Networks Signals and Systems
Prof. T. K. Basu
Department of Electrical Engineering
Indian Institute of Technology, Kharagpur
Lecture - 07
Signals Laplace Transforms (contd…)

Okay, so late later we will continue with the discussion on signals.

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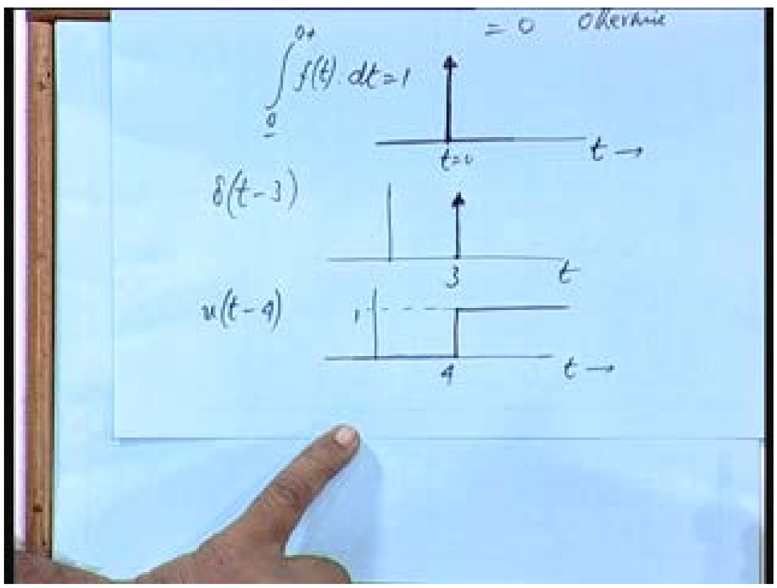

Last time we are discussing about impulse functions. Unit impulse will be denoting by delta t this is a function whose value is 1 at t equal to 0 and equal to 0 otherwise, where this function is representing the force so this is from 0 minus to 0 plus or you can write infinity it really does not matter that means the function is coming here its magnitude is very very large and duration is very very small and then it is going and this thick line represents something equivalent to this area all right.

So the integral is finite when this is 1 it is a unit impulse okay, this is time t and this is a function we normally show it by an arrow and a vertical line impulse appears at this moment t equal to 0. So delta t minus 3 represents what this is a shifted version, shifted version of the same impulse that is at t equal to 3 this function appears okay. Similarly, what will be u t minus 4, it is the shifted version of unit step okay, u t minus 4 represents a function which starts after t equal to 4 or at t equal to 4, it is unity after that before that it is 0.

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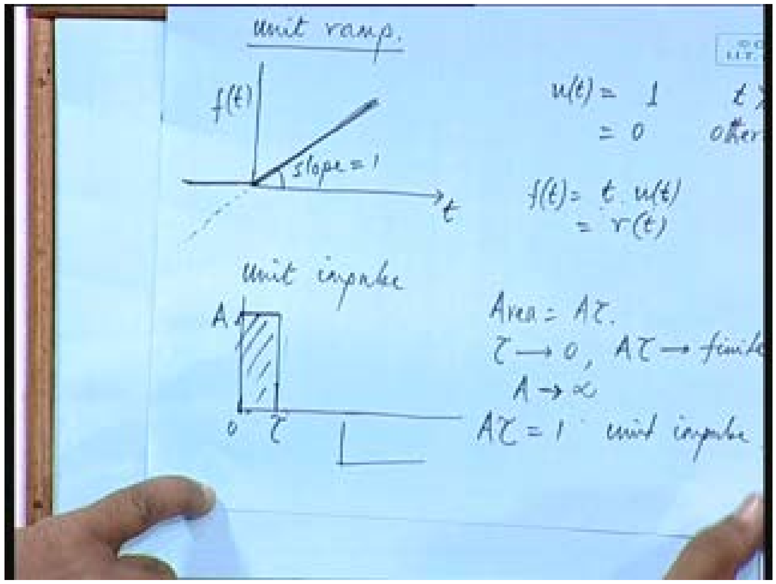

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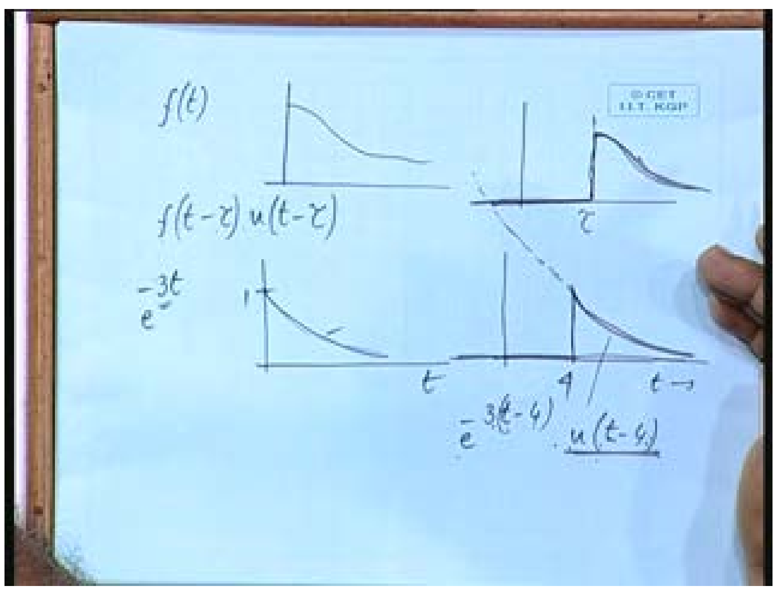

So any function if we shift f(t) is a function so like this, then what is f(t) minus tau if I want to represent the same function which is starting after an interval tau, if it is identical okay then what would be the mathematical representation of this function f(t) minus tau does it represent this f(t) minus tau represents this function on this side all right but what about this value I want to show that this is 0 till that time. So this has to be multiplied by u(t) minus tau so f(t) minus tau in to u(t) minus tau represents this function okay.

So there is a finite delay of tau, so e to the power minus 3 t what is this function like exponentially decaying okay starting with 1. So if I want to represents this same function which is shifted by 4 seconds say what should I write for this e to the power minus 3 e to the power minus 3 t minus 4 u t minus 4. Now can you tell me if I do not write this what kind of function will it be if I do not write this function this also has a value here is it not so my interest is to express this function which starts only after t equal to 4 but which is having an exponential decay rate given by these e to the minus 3 t.

So in the time domain I will write e to the power 3 t minus 4 in to u t minus 4. So you will find this u function u t minus 4 t minus tau and so on this be clubbed with this to represent a function which is starting which is starting after tau after 4 seconds and before that it is always 0 okay.

