Networks Signals and Systems

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Lecture - 08

Laplace Transform (contd…)

Okay Good morning friends, we shall continue our discussions on Laplace transform. Yesterday, we derived a transforms of some standard functions will continue with that before we go to the set of other properties I just like to mention there are function f (t) is Laplace transformable if f(t) mod e to the power minus sigma t dt, 0 to infinity is a finite quantity that is it is less than infinity if you have a function say f (t) equal to say t squared or t cubed will this property hold good for this f (t).

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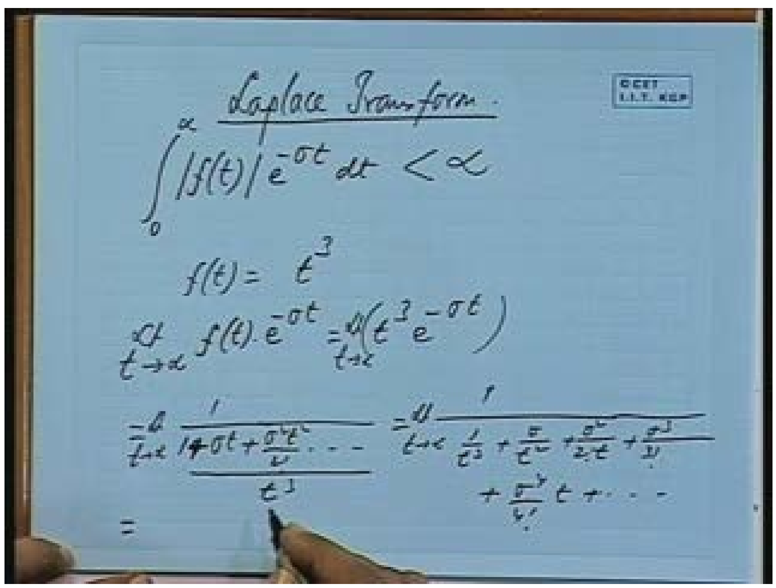

Let us see, f (t) in to e to the power minus sigma t as t_{10} ds to infinity, how much is this product t cubed e to the power minus sigma t okay limit t_{10} in to infinity, so what will be the limit. I can always write this as e to the power sigma t, I can write as 1 by e to the power sigma t so 1 plus sigma t sorry, 1 minus 1 plus sigma t plus sigma squared t squared by factorial 2 and so on and the t cube on the numerator can be brought back here, it can be brought down here. So this will be 1 by limit t_{10} in to infinity 1 by t cubed plus sigma by t squared plus sigma squared by factorial 2 t all right like that plus sigma cubed by factorial 3 t cube will get cancelled and after that you will get sigma 4 by factorial 4 t and so on okay. Now how much is this coming to 1 by t cube it will be 0, this will be 0, this will be 0, this will be finite. So it would be equal to 1 by sigma cubed by factorial 3 plus this will be infinite terms multiplied by t so it will be equal to 0 this is may be t 10 to infinity, so this will be 0.

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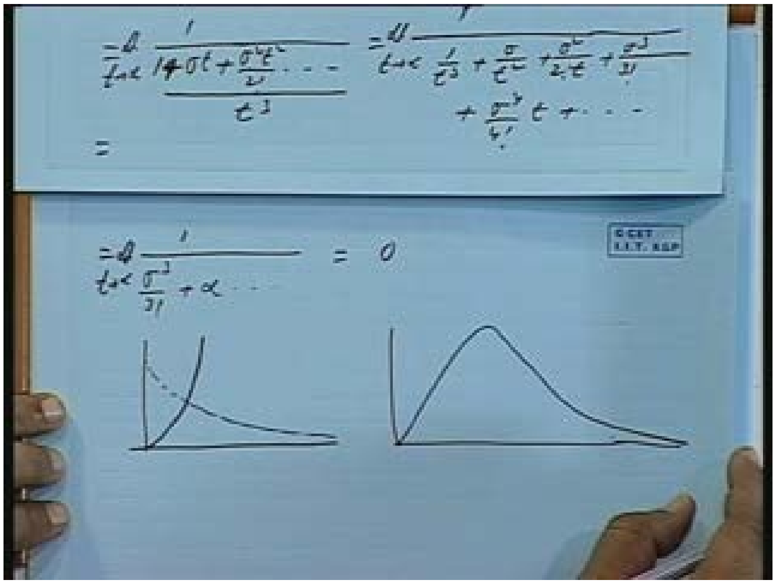

so a function now when t tends to 0, when tends to 0 this is a infinity, this is a infinity, this is a infinity, these are all 0's so 1 by infinity that is again 0. So what will this function look like t cubed is a function which is going up like this to be parallel alright and t cubed in to e to the power minus sigma t, e to the power minus sigma t is a function going like this. So the product will be starting from 0 ending at 0 so somewhere it may have a peak and then it will go like this.

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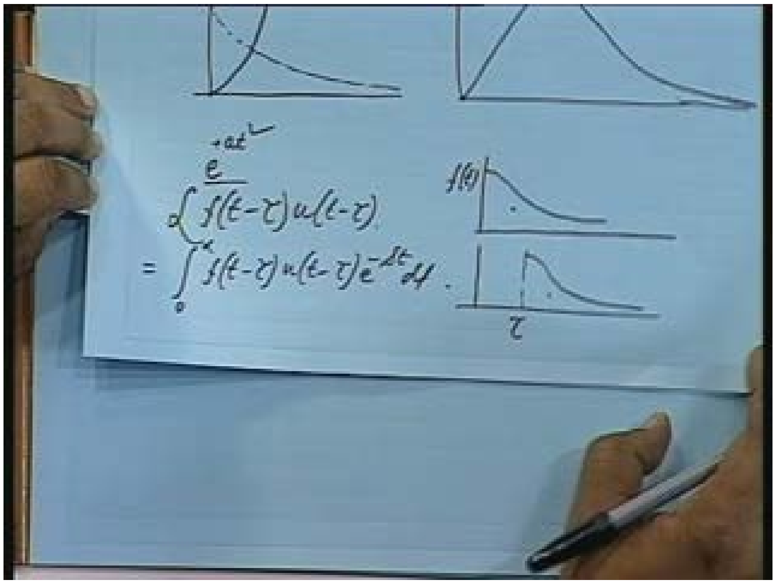

So the area will be finite this is going to 0, so area is finite. So I have just taken an example with t cube it can be show on that for any t to the power n it will be always converging to finite value and hence the integration from 0 to infinity will also give you some finite sum it will be less then infinity, so it is transformable. Functions of this kind, so e to the power minus or plus a t squared if f (t) is of this kind then you will find again by taking the limits this will not tend to a finite value. So this is not transformable but any other functions set e to the power n are functions of this kind they will be transformable.

So for any transformable functions you have to check whether this property holds good or not, f (t) may go to infinity but f (t) in to e to the power minus sigma t this integration this entire product if it goes to a finite value then and the integration if it gives you a finite value then it is transformable. Next will take up some properties of Laplace transform, Laplace transform of shifted function. Last time we discussed about some shifted functions how to represent them suppose f (t) is a function like this then f (t) minus tau in to u t minus tau is basically this same function starting after the interval tau okay.

