Networks, Signals and Systems

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Lecture - 05
Tutorial

Good morning friends, today we will be taking up some problems we will have a tutorial session today on some of the problems based on the theory that you have covered already in first year class or in the last few days, say let us take the first one a very simple one you are having a voltage source of 10 volts okay connected through a resistor 2 ohms, this is 1 ohm then again you are having another source 5 volts then 2 ohms, 6 ohms under current source at this end of 4 amperes okay. All these values are in ohms 2, 1, 2, 6 and there are 3 sources. You are ask to calculate all the currents to different branches, all the currents in all the branches and the potential at nodes, say this node I call it b and c all the currents and node potential V b and V c with respect to a reference, say this one okay, there has to be a reference whenever we specify the voltage of any particular node okay.

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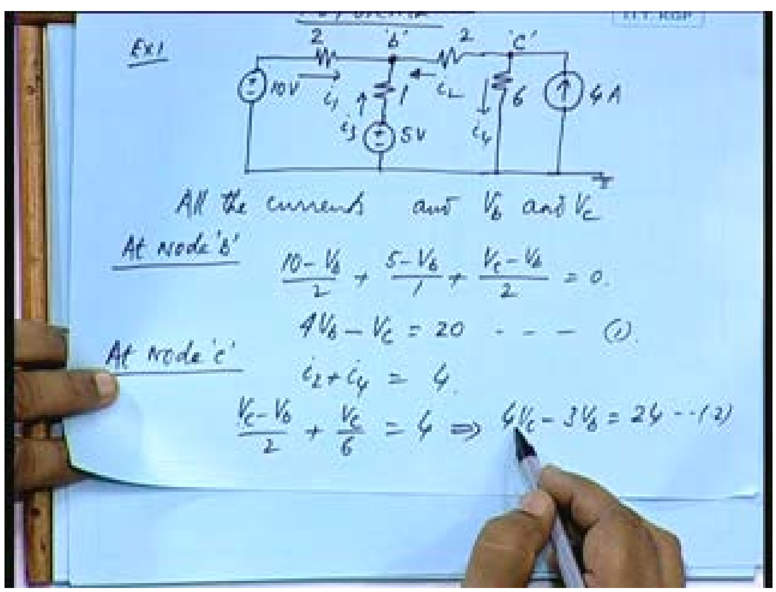

Now the easiest one will be to apply nodal equations at these nodes okay. So here if you apply nodal equation at b at node V, what shall I write? Suppose this is the current I 1m, this is the current I 2 and this is the current I 3 okay. All going inward towards the point b, it could have been taken other ways also it could have been taken away from b. So node b I 1 will be 10 minus, 10 minus V b by 2 10 minus V b by 2, it is flowing towards b 10 minus V b by 2 plus this is with respect to the ground 5 volts of 5 minus V b by 1 plus V c minus V b by 2 and that is equal to 0 okay.

So if you simplify multiplied by 2 and see if I am multiplied by 2 this will become 10 minus V b this will be 10 minus twice V b so minus thrice V b and again 1 more, so 4 V b, so 4 V b minus V c okay is equal to 20 okay, after simplifying this. Now if I apply the same nodal equation at node C what would be the currents once again if I take currents going inward or say I 4 is going this way sorry, suppose I 4 is the current going away from this, so I 4 plus I 2 is equal to 4, correct me if there is any slip I 2 plus I 4 is equal to 4 is that agreed and how much is I 2, V c minus V b by 2 thank you and I 4 is V c by 6. So that gives me finally 4 V c minus 3 V b is equal to 24, is it not just multiply by 6, 6 in to 4, 24 and manipulation here gives me 3 V c plus 1 V c 4 V c minus 3 V b okay.

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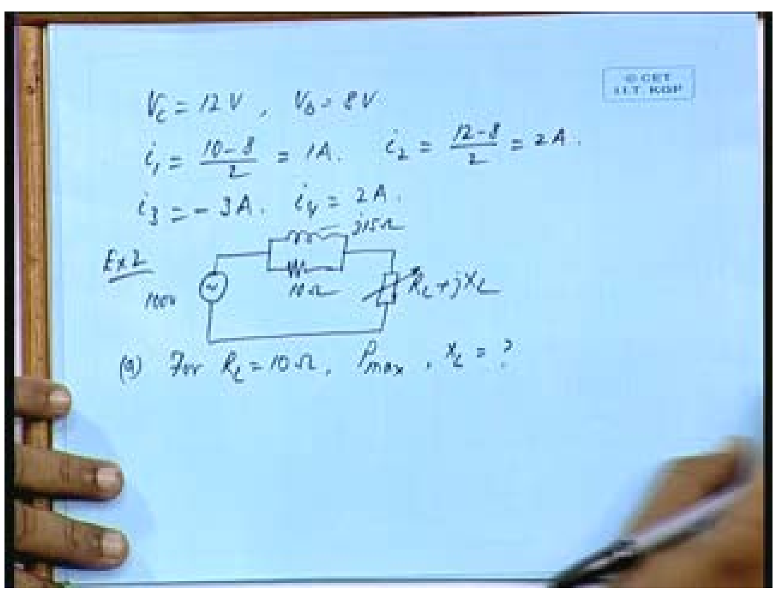

So if you solve these 2 equations you get equations are 4 V b minus V c 4 V c minus 3 V b. So you can multiply this by 4, add with this. So 4 in to 4, 16 minus 3 and 4 in to 20 plus 24, so after doing that 104 divided by 13, so that gives me 8 V c is equal to 12 and V b is equal to 8 okay. So what will be I 1 once you know V b and V c we just use this one that will give you I 1, 10 minus V b therefore I 1 will be 10 minus 8 by 2m, is it not, this is 10, this is 8 divided by 2 okay that is equal to 1 ampere. Similarly, I 2 will be 12 minus 8 by 2 this is 12 minus 8 by 2 you have already got this values so this is 2 ampere I 3 similarly, I 3 is this on 5 minus 8 divided by 1 this is at a higher potential 8.