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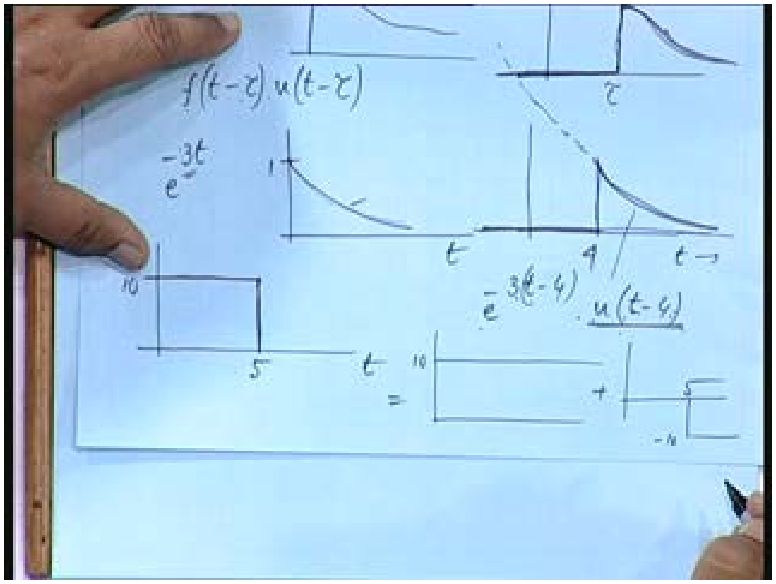

Now tell me how I represents a pulse by continuous functions suppose this is 5 seconds, this is 10 what will be the representation of this in terms of known standard continuous functions I can write this as sum of this function of magnitude 10 plus function which is minus 10 after 5 seconds okay. So what will be the mathematical form for this 10 u t then minus 10 u t minus 5 okay? So this will be representing this function okay. Let us take a few more examples what would be suppose this is 20 this is 4 seconds what be the representation of this yes, 20 20 by 4, it is having a slope of 20 by 4, 20 by 4 t. If I take 20 by 4 t this will be a continuous function but then it has to be interrupted at 4 second, 4th second that is at t equal to 4 it has to be brought down to 0. So minus now 20 by 4 in to t represents this that will be there at t equal to 4 I have to add something to counter this so that it is brought to 0.

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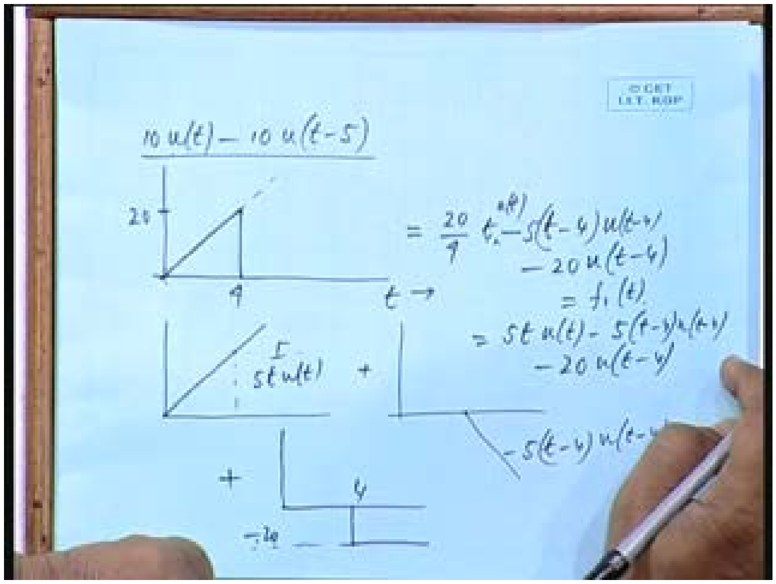

So minus minus t t minus 4 again t minus 4, t minus 4 represents what t minus 4 in to u t minus 4 u t minus 4, actually with this t also I should have written ut so and what about 20 by 4 slope, there is no slope here 20 by 4 is 5 so 5 in to t minus 4 u t minus 4 will that be all right. Let us see what it is 20 by 4 there is 5 t this is the first 1 5 t, u t okay or 5 in to r t somebody may write 5 in to r t ramp plus minus 5 t minus 4 u t minus 4 all right.

So what will be the end product this 1 will the end product be this one, all of you agree yes then anything else what is it see this is going on increasing after 5 seconds you are applying a negative ramp of the same slope, so the increment will be counted by the decrement you are off setting the increment by having a decrement at the same rate what about the existing value then it will stay there at that value. You are subtracting the increment with the decrement is it not it is like this you deposit every month 100 rupees in the bank. So after 12 months, after 12 months you deposited 100 rupees and your brother takes away 100 rupees, you are having a joint account all right.

So your addition is offset by the subtraction then how much is the money left in the bank all the time it will be remaining at the 1200, in the first area accumulated 1200 rupees. So it will be staying there at that constant value, so here this plus this will not suffice then plus you have to take out that 1200 rupees also then only it will become 0. So it will come to 0 value only when you are applying a negative step of magnitude how much 20 okay. So 20 by 4 t in to u t minus 5 in to t minus 4 u t minus 4 minus 20 u t minus 4 that has to be applied at the same time. So summation of these 3 will give you this is that all right. If I call it some f_1 (t) that is simplified from 5 t ut minus 5 t minus 4 u t minus 4 minus 20 u t minus 4 okay.

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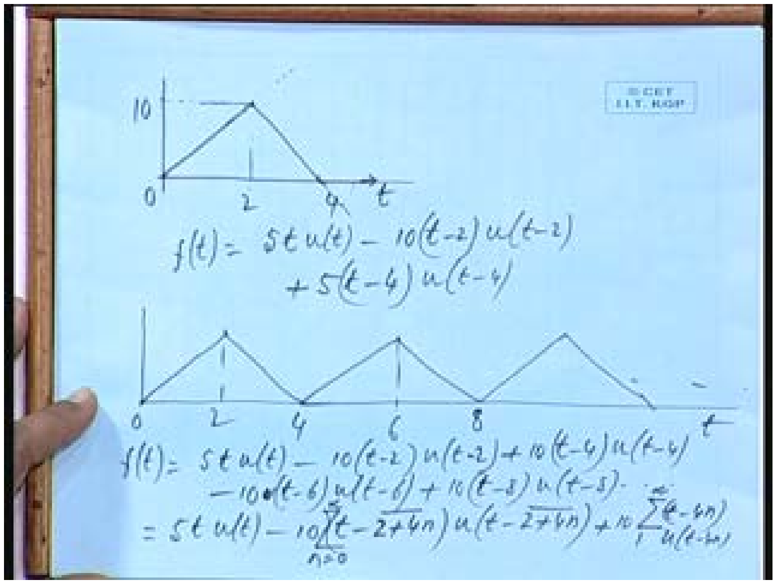

What would be a representation of this? Suppose this is 2 seconds, this is 4 seconds and this is say 10 what would be the representation of this? This is having a slope of again 10 by 2, 5, so f(t) will be 5t u(t) agreed 5t u(t) that will be taking it along this line but at t equal to 2 what I do, I give a decrement. So now tell me what should be this minus 5 minus 5 t minus 2 u (t) minus 2 it is twice the slope because something was going up you want to offset it by 5 t then it will be coming to horizontal position, another 5 t will make it this way with a negative slope. So minus ten t minus 2 u(t) minus 2 will that be all right.

