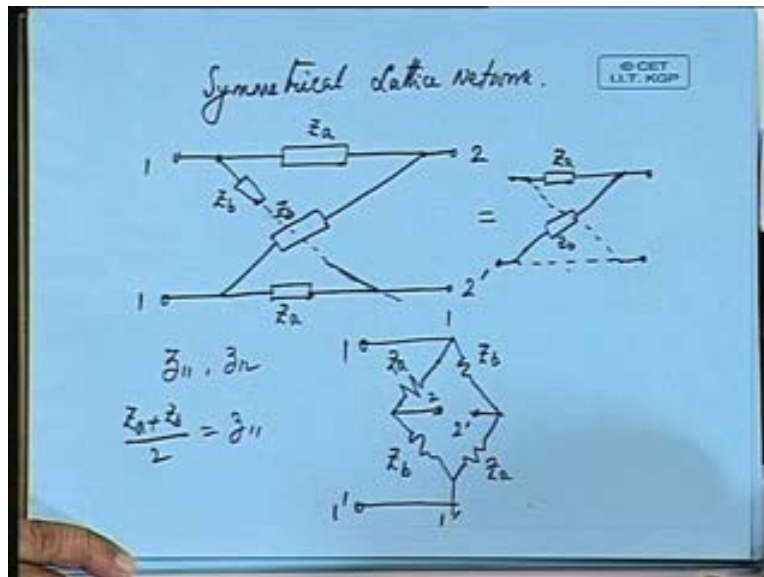


Networks, Signals and Systems
Prof. T. K. Basu
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Lecture - 34
Synthesis of 2 - Port Network (Contd...)

Good afternoon friends, today we shall be discussing about symmetrical network, symmetrical Lattice network for 2 port synthesis.

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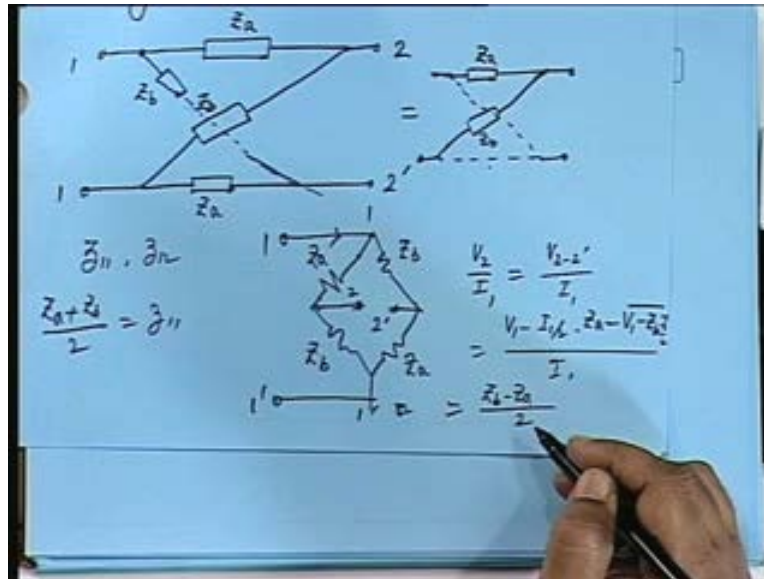


Let us take simple network function in this form Z_a is an impedance, Z_b is this impedance and similarly you have Z_a here and Z_b here. Now what will be for this network what will be Z_{11} and Z_{12} in terms of Z_a and Z_b you shall be see the symmetry, you shall be denoting this henceforth by an impedance Z_a and Z_b okay and it is understood by this dotted lines you denote the other 2 counterparts, this is Z_a and this is Z_b . So we shall be showing only the dotted lines and what will be Z_{11} and Z_{12} , if you look at this network see one side is Z_a and Z_b , so I can show from one say I can show it like this Z_a and Z_b okay the junction is one Z_a and Z_b okay, from Z_a the junction of Z_a and Z_b this is 2 that is another branch Z_b and Z_a and that junction is 2 dashed.

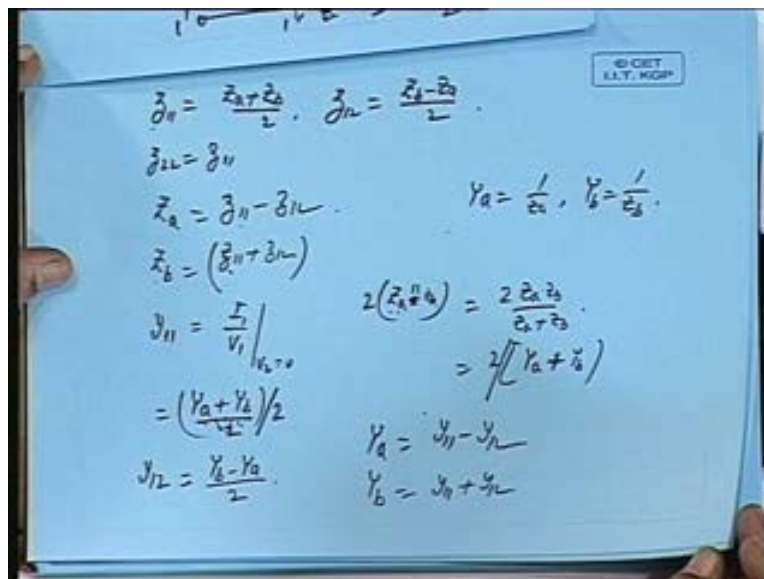
So basically this is 1 dashed if you look at it 1, 1 dashed and 2, 2 dashed this is bridge circuit where this potential across the bridge elements will be the output 2, 2 dashed. So what is Z_{11} we want by I_1 so the impedance measured from 1, 1 dashed when this is kept open, so what is the impedance Z_a plus Z_b , again Z_a plus Z_b . So it will be Z_a plus Z_b by 2 this will be Z_{11} okay what

would be Z_{12} , Z_{12} by definition V_2 by I_1 okay. So when current is I_1 what is the voltage across this, so current is I_1 this is I_1 by 2 this is I_1 by 2.

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So I_1 by 2 into Z_a is a drop here, so from here if I call it V_{22} dashed by I_1 what is V_{22} dashed V_2 is V_1 minus this I_1 by 2 into Z_a is it not and 2 dashed is V_1 minus Z_b into I_1 by 2. So that divided by I_1 so that gives me V_1 gets cancelled okay Z_b this is minus Z_b , Z_b becomes plus so Z_b minus Z_a by 2, I_1 by 2 and I_1 by 2. So Z_b minus Z_a by 2, so we have got 2 interesting results Z_{11} is Z_a

plus Z_b by 2 and z_{12} is Z_b minus Z_a by 2, z_{22} this case is same as Z_{11} okay. So if you are given this z_{11} and z_{12} as specification what will be Z_a if I add them divided by 2 I will get Z_b , Z_b is this plus this and Z_a is this minus this z_{11} minus z_{12} okay.