So what will be the Laplace transform of this in terms of Laplace transform of this function okay, what will be the Laplace transform of this function in terms of the Laplace transform of this function, you would like to find out that. Now Laplace transform of f (t) minus tau u t minus tau will be equal to integration 0 to infinity f (t) minus tau u t minus tau e to the power minus s t dt okay, by definition. So that gives me equal to 0 to infinity now between 0 and infinity if I take a function now the function starts after tau that means it is 0 here. So the integration from 0 to infinity means tau to infinity so I can break it up in to 0 to tau f t minus tau, u t minus tau e to the power minus s (t) dt plus tau to infinity f (t) minus tau, u t minus tau e to the power minus s (t) dt and this part is 0, 0 to tau is equal to 0 plus tau to infinity t minus tau u t minus tau, e to the power minus s t dt.

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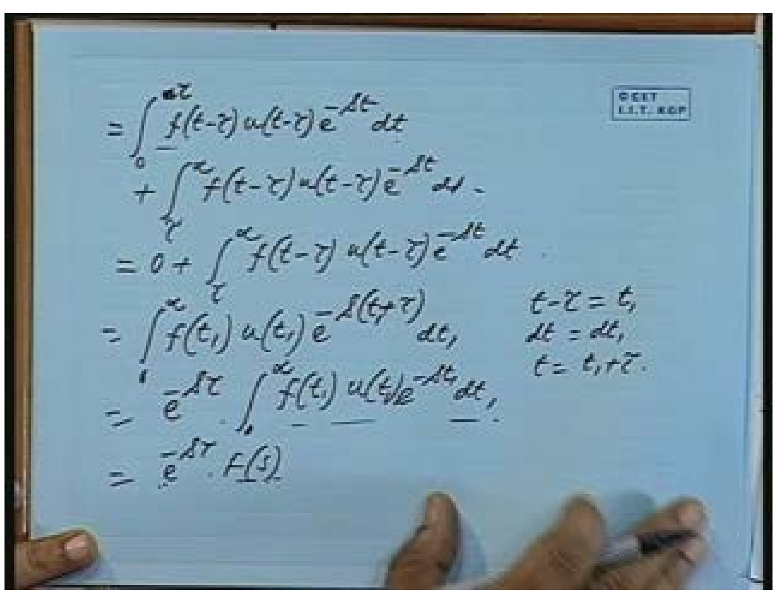

Now t minus tau I can put another variable t_1 so dt is equal to d (t_1) and t is equal to t_1 plus tau, so replace t by t_1 plus tau, so when t is tau when t is tau t_1 is 0, so this limit become 0 to infinity f (t_1) u t_1 actually f (t_1) u (t_1) from 0 it does not mean much here I can break it up in to e to the power minus s (t_1), e to the power minus s tau, e to the power minus s tau can be taken out because the integration is with respect to t_1, so thank you very much 0 to infinity f (t_1) u (t_1), e to the power minus s (t_1) d (t_1) and what is this f of s. So in terms of the original transform the shifted function has a transform that is e to the power minus s tau in to f (s) okay. So now let us derived some of those the Laplace transform of some of the shifted functions, we discussed yesterday.

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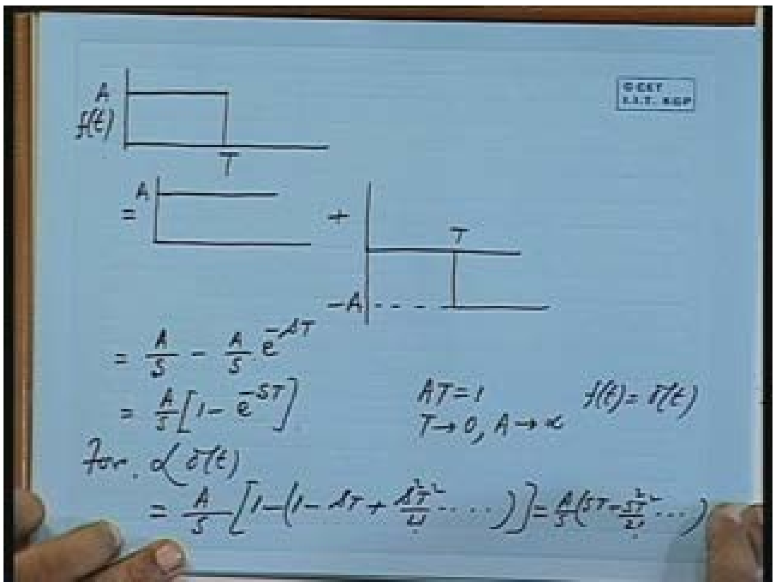

Let us take a simple example, suppose this is A, T this we wrote as equal to a function like this plus function like this minus A and plus A, is it not? So what will be the Laplace transform of the first function A by s this one A by s shifted by T, so multiplied by e to the power minus s t just now we have seen that any function if it is shifted by tau so the original Laplace transform multiplied by e to the power minus s tau, this will be the result. So this is A by s in to 1 minus e to the power minus s t okay. Now suppose we take T very very small T very very small 10 in to 0 and A is 10 in to infinity such that A in to T is 1 we get a unit impulse, is it not? When T tends to 0 and A tends to infinity but the product is finite and in this case if it is 1 then what will be this this will tend to for this f (t) is nothing but delta t okay.

So this value are f (t) so Laplace transform of delta t we want to compute will substitute that condition here A by s in to 1 minus what is e to the power minus s t, 1 minus s t plus s squared T squared by factorial 2 and so on okay. Now let me simplify here itself 1 will go A by s in to s in to T then will find 1 s will go s t squared and so on okay, A by s, s t minus s squared T squared by factorial 2 and so on. So that gives me equal to AT minus sorry As t squared by factorial 2 plus A s squared T cube by factorial 3 and so on and this AT is 1 and this is AT in to T, T tends to 0 means this may be all 0 okay T tends to 0. So Laplace transform of delta T is unity okay what would be the Laplace transform of a periodic function. So let us take periodic function like this and so on so this is of magnitude 10 this is 1, 2, 3, 4 and so on.

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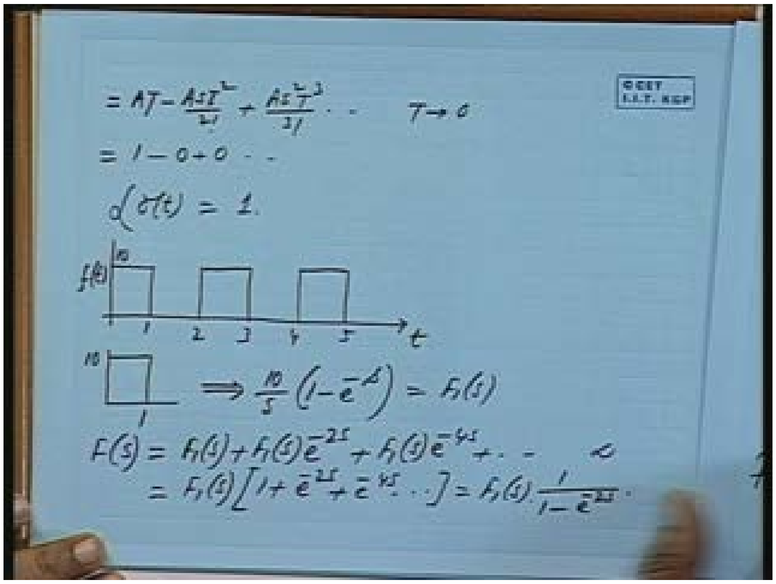

So let us take first of all the the first block which is repeated after every period, so what is the Laplace transform of this first block, could you please tell me? It will be 10 by s in to 1 minus e to the power minus 1 s is it not tau is 1. Let us call it F_1 s then what would be f (s) the same thing being repeated, so it would be F_1 (s) plus you F_1 (s) is repeated after 2 seconds so F_1 (s) in to e to the power minus 2 s plus F_1 s in to e to the power minus 4 s and so on up to infinity okay. So F_1 s if I take common will be 1 plus e to the power minus 2 s plus e to the power minus 4 s and so on which means F_1 (s) in to its a geometric series common ratio is e to the power minus 2 s. So 1 minus e to the power minus 2s, now F_1 (s) is already obtained substitute that what you get F_1 (s) equal to already known so just repeating it here therefore f s will be 10 by s 1 minus e to the power minus s, 1 minus e to the power minus 2 s okay e to the power 1 minus e s goes out, is that all right.

