**Networks, Signals and Systems**
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**Lecture - 04**
**2-port Parameters Short ckt, Open ckt and Transmission Parameters**

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Good afternoon friends, we shall continue our discussions on 2 port network. The last class we have established relations between the voltages and currents in terms of y parameters and Z parameters. Let us take a simple example of a resistive network, say these are the values given 5 ohms, 10 ohms, 10 ohms, 20 ohms and 20 ohms is just an arbitrary set of values, I have taken what would be the Z parameters say, Z 11 is a voltage at this point, at this port divided by the current when this is kept open. So Z 11 is 5 ohms plus 10 ohms in parallel with 10 plus 20, 30 so it will be 10 in to 30 by 10 plus 30. So that gives me 300 by 47.5 plus 5. So it gives me 12.5 ohms okay.

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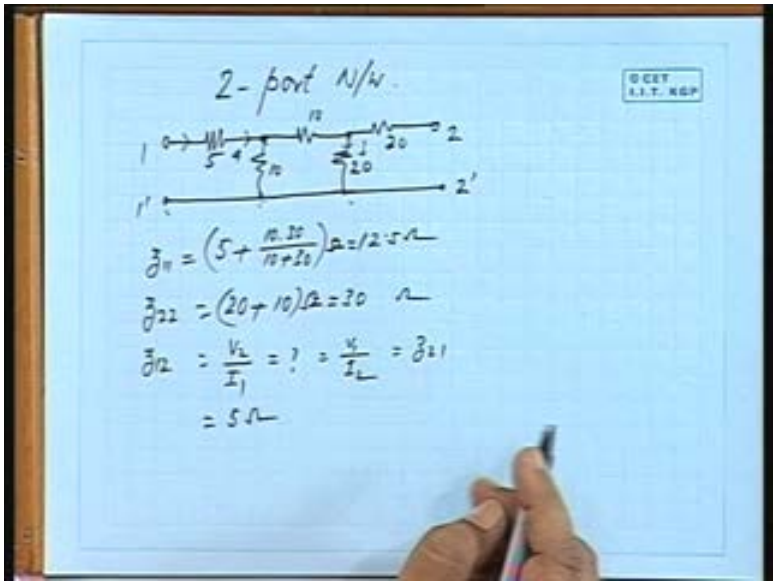


Similarly, what will be Z 22 it is the impedance from this side when this is kept open, so it is 20 plus 20 in parallel with 20, so that is 10, 30 ohms I should write this ohms this is also in ohms okay Z 12, if you want to determine it will be V 1 by I 2 when I 1 is 0. So if I keep it open send a current here, how much is a voltage choke here that means how much is a voltage here, since this is kept open this voltage is this voltage is kept open and I am forcing in a voltage from this end, so it is V 2 by I 1 and how much is that.

Now in our earlier discussion for a ladder network, it is something like a ladder structure. So if I have a unit current here okay V 2 by I 1 sorry Z 1 2 is V 1 by I 2, so I am sending in a current here say you can take 1 ampere here say, if 1 ampere flows through this then that will give me how much 20 volts drop here 20 by 21 ampere here, so 1 plus 1, 2 amperes and that will make it. It is V 2 by I 1 you can take or somebody may take V 1 by I 2 in the morning we discussed Z 12 is same as Z 21, so either way if I calculate V 2 by I 1 that is the voltage obtained here when I am sending a current here okay. So if 1 ampere current flows through this 20 plus 10, 30 volts, 40 volts by 10 ohm, so 3 ampere.

So 3 ampere plus 1 ampere 4 amperes is the current here okay, so for a unit current here 1 ampere current here 4 ampere of current will flow through this. So and how much will be the corresponding voltage here voltage is 20, 20, 20 volts, so 20 volts by 4 ampere that is 5 ohms okay. So Z 12 is 5 ohms same as Z 1 is that all right. One way go for started the reduction and get the t formation and then you will get the same values for passive network since, we can always find an equivalent t or a phi for any network. Now if I have a voltage V 1, say 10 volts and short this and record the current, current through this, how much is the current through this.