So if I have given the specifications z_{11} and z_{12} just subtract z_{12} from z_{11} you get Z_a add you get Z_b and hence synthesis, if you are given instead of this, if you are given in terms of y parameters what is y_{11} . Suppose, I call it Y_a as 1 by Z_a , Y_b as 1 by Z_b then what will be y_{11} short circuit parameters, y_{11} is I_1 by V_1 when V_2 is 0 if I short circuit this it is Z_a plus Z_b in parallel again Z_a plus Z_b in parallel. So it will be and then in series, so Z_a plus Z_b sorry Z_a parallel Z_b 2 times this, so twice Z_a , Z_b by Z_a plus Z_b okay. So if I divided by this 2 into 1 by Z_a plus 1 by Z_b , so Y_a plus Y_b okay.

So y_{11} is correct me if I am wrong, Y_a plus Y_b into 2 or by 2 into 2 sure this is 2 into this 2 into this so this is is this all right 2 by and what is this? This is parallel combination of, so what is inverse of that, so this should Y_n , Y_a plus Y_b divided by 2 okay what will be y_{12} similarly, we will find y_{12} will be Y_b minus Y_a divided by 2, you can verify for yourself okay. So what will be Y_a if I multiply by 2 and 2 into y_{11} plus 2 into y_{12} will be giving me Y_b and Y_a will be y_{11} minus y_{12} and Y_b will be y_{11} plus y_{12} almost identical relationship all right. So now the synthesis is very very simple, is it not compare to Laden network, Lattice synthesis is simpler. Now let us see there are different types of specifications for **2 port synth**, 2 port networks

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$$\begin{aligned}
 z_{11} &= \frac{s^2 + 4s}{(s+1)(s+9)} & z_{12} &= \frac{2s}{(s+1)(s+9)} \\
 &= \frac{k_1s}{s+1} + \frac{k_2s}{s+9} & &= \frac{k_1s}{s+1} + \frac{k_2s}{s+9} \\
 &= \frac{3s}{s(s+1)} + \frac{5s}{s(s+9)} & &= \frac{2s}{s(s+1)} + \frac{15}{s(s+9)} \\
 z_a &= z_{11} - z_{12} & z_b &= z_{11} + z_{12} \\
 &= \frac{1s}{s(s+1)} + \frac{7s}{s(s+9)} & &= \frac{5s}{s(s+1)} + \frac{3s}{s(s+9)}
 \end{aligned}$$

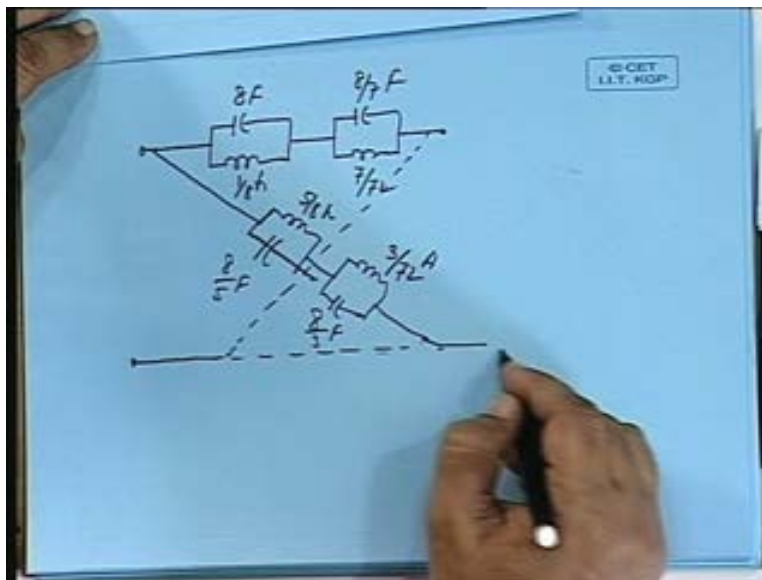
So let us synthesis by this method for different specifications. Suppose you are given z_{11} let us take an example z_{11} as sorry let us take an example z_{11} again say LC elements s cubed plus $4S$ divided by s squared plus 1 into s squared plus 6, 9, z_{12} is equal to $2S$ by s squared plus 1 into s squared plus 9 okay one thing is you can factorize this make partial fractions. So $K_1(s)$ by S

squared plus 1 plus $K_2(s)$ by S squared plus 9 okay how much is K_1 multiply by s squared plus 1 divided by S .

So this is s squared plus 4, so 3 by 8 okay S by S squared plus 1 similarly, this one will be 9, 5 by 8 s by s squared plus 9 okay z_{12} similarly, if we say $K_1(s)$ by S squared plus 1 plus $K_2(s)$ by S squared plus 9, so that gives me S squared plus 1 and divided by S , so 2 divided by 8, so 1 by 4 s by s squared plus 9 okay S squared plus 1, thank you. Then K_2 if I multiply by S squared plus 9 and cancel out S , so minus 1 by 4 S into S squared plus 9 okay. Now what will be Z_a , Z_a was z_{11} minus z_{12} so this minus this okay I can always write 3 by 8 minus 1 by 4 which is 2 by 8 so, s by s squared plus 1 into 1 by 8 okay plus 5 by 8 minus 1 by 4.

So plus 3 by 8 s by s squared plus 9 okay so both are positive real whereas Z_{12} may have a negative sign but you have to see whether after combining everything remains positive or not, Z_b similarly, Z_b was a z_{11} plus z_{12} , pardon 5 by 8 yes, minus of Z_a where did I write Z_a , Z_a is z_{11} minus z_{12} so 3 by 8 minus 1 by 4 okay 5 by 8 plus 1 by 4, so 2 by 8, 7 by 8 thank you, thank you. Similarly, Z_b will be z_{11} plus z_{12} so 3 by 8 plus 2 by 8, 5 by 8 s by s squared plus 1 and if I add this with this, this will become 3 by 8 okay s by s squared plus 9. So what does it look like with this specification I can realize a network, this is an LC network, LC parallel a value 1 by 8 Henry and when s is made very large 1 by 8, 8 farads is it all right. Again 7 by 72 divided by 8 by 7 farads is it all right this is Z_a similarly Z_b will be an inductance and a capacitance, these 2 elements are identical.