So minus 3 ampere is that alright is a this is at a potential of 8 volts this is 5, so current will be in this direction and I 4 similarly will be 2 ampere 12 divided by 6 okay. Let us take another example, say this is the problem that we that just now discussed is based on simple nodal equations will take up a problem on maximum part transfer theorem which we have studied in first year class, you are having a source of say 100 volts voltage source of 100 volts, you are having an impedance at the load end RL plus jxl which is variable and there is a reactance plus j 15 ohms okay.

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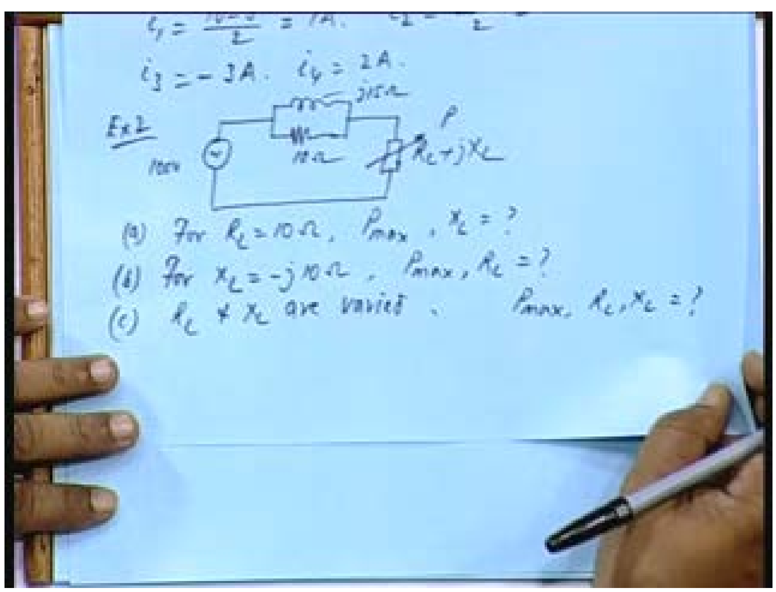

There are 3 conditions under which you are ask to calculate the condition for maximum power transfer A for RL is equal to 10 find P max that means you are varying only XL, RL is fixed only XL is varied, what would be the value of the maximum power that you can get here find P max and corresponding XL okay for this maximum power condition. Second condition is for XL equal to minus j 10 ohms that means you are keeping the reactance or the load fixed minus j 10 ohm that means a capacity reactance you have fixed what would be the value of P max when I say P max means load power okay this is P and the corresponding value of RL okay these RL means corresponding to this maximum power and thirdly, when both RL and XL are varied okay what would be P max and corresponding RL, XL okay.

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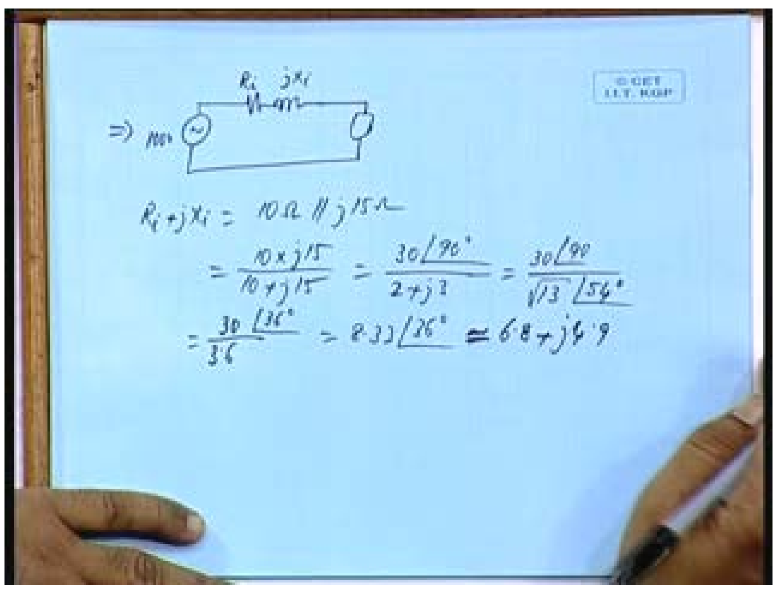

So the problem has got 3 sections, 3 different situations, so let us take the first one before we go to attack this problem we can think of reducing this parallel combination in to a single form. So what would be the value of this I can reduce this network to a source and how much is this r, I will it ri and j x i, so how much is that ri plus jxi will be the parallel combination of 10 ohms in parallel with j 15 ohms okay that is 10 in to j 15 divided by 10 plus j 15 okay, you can take out 5. So 2 into 15, 13 j can be shown by an angle 90 this is 2 plus j 3 okay I will make very approximate calculations and simultaneously, I will tell you how to make simple computations from without using a calculator, how much is 2 plus j 3, 2 squared plus 3 squared 13 and 13. So square root of 13 and an angle of tan inverse 3 by 2 tan inverse 1.5, how much is tan inverse 1 45 degrees tan inverse 2, tan inverse 263 degrees okay 62 point something so you interpolate tangent curve if you know what will be the value between 45 and 63, 45 and 63, it is about 54.

So it will be somewhere closed to 54 degrees let us take very crude simplification approximation 54 degrees. So how much is it, root over of 13 how much is 36 squared all these square numbers say root 13 and 1300, this square roots except for the decimal difference, it will be same, 1300 square root of 1300 square root of 35, how do calculate square root of the square of 35 last 5 square and 5 square and then 3 into 4, 3 in to 4. So 1225 add 135 and 136, so that gives you 1296 that is 36 square next square so 1296 is approximately 39 well, so it will be 36, so 3.6, so 30 divided by 3.6 and an angle 90 minus 54.