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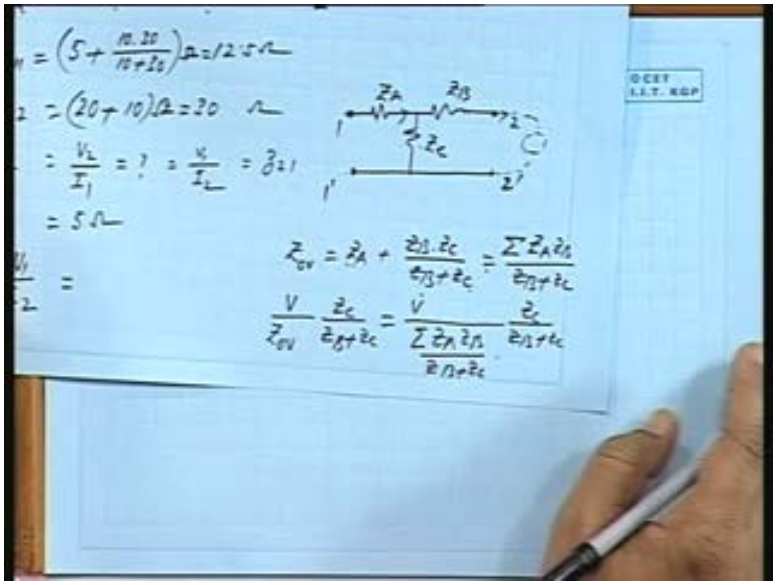

Suppose this is Z A, this is Z B, this is Z C okay, when I am shorting this and measuring the ratio of voltage to current, how much is the voltage V 1 by I 2 when this is shorted, when this is shorted Z A plus combination of Z B, Z C is the overall impedance. So overall impedance is Z A plus Z B, Z C by Z B plus Z C okay and how much is the current flowing through this, voltage by Z over all this is, this current and a fraction of that is flowing through this what fraction is it, it is the other impedance Z C by Z B plus Z C is that all right and how much does it come out to be V by Z over all is this.

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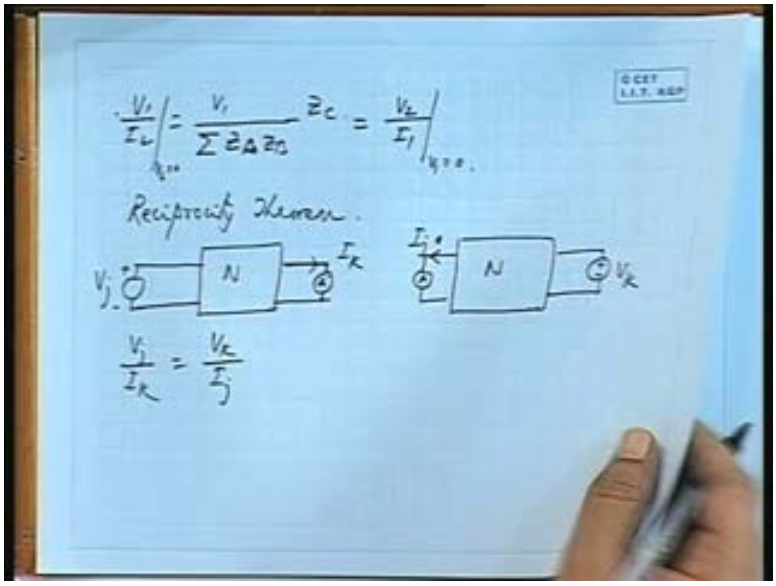


So I can write this Z A, Z B, Z A, Z C plus Z B, Z C, so I will write in this from sigma Z A, Z B which means all such terms taken together by Z B plus Z C okay. So V by sigma Z A, Z B by Z B plus Z C in to Z C by Z B plus Z C, so Z B plus Z C gets cancelled okay so that gives me V by this is this okay. So V 1 by I 2 is V 1 by sigma Z B, Z A, Z B and multiplied by Z C is that all right, Z C is this and these are all combination terms if I apply a voltage here and short circuit this measure the current, what will be this voltage by this current, this voltage by this current, it will be same Z C divided by the same term, is it not from symmetry.

So is also equal to V 2 by I 1 okay V 1 by I 2 when V 2 shorted and V 2 by I 1 when V 1 shorted okay that means in any multiport network if I inject a voltage at one way, a way terminal pair and if I short circuit any other port pairs all right, any other port and measure the current there then the voltage current ratio is same if I interchange the source and the shorting terminals all right that means the impedance, it is an impedance basically V 1 by I 2 this is the expression for an impedance, this is an impedance. The voltage applied at this port and the current measured at this port, the ratio is same as when we get the same similar ratio of this voltage and this current, this is known as reciprocity theorem, response to excitation is constant response to excitation is constant between 2 ports.