So this plus this will that give me this or this will continue continue is it not so that has to be again off set by plus plus how much 5 plus 10 10 t because it was going up then to bring to this position you had given minus 10 t is it not then again you had to turn it to that position it has to be plus 10. I want to make it horizontal if I give plus 10 t what happens, it will rise again it will start rising okay I am coming to that in the next problem. So here it will be plus 5 t minus 4 4 u (t) minus 4 okay. Now the next question is suppose I have triangular function which is repeat it which is repeating and so on.

So what will be the expression for 0, 4, 8 and so on, this is 6, this is 2 and so on. So what will be this f(t) will be 5t u(t) okay minus 10 t minus 2 u t minus 2. Now plus plus 10 t minus 4 u t minus 4 very good, then minus minus, how much? So it is alternative plus 10 and minus 10 at an interval of 2 seconds okay. So minus 10 t minus 6 u (t) minus 6 plus 10 t minus 8 and so on. So generalize it u t minus 8 and so on. First 1 you keep aside 5 t u (t) and after that minus 10 t minus 2 minus 10 t minus 6 and so on.

So minus 10 t minus I can write 2, 6, 10, 14 so t 2 plus 4 n, n is an integer u t minus 2 plus 4 n okay, where this summation n varying from 0 to infinity. So n equal to 0, 1, 2, 3 and so on similarly plus 10 again you put a sigma t minus t minus 4 n, t minus 4 n, n varying from in to u t minus 4 n n varying from 1 to not 0, 1 all right 1 to infinity is that all right because of first 1 is 5 t u(t). So this will be the general expression for a periodic function.

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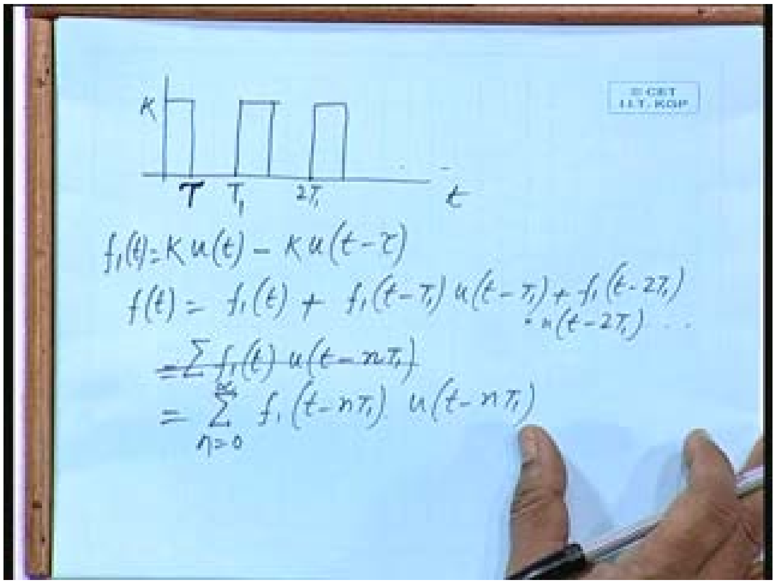

Similarly, if I have a pulse train you are gradually going towards a periodic function, pulse train of width tau and period t, T_1 there are likely chances of confusing tau with T_1 t so I am putting T_1 and this is our magnitude say k and it is appearing periodically what will be the expression for this type of function, for this type of function. For the first pulse what is the expression k u t minus k u t minus tau okay. This is if I call it f_1 (t) this is the first 1 the same thing repeats after T_1, T_2, T_2 is 2 T_1 and so on so ft can be written as f_1 (t) plus f_1 (t) minus T_1 in to u t minus T_1 okay and so on in to u t minus $2T_1$ and so on.

So it can be written as f_1 t u t minus n T_1 is it f_1 t or f_1 t minus n T_1 yes f_1 t minus n T_1 u t minus n T_1 submitted over n, 0 to infinity. So that represents a periodic function and you can substitute for f_1 (t) this 1 okay so every time tau will also appear here okay, n T_1 minus tau okay. When I incorporate this 1 of the terms will be this is having 2 component so you have to write like this. Another example I would like to take up what is the representation of this function 0 suppose this is 10 and this is say 10 seconds. This is a sinusoid just 1 half of a sinusoid and that the function remains 0 after that okay after half the cycle it remains 0.

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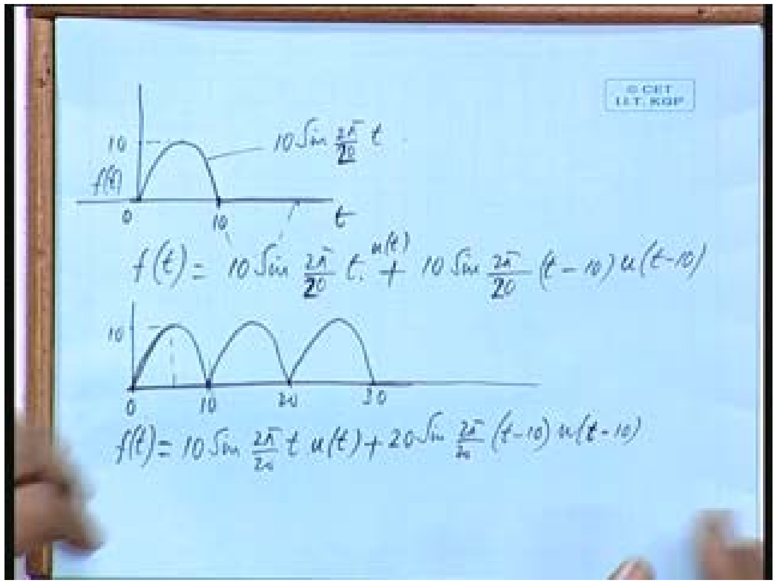

So this can be written as this portion is 10 sin twice phi by 10 t okay only this portion is following this so how do I write f(t) if this is f(t), f(t) if I write a function like this then it will continue but I am taking only 1 part of it. I have shown the expression for this only this part for this part but actual function is like this is it not if I write like this only then it will continue okay so what should be f(t) 10 sin 2 phi by 10 t minus what what do it {sub} (00:22:58) what do I subtract 10 sin 2 phi by 10 t minus 10 I subtract in to u do you all agree, do you all agree if I subtract this from here what do I get. Every time you are having subtraction in your mind whenever the function is terminated all right you are applying a negative function of the same magnitude and trying to offset the existing function and making it 0 but you see this function is continuing like this see if I just add these function at this point then that will be countering it basically the delay is 10 seconds all right.

So if I add this function itself which is delayed by 10 seconds the positive part will be countered by the negative part all right. Here the negative sign need not come is that clear therefore if I have a repeated function like this what is it actually rectified, full if rectifier output, rectified sound sign sign if so what will be the expression for this the same magnitude say 10 and was this all right, yes 10, 2 phi by 20, 10, 2 phi by 20 basically this is not the period this is the period 20 so 2 phi by 20, 2 phi by 20 okay. Now suppose this is 10, 20, 30 now you tell me what will be f (t) like what should I write? 10 first part sin 2 phi by 20 in to t mind you for such functions unless it is a delayed function I am not writing u (t) it is understood that we are not considering the negative part of time.