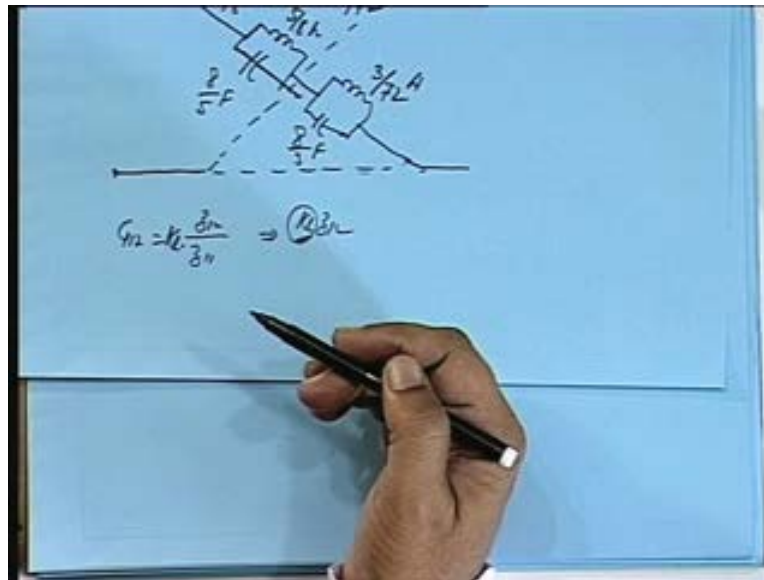
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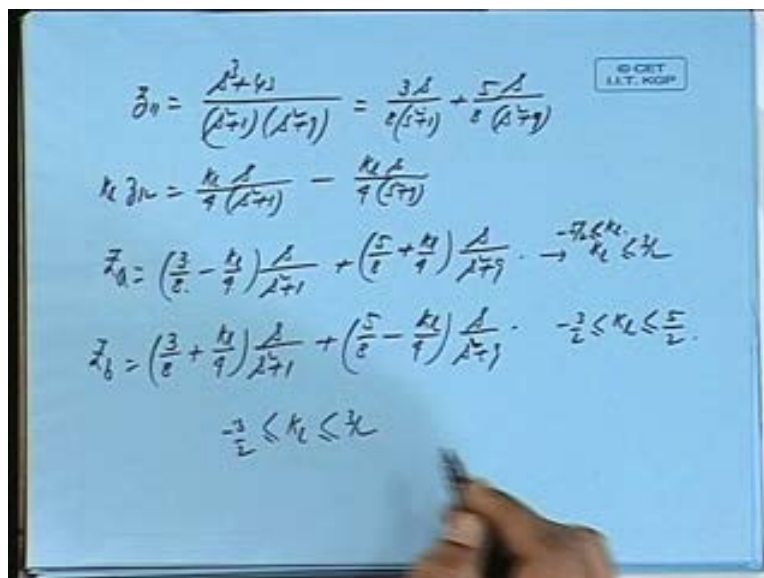
So this one will be Z_b is 5 S by 8 S plus 8 by 5 farads okay and 5 by 8 Henry. Similarly, this one 3 S by 8 s squared so 8 by 3 farads and 3 by 20, 3 by 72 okay Henry. So these are the elements values there are large number of elements coming but network is very easy to realize. Next we go for a situation when z_{11} minus z_{12} , z_{11} plus z_{12} , these things are made basically z_{12}

contributes to the function G_{12} , is it not it is G_{12} gain which will be decided by z_{12} by z_{11} . Now I can always multiply this by a constant because poles and 0s will not get disturbed by that constant it is something like an amplifier we can put later on which can take care of whatever gain is to be adjusted. So instead of z_{12} sometimes we go for K_L times z_{12} , if I make K_L times z_{12} then it will be z_{11} minus K_L times z_{12} , z_{11} plus K_L times z_{12} okay.

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So all these will be multiplied by a factor K_L and you can adjust the value of K_L such that every time whether it is z_{11} minus z_{12} and z_{11} plus z_{12} all of them will be positive real okay because there may be a negative sign here. So sometimes we will be adding some times will be subtracting so some of them may turn out to be negative if we do not have a factor K_L , so K_L is an adjustable parameter. So G_{12} this is z_1, z_{12} by z_{11} and we can adjust K_L so we will write K_L as a multiplying factor. So for the same problem for this problem, what is the maximum value of K_L so that we can realize a Lattice network okay.

So we will take up these values now z_{11} is $s^3 + 4s$ divided by $s^2 + 1$ into $s^2 + 9$ which has given me factors like 3 by $8s$ by $s^2 + 1$ plus 5 by $8s$ by $s^2 + 9$. We will very soon see the effect of K and K_L times z_{12} will be K_L times 1 by $4s$ into $s^2 + 1$ okay minus K_L by $4s$ into $s^2 + 9$ okay. So Z_a, Z_a is z_{11} minus this so 3 by 8 minus K_L by 4 into s by $s^2 + 1$ plus 5 by 8 plus K_L by 4 okay into s by $s^2 + 9$, Z_b will be 3 by 8 plus K_L by 4 into s by $s^2 + 1$ plus 5 by 8 minus K_L by 4 into s by $s^2 + 9$. Now for this to be positive real what should be the range of K_L you vary from say minus to plus if you had it then this has to be always positive such 3 by 8 is equal to K_L by 4 that is the limiting situation when this becomes K_L is equal to 3 by 2 okay if K_L becomes more than that then this will become negative.

So K_L should be less than equal to 3 by 2 and here K_L by 4 I can take it to minus 5 by 8 till that time it will be positive that means minus 5 by 2 less than K_L so these are the 2 limits for K_L for Z_a to be positive real. Similarly, for Z_b it will be minus 3 by 2 K_L and 5 by 2 , now if you use both the restrictions then K_L should be minus 3 by 2 , 2 plus 3 by 2 okay. So you can choose any of these limiting values and then see what will be the network like suppose we choose K_L is equal to 3 by 2 .

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Handwritten mathematical derivations on a whiteboard:

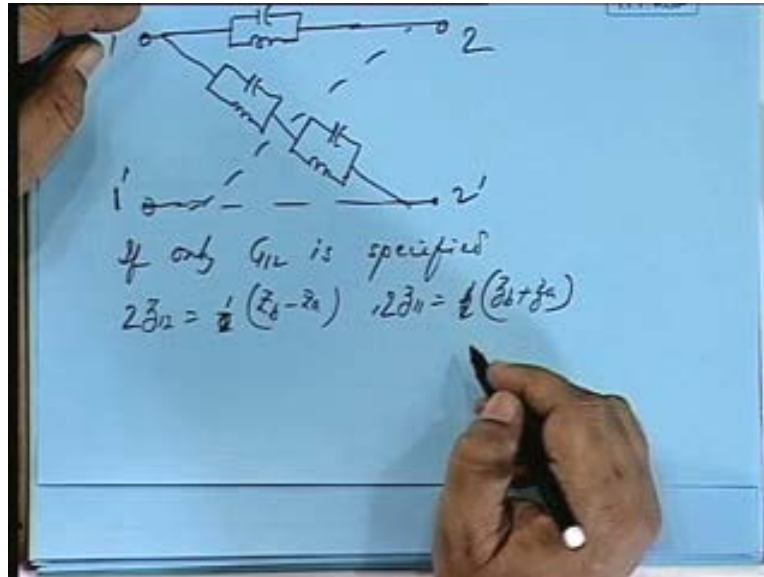
$$= \left(\frac{3}{8} - \frac{K_L}{4}\right) \frac{s}{s^2+1} + \left(\frac{5}{8} + \frac{K_L}{4}\right) \frac{s}{s^2+9} \rightarrow -\frac{3}{2} \leq K_L \leq \frac{3}{2}$$