Okay 36 degrees very close to 36 degrees okay so 30 by 36 is nothing but 5 by 6. So after change of decimal it will be 50 by 6, 8.33 okay and an angle of 36 degrees still you can simplify how much is R plus JX 36 degrees if you take it 37 degrees okay. So that is cause is point 8, so point 8 in to 8.33 okay point 8 in to 8.33 and point 6 into 8.33, is it not point 6 plus j point 6, point 8 plus j point 6 that gives you approximately 36 degrees okay, it is a crude approximation anyway let us put those values. So that gives me 6.8 which is always an approximation plus j 4.9 alright. So for maximum power condition when you do not have any scope to change the resistance then I will try to maximize the current and that I can have RL is fixed, RL is fixed, I am having only XL which can be varied. So what should be the value of XL, when it perfectly matches with this and there is a cancellation I get maximum current.

So maximum current under this condition will be 100 volts divided by ri plus RL which means 100 divided by ri was calculated to be 6.8 approximately and RL is 10. So 100 by 16.8 okay 100 by 16.6 is how much 1.66 approximately 6, so it may close to 6, 5.9 amperes okay. So P max will be I max squared in to RL and RL is 10 whatever be that so it is pretty close to say 355 approximately.

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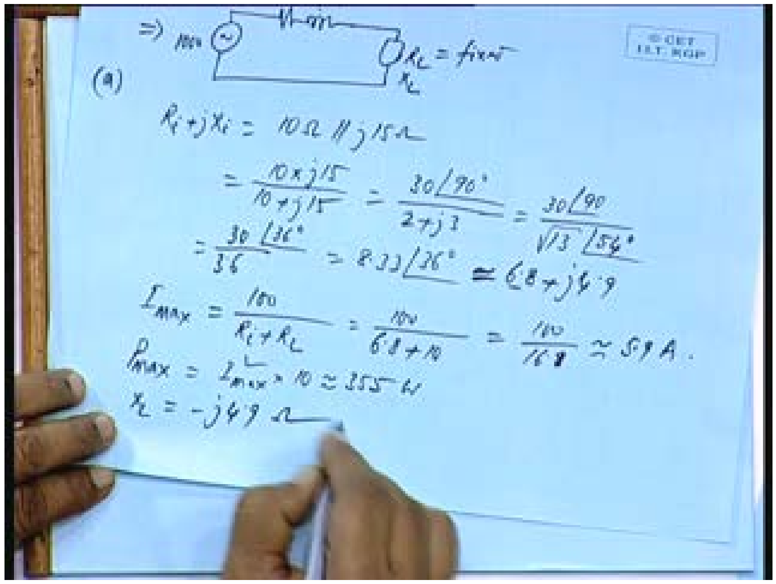

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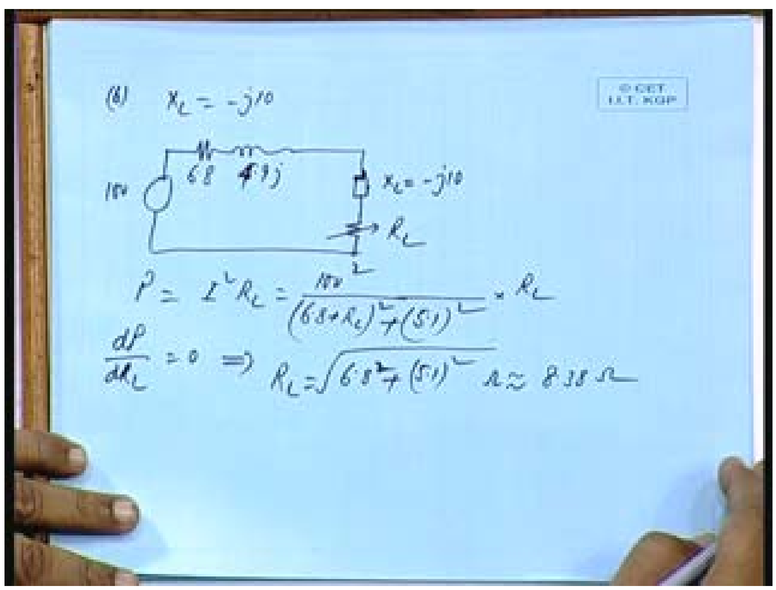

So many watts okay 355 watts this is approximation. Let us take the second case and what is the value of corresponding XL, XL should be minus j 4.9 is that alright second condition is when you are having XL as minus j 10. So how much will be the corresponding value of resistance you are having this is 6.8m, this is 5.9, sorry 4.9, 4.9 j okay then you are having a reactance okay, a reactance XL is equal to minus j 10 that is the capacity reactance and the resistance which is varied okay. So what should be the condition for getting maximum power here, P is equal to I squared RL which means 100 divided by how much is the current 6.8 plus RL.

So if you square it 100 square by these squared plus total reactance which is minus j 10 plus 4.9 so 5.1 squared is that alright multiplied by RL. So if you differentiate this with respect to RL itself what would be the condition dp by drl equal to 0, you will find, you will find after diff differentiating that this resistance should be equal to the rest of the impedance, rest of the impedance should be equal to RL, is that alright. So RL should be equal to 6.8 squared plus this difference that is 5.1 squared whole thing and root it is that alright.

