/31

Well, the main objective of this lesson is to explain the concept of an ideal transformer. That is, once you through this lesson, then you should be able to explain the concept of ideal transformer. Develop the equation or the equivalent circuits, models for single phase transformers and three phase transformers and three winding transformers.