So far is the first part is concerned otherwise I should have to be more precious I should have written it multiplied by u (t) the first 1 okay then it will be precise okay, it does not exist on this side either sot in to u (t) now what will be this function what be the representation 2 phi by 20 t in to u(t) is this one plus 20 sin 2 phi by 20 t minus 10 u (t) minus 10 what does it give me if I add plus 20 then it will be this one okay. Then again add if I take this plus this what happens negative let us draw and then see this plus this gives me what, this one is 10 plus there is another sinusoid that I am adding after half the cycle of magnitude twice okay because of the space here I am so it is like this and so on so if I add this with this what happens it would be this then minus 10 and plus 20.

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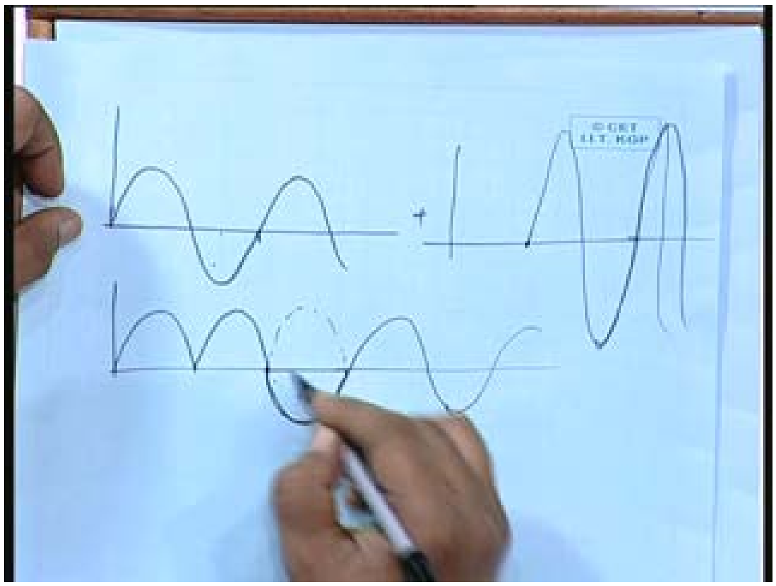

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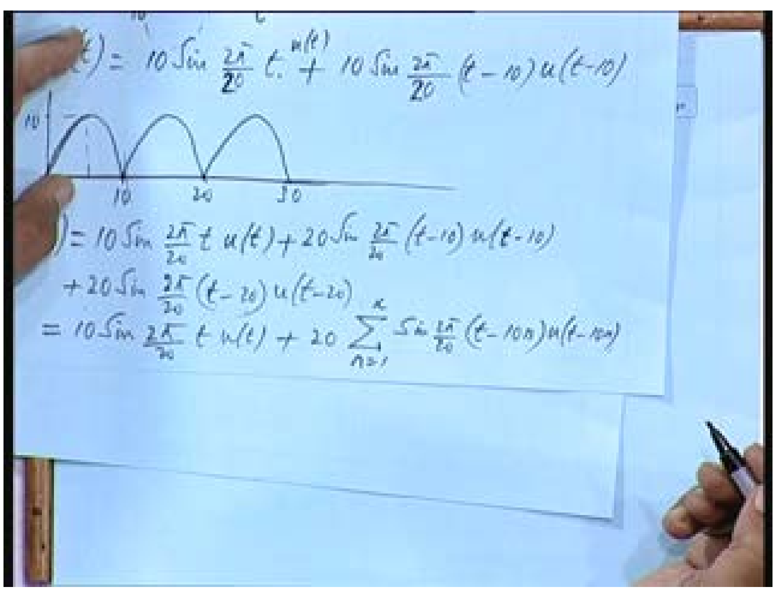

So this will be plus 20 then after this point this is plus and this is minus 20 so it will become is it not so at this point again I have to add sir plus 20 plus 20 to cancel this as well as add another 10 is that all right. So plus 20 sin 2 phi by 20, t minus 20 u (t) minus 20 and by the similar logic you keep on adding 20 sin 2 phi by 20, t minus 30, t minus 40, u (t) minus 30, u (t) minus 40 and so on. So the general term will be if I now write this as 10 sin 2 phi by 20 t in to u (t) plus 20 summation sin 2 phi by 20 t minus 10 n where n is an integer integer varying from 1 to infinity u(t) minus 10 n is that all right, u (t) minus 10 n. So you know how to represent periodic functions where the periodic function a will be consisting of in your shifted version of some continuous standard functions.

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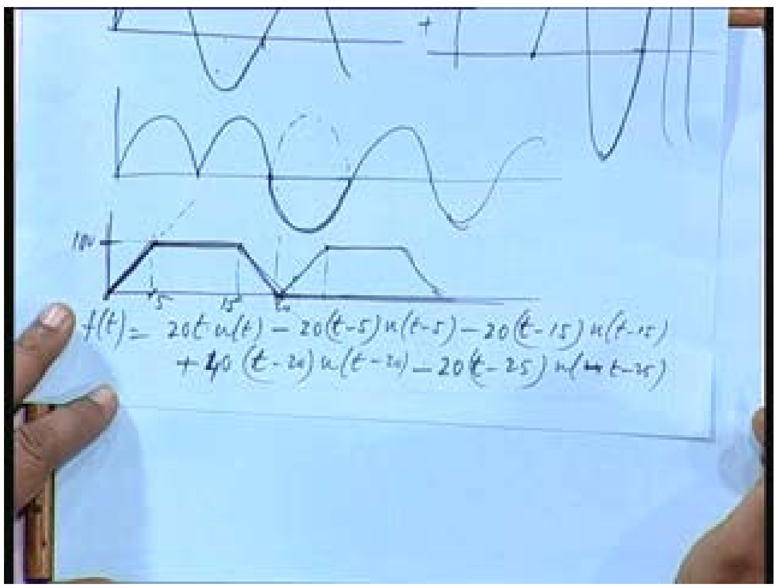

If you have a trapezoid, what would be the representation of this function or if it is repeated a repetitive function like this. So will be interested in finding out first of all the first period and then keep on repeating okay. So here it will be if the slope is k then k in to t all right whatever be the slope accordingly f (t) will be, suppose this magnitude is 100 this is 5 then it will be 100 by 5 20 t u (t) is it not, it would have continued like this. So at t equal to 5 we apply a negative slope of the same value then it remains there. So minus 20 t minus 5 u (t) minus 5, now I do not have to add anything else because I am not bringing it to 0 like the earlier case it remains there then at this point suppose this is 15 and this is 20 then minus at this point I apply again a negative slope depending on the slope here I have put identical values that is 50 in to 25 seconds range 0 to 5 was also 5 seconds were the slope will be same.

So it will be minus 20 t minus 15 u (t) minus 15 and then it would have continued like this I can take this 1 block and then keep on repeating it all right. If I do that then at 20 I should bring it to 0, so it was otherwise continuing so at 20 what should I do plus 20 t minus 20, u (t) minus 20. So these 4 functions collectively will give me the first problem that means the function remains constant after this it goes like this and then remain constant at 0 value, if it remains constant at 0 then that will be giving you the first period. Now you keep on repeating this this is one approach the other is at this point also I add twice the slope again at this point I apply apply what minus 5minus 5t, 20t okay.