$$= \left(\frac{3}{8} + \frac{K_L}{4}\right) \frac{s}{s^2+1} + \left(\frac{5}{8} - \frac{K_L}{4}\right) \frac{s}{s^2+9} \rightarrow -\frac{3}{2} \leq K_L \leq \frac{3}{2}$$

$-\frac{3}{2} \leq K_L \leq \frac{3}{2}$
 Select $K_L = \frac{3}{2}$
 $Z_a = \frac{s}{s^2+1}$
 $Z_b = \frac{3}{4} \frac{s}{s^2+1} + \frac{1}{4} \frac{s}{s^2+9}$

Let us choose K_L equal to 3 by 2 plus 3 by 2 then what happens if you choose K_L equal to plus 3 by 2 first term becomes 0, so this element will not figure in Z_a only thing accordingly this value will change so 3 by 2, 3 by 8 plus 5 by 8, so Z_a is equal to s by s squared plus 9 is that so 3 by 8 plus 5 by 8 Z_b in that case becomes 3 by 2 means 3 by 8, 3 by 2, so 3 by 8 plus 3 by 8, 6 by 8, 3 by 4 okay 3 by 4 s by s squared plus 1 and this one 5 by 8 minus 3 by 8, so one fourth, 1 by 4 s by s squared plus 9.

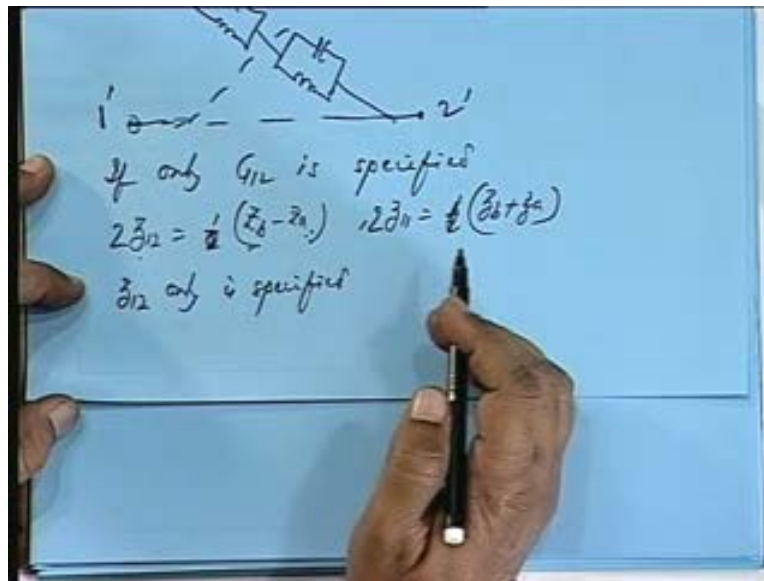
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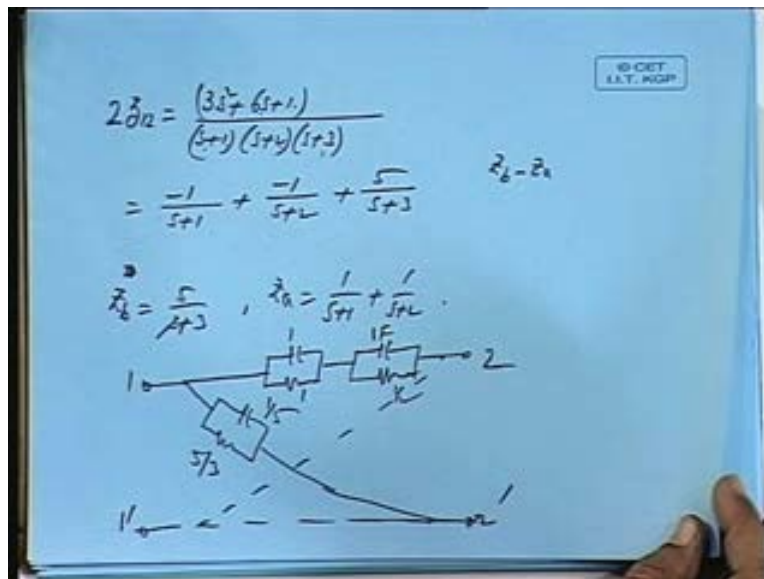
So in Z_a you have just one LC network in Z_b you have to such such elements is at all right. You can calculate the values okay see earlier we had 1 parallel bank here again one another trans circuit here. So this one again here similar elements will be coming, so these we are eliminating by taking K_L equal to that limiting value 1 of the limits if you take K_L equal to the other limit minus 3 by 2 minus 3 by 2 minus 3 by 2 then in Z_b this will get eliminated. So Z_b will have one less element whereas Z_a we will have 2 elements. So this is how we realize the networks for Z_a and Z_b , let us take another example okay, let us take another specification Z_a and Z_b we have seen the specifications in terms of G_{12} , z_{11} and z_{12} and G_{12} with a gain factor K_L okay.

Now if you are given only G , G_{12} if you are given only G_{12} , if only G_{12} is specified, only G_{12} specification means you are okay z_{12} is how much half of Z_b minus Z_a and z_{11} is half of Z_b plus Z_a okay or twice this equal to this and twice z_{11} is equal to this. Suppose you have specified let us come to G_{12} specifications just see little later if only z_{12} specified, if you are given only z_{12} then I can always adjust Z_b and Z_a in such a way that Z_b minus Z_a corresponds to z_{12} and both Z_b and Z_a are positive real functions okay. So we can have Z_b and Z_a as positive real functions, so z_{12} you are given $z_3(s)$ squared plus 6 s plus 1 mind you z_{12} need not be a positive real function divided by s plus 1 into s plus 2 into s plus 3 okay.