Let us take another example, periodic function once again like this this is 10 this is minus 10 okay 1, 2, 3, 4 and so on what would be the Laplace transform of this function? Can you write in 1 row, what is this? F_1 (s) is see if it is a same block being repeated with alternate signs then there is no problem I can take this as the primary block, first initial block whose Laplace transform is already known then what will be f (s) will be F_1 (s) in to 1 then minus e to the power it is shifted by 1 second and then its sign is changed so e to the power minus s plus e to the power minus 2 s minus e to the power minus 3 s plus e to the power minus 4 s and so on, is that all right. So how much is this common ratio is now minus e to the power minus s so it will be F_1 (s) by 1 plus e to the power minus s, so that is 10 by s_1 minus e to the power minus s by 1 plus e to the power minus s very familiar form know what is it?

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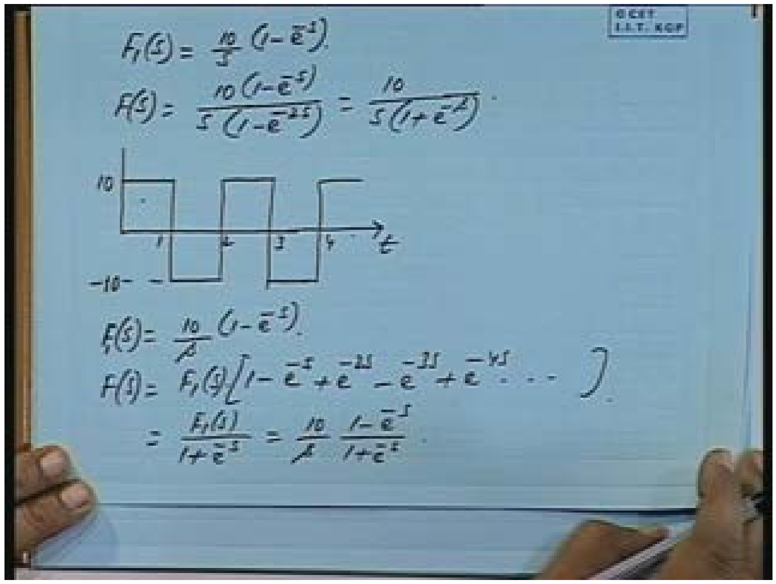

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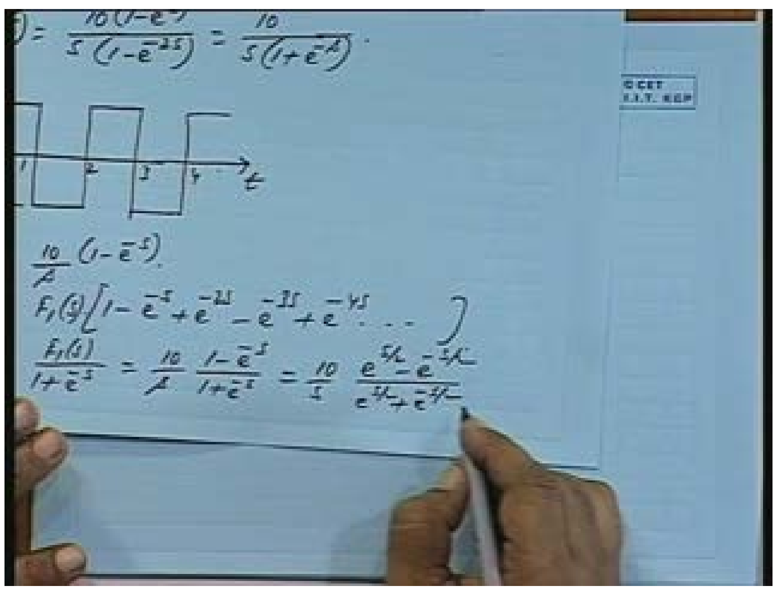

Can I can I not take e to the power s by 2 multiplied throughout by e to the power s by 2, so it will become e to the power s by 2 minus e to the power minus s by 2 divided by e to the power s by 2 plus e to the power minus s by 2. So this will be not 2 j why it j I am not putting j omega and all that this is simply cos this is sin or sin and cos, so it will be 10 so it will be 10 by s tan hyperbolic s by 2.

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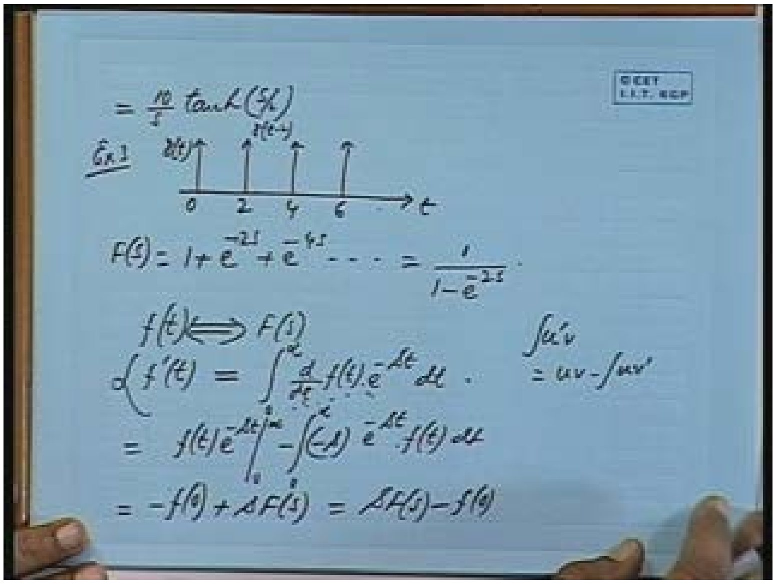

Okay another example, if I have a pulse train 0, 2, 4, 6 and so on this is delta t, this is delta t minus 2 and so on. So what would be f s for this for the first one it is 1 for the next one it is 1 in to e to the power minus 2 s plus e to the power minus 4 s and so on so it will be 1 by 1 minus e to the power minus 2 s is that alright if I have alternately coming say plus 1 minus 1 delta t minus delta t plus delta t minus delta t, we shift then only change will be here okay plus, very good. Now you have another important property suppose Laplace transform of f (t) is given as f (s) what would be the Laplace transform of f dash t, where f dash denotes the first derivative what will it be. So by definition this will be 0 to infinity d dt f (t) e to the power minus st dt.