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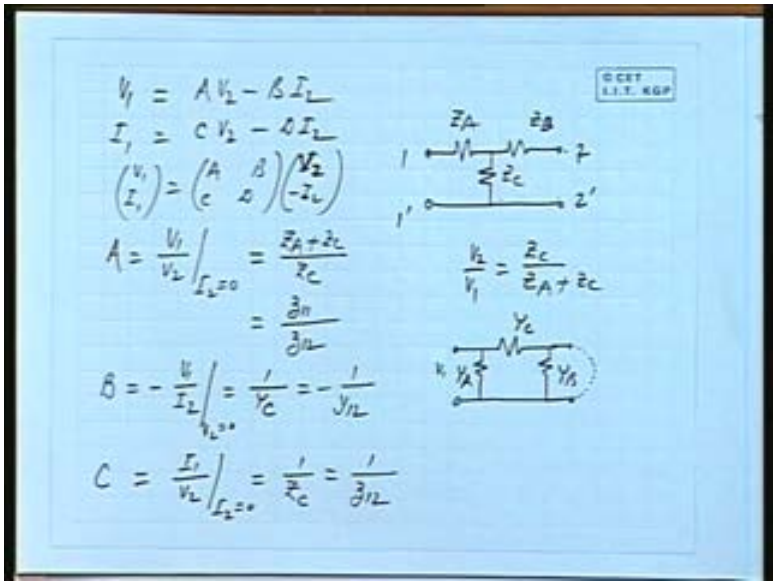


So in general will write for an N port, this is V j, this is shortened, this is an ammeter okay, this is I k, this port is shorted and here we apply a voltage V k, this current is I j, V j by I k is equal to V k by I j okay. Next we go to the transmission parameters, we write the equations like this, V 1 equal to A times V 2 minus B times I 2, I 1 is equal to C times V 2 minus D times I 2 so or in metrics form V 1, I 1 equal to A, B, C, D, I 1 minus sorry V 2 minus I 2 okay. So what would be A, V 1 by V 2 when I 2 is 0 is that all right that means once again refer to a t network this is say Z A, Z B and Z C, what does it give you, what is V 2 by V 1, we have seen V 2 by V 1 in this case V 2 is on the open circuit condition, this ZC by ZA plus ZC is it not, A is just inverse of this is it not, you can write in terms of Z 11, Z 12, Z 22 how much is ZA plus ZC if you remember Z 11 and ZC was Z 12 if you just refer to the earlier discussions. So A can be written as Z 11 by Z 12.

Similarly, what would be B, B is minus of V 1 by I 2, B is minus of V 1 by I 2 and what is V 1 by I 2, V 1 by I 2 when V 2 is 0 that is short circuit condition. So on the short circuit condition what should I write in terms of I that will be better let me write in this form. So V 1 by I 2 that means I am shorting this okay, if I short this voltage V 1 is applied across Y C, Y B is redundant. So V 1 by I 2 is how much, I 2 is in the opposite direction, so minus and minus and minus will

make it plus, so it is just Y C okay, so it is 1 by YC and what is YC in terms of Y parameters minus of Y 12 if you remember is it not this was the transfer admittance C how much is it I 1 by V 2, when I 2 is equal to 0 when I 2 is equal to 0 that means again open circuit condition all right and the open circuit condition I 1 by V 2.

So when I 2 is 0 keep this open send a current of 1 ampere how much is V 2, Z C into Z C, so 1 by Z C and what is Z C in terms of Z parameters Z 12, Z 12 you remember whenever we have open circuit parameters will refer to an equivalent star all right t connection whenever we have short circuit parameters will immediately refer to a delta then it is easier to link up Y 11, Y 12, Y 22 similarly Z 11, Z 12, Z 22 in terms of these I can immediately write okay, could someone tell me what will be D, D is again I 1 by I 2 under I 1 by minus I 2 when V 2 is 0 all right. So I will refer to the short circuit parameters.

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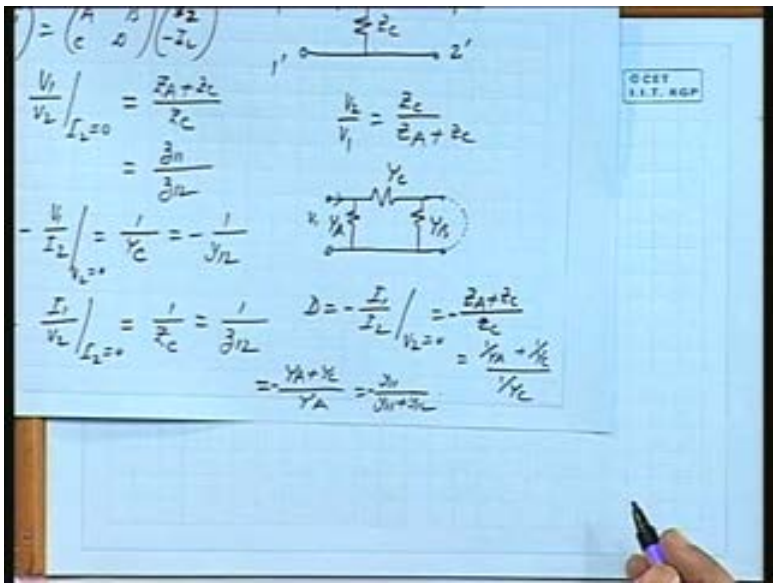

So I short this and how much is I 1, I 1 is this current and this entire current through YC will flow through the short circuited path so I 2 flows through this. So it is this current by this current, so what is this current what fraction of the main current is this YA by YA total by C, YC, yes, yes, 1 by YA by if you talk in terms of impedance say it is ZA by ZA plus ZC, is it not. So I 1 by

I 2, I 1 is a higher current, so it will be ZA plus ZC by ZC which means what ZA is 1 by YA 1 by YB, 1 by YA plus 1 by YC by 1 by YC. So how much is it? How much does it come to Y A plus Y C, YA plus YC by YA, YA by YA and what is YA plus YC, Y 11, will that be a negative sign?