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So we write twice Z_{12} as this it is a twice Z_{12} we just specified, so break it up S plus 1 what is it minus 1 by S plus 1 plus S plus 2 if I write minus 1 by S plus 2, S plus 3 how much is it S plus 3, 5 by S plus 3 okay you can see S plus 1 if I make S plus 1 equal to 0 and S is equal to minus 1, so 3 into 1 minus 6 plus 1, so 3 plus 1, 4 minus 6, minus 2 divided by 1 into 2 all right. So that gives me minus 1 by S plus 1 similarly, this one. So this is equal to Z_b by Z_b minus Z_a , so this is this is with a minus sign is with Z_a .

So this I can put in Z_a this I can put in Z_a this is in Z_a , so Z_b I can always choose as 5 by s plus 3 so simple and Z_a as 1 by S plus 1 plus 1 by S plus 2, 5 by s plus 3 one fifth farad and 5 by 3 ohm resistance and here 1 by S plus 1 is 1 farad with 1 ohm resistance, 1 by S plus 2 with half ohm resistance, 1 farad with half ohm resistance. So this is Z_a and Z_b okay, so if I have given only this specification it is very simple you can choose any value of Z_a and Z_b , next Z_a by Z_b . I can calculate what is G_{12} , G_{12} is z_{12} by z_{11} and z_{12} in terms of Z_a and Z_b , Z_b minus Z_a divided by 2, divided by 2 get cancelled Z_b plus Z_a okay.

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The image shows a person's hands writing mathematical equations on a blue board. The equations are:

$$\frac{Z_a}{Z_b} = \frac{1 - G_{12}}{1 + G_{12}} = \frac{p(s) - q(s)}{q(s) + p(s)} = \frac{m_1 + n_1}{m_2 + n_2}$$

$$G_{12} = \frac{z_{12}}{z_{11}} = \frac{Z_b - Z_a}{Z_b + Z_a} = \frac{p(s)}{q(s)}$$

$$Z_a = \frac{1 - G_{12}}{1 + G_{12}}, \quad Z_b = \frac{1 + G_{12}}{1 - G_{12}}$$

$$Z_a = \frac{1 + K \frac{m_2}{m_1}}{1 + K \frac{n_2}{n_1}}$$

So from there if I make **component by** this will become 1 minus G_{12} by 1 plus G_{12} okay, if G_{12} is given as some $p(s)$ by $q(s)$ then it will be $p(s)$ plus sorry $q(s)$ minus $p(s)$, $q(s)$ minus $p(s)$ divided by $q(s)$ plus $p(s)$ okay by depending on the nature of this 1 minus G_{12} sometimes Z_a may match with this Z_b may match with this or Z_a may match with 1 by this and Z_b may be 1 by 1 minus G_{12} . So one may choose Z_a as 1 minus G_{12} , if it is a positive real function or Z_a as 1 by 1 minus G_{12} and Z_b sorry 1 by 1 plus G_{12} , Z_b as 1 by 1 minus G_{12} okay.

Suppose, when you write **m S** G_{12} as $p(s)$ by $q(s)$, Z_a by Z_b you write like this and you get this as an even part plus an odd part similarly, even part and then odd part if it so happens either m_1 , m_2 is a constant or n_1 by n_2 is a constant that means either the odd part or the even part of the numerator is a constant times the corresponding part of the denominator or the even part of the numerator is a constant times the even part of the denominator then something very interesting comes Z_a by Z_b I can write this as 1 plus n_1 by m_1 divided by 1 plus K times n_2 is K times n_1 , n_1 by m_2 okay.

So this is odd by even, this can be an LC network this also odd by even another LC network, this is a resistance, this is a resistance. So it will be like this a resistance and an LC network series inductance, series parallel inductance, capacitance. Similarly, this side will have a resistance and

similarly another LC network I could have done it the other way round also if m_1 is some constant times n_2 or m_2 is constant times n_1 , I can divide it by n_1 sorry sorry that is what we have done if n_1 and n_2 there be a this relation I made a made a mistake sorry, correct okay, so I can get a similar expression.

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Handwritten mathematical derivation on a blue background:

$$G_{12}(s) = \frac{s^2 + 1}{s^2 + 5s + 4}$$

$$q(s) - p(s) = (s^2 + 5s + 4) - (s^2 + 1)$$

$$\frac{Z_a}{Z_b} = \frac{s^2 + 3}{2s^2 + 5s + 5} = \frac{3 + 5s}{(2s^2 + 5s) + 5}$$

$$= \frac{1 + \frac{3}{5}s}{1 + \frac{2s^2 + 5s}{5}}$$

$$Z_a = 1 + \frac{3}{5s}$$

$$Z_b = 1 + \frac{2s^2 + 5s}{5}$$

Diagram showing a series combination of a resistor (1 ohm) and a capacitor (5/3 F) in parallel with a series combination of an inductor (2H) and a capacitor (5/5F).

So let us take one example $G_{12}(s)$ is equal to $s^2 + 1$ by $S^2 + 5s + 4$ okay so $q(s)$ plus minus $p(s)$ will be $s^2 + 5s + 4$ plus minus $s^2 + 1$ which gives me there will 2 polynomials. So Z_a by Z_b , Z_a by Z_b will be first is a negative say $s^2 + 3$ divided by $2s^2 + 5s + 5$ okay. So if you separate out the even part and odd part $3 + 5s$ divided by $2s^2 + 5s + 5$, so these 2 are constant I mean proportional ratio is 1 so divide throughout by $5s$, so I can write this as $1 + \frac{3}{5s}$ divided by $1 + \frac{2s^2 + 5s}{5}$ okay.

So Z_a I can write as 1 ohm resistance plus $\frac{3}{5s}$, so 1 ohm resistance and a capacitor of $\frac{5}{3}$ Henry, farad and the Z_b as 1 ohm and then $2s^2 + 5s$, so it is an LC parallel combination, okay. You can find out the values of L and C, Z_b is $1 + \frac{2s^2 + 5s}{5}$ it is a basically a series combination, so L and C sorry this is the series combination okay. Let us take up 1 or 2 more examples it will be easier to follow. Let us take this problem z_{12} is equal to $3s^2 + 5s + 1$ divided $s^2 + 2s + 5$ into $s + 3$ into $s + 5$. So how do you find out the Lattice method its simple z_{12} we have seen Z_b minus Z_a . So let us break it up into partial fractions K_1, K_2, K_3 sorry S plus similarly, $S + 5, 25$ into $320, 75$ minus 5 into $5, 25$ so $50, 51$ okay divided by 3 into 2, 6 so this is 19 by 2 okay.

