Networks Signals and Systems

Prof. T. K. Basu
Department of Electrical Engineering
Indian Institute of Technology, Kharagpur

Lecture - 30

Tutorial

Good morning friends. Today we will have a tutorial exercise on some of the topics that we have covered so far. I will take up a few examples and discuss, there is a problem, first example is very simple one. You have a bridged capacitive circuit the values are 2 farads, 1 farad, 4 farads and 1 farad, what should be the equivalent capacitances c_1, c_2 and c_3 okay.

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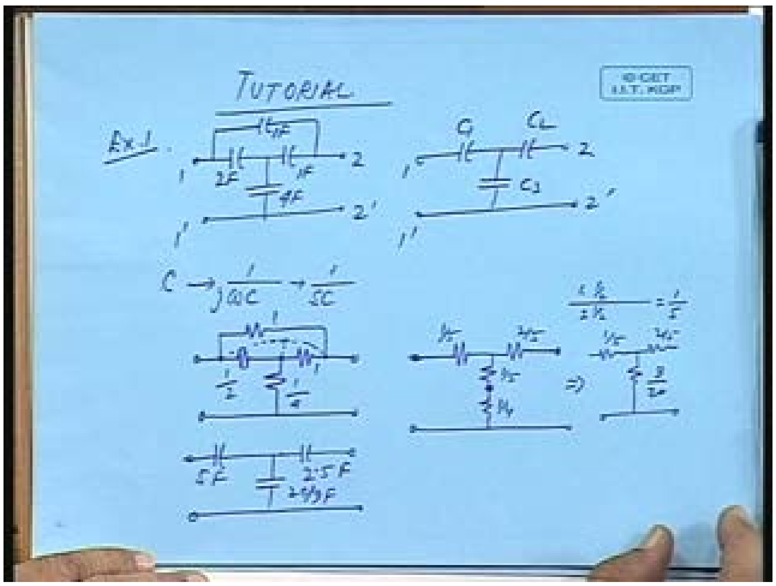

Now in this case, as you know a capacitor is having an impedance if you talk in terms of sinusoids it will be 1 by omega C, it is a j here if you talking Laplace domain it will be 1 by Sc that means the impedance is inversely proportional to C. So I can take these capacitances to the equivalent impedances or even resistances with the values which will be just inverse of these something like half ohms, one fourth ohm, 1 ohm and 1 ohm, I can replace it by an equivalent impedance circuit.

Now after this you can get equivalent one by a star delta conversion see if this is a delta then I can have a star like this. So these 3 nodes are these 3, so what will be the value of this 1 into half divided by 1 plus half plus 1, so 1 into half divided by 2 and half, so that gives me 1 by 5, 1 by 5 similarly, this one will be 1 into 1 by 2 and half, so 2 by 5 and similarly this is 1 into 1 and this one will be again 1 into half, so this one will be 1 by 5 this is 1 by 4. So you get 1 by 5, 2 by 5 and 1 by 5 plus 1 by 4, 9 by 20. So take the inverse of these that will give you the equivalent capacitance 5 farads, 2.5 farads and 20 by 9 farads okay. So these are the 3 equivalent capacitances okay, it is so simple.

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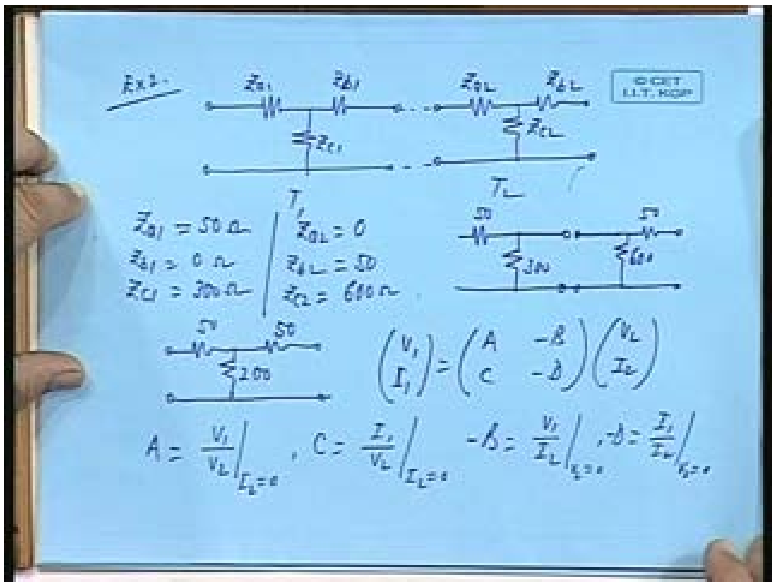