So there be a negative sign and Y 11 divided by YA, YA is Y 11 plus Y 12, Y 11 plus Y 12, so this will be D is that all right. So far we have expressed the A, B, C, D parameters in terms of Y or Z parameters. Now I leave it as an exercise to all of you to will take it up in the tutorial class also later on, will cross check write Y and Z in terms of A, B, C, D. Now there write any set in terms of any other set make a table Y Z and in some books, they write metrics A that means A, B, C, D okay Y, Z A, Y, this is 1, I mean its identical this so this need not be filled up, Z in terms of Y it will be basically Y metrics inverse.

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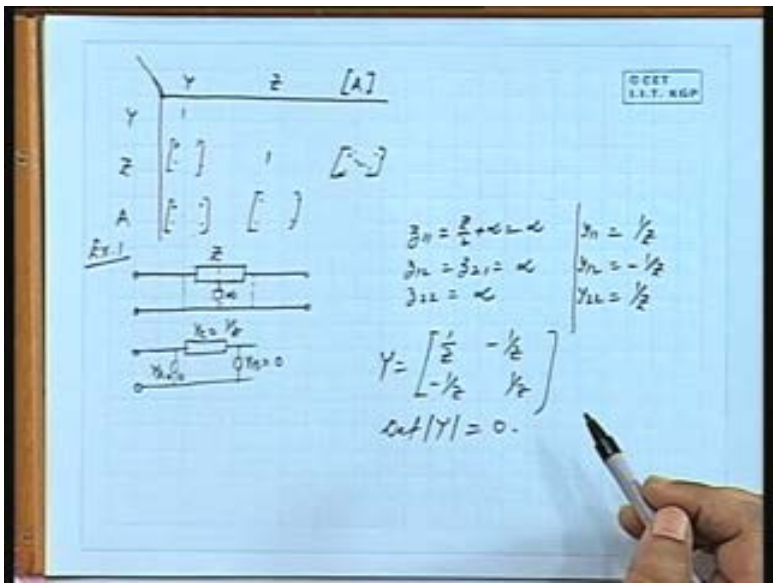


So whatever is the determinant and then all those you compute and then substitute here Z in terms of Z, it is same, Z in terms of A this is what you have to do, A in terms of Y and Z that we have already seen. Now I have written A in terms of Z 11 and Z 12, can you write totally in terms of Y's Z 11 you can write in terms of Y, Z 12 you can write in terms Y. So it should be

finally converted to all Y's similarly, I must have A in terms of all Z's as well as all Y's similarly B also from here you can also write in terms of Z because you know the relation between Y and Z similarly, C and D. So you complete both these metrics what will be A, B, C, D in terms of Y's what will be A, B, C, D in terms of Z's okay similarly, Z in terms of A's, B's and so on what would be the values in terms of A's and B's.

So complete this table okay. Now let us take up 1 or 2 simple examples there is a network very simple network first example is a network consisting of an impedance like this, this impedances are not there were open, what would be the Z parameters Z 11 equal to Z 12, Z 22. Now could you please tell me, what would be Z 11, Z 12 and Z 22 similarly, what will be Y 11, Y 12 and Y 22 for this network. Now let us see what will be Z 11, so will be completing this network either in terms of a phi or a t, for Z it is convenient to go for a t network. So I will break it up into 2 halves and then I will put 1 impedance here whose values infinity as if the midpoint has been connected to the ground to this reference line by an infinite impedance okay.

So this is Z by 2, Z by 2 an infinity, so how much will be Z 11, it will be Z by 2 plus infinity is infinity, what about Z 12, this one is also infinity Z 22, Z by 2 plus infinity. So all the 3 will be infinity, so far as the Z parameter are concerned what about Y? Once again here, I can assume this to be a phi network where this admittance is Y B is 0 infinite impedance means admittance is 0, YA is 0 and this is YC which is 1 by Z okay. So how much will be y 11, Y A plus Y C that is 0 plus 1 by Z. So this is 1 by Z similarly, Y 22 will be 0 plus 1 by Z okay and Y 12 is minus 1 by Z all right. This is something very interesting; the Y metrics parameters the Y parameters are all finite where as the Z parameters are all infinite.