Let us write then so you are adding here 40 okay 40t then what will be the next plus or minus minus should it be 40 or 20 see the after adding 40 you are getting a net increment of 20, so that 20 has to be offset so t minus 25, u (t) minus 25 and so on. So you keep on writing further terms so this is the other approach so if you are adding 40t here that will be giving me an overall function of this kind so I have to bring it down only 20 t not by 40t here okay.

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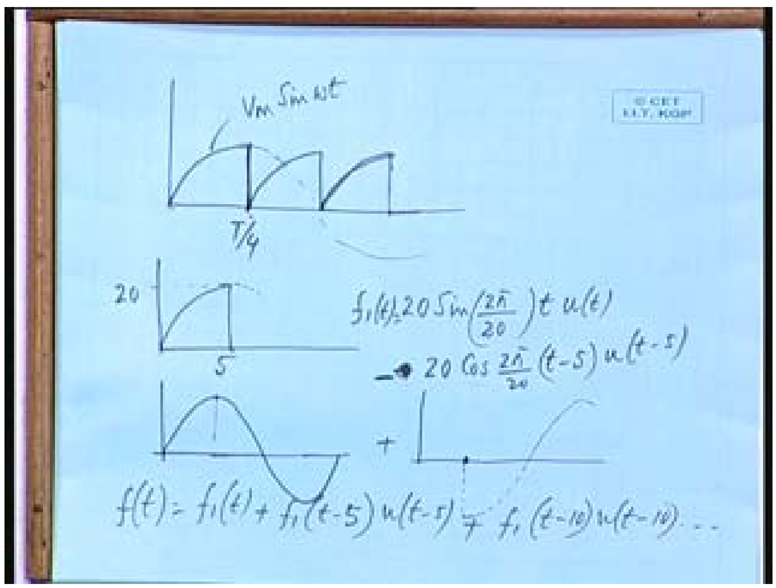

Sometimes you are given truncated sign waves this is the kind of function you get in a capacitor discharge or capacitor charging and discharging circuit. If the discharge is very fast this gives rise to basically the saw tooth function is it not sinusoid basically exponentially increasing functions. Suppose these are part of sinusoid that means 1 quadrant 90 degree, 0 to 90 degrees we take this is say in this form V_m sin omega t okay. This is a part of V_m sin omega t function but then that is truncated and that is being repeated what will be the function representation.

So up to this is t by 4 one 4th of the time period, so up to this t by 4 whatever will be the time it will be following this. So let us get the representation of this suppose the original period is suppose this is 5 seconds this is 20 then it is 20 sin how much 2 phi by 20, 20 because 4 times 5 will be the time period in to t in to u (t) this is the first part but then at 5 that is at t equal to 5 what do I add 20 what do I add after t equal to 4.

Let us see what we have to add this is one function plus addition of what will give me this value 0 from this point onward okay and what is this function cos very good cos 2 phi by 20 and then cos shifted by am I all right by writing this should it be plus or minus cos 0 would have been plus so it will be minus u (t) minus 5 that has given me only the first one okay and then you keep on repeating it with an interval of 5 seconds, the period is 5 seconds okay. So if you can represents this as f_1 (t) as the first block then the desired f (t) will be f_1 (t) plus f_1 (t) minus 5 in to u (t) minus 5 plus f_1 (t) minus 10, u (t) minus 10 and so on okay. We shall be using this at a later stage when we use Laplace transform a functions such repetitive functions we will see how to take care of this for determining the Laplace transform for transience we for the transience in networks, we use Laplace transform very extensively.

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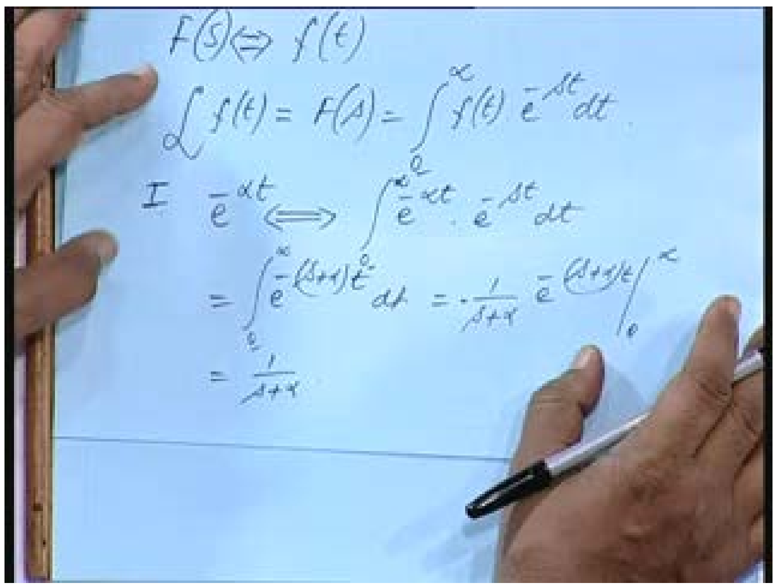

So today I just give you an introduction to Laplace transform, we shall take it up in greater detail in the next class when you go for some kind of a transform. Now transformations are required for simplification of certain competitions, for example when you take the products of 2 quantities, the quantities have to be multiplied the computational rigger involved may be privative. If you take logarithm logarithm is basically transforming the quantity in to a different domain all right then the multiplication operation becomes very simple you will add there and then you go for the inverse transformation that is taking antilog you get the product of the multiplication. So in the normal algebraic operations a in to b which is equal to c if you take in the transform domain it becomes log a plus log b which is equal to log c. So in the transform domain you add them the operations are also changing multiplication becomes addition and then after addition if you take inverse transform you get back the original product.

Similarly, in Laplace domain what we do in transient analysis you have seen you have differential equations governing inductance, capacitance, resistance all other elements in a circuit governing the dynamics of the system. They will have 2 types of solutions complementary solutions, particular integral, you also put the boundary conditions at the end to determine the coefficients the constants. Now all these can be computed in 1 row that is you get both the transient as well as the steady state solutions you get all the constants also evaluated right in the beginning you do not have to put it at the end you can evaluate them right from the beginning.

So that is an advantage you do all the competitions together and for higher order differential equations specially when you have more than 2 or 3 variables it becomes very difficult to solve differential equations. Suppose you are having number of loops and you have number of differential equations in involving currents in different loops, voltage sources. So all these can be simplified a differential equation is transformed in to an algebraic equation in to an algebraic equation by Laplace transform. So in the transform domain equations become very simple with simple algebraic variables like XY, so we use an operator s so it will be s in terms of s you get all the equations, s is the variable. You solve for it then you go for inverse transformation to get the solution in the time domain this is the beauty of this transform.

Now by definition we write F (s) as the Laplace transform of the time domain function f (t) okay and this is given by a curly L f (t) that is Laplace transform of the function f (t) is equal to F (s) that is 0 minus infinity f (t) e to the power minus s (t) in to d (t). We define the function like this so we will take from this very definition the very typical signals that are used in the literal circuits and determine their Laplace transform okay. Let us take a function e to power minus alpha t as the function f (t) what will be its Laplace transform. So e to the power minus alpha t in to e to the power minus s (t) dt, 0 to infinity when you write 0 minus that means the function is taken only from t equal to 0 before that it is not existent all right.