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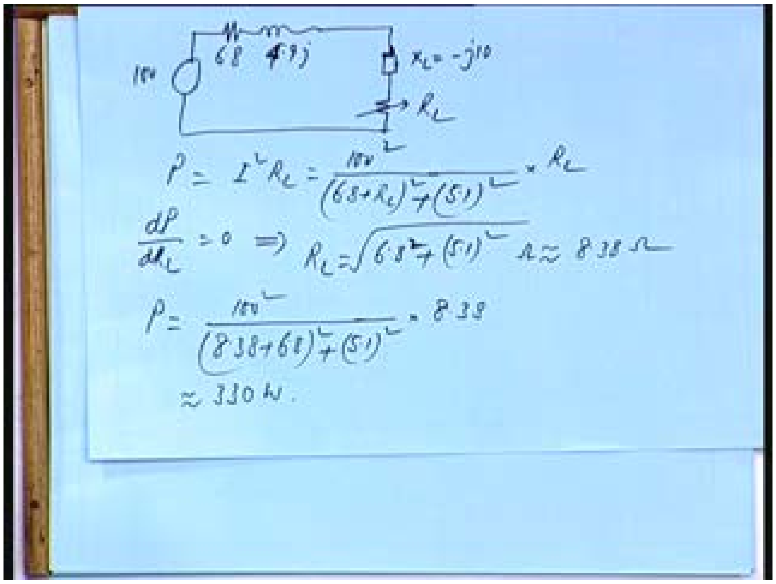

So that gives me approximately 8.38 ohms, this is so many ohms okay, this is an approximate calculation all my calculations here will be approximate. So method is more important, so P will be corresponding P will be therefore how much will be P 100 squared divided by now you substitute the values 8.38 plus 6.8 whole squared plus 5.1squared in to RL which is 8.38 that comes to approximately 330 okay and the last situation is when both RL and XL are varied and all of you know because this is the relationship that you normally derived in classes that is what would be the maximum power transferred to the load, the condition is load impedance should be complex conjugate of the internal impedance.

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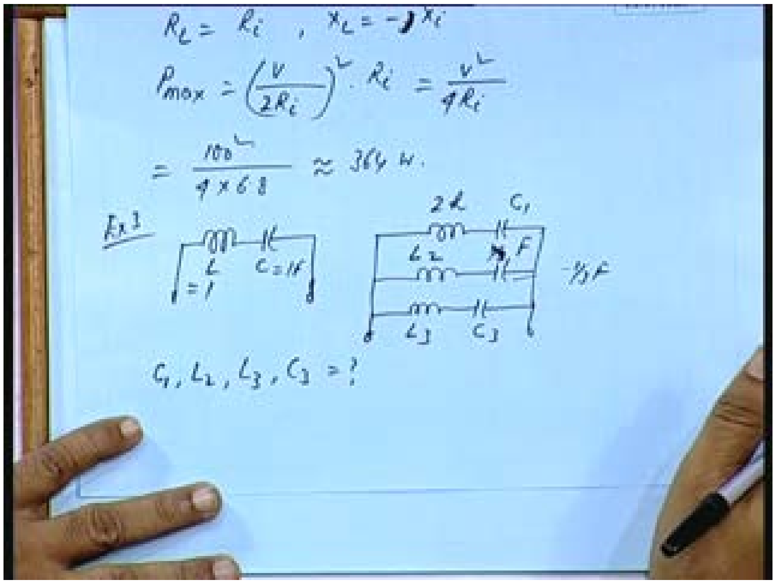

So RL should be equal to ri and X L should be equal to minus of sorry minus of X I, j is implied, so P maximum will be voltage by how much is the total impedance it is R I plus R L which is twice arrive in to R I, R I is equal to R L. So that will be V squared by 4 R I, so that is 100 squared divided by 4 in to 6.8 is that alright. So that will be the power approximately 364 watts, okay. Let us take another example you find there is a variety, you might have come across this type of problem this L is equal to 1 Henry C is equal to 1 farad, we find that is a parallel combination of 3 such elements, this is 2 Henry to be coupled with a capacitors C 1, this is one third farad, one third sorry let me write it here, one third farad to be coupled with some L 2 and another pair L 3, C 3, what would be these values of inductances and capacitances C 1, L 2, L 3 and C 3 such that the impedance in here and impedance in here will be same for all frequencies okay that means this is equivalent to this.

Now if it is equivalent then under all conditions, the impedances will be same. So I can take some specific values and equate the conditions alright. Let us see in general there is a resonance frequency for this impedance, there is a resonance frequency for this, there is a resonance frequency for this alright since all of them say I can call them Z 1, Z 2 and Z 3.

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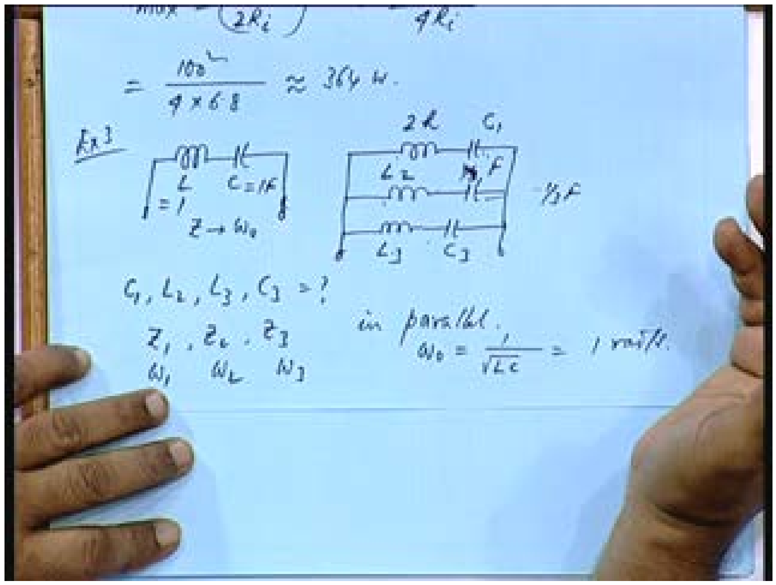

So Z 1, Z 2 and Z 3, they are in parallel at resonance corresponding to this, the resonance frequencies are say omega 1 omega 2 and omega 3 okay at resonance and similarly, for this the impedance is say Z and corresponding resonance frequency is say omega naught, how much is omega naught 1 by root LC that is equal to 1 because L is 1, C is 1 okay 1 radiant per second. Now this one will exhibit a resonance at omega 1 alright at resonance, what is the impedance here, 0, so what is the parallel combination giving me, 0.