Next we are given a problem on transmission parameters of 2 cascaded sets. The problem is very simple one, this is Z_{a1}, Z_{b1}, Z_{c1} cascaded with Z_{a2}, Z_{b2} and Z_{c2}, what will be the total A, B, C, D parameters of the total combination, what will be the A, B, C, D parameters of this total combination. The values given are Z_{a1}, Z_{a1} equal to 50 ohms, Z_{b1} equal to 0, Z_{c1} is equal to 200 ohm, 300 ohms, Z_{a2}, Z_{b2}, Z_{a2} is 0, Z_{b2} is 50 ohms and Z_{c2} is given as 600 ohms, what be the A, B, C, D parameters. Let us draw this network first, this is 50 ohms, this is 300, this is 0, this is 0, this is 600 and this is 50. You see the network is very simple, it is a phi network if I combine them it will be 50 ohms 600 and 300 in parallel that gives me 200 ohms and 50 ohms. So A, B, C, D parameters if we remember V_1, I_1 we are writing in terms of A, B, C, D as V_2, I_2.

So A is V_1 by V_2 when I_2 is 0 similarly, C is I_1 by V_2, when I_2 is 0, B is minus B is V_1 by I_2 when V_2 is 0 that is under short circuit condition and similarly D is minus D is I_1 by I_2 when V_2 is 0. So from this diagram it is very simple V_1 by V_2, this is 50 plus 200 and this is 200, so fif 250 by 200, A is 250 by 200 that gives me 1.25. Similarly, B you can find out V_1 by I_2 when this is shorted, when this is shorted we apply a voltage V here what is the current that is flowing through this. So B comes out to be 100 and 12.5 ohms, C comes out to be 5 into 10 to the power of minus 3 and D is once again 1.25. These are very simple relationships all of you can determine this. Next we have a question determine for a for a network like this, couple circuit having 3 coils wound on wound on this steel structure 1, 1 dashed, you are sending a current I_1 this is 2, 2 dashed shorted through a resistance R_2 and similarly you are having 3, 3 dashed there is resistance here R_3.

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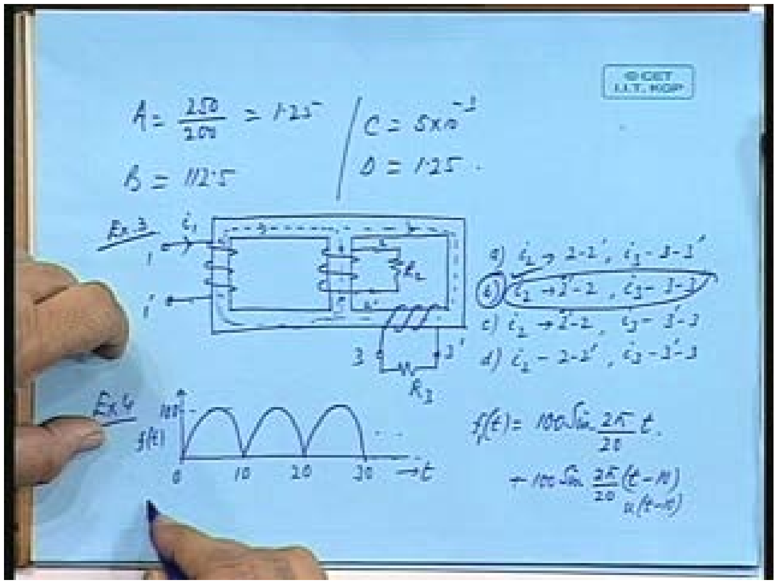