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$$z_{12} = \frac{3s^2 + 5s + 1}{(s+2)(s+3)(s+5)}$$

$$= \frac{k_1}{s+2} + \frac{k_2}{s+3} + \frac{k_3}{s+5}$$

$$= \frac{1}{s+2} + \frac{13}{2(s+3)} + \frac{19}{5(s+5)}$$

$$\Rightarrow z_{12} = \frac{2}{s+2} - \frac{13}{s+3} + \frac{19}{s+5}$$

$$Z_b = \frac{2}{s+2} + \frac{19}{s+5}, \quad Z_a = -\frac{13}{s+3}$$

So what should be Z_a , Z_b minus Z_a is z_{12} , is it z_{12} is how much twice or twice z_{12} , so twice z_{12} we have to calculate these 2 times these 2 by S plus 2 minus 13 by S plus 3 plus 19 by S plus 5. So Z_a and Z_b , Z_b will correspond to this 2 by s plus 2 okay plus 19 by s plus 5 we can put this one and Z_a is whatever is as I said you the negative sign 13 by S plus 3 okay.

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$$= \frac{k_1}{s+2} + \frac{k_2}{s+3} + \frac{k_3}{s+5}$$

$$= \frac{1}{s+2} + \frac{13}{2(s+3)} + \frac{19}{5(s+5)}$$

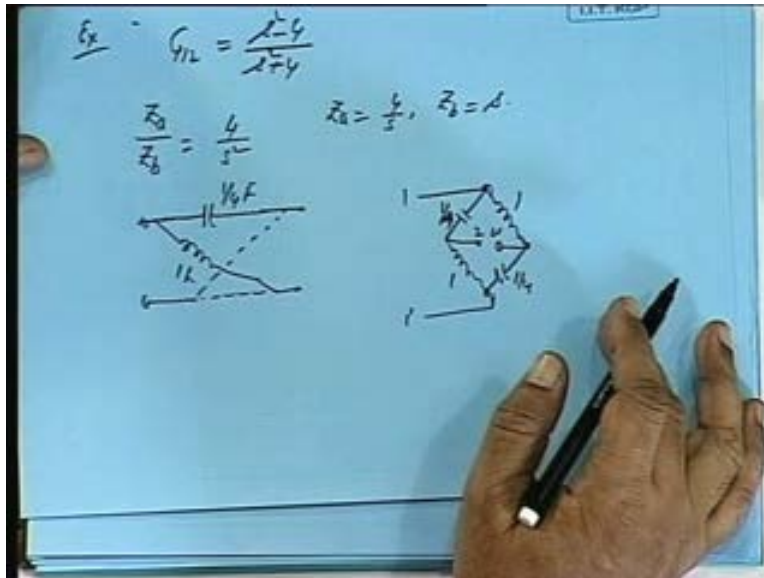
$$\Rightarrow z_{12} = \frac{2}{s+2} - \frac{13}{s+3} + \frac{19}{s+5}$$

$$Z_b = \frac{2}{s+2} + \frac{19}{s+5}, \quad Z_a = -\frac{13}{s+3}$$

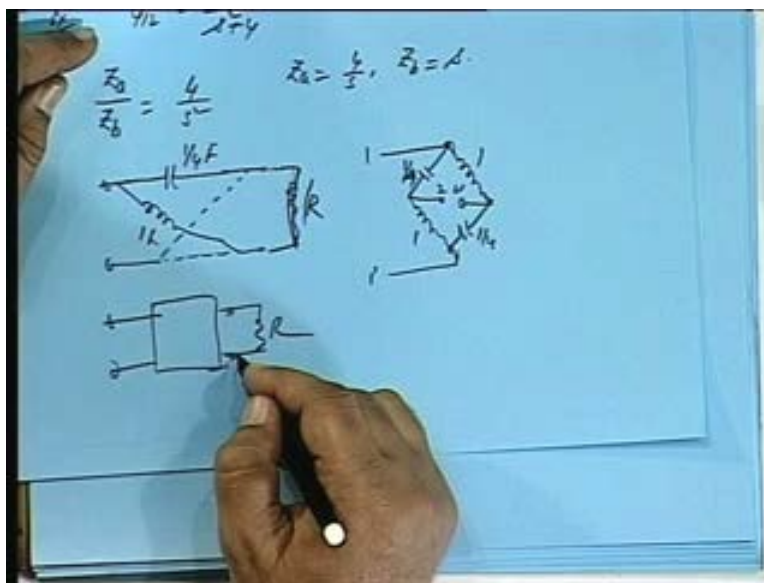
So 2 by S plus 2 is an RL network or RC sorry RC a value 1 ohm and half a farad and then again so this is 1 by 19 and 19 by 5 okay and this side Z_a , 13 by S plus 3, so 13 by 3 ohms and 1 by 13

farad, so this will be the Lattice. We had given another example G_{12} is equal to $s^2 - 4$ by $s^2 + 4$ okay so what is Z_b the Z_a or Z_a by Z_b how much is that Z_a by Z_b this minus this 4 by s^2 is it all right 8 by twice s^2 4 by s^2 . So what should we take Z_a , I can write as 4 by s and Z_b as s .

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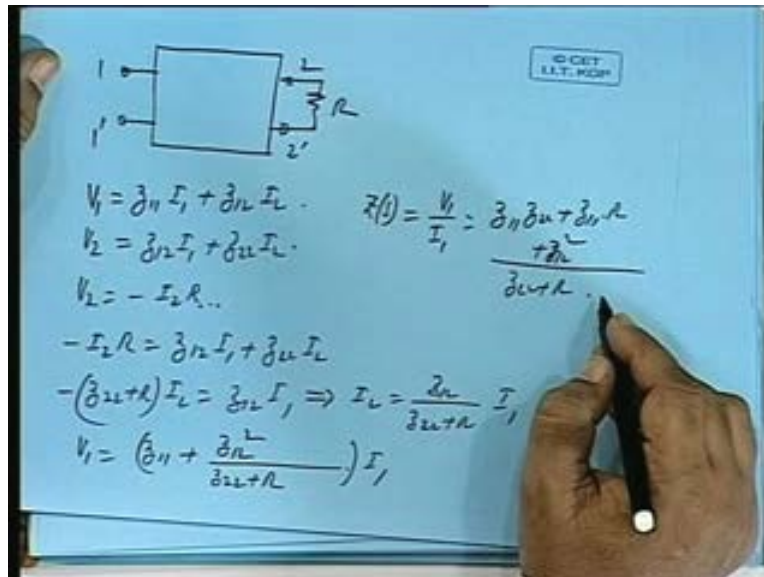
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So you have Z_a as a capacitor of 1 by 4 farad and Z_b as an inductor 1 Henry or in other words we are having a capacitor and an inductor, inductor and a capacitor, is that all right 1 Henry and one

fourth farad. So for this network the gain function will be this, this is very interesting distribution okay. Now this is the basic background that I have given for 2 port Lattice synthesis, now we have normally the impedance, the networks will not be necessarily terminated by infinite impedance, they can be terminated by a fixed impedance, sometimes they are terminated by fixed impedance. So if they are terminated by fixed impedance what is that relationship with the output and input that is for a terminated network which is terminated by a constant resistance R not in this example you have a 2 port network and if we have terminating resistance R then what will be the relationship between input and output okay.