There is no finite Z metrics where as there is a finite y metric, what does it mean? It is inverse of this. So inverse does not exist it is a it is going to give a singular metrics. Let us see Y metrics is 1 by Z minus 1 by Z minus 1 by Z, 1 by Z, what is the determinant 1 by Z square minus 1 by Z square 0. So the determinant is so Z does not exist, I hope this is clear. Let us take up another example, now we have just 1 shunt element what would be the Z parameters and Y parameters the Z 11 is how much, so I can once again put a fictitious element here whose impedance is 0, so Z A is 0, Z B is 0 and ZC is 1 by Y.

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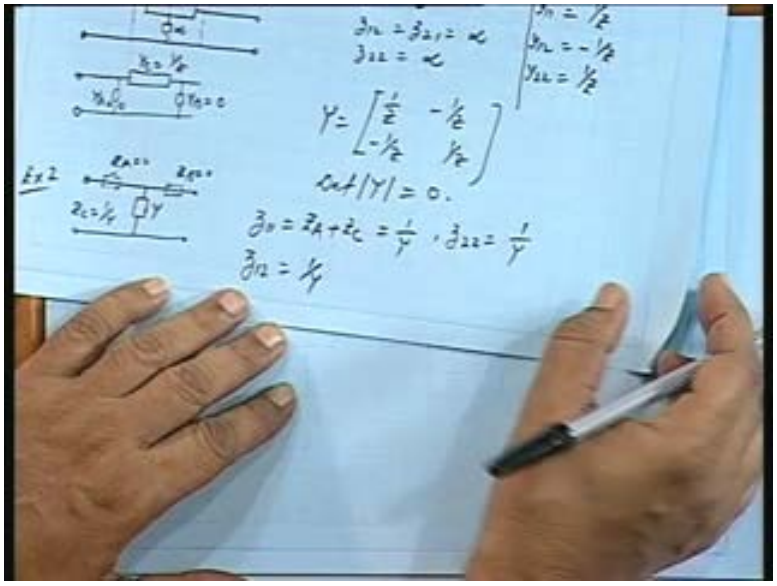

So how much is Z 11, ZA plus ZC is it not that is equal to 1 by Y, what about Z 22 similarly, 1 by Y okay and Z 12 yes, Z 12 will be again no Z 12 does not have a negative sign, it is y 12 in the short circuit case only you get negative sign in the impedance terms you do not get a negative signs, so Z 12 is just this impedance which is 1 by Y.

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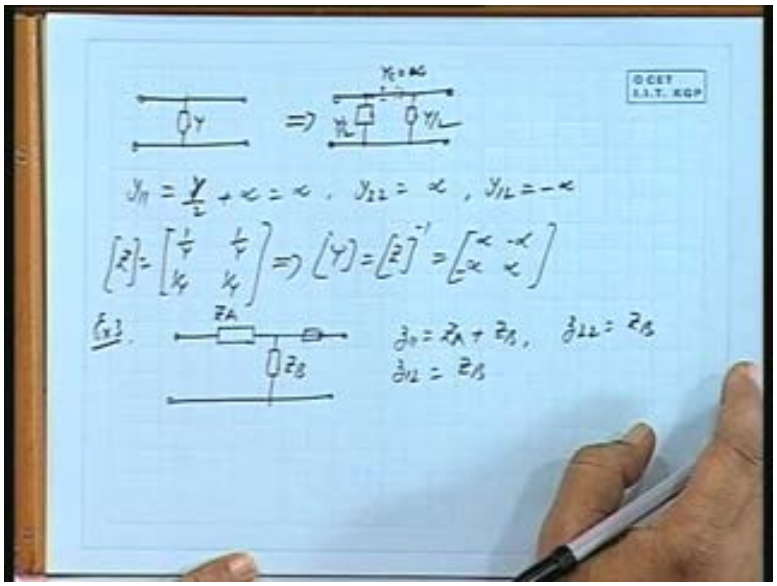

So now the Z metrics is finite we can imagine its inwards does not exist because you can see the determinant will be 0 it will be Y, Y, Y, Y a 1 by Y 1 by Y 1 by Y 1 by Y. So that will give you 0, so that can be verified from here if I draw the equivalent phi this Y, I can distribute into Y by 2 and Y by 2 okay and in between I put Y C equal to 0, 0 admittance, sorry infinite admittance is a short circuit. So infinite admittance, so what will be Y parameters then y 11 is Y by 2 plus this one which is infinity capital Y by 2 plus infinity. So that is infinity, what about y 22 similarly again Y by 2 plus infinity.