So if you integrate by parts alright this you can take as integration of u dashed v is u v minus integration u v dashed okay. So if I take this as this as v okay this can be differentiated this can be integrated so it will be f (t) if I integrate this in to v, e to the power minus s (t), 0 to infinity minus derivative of this will become minus s in to e to the power minus s t and integration of this is f (t) dt so this 1 will give me if I put infinity this becomes 0 minus if I put 0 it will be minus f (0) minus and minus will make it plus and integration of f (t) e to the power minus s t dt is f (s). So it is s in to f (s) where f (s) is the Laplace transform of original function f t minus f at 0 okay.

Now we shall derive the Laplace transform of integrals of functions from here. You can go from the basic definitions there is another way of looking at it. Let us take f (t) as some f_0 (t), df (t) by dt as df_0 by dt as some f_1 (t) therefore d square f (t) by dt squared is df_1 t by dt as f_2 (t) okay. So f_0 (t), f_1 (t), f_2 (t) mean they are all successive derivatives and corresponding Laplace transforms we denote as capital F_0 (s), F_1 (s), F_2 (s) and so on. So just now we have derived F_1 (s) is what derivative F_1 (t) its Laplace transform is F_1 (s), so it will be in terms of F_0 (s), f_0 (s) minus f_0, f_0 at 0 is that alright.

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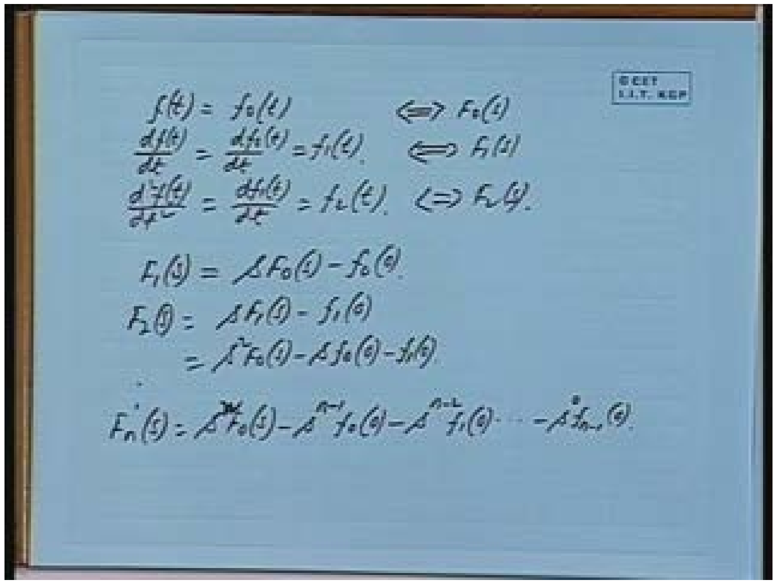

Similarly, F_2 (s) will be s in to F_1 (s) minus f_1 at 0 if I substitute F_1 (s) here it will be s square f_0 (s) minus s minus f_{10} that means if I take the nth derivative and its Laplace transform will be in terms of the original Laplace transform s to the power n sorry this n minus s to the power n minus 1 f_{00} minus s to the power n minus 2 f_{10} and so on minus s to the power 0 when I am calculating f_n then f_n minus 1 0 okay is just by induction. We have gone downward from f_0 derivative is f_1 next derivative is f_2 etcetera if you go up it will be integration all right. So these equations can be just a little bit of manipulation I can write on this side F_1 s is s f_0, so F_0 (s) is how much in terms of F_1 it will be F_1 (s) by s plus f_{00} by s okay.

So sorry, if you are given the Laplace transform of F_1 in terms of that that is f_1 (t) is known in terms of that can you compute the Laplace transform of its high order function by high order I mean the integral lower order means derivative, so F_0 s that is the integral of f_1 (t) what will be its Laplace transform. So F_0 (s) will be the Laplace transform of the original function divided by s. Now plus what is f_{00} the integral evaluated at t equal to 0 what does it mean? So it is like this if you have a current expression for example, current is for example, in a capacitor 1 by c integral I dt, 0 to t is the voltage at any instant if I take if I have to take the Laplace transform of this given the Laplace transform of I (t) as I (s) what will be v (s), v (s) is integral of I is it not. So if I have to compute v (s) it will be 1 by c in to integral of this means original Laplace transform divided by s then plus plus what is it I_0 what is f_0 it is integral of that so integral of I dt okay divided by s and this is to be evaluated at 0 from where do we start, so 0 minus to 0 plus or basically whatever has been there before the counting of time before t equal to 0 so that gives me integral of I dt is what charge q, is it not?

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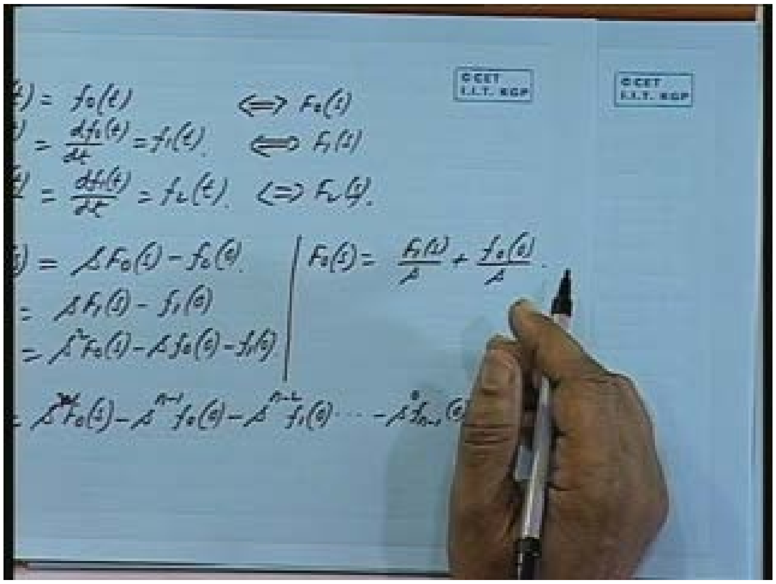

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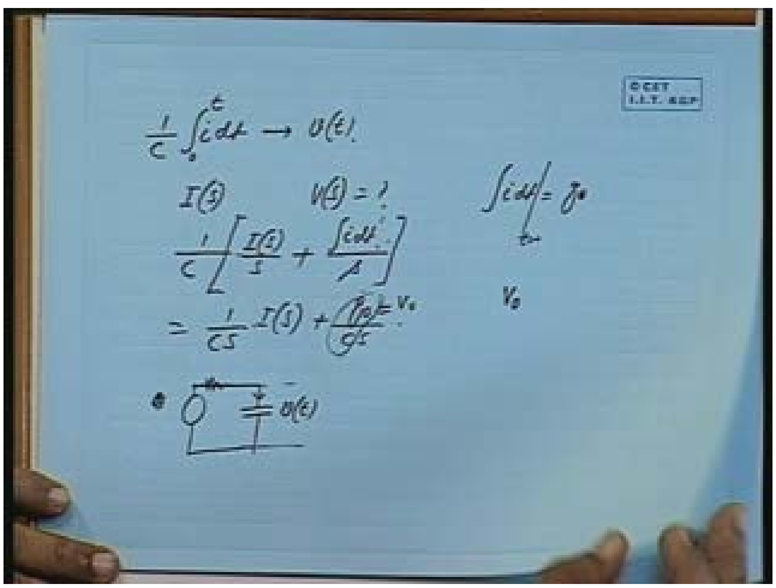

So integral I dt means this is to be evaluated at t equal to 0 means the initial charge q_0 alright mind you it is not initial value of i initial value of integration of i, when you take the integration so initial value of not f_1 f_0 means integration of f_1 alright. If the initial condition is given as 0 then there is no problem, so 1 by c s in to I (s) plus if I call it q_0 by c s, q_0 by c is nothing but initial voltage across the capacitor, so V_0 by s okay. So when we evaluate for example when we evaluate the voltage across a capacitor you are energizing by this that can be a resistance also you want to measure the voltage across this capacitor then its initial voltage plus at any instant if current is I_t if you want to deal with this problem in the Laplace domain then I (s) in to 1 by c s this total sum will be the voltage across the capacitor, initial voltage across the capacitor plus the drop due to this is the total voltage at any instant.