Any of the arms, if you find that the impedance is 0 that means there is a direct shot. So these 2 will become redundant, so the impedance is 0 at omega 1 similarly impedance is 0 at omega 2 this is 0 at omega 3 but this is 0 at omega naught when is this possible, when they are equivalent omega not must be equal to omega 1 omega 2 and omega 3 that means all the 3 frequencies must be same same as omega naught, is it not. So if you impose that condition then what is omega 1 is 1 by root L 1, C 1 say okay. So L 1, C 1 must be equal to L C equal to L 2, C 2 equal to L 3, C 3 okay so from the first one L 1 is 2, C 1 is to be calculated, so 2 in to C 1, L C is 1, we have just now seen L C is 1. So that gives me C 1 is equal to half farad similarly, L 2, C 2, L 2 in to one third is also equal to 1 L C hence L 2 is how much, 3 Henry okay.

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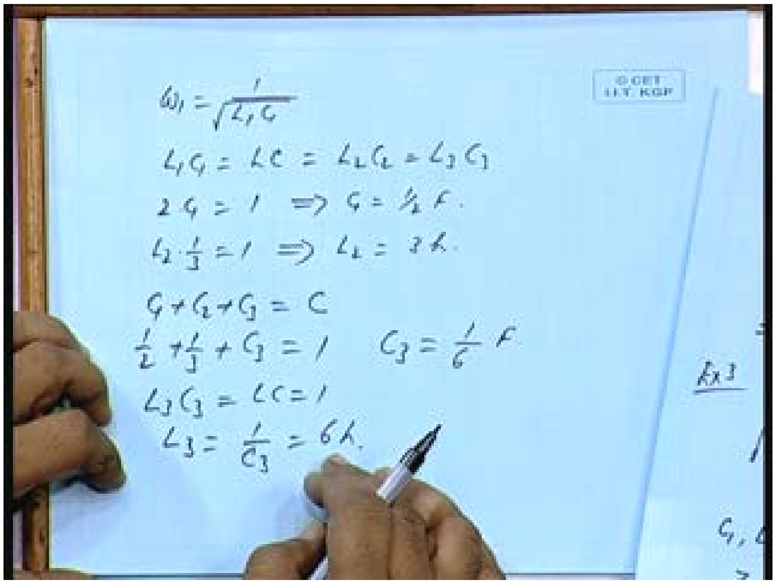

Now if I take a very very small frequency, very small frequency, how much will be this. This will be omega in to L 10 in to 0, so it will be practically capacitive it will be10 to 1 to 1 by mega C if you forget about the j part because everywhere the j will get cancel. So this impedance will be capacitive 1 by C in to omega what about the net value here this will be short, this will be short, this will be short, omega is 0 practically. So there are 3 capacitances in parallel, so C 1 plus C 2 plus C 3 and that should be equal to C. So C 1 plus C 2 plus C 3 equal to C, now C 1 is already known half, C 2 is known that is given as one third and C 3 to be determined and C is 1, so how much is C 3, 1 by 1 by 2 plus 1 by 3 is 5 by 6.

So 1 minus 5 by 6 is 1 by 6 farad okay and we also know L 3, C 3, L 3, C 3 should be equal to LC that is equal to 1. So L 3 will come out to be 1 by C 3, so 6 Henry's alright I could have also taken a very high frequency when the capacitors will be short and they will be parallel combinations of 3 inductances L 1, L 2, L 3 and that should be equal to 1. You will find that will also give you 1 by omega L 1, 1 by omega L 2, 1 by omega L 3 is equal to 1 by omega L. So if you equate L 3 will also come out to be 6 Henry in that network, so either you take a very low frequency or very high frequency and then you equate the corresponding impedance elements okay sorry.

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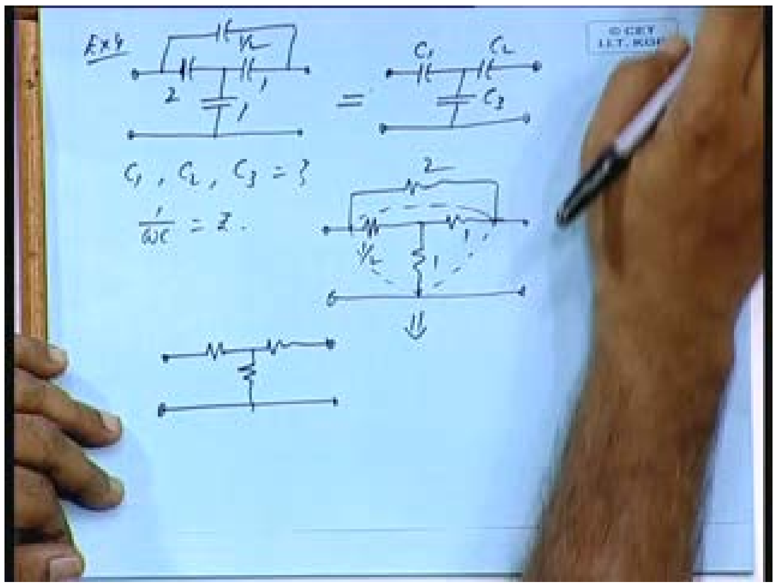

Now let us take another example okay I will take the next page. You are given this is a clearly simple network, the values here are in farads 2 farads, 1 farad, 1 farad and half a farad okay this is equivalently expressed as a star C 1, C 2, C 3, what would be the values of C 1, C 2 and C 3, C 1, C 2 and C 3, how much will be the values. Now as you know the impedance of a capacitor is 1 by C, 1 by C in to omega okay. If we replace these impedances these capacitances by their corresponding impedances then it will be proportional to 1 by 2, 1 by 1, 1 by 1 and 1 by half
Okay. So, as if there are resistances quality impedance or resistance of magnitude half just inverse of these 1, 1 and 2 okay.