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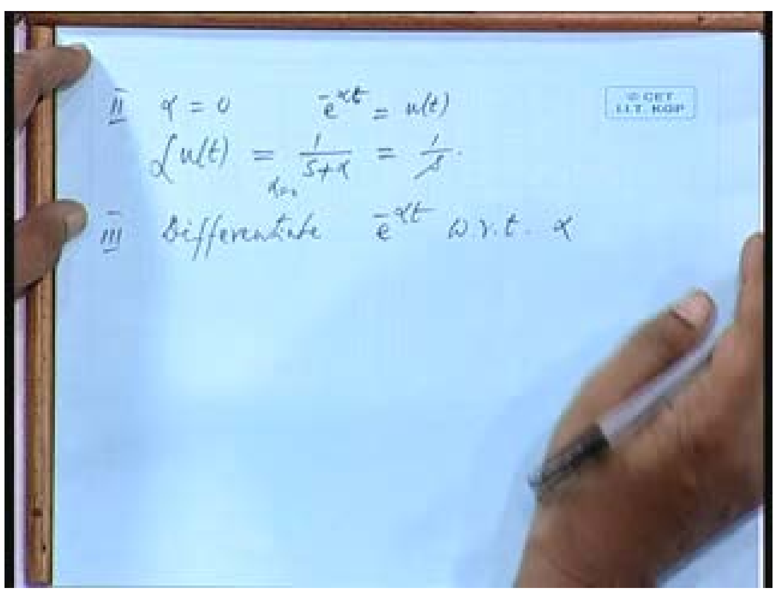

So you take up only those functions which exist only the positive region of time, this is s plus alpha that will give me 1 by s plus alpha with a negative sign and e to the power minus s plus alpha in to t 0 to infinity, at infinity these becomes 0 at 0 this becomes 1 since it is in the lower limit so that will be associated with a negative sign, negative and negative will become positive so it will be 1 by s plus alpha okay. Now suppose alpha equal to 0 then e to the power minus alpha t that we are considering between 0 minus to infinity becomes u (t) because we are considering only in that range so it becomes ut so what about the result put alpha equal to 0 that gives me 1 by s okay then let us differentiate e to the power minus alpha t with respect to with respect to alpha. So e to the power minus alpha t has given me a transform of 1 by s plus alpha alpha is a common term all right.

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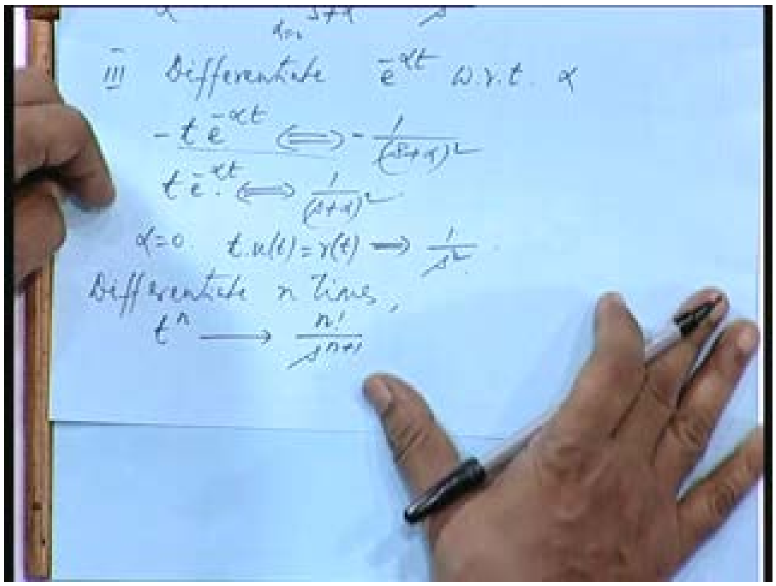

So let us differentiate with respect to alpha, the left hand side will be minus t in to e to the power minus alpha t and on this side it will be minus 1 by s plus alpha Whole Square minus goes out. So t in to e to the power minus alpha t will be having a transform 1 by s plus alpha whole square put t equal to 0 if I put t equal to sorry alpha equal to 0 again in this case and on this side it is t in to u (t), u (t) is implied here because we are always considering only the positive region of time t in to u (t) that is the ramp ramp function r (t) sorry, its transform is 1 by s square differentiate once again with respect to alpha this will be t cube on this side it will be 2 by s cubed.

So if I differentiate n times differentiate n times on this side and then make alpha equal to 0 differentiate this one n times and then at the end you put alpha equal to 0 on this side you have t to the power n, this side it will be factorial n by s to the power n plus 1, see here it will be 2 by s cube next stage then 2 in to 3 by s to the power 4 and so on let us put alpha equal to a plus jb okay then e to the power minus a plus jb in to t will have a Laplace transform of 1 by s plus a plus jb. I have just replaced alpha by a plus jb the first what is this e to the power minus at cosine bt plus j sin bt okay so on this side if I separate out the real and imaginary parts will be s plus a by s plus a whole square plus b squared minus jb by s plus a whole square plus b squared equate the real parts and the imaginary parts then e to the power minus at cosine bt will give me s plus a by s plus a whole squared plus b square, e to the power minus at sin bt will be b by s plus a whole square plus b square see e to the power minus at in to cosine bt is a damped sinusoid starting with a maximum value.

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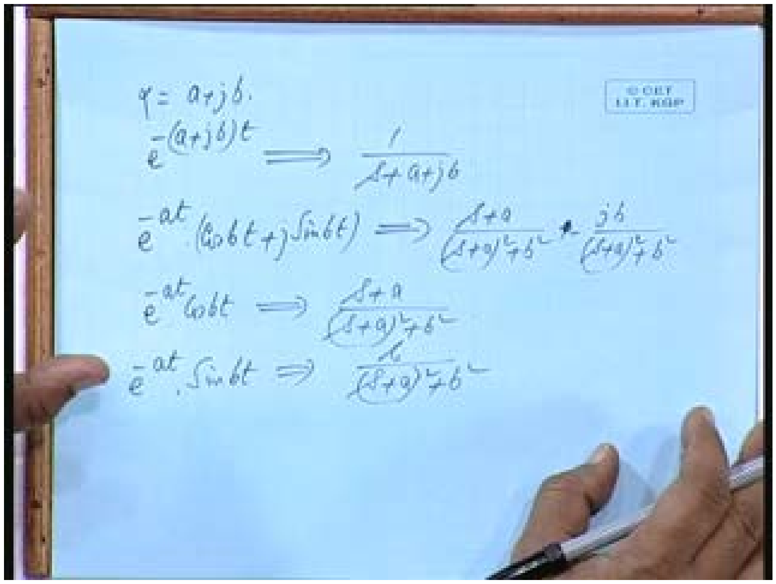