Now the question is there are 4 statements given which one is correct I_2 when the current is flowing through this winding 1, 1 dashed in this direction I_1 then the current through R_2 and R_3 will be one possibility is I_2 will be 2, 2 dashed externally, 2 to 2 dashed and I_3 will be 3 to 3 dashed, 3 to 3 dashed okay b, I_2 is 2 dash to 2 and I_3 is 3 to 3 dashed c, I_2 is 2 dash to 2 and I_3 is 3 dash to 3 the 4th possibility is I_2 is 2 to 2 dashed and I_3 is 3 dash to 3 okay. So we have to see whenever there is a current flowing through I_1 what is the sense of the flux flowing through this.

Suppose, you take the current in this direction all right it is coming out like this. So what will be the North Pole, North Pole will be in this coil it will be in this direction and here it is coming out. So this is south so flux will be flowing like this okay like this. So in R_2 suppose the current is from 2 to 2 dashed, this is kept open, this is kept open 2 to 2 dashed then which direction this current will send the flux. If you look at it 2 to 2 dashed that is current is going like this then the current is flowing in this direction okay from bottom to top it is in this direction that means this will be a North pole this will be a South pole when it is going like this it is North pole. So that means it will try to send the flux like this and like this. So it will be adding this flux that is not possible if I excite this now, if I excite this now and if the current flows like this then this will try to add to the flux that is already being established by this okay. So that goes against the very basic principle of conservation of energy.

So this will be trying to if the current flows in this direction this will try to oppose the flux. So this one will try to if a current I_1 flows through this it will try to flow it will try to send a flux like this then this current has to be in this direction. So it will be 2 dash to 2, so 2 dash to 2 there are 2 possibilities what will be the current through this. Similarly, if you go through the same argument here if a current flows like this what will be the direction of the flux will find if it is 3 to 3 dashed, 3 to 3 dashed okay then the flux will be opposing the flux that is being established by I_1 okay.

So this will be the correct direction of the currents that is this current will be from to 2 dash to 2 this current will be 3 to 3 dashed. So this is the correct answer next you take up another example write the expressions for the functions and so on. It is a periodic function 0, 10, 20, 30 and so on, this is 100, it is a rectified sin wave, what would be the expressions for the current f(t) current or voltage whatever is right in terms of continuous functions, regular functions and what will be the corresponding Laplace transform.

Now f(t) if I take only the first period, it is a part of 100 sin 2 phi by t where, t is basically this is the period 20 okay, it is this period, if I write like this then it will represent the function which continues if I add another function to that I call this part as only 1 period then if I add another sinusoid which is delayed by 10 seconds 100 sin 2 phi by 20 into t minus 10 seconds into u(t) minus 10 then this plus this will be giving me a net function of, see this is the first part plus a function that i am adding from here, so this plus this will give me this will get cancelled with this the first okay. It will keep on cancelling hence forth after 10 seconds. So if I add this with this this is a delayed function represented like this I will get this 1 which I am calling as $f_1(t)$ okay. So somebody may write f(t) as $f_1(t)$ plus KT where, T is 10 seconds and write like this summation it will represent this function okay where, K is any positive integer.