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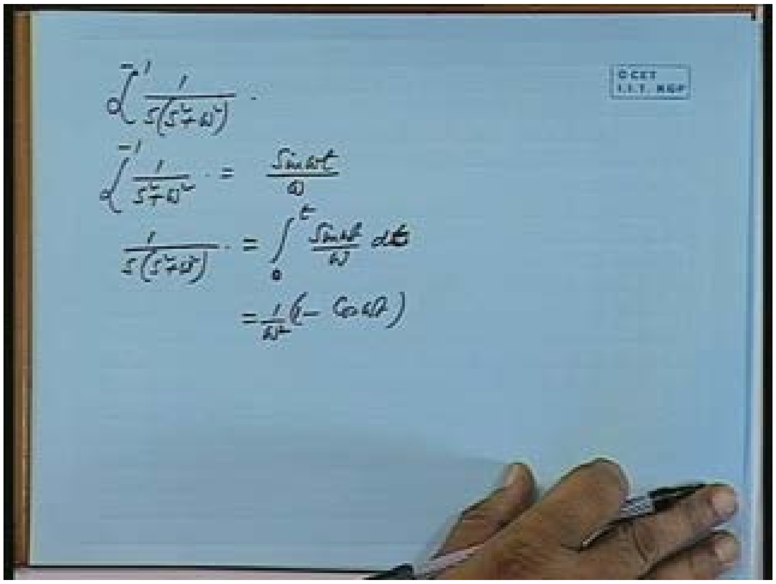

Now we will take up 1 or 2 small examples, then again will come to some other properties what would be Laplace inverse of say 1 by s in to s squared plus omega square 1 by s square plus omega squared what is the Laplace inverse, if you remember this is sin omega t what is the Laplace transform of sin omega t omega by s square plus omega square, so omega is not there so I will just write sin omega t by omega okay and just now we have seen if the initial conditions are not given otherwise mention say it will be treated as 0. So what would be therefore Laplace inverse of 1 by s in to s squared plus omega square 1 by s in to s square plus omega square it will be integration 0 to t, sin omega t by omega sorry dt, is it not? So that will give me 1 minus cos omega t by omega okay 1 comes because you are putting limit on sin cosine omega t. So this there is another omega so while integrating you will get omega squared sin omega t will give me another omega. So 1 by omega square 1 minus cosine omega t is that okay one may do it by partial fractions I can write this as A by s plus b s by s square plus omega square. Let us see how much is A multiply by s put s equal to 0.

Let it be 1 by omega squared how much is B multiply by s squared plus omega square divide by s, so will be and then put s square plus omega square equal to 0. So if I am multiply by s square plus omega squared this will go on the left hand side will be 1 by s then this will be s square plus omega square in the numerator and here it will be b s and then if I divide by s this will be s square this will be s square. So it will be 1 by s square on this side on this side it will be A in to s square plus omega squared plus B and now, I am making s square plus omega square equal to 0.

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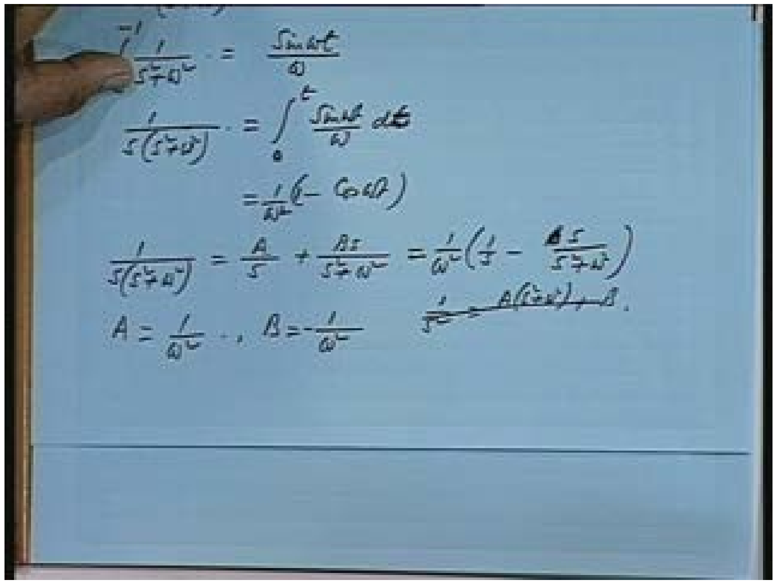

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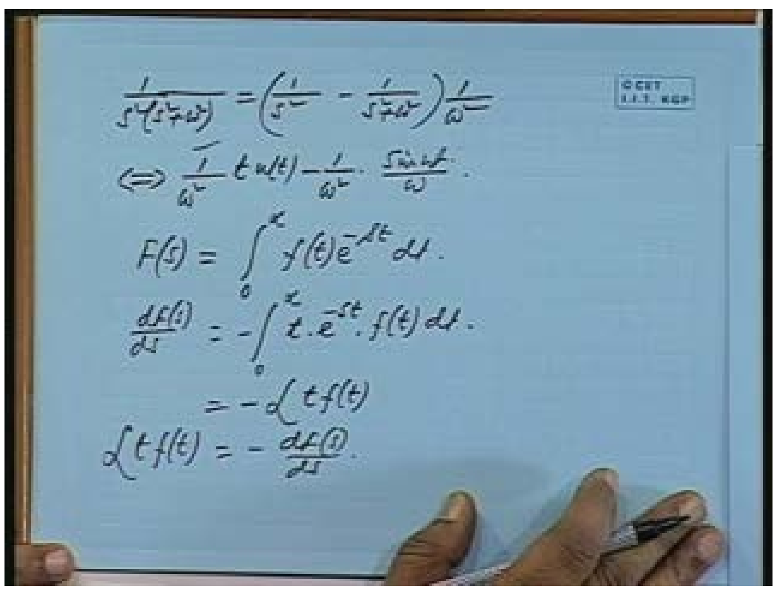

So that means 1 by minus 1 by omega square all right that will be B. So this one will be 1 by omega squared in to 1 by s minus B means 1 by omega square. So s by s square plus omega square. Again, if you take the Laplace inverse it get the same result, so you can do it by either method alright making use of the property of integrating a function or making partial fractions. Similarly, you can solve for1 by s square in to s square plus omega square. So if you know the result of this once again divide by s means once again an integral function, so you integrate this one again you will get the Laplace transform partial fraction will be simpler, is it not? will be 1 by s square minus 1 by s square plus omega square. So that will give me s square plus omega square minus minus s square.