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Let us take the original relationship between voltages and currents V_1 is equal to z_{11} , I_1 plus z_{12} , I_2 and V_2 is equal to z_{12} , I_1 plus z_{22} , I_2 this is a general relationship when it is terminated by a resistance R then V_2 is also equal to minus I_2 times R alright, conventionally we take the currents to be positive when they are in 1 but when I terminating it then I_2 is in the negative relation, so V_2 becomes minus I_2 into R okay. So what will be the impedance from this side, I will call it $Z(s)$ impedance seen from this side that is V_1 by I_1 , when this is terminated, so let us substitute it here minus I_2 into R is equal to z_{12} , I_1 plus z_{22} , I_2 if I take I_2 on this side then minus z_{22} plus R into I_2 , z_{12} , I_1 pardon, I_2 and z_{22} , I_2 , see I am talking this equation this is equated to this so minus $I_2 R$ is z_{12} , z_{12} and z_{21} there identical that we have seen for passive bilateral network it is same okay.

So I_2 , I can write as z_{12} by z_{22} plus R into I_1 substitute this in the first equation what you get V_1 is equal to z_{11} plus z_{12} into I_2 which is z_{12} squared divided by z_{22} plus R whole into I_1 okay. So $Z(s)$ which is V_1 by I_1 , V_1 by I_1 is nothing but z_{11} if I sum multiply z_{11} , z_{22} plus z_{11} R plus z_{12} square whole thing divided by z_{22} plus R okay. If we write like R is equal to 1 ohm, if we find that in a normalized equation if R is 1 ohm and the impedance seen from this side is also 1 ohm then what happens. If V_1 by I_1 is also 1 ohm plus $Z(s)$ is equal to 1 ohm when R is 1, what we

get Z_{11} , V_1 by I_1 is equal to z_{11} plus z_{12} square by z_{22} plus 1 equal to 1 or z_{11} , z_{22} plus z_{11} plus z_{12} square is equal to z_{22} plus 1, am I alright. So what we get z_{11} into z_{22} plus now if you have a symmetrical network where z_{11} is equal to z_{22} okay, is it alright z_{11} and z_{22} the same. So this is z_{11} square plus there is something I have made a slip, z_{11} square just check-up there was a negative sign somewhere, so I_2 was negative so this one will be z_{11} , z_{11} minus z_{12} square is it alright. Sometimes such mistakes will put to to a challenge when you do not get that desired result.

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Handwritten mathematical derivation on a blue board:

$$Z(\beta) = 1, \text{ and } R = 1,$$

$$\frac{V_1}{I_1} = \frac{z_{11} + \frac{z_{12}^2}{z_{22} + 1}}{1} = 1$$

$$\text{or, } z_{11} z_{22} + \frac{z_{12}^2}{z_{22} + 1} = z_{22} + 1$$

$$\frac{z_{12}^2}{z_{22} + 1} = 1 + z_{22}$$

$$z_{11} = \frac{z_a + z_b}{2}, \quad z_{12} = \frac{z_b - z_a}{2}$$

$$\frac{(z_a + z_b)^2}{4} = 1 + \frac{(z_b - z_a)^2}{4}$$

$$\underline{z_a z_b = 1} \quad \boxed{z_a z_b = R^2}$$

So minus z_{12} square or z_{11} square is equal to 1 plus z_{12} squared okay, what is z_{11} , Z_b plus Z_a by 2 what is z_{12} , Z_b minus Z_a by 2. If I substitute here z_{11} squared that is Z_b plus Z_a by 4 is equal to 1 plus Z_b minus Z_a squared by 4 equate Z_a square Z_b square will get cancelled, so we are left with twice Z_a , Z_b so Z_a , Z_b by 2 okay. If I bring it to this side then a plus b whole square minus a minus b whole square that will be $4ab$ that divided by 4 is just ab okay, thank you Z_a , Z_b comes out to therefore 1. So if you want that the network we are done it to have a network which if tenanted by a resistance R , let us take a normalize equation 1 ohm then the impedance seen from this side is also 1 ohm what will be that Z_a and Z_b now Z_a and Z_b must be at this relation Z_a , Z_b is equal to 1.

So in the next class we will start from here. This is for a normalised relation if it is equal to any R , Z_a , Z_b should be equal to R square. So Z_a , Z_b should be equal to R square that you can see from here itself if you would have put R here, R here, you would have got R square. So in general Z_a , Z_b should be equal to R square once you select Z_a automatically Z_b is fixed for such lattice network committing impedance R will generate and impedance of R from the looking inside, thank you very much.

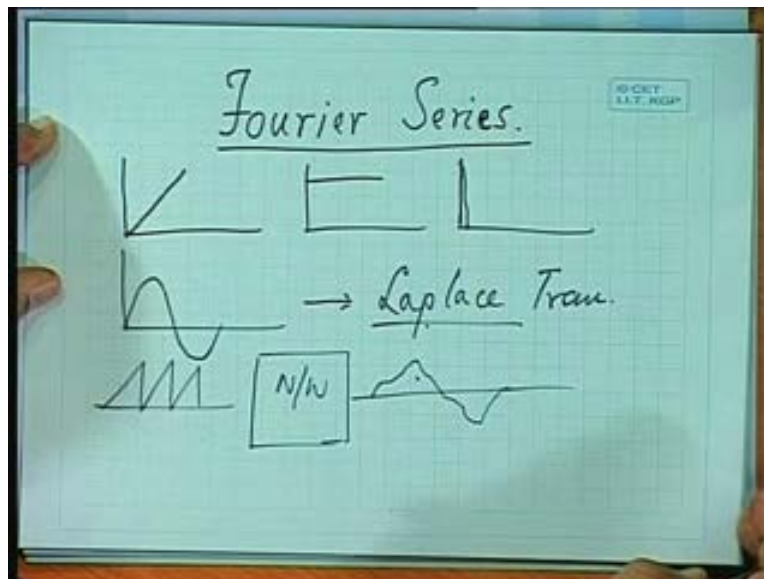
Preview of the next lecture

Lecture - 35

Fourier series

Good afternoon friends, today we shall taking up Fourier Series. So far we have discussed about network functions and their behaviour under different conditions with different types of excitation.