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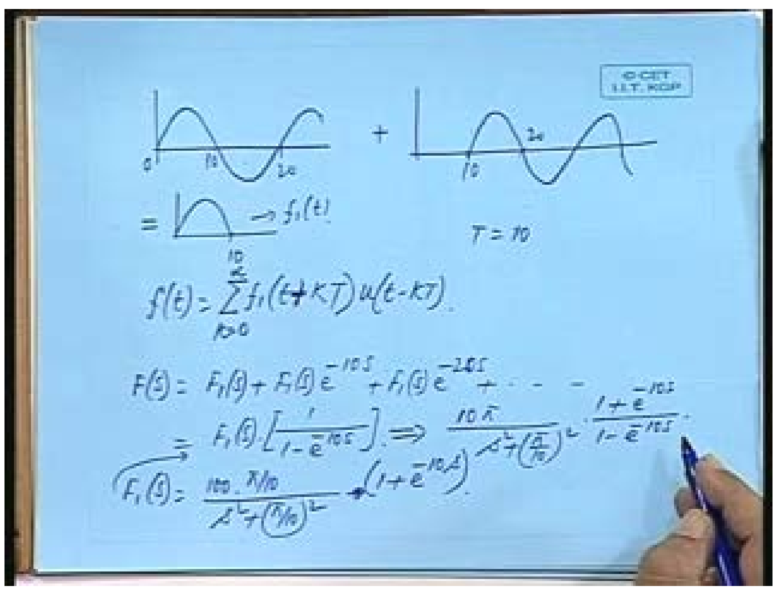

So it will represent this function of course into u(t) minus KT okay the correct representation. So this is f(t), now 2 phi by 20 can be written as phi by 10 what will be the Laplace transform of this then let us again start from the beginning Laplace transform of this function if I call it $F_1(s)$. So it will be $F_1(s)$ plus law plus transform of this shifted by 10 seconds $F_1(s)$ into e to the power of minus 10 S plus $F_1(s)$ into e to the power of minus 20s and so on which means $F_1(s)$ into 1 by 1 minus e to the power of minus 10s and what is $F_1(s)$, $F_1(s)$ will be Laplace transform of this which is 100 into sin 2 phi by 20 means phi by 10 divided by s squared plus phi by 10 whole squared, this is the Laplace transform of this part and for this part the same thing shifted by 10 seconds. So it will be this plus this into e to the power of minus 10 minus 10s so combining the 2 I will get.

So this is $F_1(s)$ so substitute here so that gives me 100 into phi by 10 that gives me 10 phi divided by S squared plus phi by 10 squared into 1 plus e to the power of minus 10s by 1 minus e to the power of minus 10s somebody may write this as cot hyperbolic 5S, e to the power of minus 5S, if I multiply, so cot hyperbolic 5s okay, so this will be the Laplace transform. Similarly, there is another function, what will be the expression for this type of periodic function, it is pretty simple 0, this is 5, 2, 4, 6, 8, 10 and so on this is minus 5.

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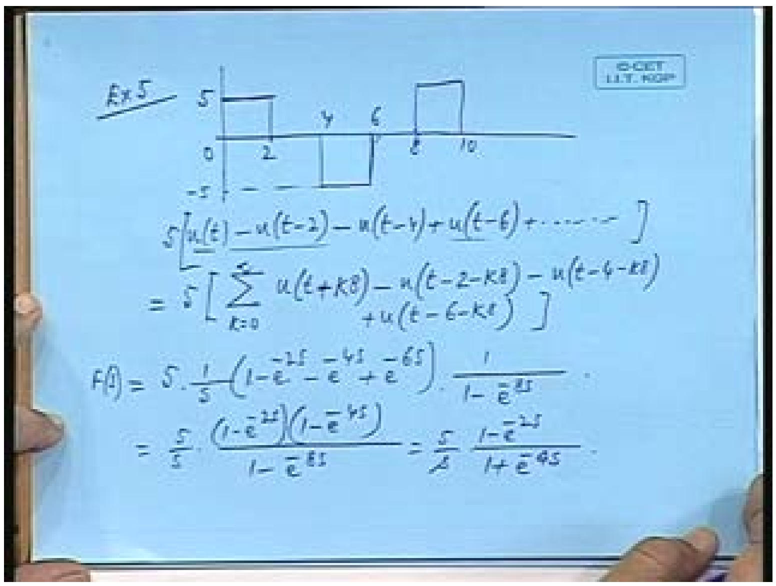

Obviously, the first block can be written as 5 into u(t) and then at 2 seconds I apply a negative step and u t minus 2, so that gives me the first block again at 4 it is negative so minus u t minus 4 and at 6 I apply u t minus 6 a positive step and that completes1 period up to 8 seconds and then it keeps on repeating. So show 1 period if I show 1 period and then show it as a repetition of the same function this will be written as 5 I can write this as summation u t minus 2 okay u(t), so plus after 8 seconds again it is plus u(t).