So I will divide by omega square so corresponding inverse will be 1 by s square is Laplace transform form of what ramp function t in to u t okay and this 1 1 by s square plus omega square sin omega t by omega so it will be 1 by omega square t u t minus 1 by omega cube sin omega t okay. I need not write u (t) all the time I told you if you write t so long as you understand if it is a ramp function it is okay. Another important property will derive now so by definition we have f (t) e to the power minus s (t) dt. The common term is s common variable is s if I differentiate with respect to s what do you get if you differentiate with respect s it will be t in to e to the power minus s (t) with a negative sign f (t) dt and what is this Laplace transform of t, f (t) with a negative sign.

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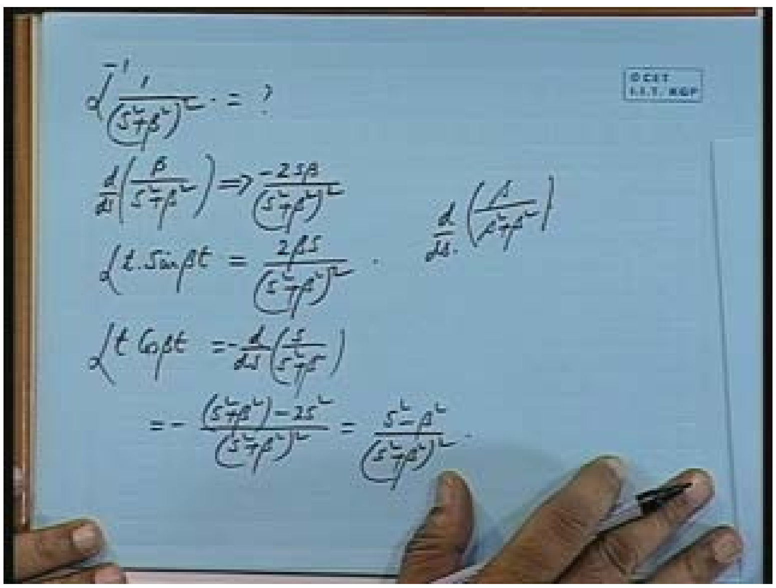

So Laplace transform of t in to f (t) is minus d f (s) by ds okay minus d f (s) by ds again we take a few examples, what will be Laplace inverse of this 1 by s square plus B squared if I take the derivative with respect to s what is it, derivative of this 1 by s square plus beta beta squared yes, could you please tell me 2s minus 2s okay. So if I put a beta here all right okay t into this is corresponding to sin beta t is it not t in to sin beta t, t in to sin beta Laplace transform of t in to sin beta t is how much then t in to sin beta t, t in to f (t) is d f (s) by ds with a negative sign alright.

So this is corresponding to sin beta t, so t in to sin beta t should be equal to derivative of this with respect with an negative sign, so derivative is this. So this is nothing but 2 beta s by s square plus beta square whole square is it not is that alright t in to cosine beta t what will the Laplace transform cosine beta t is s by s square plus beta square if you take the derivative of this how much is it? So d ds of with negative sign d ds of s by s square plus beta squared how much is that d ds of s by s square by beta square s square plus beta squared in to derivative of this minus 2 s squared divided by s square plus beta square whole squared.

So that gives me s squared minus beta square divided by s square plus beta square whole square. Okay the other 1 is t in to sin beta t_2 beta s by s square plus beta square whole square okay. So I can make any combination t in to cosine beta t plus minus t in to sin beta t Laplace transform of this will be s square minus b is beta squared by s square plus beta square whole squared plus minus 2 beta s by s square plus beta square whole square okay.

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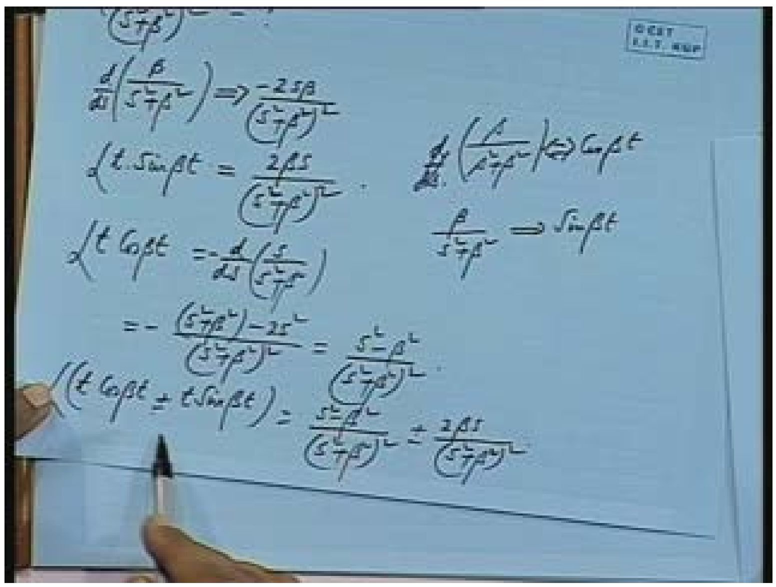

Now you already know beta by s square plus beta square corresponds to sin beta t and this one corresponds to cos beta t. So by making manipulations there are 4 results now 1 is this plus this this minus this so I have already got 4 such Laplace transforms by manipulating you can find out the Laplace transform of this because I have break it up in to see, if I make partial fractions or manipulate these I can get the inverse of this I leave it as an exercise you do it yourself okay, not very difficult. Next we take up application of Laplace transform in simple network problems, suppose you consider a simple RL circuit there is a DC source of voltage V and the switch is put on at t equal to 0 initially this is uncharged what would be the expression for the current i (t) you write V you are suddenly switching on. So it will be v u (t), V is a magnitude of voltage you are applying a step voltage and if I is the current when it is Ri, I am not writing i (t) it is understood i is a time varying quantity l di by dt if I take Laplace transform on both sides it will be V by s on this side it will be r is plus l di by dt and what is the Laplace transform of the derivative function s times I (s) minus I (0) if I assume the initial condition to be 0 then I_0 is 0 so that will not come.

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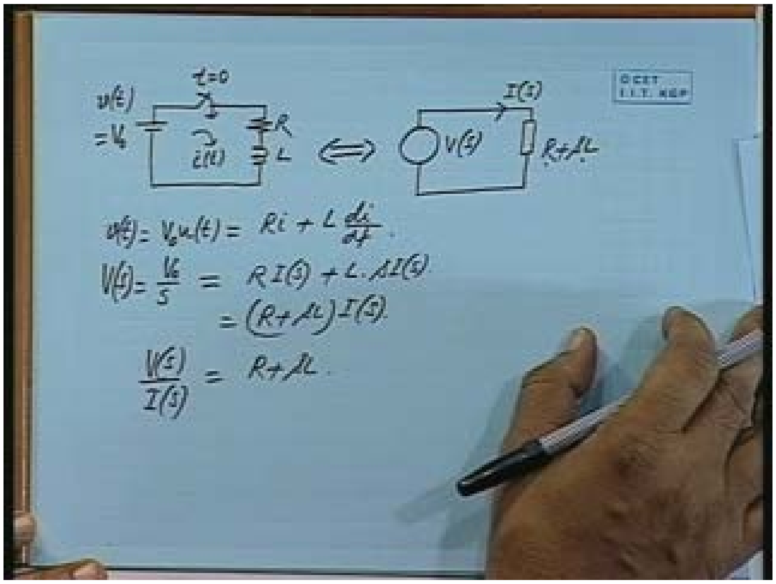