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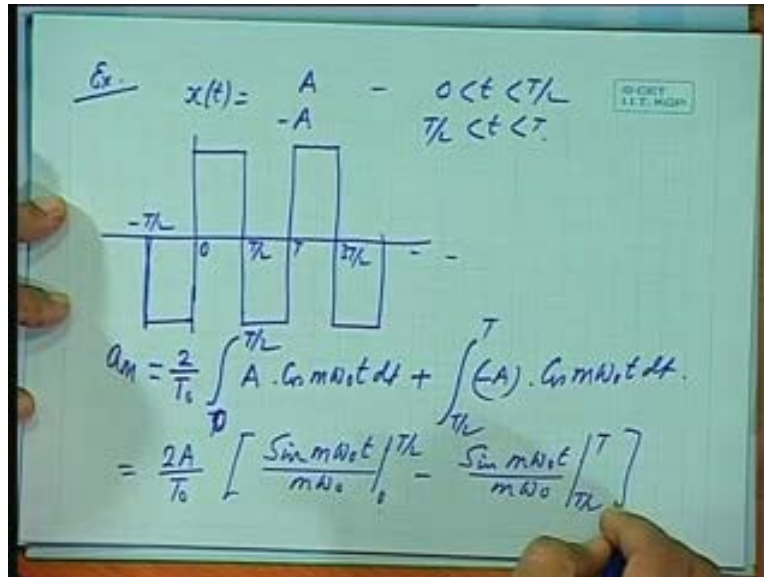


For example, a ramp or a step or an impulse or a sinusoid like this we have taken up different types of functions and you have seen the behaviour of networks and we have used so far in the time domain, the response has been evaluated by using Laplace Transform okay. Suppose you have given a network and you have a periodic function then you take the Laplace transform of this take the network function and then you can evaluate the response the response will also be periodic or something like this. Now the response that we get taking Laplace transform of the function is giving me a total solution that is say total solution means including the transients suppose the responses like this. After sometime it will be stabilizing to a steady value if you are interested in getting the **steady sol** steady state solution. We are not interesting in the transient part we are interested only in the steady state solution then there is an advantage of using the frequency response technique.

We have studied so far the response due to a particular frequency okay. Now if you are giving a periodic function comes it of this type, if you are giving a periodic function as input it is possible to resolve this into large number of sinusoidal functions. Our aim here is to resolve that and then find out the response corresponding to each elementary component of those functions sinusoidal functions and the response due to each of them you calculate independently then use supervision

theorem take the total sum that will be the response due to any periodic function. So for periodic pass signals, we mean pass signal means signals with finite average path.

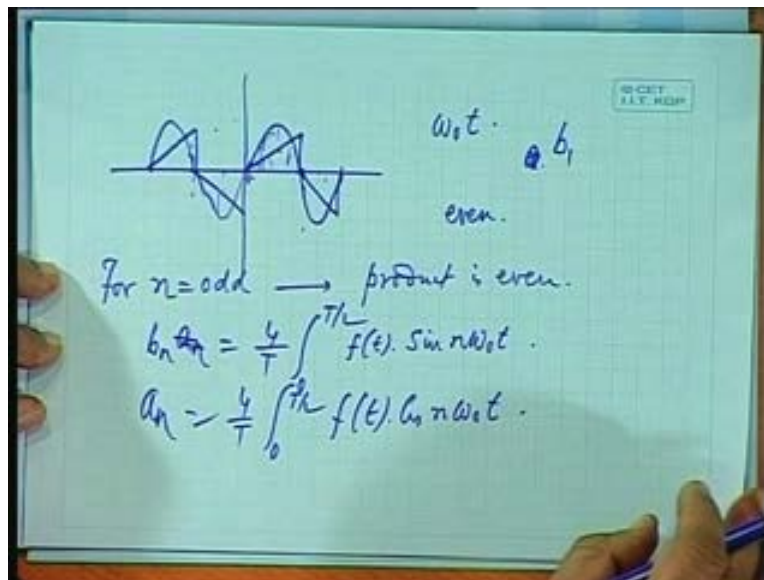
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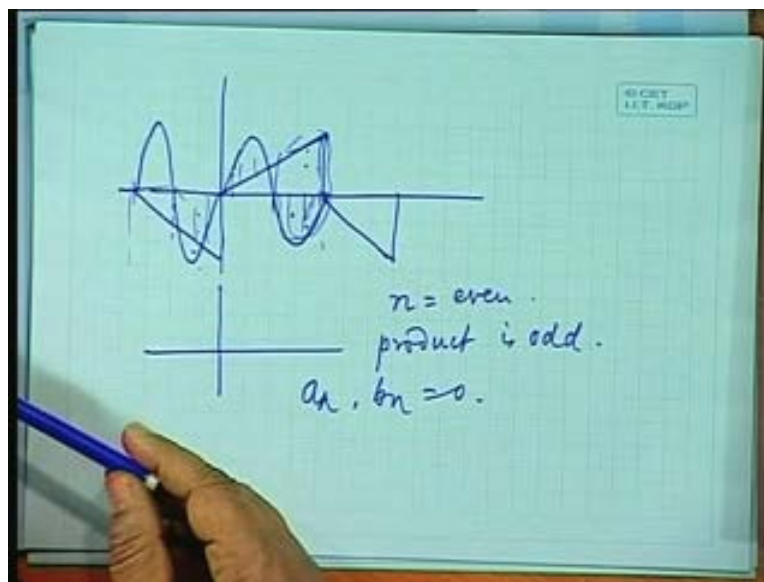
Suppose $x(t)$ is a function which is given as A in this range, T by $2T$, T that means is a kind of function. We define periodic functions only in one period that is good enough like this okay, this is $0, T$ by $2, T, 3T$ by 2 and so on is minus T by 2 and so on, what will be the Fourier coefficients for such a function, a_m will be 2 by $T_0, 0, 2$ okay we can write 0 to T by 2 A it is magnitude is A into cosine m omega not t dt minus sorry, plus T by 2 into T and this becomes minus A cosine m omega not t dt. If you integrate $2A$ by T naught will come it is a cos term and here you get sin of m omega naught T by m omega naught, 0 to T by 2 then with a negative sign, sign m omega naught T by m omega naught, T by 2 to T to an. So it will be 4 by T_0 to T by 2 that function f_t and $\sin n$ omega naught t where n is odd and similarly, this is b_n and a_n will be similarly 4 by $T, 0$ to T by $2, f(t)$ cosine n omega naught T okay. If we take n even then what happens let us see.

Now you are having between this and this point say something like double harmonic. Here you see this is one period, this is positive and positive but here it is negative and positive and the this is negative but this is a large value and this is, this much value its counterpart is here where, both are negative say this component the product of this part, this zone is equal in magnitude to the product of this but here it is positive into negative, here it is negative and negative. So there will be cancelling.

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So this is an odd function, so if n is even product is odd, this product is odd and hence the integral is 0 okay. So a_n and b_n will be 0 for n even for this type of half wave symmetric function alright. So thank you very much in the next class should be continue with this a few more interesting features of Fourier series and some standard functions will take for evaluating the Fourier series, thank you.