Okay so u t plus k_8, 8 minus u t minus 2 minus k_8 okay minus u t minus 4 minus k_8 plus u t minus 6 minus k_8 where, k varies from 0 to infinity this will be the representation of a periodic function, what be the Laplace transform of this? it is 5 into you take just 1 period and that keeps on repeating, so for 1 period F(s) will be for 1 period, it is 5 into 1 by S into 1 minus e to the power of minus 2s minus e to the power of minus 4s plus e to the power of minus 6s, this whole thing into 1 by it is repeated 1 minus e to the power of minus 8s, is it not this is something like a earlier $F_1(s)$ and multiplied by this that will give you total F(s) so 5 by S_1 minus e to the power of minus 2 S can be taken common.

So in bracket we get 1 minus e to the power of minus 4s that is equal to 5 by s into 1 minus e to the power of minus 4s, e to the power of minus 4s and 1 minus e to the power of minus 8s, there is a common factor, so that gives me 1 plus e to the power of minus 4s. So this we call Laplace transform of this function okay. Next we have a question on Laplace transform again determine the initial and final values of the function whose Laplace transform is given as s plus 4 into s plus 70 divided by s plus 1 into s plus 5 into 2s plus 7.

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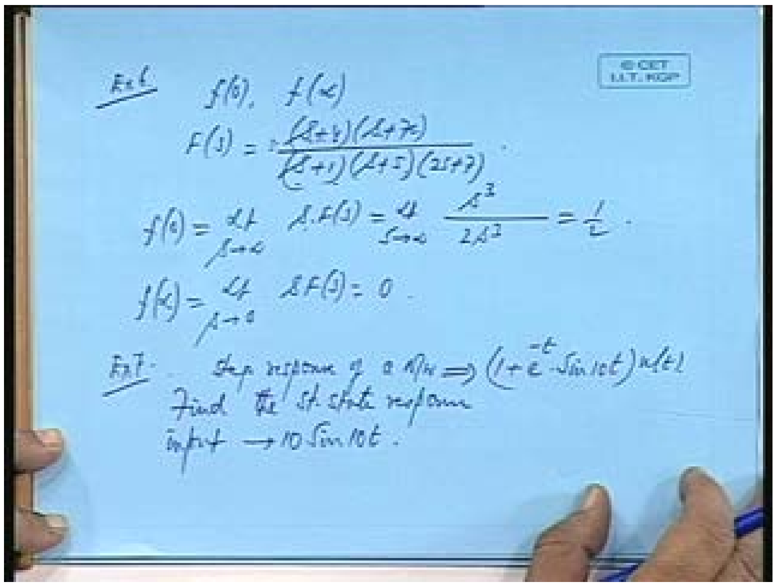

Obviously, f_0 we compute as limit s standing to infinity s into F(s), so if I multiply by s and then make s standing to infinity only the highest power of s has to be computed, so limit s standing to infinity if I leave only the highest power will be s cube after multiplying by s, s into s into s divided by s into s into 2s. So that gives me half similarly f at infinity will be limit s standing to 0 s F(s) if I multiply by s and then put s equal to 0 all these terms will give me 4, 70, 1, 5, 7 but this s will give me 0, so this is 0.

Next example the step response of a network is 1 plus e to the power of minus t sin 10t into u(t), find the steady state response the steady state response when the input is 10, sin 10t that means if I have instead of e to the power of minus t that is at decaying sinusoid if I have a steady sinusoid what will be the response. Let us see, let us determine the transfer function the Laplace transform of the output is for the step input, step response is 1 by S plus 1 by s plus 1 whole squared plus 10 squared is not 10 by s okay, one may write this as s squared plus twice s plus 100. So s squared plus 2s plus 10s, so 12s plus 100 divided by s squared plus 2s plus 101, okay s squared plus 2 s plus 1, so this is a this is also 101 alright multiplied by s.