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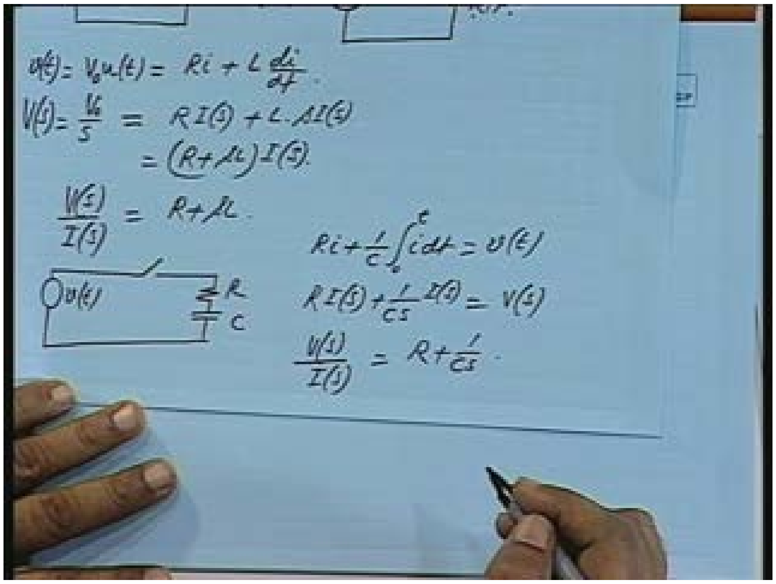

So it is R plus sL in to I (s) alright so like you are having voltage equal to impedance in to current in simple AC circuit. Here, also the applied voltage in the transform domain this is equal to R plus sL in to current in the time domain Laplace domain. So in the transform domain I will write V is 0 excuse me v_0 mean a specific value and v (s) is the Laplace transform of v (t) in this case it is V_0 in to u (t) okay. So v (s) by I (s) that means in the transform domain voltage function by current function is equal to R plus sL in the transform domain. So a resistance therefore these circuit I will write as v s the voltage source and an impedance which is R plus sL and the current is I (s) so an impedance is given by R plus sL for R and L in a similar manner we can show if there is an if the there is a capacitance, if there is a voltage source here v (t) Ri plus 1 by c integral I dt is equal to the applied voltage if I take the Laplace transform will be R in to I s plus 1 over c in to s in to I (s) once again if I assume the capacitor to be initially uncharged then that q_0 by c (s) will be 0, so this side it will be v (s) or v (s) by I (s) will turn out to be R plus 1 by c (s).

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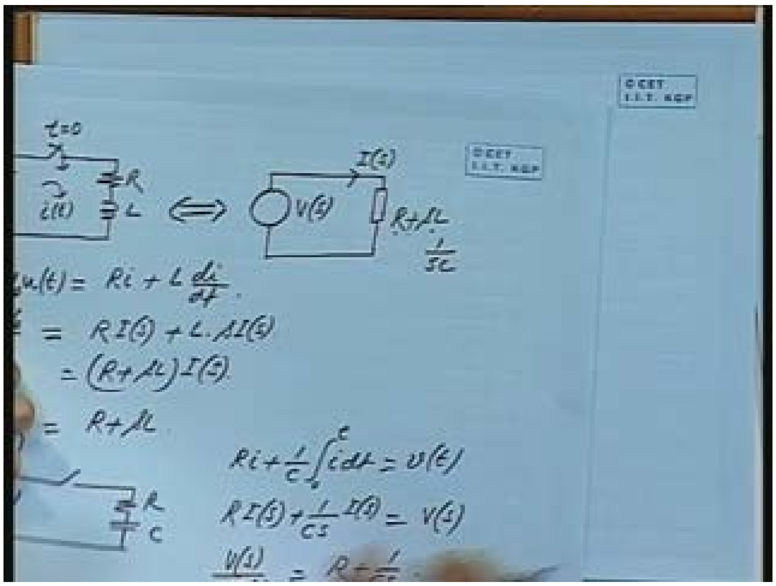

So if it is an inductive element it will be R plus sL if there is a capacitive element present here then it will be equivalently represented by 1 by sc. So if you have RLC wherever there are resistances you replace it by you return it that is resistance should be returned as it is inductance will be replace by sL capacitance will be replace by 1 by s c and in all network configurations next part will be using the transform domain variables are s_{11} by s c sorry if you do this then any network problem can be solved. Let us take 1 or 2 more examples, you have a source here a resistance here. Okay therefore, we shall replace this entire circuit by an equivalent v_1 (s), R_1 you can show it like this or a symbol like this can also be used for a generalized impedance z (s) it can be R it can be 1 by sL, 1 by sc it can be sL or any combinations this is R_2 plus sL_2.

Similarly, this 1 is R_3 plus 1 by sc_3 and this side you have got V_2 (s). Now for calculating the current through any branch or potential at any node you can apply supposition theorem, nodal analysis, thermos theorem, any theorem you want by treating these as simple algebraic operational impedances all right, treating s as an algebraic operator and treat this as it is and then when you get the expression for current I_s take the inverse you get the current I (t). So if I am interested in the current through this I_2, I_2 (s) what will be I_2 (s), we can go for nodal analysis.

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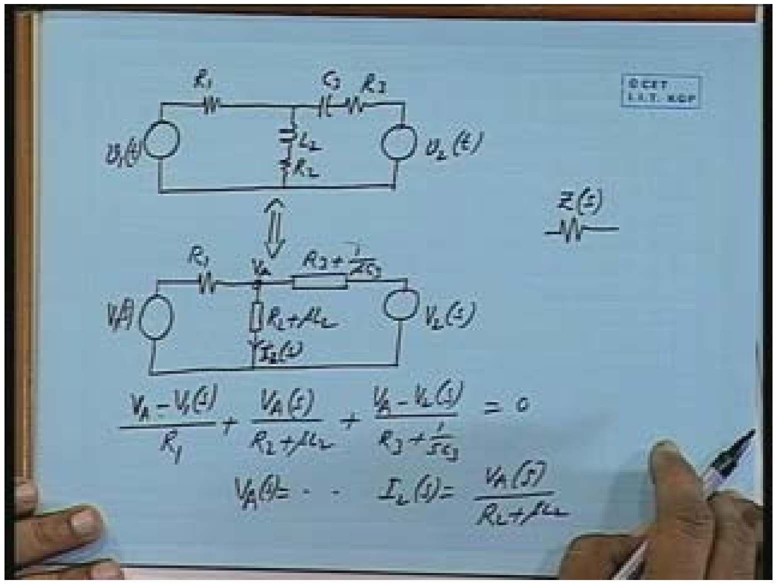

So let this potential be V okay V_A, so V_A minus V_1 (s) by R_1 plus V_A by R_2 plus sL_2 plus V_A minus V_2 s by R_3 plus 1 by sc_3 equal to 0. Now V_A, V_1 (s) and V_2 (s) will be given to you it may be stay function ramp function or any other function so put those values solve for V_A, V_A means V_A (s) okay once you know V_A (s), then V_A (s) divided by R_2 plus sL_2 therefore I_2 (s) will be V_A (s) by R_2 plus sL_2 okay. So far you have got the solutions in terms of s now you make partial fractions expand it partial fractions and then take the inverse you will get the current I_2 okay while making partial fractions sometimes you may come across multiple poles at a particular point say

You want to find out the partial fractions of say the denominator is s plus alpha 1 in to s plus alpha 2 and so on but s plus some alpha k is having n number of poles that is 3, 4 multiple roots then this can be written as A_1 by s plus alpha 1 plus A_2 by s plus alpha 2 and so on valuation of A_1, A_2 etcetera is very simple straight forward multiplied by s plus alpha 1 then make s, s plus alpha 1 equal to 0 will get A_1 and so on but this 1 will be some ak_1 by s plus alpha k_n plus ak_2 by s plus alpha k_n minus 1 and so on. Evaluation of this becomes a little difficult straight forward if you apply that technique then you will be able to get only a k_1.