So infinity and what about Y 12, Y C that is minus infinity y 1 2 will have a negative sign of the actual admittance. So it is 1 by Y, 1 by Y, 1 by Y, 1 by Y this was our Z metrics all right. So inverse of that Y metrics will be Z inverse okay that is infinity, infinity, infinity, infinity okay because the determinant does not exist other way is also it is verified from here. A third example will take an L connection Z A and Z B what will be Z 11, Z 11 will be Z A plus Z B, Z 12, Z B Z B and Z 22 also Z B, I can always assume a 0 impedance value here.

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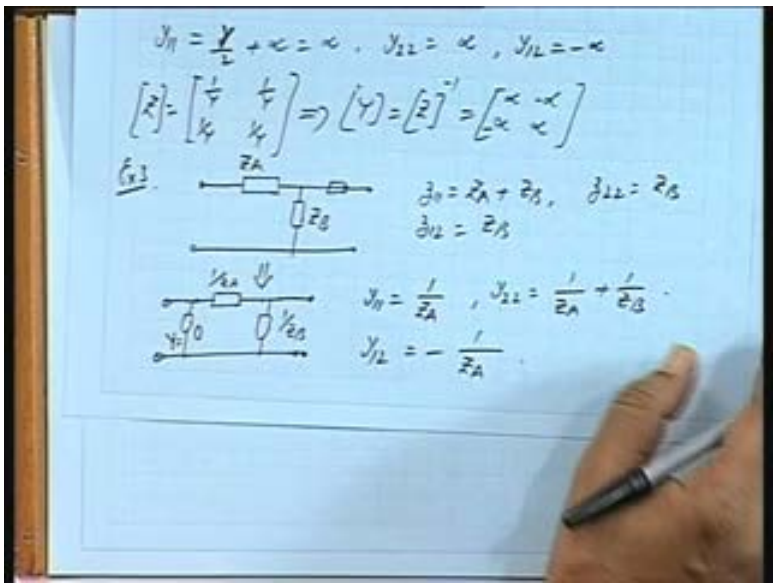

So this plus this will be Z B itself okay now if you are ask to calculate the Y parameters for this then I will take an equivalent phi. So this is 1 by Z A admittance, this one, this admittance is 1 by

Z B and what about this admittance, infinite impedance that means the admittance is 0 okay. Now you calculate the Y parameters what will be Y 11, Y 11 is this plus this, so 1 by Z A okay Y 22, 1 by Z A plus 1 by Z B thank you and Y 12 minus 1 by Z A minus 1 by Z A is that all right you will find if I take the inverse of this metrics I will get these segment, elements of this segment takes will be like this. Now sometimes we are given a network in this form sub breached network, 2 port network which is breached. This can be conceived as 2, 2 port networks which are connected in parallel okay as if I am having this one I call it network a plus network b, this is only an example, it can be anything if I can break up that network in to 2 separate networks like this which are put in parallel.

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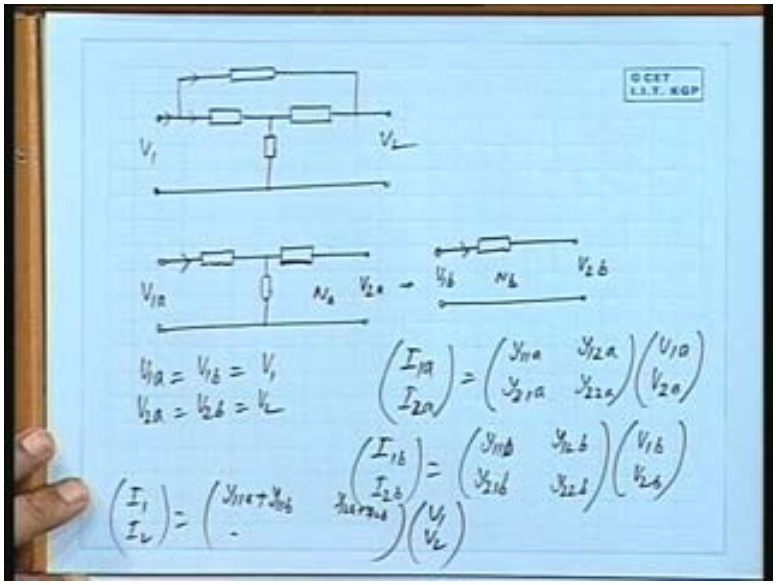


When I say the 2 networks are in parallel what are the common features, what are the things which are common the voltage applied here V 1 and V 2 are same if I call it V 1 A and V 2 A for this network V 1 B and V 2 B then V 1 A equal to V 1 B is equal to V 1 and V 2 A is equal to V 2 B is equal to V 2. This voltage is same as this and this, this voltage is same as this and this okay.