Now it is a t network which we can replace by an equivalent phi that is from a star to delta then these 2 elements will come in parallel and then you will get an equivalent t once again if we express those elements of that t network in terms of the capacitances will again take the inverse. So will get back C 1, C 2, C 3, so if you just compute if you just compute the equivalent of this star I just write in one shot it finally gives me I hope you all remember the relationship between star and delta elements okay.

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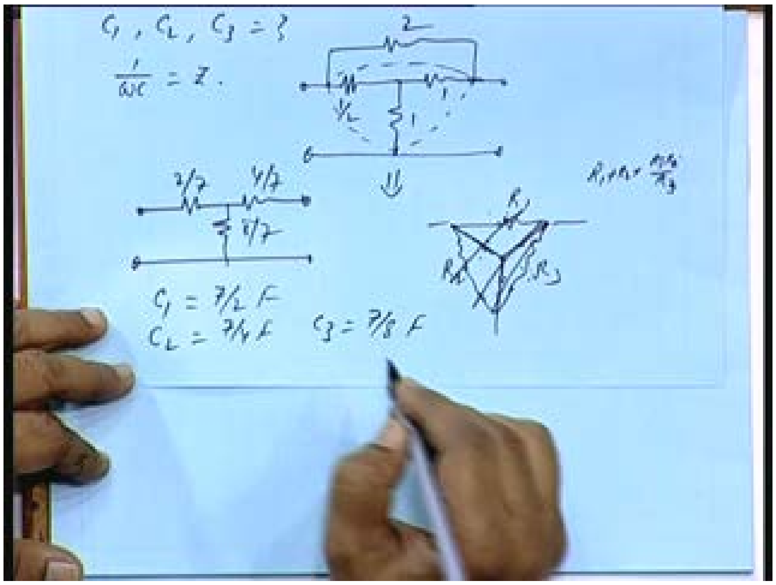

You know how to remember if you want to go from delta to star suppose, this is R 1, R 2, R 3. So the one which will be in the lap of R 1 and R 2 will be R 1, R 2 divided by their sum similarly, R 1, R 3 divided by the sum will be this alright and the reverse will be suppose you are given the star elements if you want to calculate the delta element. So suppose this is the element that the element oppose to this just opposite of this will be equal to this sorry from star to delta.

So what should I take, this one plus this one plus this in to this divided by this okay. It is like R 1 plus R 2 plus R 1, R 2 by R 3 that means you take the product of 2 at a time and then their sum R 1, R 3, R 2, R 3 plus R 1, R 2 and then divide by the element that will come opposite to this okay. So, this one comes out to be 2 by 7, 4 by 7 and 8 by 7 okay. So what is the value of C 1, 7 by 2 farads, C 2, 7 by 4 farads and C 3 will be 7 by 8 farads, is that alright. So these 3 elements have just inverse of these resistances. Next, we go to the computation of 4 terminal network parameters or 2 port parameters I will take a simple example sorry, this is a 5 ohm resistance, this is a 10 ohm resistance, this is 10 ohm, this is 20 ohm, this is 20 ohm,. I do not recollect whether discussed the parameters the last class within a example.

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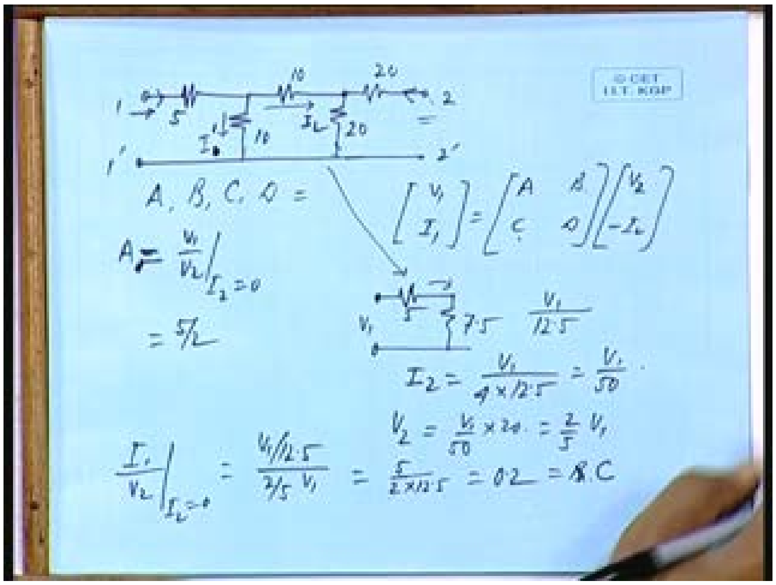

So what would be this particular numerical problem we had discussed okay. So we have calculated the parameters Z 11, Z 22, Y 11, Y 22 okay what would be the A, B, C, D parameters for this A, B, C, D parameters for this, how do calculate A remember what was written as A, B, C, D, A, B, C, D this was V 2, V 2 minus I 2 and V 1 and I 1. So A was, A was V 1 by V 2 and I 2 is 0, I 2 is 0 means keep it open this is I 2, this is I 1 and I 2 is 0 means keep it open and then measure this voltage, this voltage you apply a voltage V 1 here, what would be the voltage and then take the ratio V 1 by V 2.