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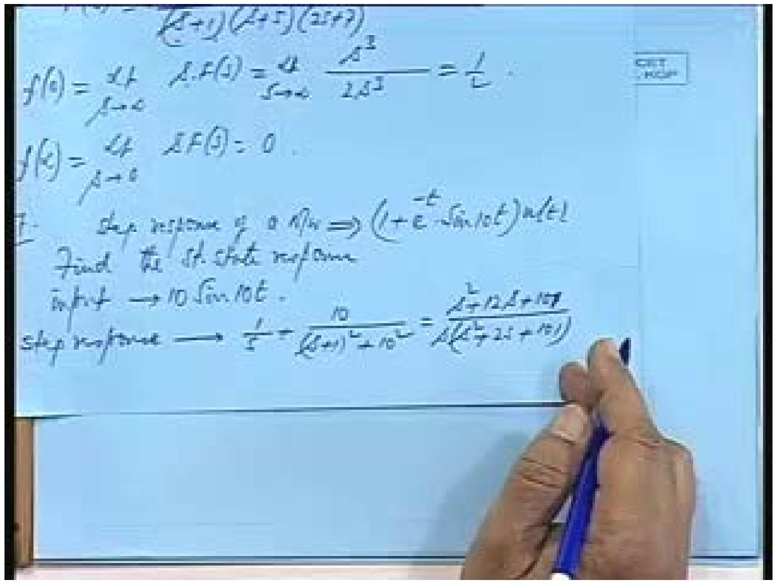

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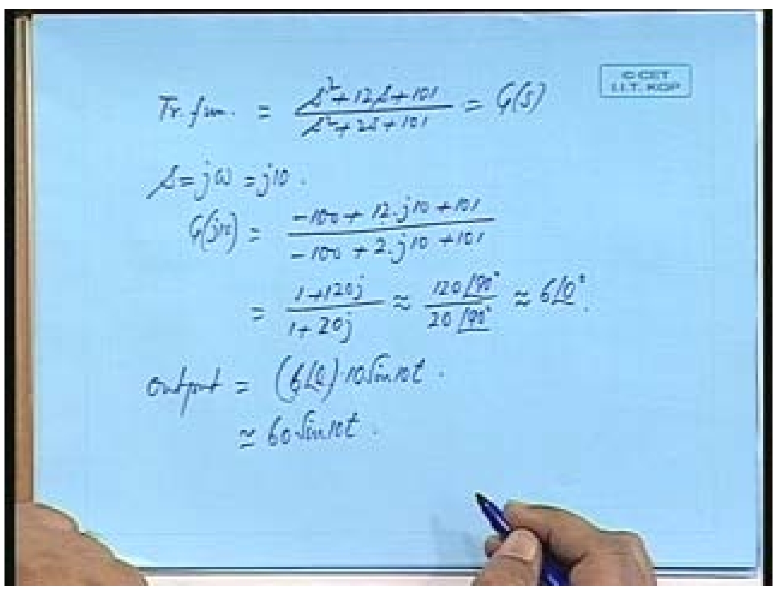

So the transfer function can be written as output by input, input is 1 by s, so if I divide this by 1 by s that comes out to be s squared plus 12s plus 101 divided by s squared plus 2s plus 101 okay. This is the step response, so if I divide the response by the input given to find out the input was step input so Laplace transform is 1 by s, so after dividing by 1 by s we get this. Now we want to determine the steady state response due to a sinusoidal input, so for a sinusoidal input we straight away write s equal to j omega, omega here is 10. If I substitute here therefore G, I call it G(s), so G (j10) will be minus 100 minus omega square, so minus 100 plus 12 into j10 plus 101 divided by minus 100 plus 2 into j10 plus 101 that gives me 1 plus 20j divided by 1 plus sorry 120j divided by 1 plus 20j, one is very very small compared to 120. So this is approximately 120 and an angle very close to 90 degrees.