Therefore for this network if we write the equation say I 1 a and I 1 b then I 1 a plus I 1 b will be total I 1 okay. So V 1, I 1 a, I 1 b sorry, I 2 a can be written as Y 1 1 a, Y 1 2 a, Y 2 1 a and Y 2 2 a in to V 1 a and V 2 a okay which is nothing but V 1 and V 2 okay, this is V 1 and V 2. Similarly the second network is I 1 b, I 2 b equal to y 1 1 b, y 2 1 b, y 1 2 b, y 2 2 b, V 1 b, V 2 b since, this vector is same as this vector so and these elements can be added together to generate the total current vector this plus this. So I 1, I 2 which is nothing but this plus this will be equal to Y 1 1 a plus Y 1 1 b similarly, Y 1 2 a plus Y 1 2 b and so on that means element wise you add the values of Y parameters and then this vector is same as this. So I will write straight away V 1, V 2 that means the admittance matrices are added together for the 2 sub networks when they are in parallel is that all right. This is quite common, you can have a bridged t network, you can have a bridge t network.

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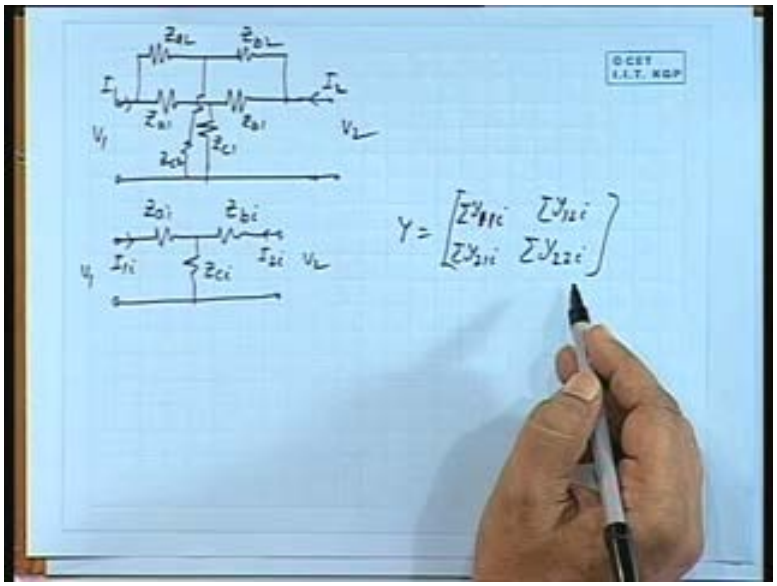

So if I ask you say this is Z a, Z b, Z c, I will call it Z a 1, Z b 1, Z c 1 okay another set Z a 2, Z b 2 and Z c 2 if I am given a network like this, what would be the voltage current relations, this is I 1, this is I 2, how do I write like that. This can be written as 2 separate networks 1 and 2, Z a i, Z b i where I is taking 2 values I 1 and I equal to 2, Z c i corresponding I 1 I, I 2 i. I can write like this same voltages V 1 and V 2 I need not write V 1 I because voltages are same V 1 I, V 2 I

etcetera should be same as V 1, V 2 therefore, from here you write the Y parameter values okay you have to just go for star to delta transformation or by the earlier methods or circuit by applying short circuits you can write in terms of these impedances once you know in terms of Z a I, Z b I, Z c I you write and then substitute the values the exact values for these for that 2 cases and then add them together.

So Y metrics will be ya sorry Y 11 I summation over I, Y 12 I summation Y 21 I summation these are identical Y 22 I summation okay. So this is a twenty network which can be reduced from a star to a delta and directly you can write this star delta conversion you have done. So basically star to delta you have to do you have to write that intense.

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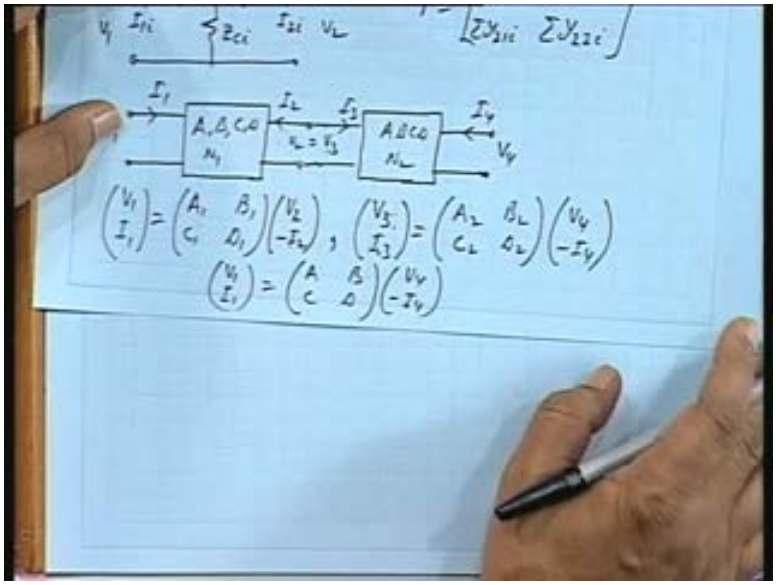