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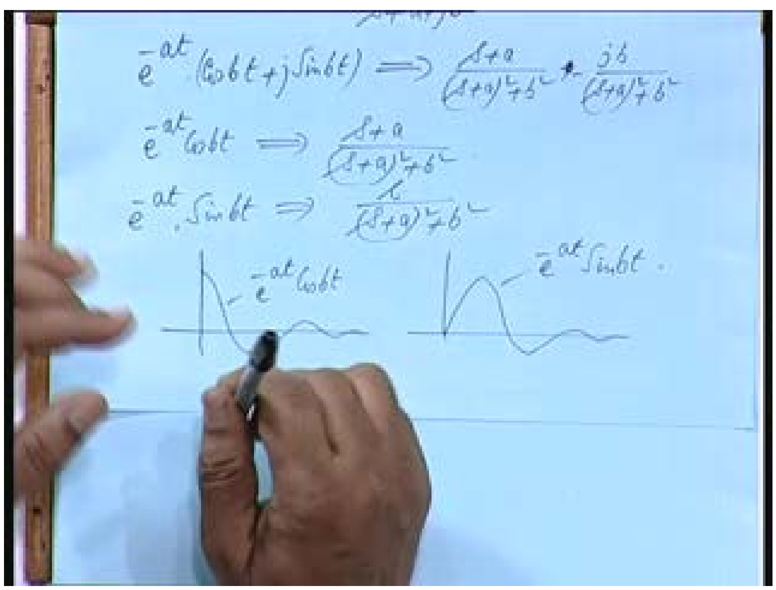

This is e to the power minus at cosine bt similarly, e to the power minus at sin bt it is like this you take a pendulum give it an oscillation all right give it a displacement and then let it oscillate it will oscillate and die down is it not and you have a recorder all right you just record the image of this point, the bob then it will be seen damped sinusoid if you start your camera when it is in the maximum position then it will be a cosine function all right you move that move the paper it will be discovering this if you start from the central position then it will be this 1 okay so both are representing basically a damped sinusoid. So today we will stop here for today we will continue with this in the next class.

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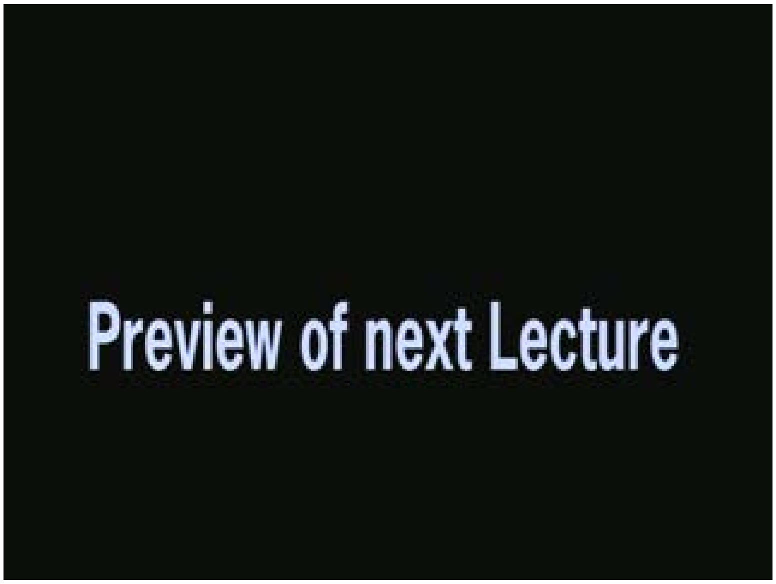

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Okay good morning friends, we shall continue our discussions on Laplace transform yesterday we derived the transforms of some standard functions we will continue with that before we go to the set of other properties I just like to mention there are function f (t) is Laplace transformable if f (t) mod e to the power minus sigma t dt 0 to infinity is a finite quantity that is it is less than infinity.

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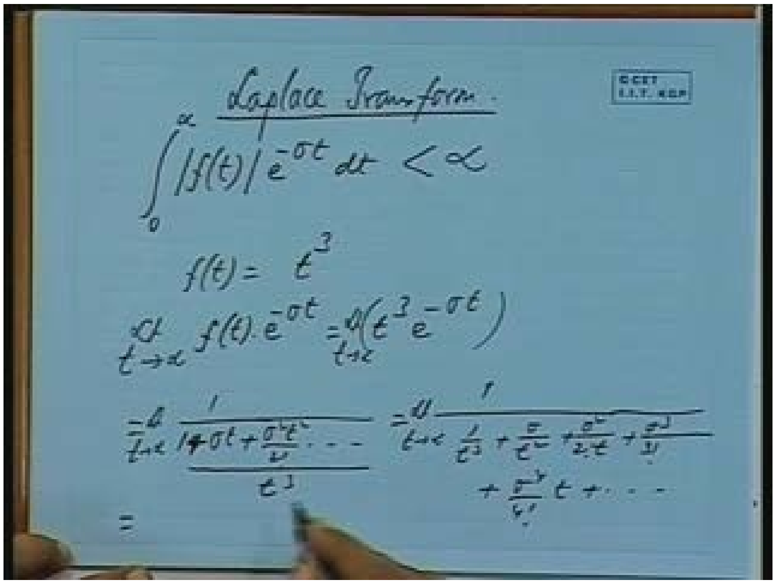

If you have a function say f (t) equal to say t squared or t cubed will this property hold good for this f (t) let us see f (t) in to e to the power minus sigma t as t tends to infinity how much is this product? t cube e to the power minus sigma t okay limit t 10 in to infinity. So what will be the limit we can always write this as e to the power sigma t I can write as 1 by e to the power sigma t, so 1 plus sigma t sorry 1 minus 1 plus sigma t plus sigma square t square by factorial 2 and so on and the t cube on the numerator can be brought back here. It can be brought down here so this will be 1 by limit t 10 in to infinity 1 by t cubed plus sigma by t squared plus sigma squared by factorial 2 t all right like that plus sigma cube by factorial 3 t cube will get cancelled and after that you will get sigma 4 by factorial 4 t and so on okay. Now how much is this coming to 1 by t cube it will be 0 this will be 0 this will be 0, this will be finite let it would be equal to 1 by sigma cube by factorial 3 plus this will be infinite terms multiplied by t.

So it will be equal to 0 this is may be t ten to infinity so this will be 0, so a function now when t tends to 0, when t tends to 0 this is a infinity, this is a infinity, this is a infinity these are all 0's. So 1 by infinity that is again 0 so what will this function look like t cubed is a function which is going up like this to be parallel alright and t cubed in to e to the power minus sigma t, e to the power minus sigma t is a function going like this. So the product will be starting from 0 ending at 0 so somewhere it may have a peak and then it will go like this. So the area will be finite this is going to 0.