So for evaluation of a k_1 you multiplied by s plus alpha k to the power n both sides, so f (s) is multiplied by this and then put s equal to minus alpha k okay. This product let me call it some q (s) evaluated minus alpha k then A_{k2} will be derivative of this q s and then evaluate at s equal to minus alpha k similarly A_{k3} will be 1 by factorial 2, d square q (s) by ds square, second derivative we take and then substitute s equal to minus alpha k and so on third derivative it will be factorial 3 and so on so you keep on taking the derivatives of this product and then evaluate the function at s equal to minus alpha k that will give you successive residues. Once you know that you can expand you can find out the inverse okay so will stop here for today. Thank you very much you try some other problems at home and will take up some numerical problems in the next class.

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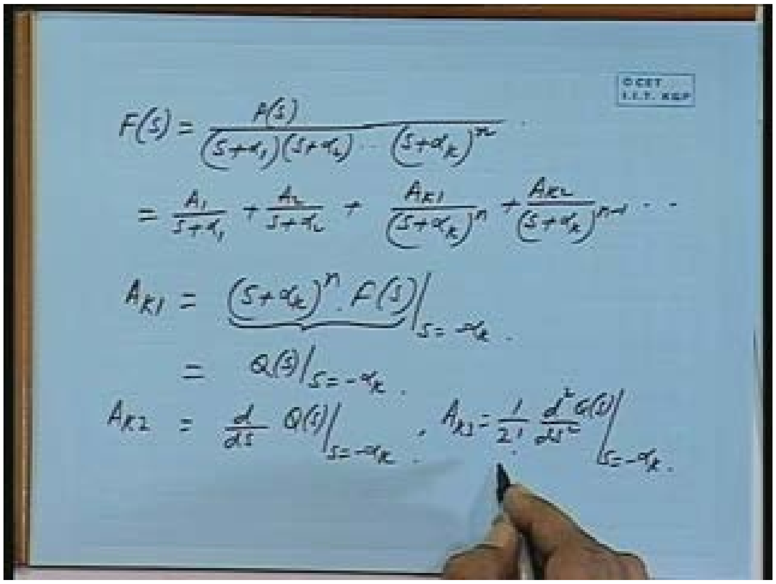

Preview

Lecture - 09

Tutorial on Laplace Transform-Application to Circuit Problems

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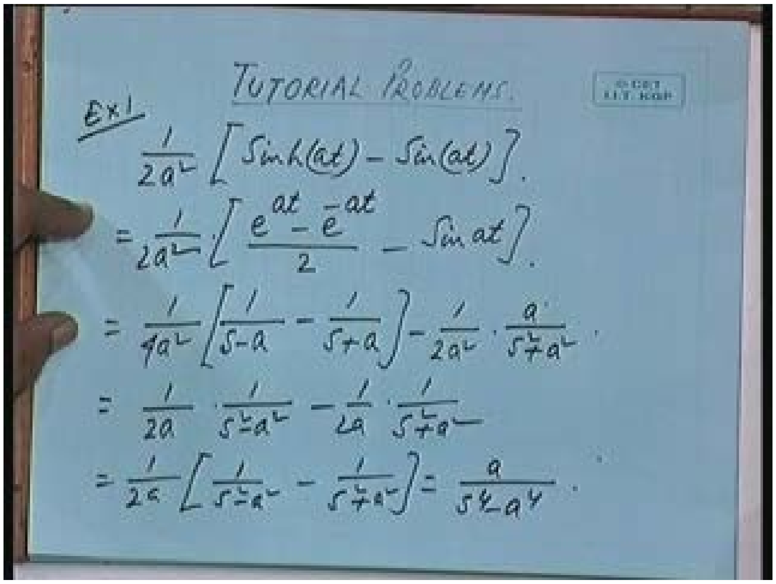

Okay Good afternoon friends, will continue with Laplace transform. Today will have some tutorial session, will be solving some problems employing Laplace transform. Let us take simple functions what be the Laplace transform of 1 by 2 a squared sin a t minus sin at, what be the Laplace transform of this? So this 1 can be written as 1 by 2 a square what is sin i e to the power a t minus e to the power minus a t divided by 2 minus sin a t. So it is pretty simple 1 by 4 a square e to the power at will be 1 by s minus a minus 1 by s plus a is that all right minus 1 by 2 a squared sin a t will give you a divided by s square plus a square, is that okay? So if I add these 2 it will be s plus a minus s plus a so twice a, so that will be 1 by 2 a into s squared minus a square minus here 1 a goes, so 1 by 2 a 1 by s squared plus a square.

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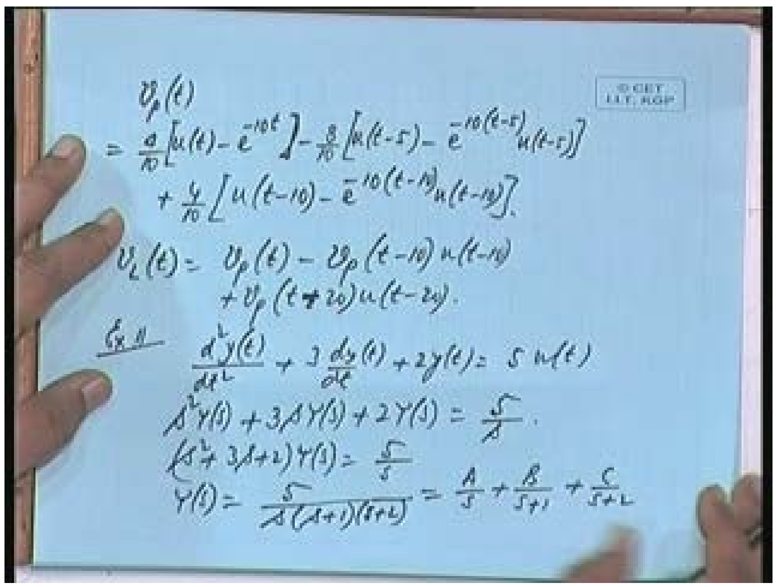

So 1 by 2 a I can take out it will become 1 by s square minus a squared minus 1 by s square plus a squared and that gives me s square plus s squared minus s square. So twice s squared twice a will go, so a by s to the power 4 minus a to the power 4 okay plus 2 times y (s) is equal to 5 by s so it is s squared plus 3 s plus 2 in to y s which is 5 by s. So how much is y (s) 5 divided by s in to s square plus 3 s plus 2 can be written as s plus 1 in to s plus 2. Okay I can write it as A by s plus B by s plus 1 plus c by s plus 2 and taking the inverse of these 3, I will get the final y (t) as A into u t plus B in to u t the power minus t plus c in to u t the power minus 2 t where A B C residues can be calculated from here by standard partial fraction technique, okay. So we will stop here for today we will take it up in the next class with further examples using Laplace transform for network problems. Thank you very much.