Next suppose you have 2 such 2 port networks in cascade that is they put in cascade like this, I 1 V 1, I 2, V 2 again this is this I 2 becomes I 3, I 4, I 2 is minus I 3 or I 3 is minus I 2 okay. Suppose this is the first set A, B, C, D for network 1 A, B, C, D for network 2. So I will call it A 1, B 1, C 1, D 1, A 2, B 2, C 2, D 2. So V 1, I 1 is equal to A 1, B 1, C 1, D 1 in to V 2 minus I 2 okay then this same V 2 is appearing here. So V 2 is equal to V 3, V 3, I 3 equal to A 2, B 2, C 2 D 2 in to V 4 minus I 4.

Now what would be the relation between V 1, I 1 and V 4, I 4 if V 2 is equal to V 3 and I 2 is minus I 3. So can you try that you will find the A, B, C, D matrices I just multiplied that will give you the overall A, B, C, D matrix. So I 1, V 1, I 1 in terms of A, B, C, D, V 4 minus I 4. So what is this A, B, C, D is it the product of these 2 matrices you have to establish that do that you can see V 2 minus I 2 is same as V 2 is equal to V 3, I can write V 3 to I 3 this is same as this, is it not because minus I 2 is I 3 we can replace that so you can replace it, so it becomes this in to this in to this.

So A, B, C, D matrices will be just multiplied now in a practical situation you have say in a transmission system you have a transmission line of some dimension some height okay then you change the height there is some change in the line parameters so A, B, C, D parameter values also change and if they put in cascade all right there is no taping of power as such but there they are put in cascade even if there is some taping it really does not matter because that would be giving you some shunt element only, so overall A, B, C, D parameter will be the product of these 2 A, B, C, D matrices.

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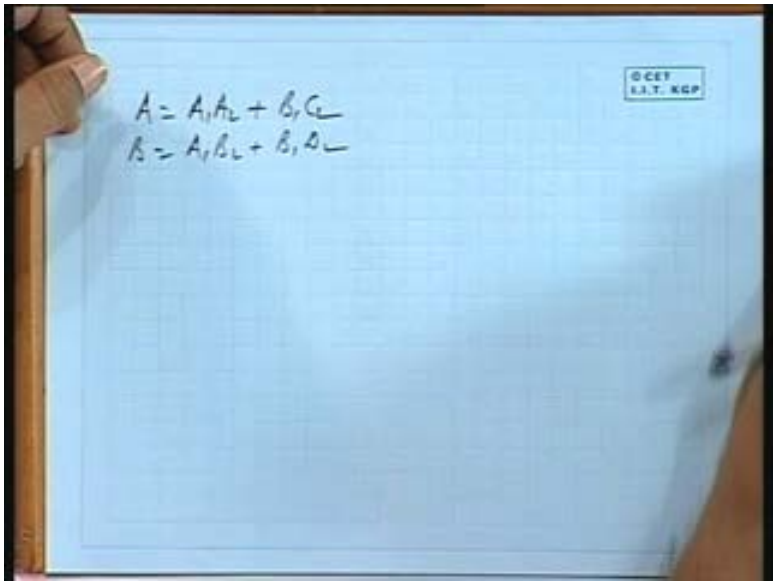

So A will be over all A will be A 1, A 2 plus B 1, C 2, B will be B will be this multiplied by this A 1, A 2 plus B 1, C 2 then this multiplied by this so A 1, B 2 plus B 1, D 2, A 1, B 2 plus B 1, D 2 mind you this coefficient, this element A is the ratio of 2 voltages. So it is a dimensionless constant, dimensionless parameter okay. Now then it will have a magnitude does it have an angle it may have an angle if the impedances are complex normally in a transmission line we have RLC, so impedances are in a complex form so the ratio can be a complex constant dimensionless but it may have an angle.

So here also we find it is the product of 2 dimensionless quantities, it is this B 1 what was the dimension of B 1 set B, B is the ratio between voltage and current that is impedance and C is current and voltage that is admittance. So B and C product will also be a dimensionless quantity, one is more the other is low. So this conforms to the dimension of this similarly, B impedance this one is impedance multiplied by a constant, this also impedance multiplied by a constant because D has the dimension of a it does not have any dimension, it is the ratio of 2 currents so similarly, C and D you can write.

So will stop here for today and next will take up will discuss something about hybrid parameters and then will take up some numerical problems. So the next class will be a tutorial class on whatever we have covered so for will have some problems, okay thank you very much.