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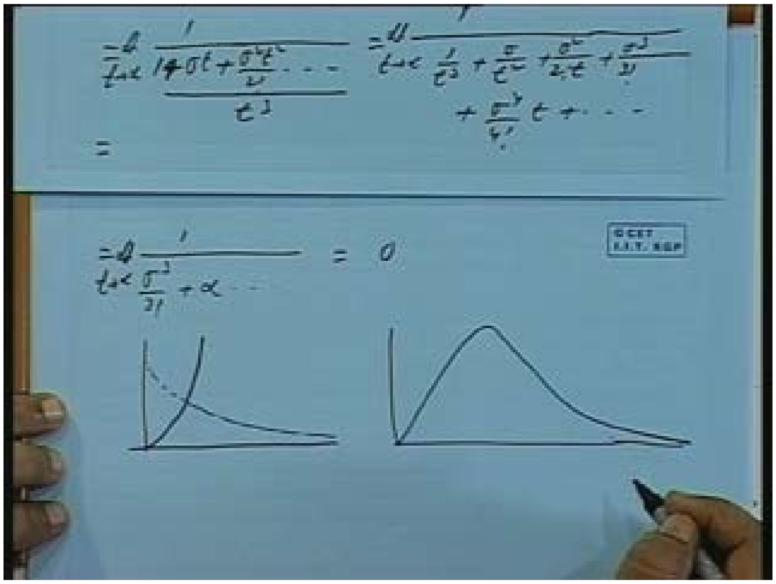

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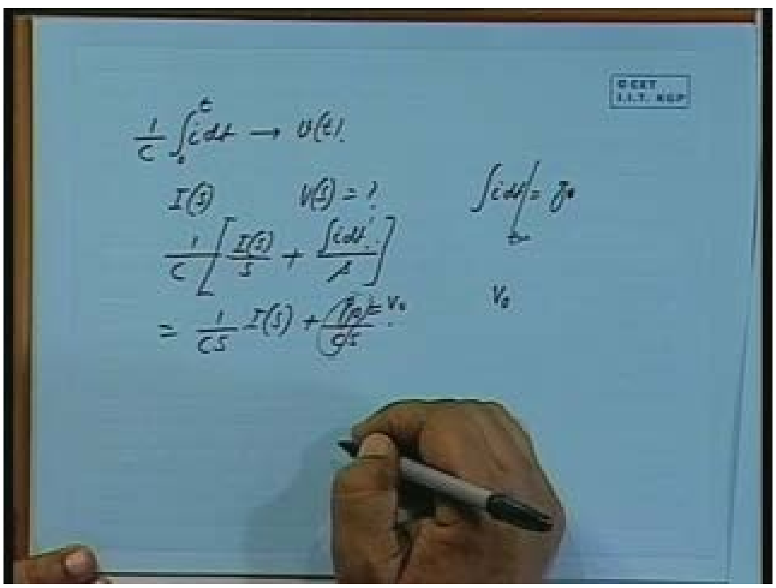

So area is finite if you have a current expression, for example current is for example in a capacitor 1 by c integral I dt, 0 to t is the voltage at any instant if I take if I have to take the Laplace transform of this given the Laplace transform of I (t) as I (s) what will be V (s), V (s) is integral of I is it not so if I have to compute V (s) it will be 1 by c into integral of this means original Laplace transform divided by s then plus plus what is it I_0, what is f_0 it is integral of that. So integral of I dt okay divided by s and this is to be evaluated at 0 from where do we start, so 0 minus to 0 plus or basically whatever has been there before the counting of time, before t equal to 0. So that gives me integral of I dt is what charge q is it not, so integral I dt means this is to be evaluated at t equal to 0 means the initial charge q_0 all right mind you it is not initial value of I, initial value of integration of i.

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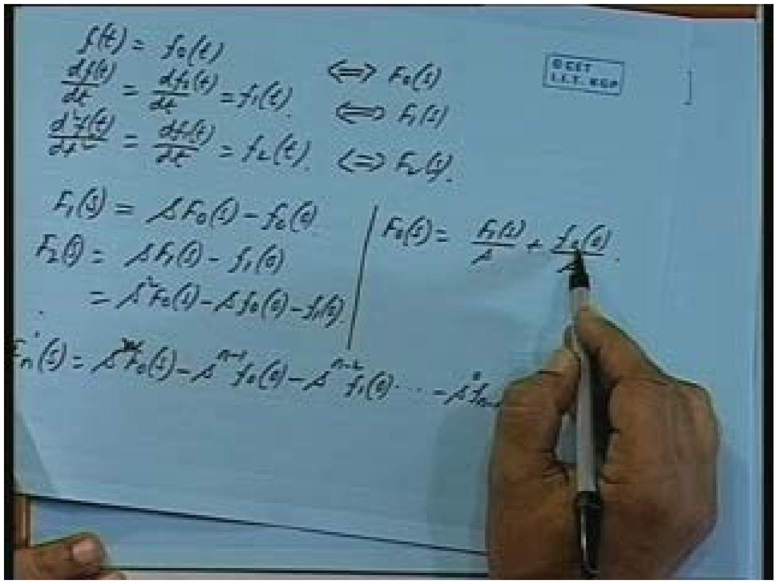

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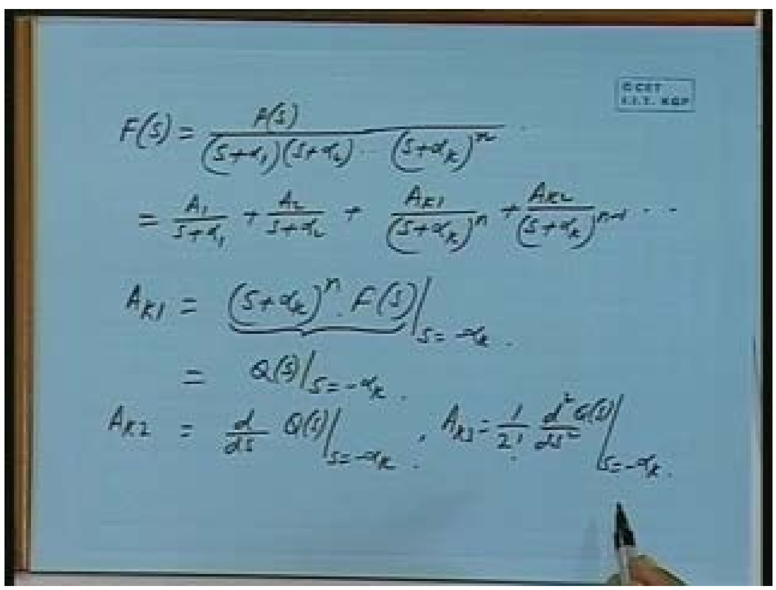

When you take the integration, so initial value of not f_1 f_0 means integration of f_1 all right. If the initial condition is given as 0 then there is no problem, so 1 by cs in to I (s) plus if I call it q_0 by cs, q_0 by c is nothing but initial voltage across the capacitor. So V_0 by s Evaluate at minus alpha k then A_{k2} will be derivative of this Q_s and then evaluate at s equal to minus alpha k. Similarly, A_{k3} will be 1 by factorial 2, d square Q_s by ds square second derivative you take and then substitute s equal to minus alpha k and so on third derivative it will be factorial 3 and so on.

So you keep on taking the derivatives of this product and then evaluate the function at s equal to minus alpha k that will give you successive residues. Once you know that you can expand you can find out the inverse okay. So we will stop here for today. Thank you very much, you try some other problems at home and we will take up some numerical problems in the next class.