So let us see if I keep it open the voltage here is this voltage, now how much is this voltage. So before that you have to calculate the equivalent impedance were this is 20 plus 10, 30 in parallel with 10. So 10 in to 30 divided by 10 plus 30, so if I draw a simplified equivalent this is 5, this is combination of 30 and 10, so 7.5, 7.5. So how much is the current it will be V by 12.5 what fraction of this current flows through this 20 plus 10, 30 ohms and 10 ohms.

So it will be 10 by 10 plus 30 that is one 4^{th}, 10 by 40 means one 4th of this. So if I call it I 2 this is I 1 then I 2 is V by 4 in to 12.5. So V by 50 okay that is a current here and how much is the drop here therefore across 20 ohm resistance therefore V 2 will be V by 50 multiplied by 20. So 2 by 5 in to V 1 okay therefore V 2 by V 1, V 1 by V 2 will be 5 by 2, is it not under a open circuit condition. So similarly you can calculate B, C, D. So C is also under that open circuit condition when I 2 is 0, it is a relation it is a ratio between I 1 and V 2.

So how much is I 1, I 1 means sorry, this is I 1, I, that is the net current that is flowing from here I should not write I 1, let me call it I dash under the same open circuit condition how much is the current send from here that I have just now computed here V by 12.5 so how much is the ratio I 1 by V 2 under open circuit condition, how much is I 1, V 1 by 12.5 and how much is V 2 is 2 by 5V 1 is that alright. So V 1 gets cancelled, this is 5 by 2 in to 12.5, how much is it, this is 25, 5 by 25, so 0.2.

So V is 0.2 similarly sorry this is C, C, C is 0.2, B you can compute when a we when we make V 2 equal to 0 that is apply short circuit and again measure the voltages currents from the sending end and the receiving end current. The short circuit current it will change its direction, so negative sign will have to be taken properly you will get the ratio B and D okay. So I leave it as an exercise to be completed by you at home.

I will take another interesting problem with active elements or dependent sources in the circuit. Suppose we have a current source which is 3 times V R 2 that is R 2 is this element having a resistance value of 20 ohms R 3 is also 20 ohms R 4 is equal to 10 ohms and R 1 is equal to 10 ohms, this is 1, 1 dashed 2, 2 dashed okay and this is V R 2 assuming this to be plus and this to be minus the voltage across R 2 is V R 2 and there is a dependent source, current source whose magnitude is 3 times V R 2 flowing in this direction alright what it would be say A, B, C, D parameters.

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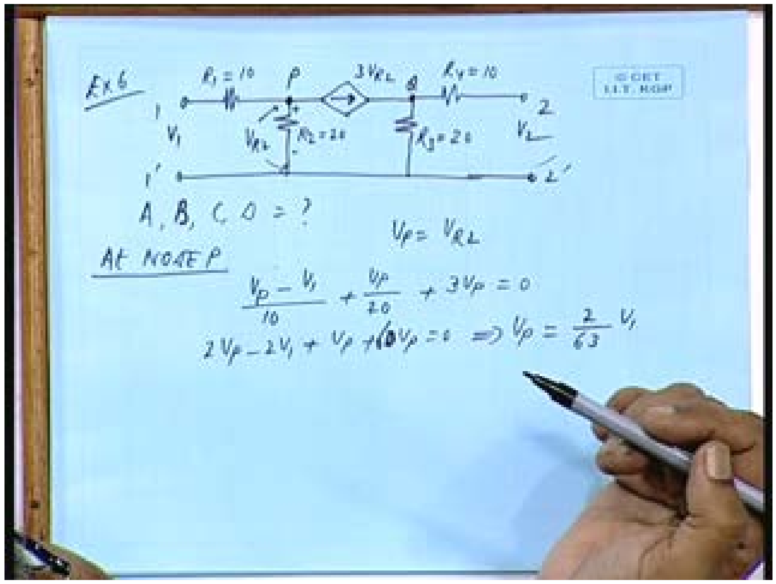

Now let us write these nodes as P and Q most of the time will find nodal analysis helps a lot in simplifying the equations and you can get the result very quickly. Let us apply at node P, the nodal equations and node Q, what would be the equation here suppose this is the voltage V 1, this is voltage V 2, V R, this is the voltage V P equal to V R 2 is it not, is the voltage across the resistance V R 2.

So V 1 okay V P minus V 1 by 10 plus V P by 20, V P by 20 plus 3 V R 2, 3 V P equal to 0, is it not? This is the current here, so if you multiplied by 20, so twice V P minus twice V 1 plus V P plus 3 V P equal to 0 that gives me V P equal to in terms of V 1, in terms of supply voltage 20 in to 3, it is 60, now 3 in to 20, 60 you must be have watched out 60 plus 2 plus 1, 63 V P is equal to twice V 1. So 2 by 63 V 1, so this potential is fixed so long as your supply voltage is V 1 this potential is fixed irrespective of whatever you put on this side is very interesting.

Next at Q, so current through the current source is 3 times V R to 3 times V P, so that is also current source okay this is current dependent current source ICCS. So I need not write that ICS is 3 times V R 2 that is 2 by 21 multiplied by 3 that gives in to V 1 okay. Now if you keep it open, if you keep it open, what is V 2 we are going to measure, we are going to measure V 2 the ratio between V 2 and V 12 calculate A, is it not under the open circuit condition.

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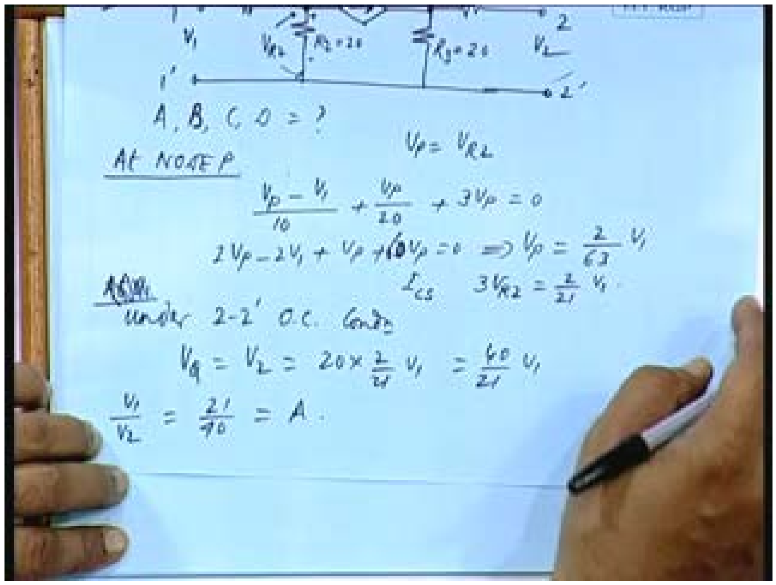

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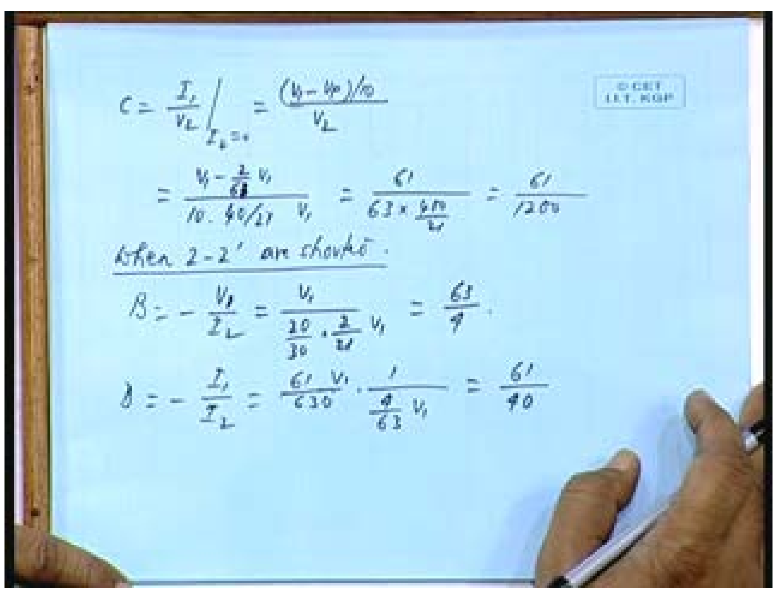

So how much is that when I keep it open I 2 is 0, I will compute that first under 2, 2 dashed OC condition, how much is this, how much is this voltage current in to R 3. So V Q is equal to V 2 is equal to 20 in to and how much is that current 2 by 21, V 1. So 40 by 21, V 1. So how much is V 1 by V 2, 21 by 40 is it not and that is equal to A, is it not, A is this voltage by this voltage under open circuit condition, under open circuit condition what are the measurements that you make A and C, is it not A and C will be under open circuit condition I 20.

So how much is C, C is I 1 by V 2 when I 2 is 0 and this current by V 2, how much is that, how much is that, how much is I 1, it is V 1 minus this potential V P which is already calculated as 2 by 21 of V 1 sorry 2 by 63 of V 1, so I 1 is V 1 minus V P divided by 10 ohms and divided by V 2 and V 1 minus 2 by 63 V 1 is it alright okay. So 2 by 61 V 1 divided by 63, 63 thank you, 2 by 63 divided by 10 in to V 2 we have computed as V 2, how much was V 2, 40 by 21, 40 by 21, 40 by 21 V 1, 40 by 21 V 1.So if you simplify this 61 by 63 in to 400 divided by 21, so 61 by 3, 4 the 12, 1200.

So this is C when 2, 2 dashed this pair 2, 2 dashed this terminal pair is shorted 2, 2 dashed are shorted, what would be B, B is minus V 1 by I 2, is it not you short this current flows in the opposite direction I 2 conventionally is positive in this direction. So I 2 is negative and you are applying a voltage here, how much is that whether you short circuit or open circuit, this current current supplied here is fixed that is what we found here, is it not for the node B we got this equation from where this potential is fixed alright and this current says this current is a constant one, I mean dependent on only this potential, this potential is a fraction of this potential.

So this current is fixed is a current controlled current source, voltage controlled current source, so this one will be V 1 by I 2 this is shorted, so what fraction of this current flows through this side 20 divided by 20 divided by 30, is it not. So 20 divided by 30 of this current which is 2 by 21, V 1 is that alright. So how much does it come to 63 by 4 okay. Similarly, D will be minus I 1 by I 2 okay and how much is that how much is I 1, you have already seen I 1 is this current which is V 1 minus V P divided by 10. So that is what we have already calculated earlier 61 by 63, is it not 61 by 63 divided by 10…. 61 by 63 is given. So V 1 is it not divided by 10 ohms so that makes it 630 in to how much is I 2, I 2 we have just now seen 4 by 63 V 1 just in the previous step. So that gives me 61 by 40 okay. So A, B, C, D parameters you have computed for 4 terminal network with sources, dependent sources the last one that I would like to take I think, I will stop here, I have another class now. So will take it up in the next class okay before we start the lecture will take up 1 or 2 more examples, thank you very much.