Similarly 20 plus 1 plus 20j that is that will also give me a magnitude close to twenty and an angle very close to 90. So both these angles almost 90 degrees will give me approximately 0 angle, magnitude is 6. So what will be the output steady state output, it will be input multiplied by this 6 angle 0, 10 sin 10t, this is not the correct way of writing I just wanted to explain. So the magnitude gets multiplied by 6 angle gets shifted by this angle theta which is 0 approximately so it will be 60 sin 10t, this will be the response, steady state response okay. Next we have another example, you are having okay.

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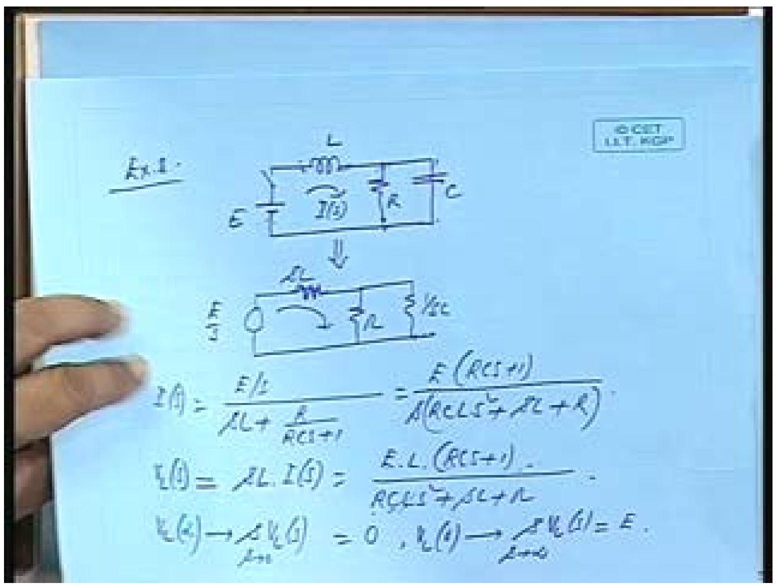

Let me use this page, you are given a simple circuit, you are asked to calculate I(s), this is a DC source I(s) and then the voltage is across R and C, at initial and final condition all right. Initial and final values of the voltages across the 3 elements okay and then what will be the critical value of R okay, for critical tamping what be the value of R for critical tamping.

So let us compute the impedance this can be replaced by E by S, SL I should write a general impedance R, 1 by SC, assuming no initial charge is said here, no initial current was there. So I(s) will be E by S divided by the total impedance SL plus combination of this will be R by RCS plus 1. So if I combine will be E into RCS plus 1 divided by RC, RCLS square plus SL plus R okay into S, this will be the expression for I(s), what will be the voltage across the inductor, voltage across the inductor, it will be SL times I(s), is it not. So S will get cancelled, so it will be E into L into RCS plus 1 divided by RCLS square plus SL plus R.

Now if I want to calculate the steady state value and that is V_L at infinity then I should calculate SV_LS and then put s_{10} into 0. So if I multiply by s this quantity and put s equal to 0, the whole thing will become 0. If I want to compute V_L at 0 then I will compute s times V_LS at s_{10} into infinity and how much is that if I multiply by S, if I multiply this by S and then make s_{10} into infinity highest power of S will have to be taken, so it will be S squared into RLCE and here it is RLCS square. So this terms will get cancelled we will be getting only E. Basically at start the entire voltage appears across the inductor obviously, what will be the voltage across this combination mind you, voltage across the resistance and voltage across the capacitance will be same. So you need not find out these values separately the voltage across the resistance when this is having maximum value in the initial condition, this will be 0 at t equal to 0, this is equal to E, so this will be 0 at t equal to infinity this is 0. So this will be E.

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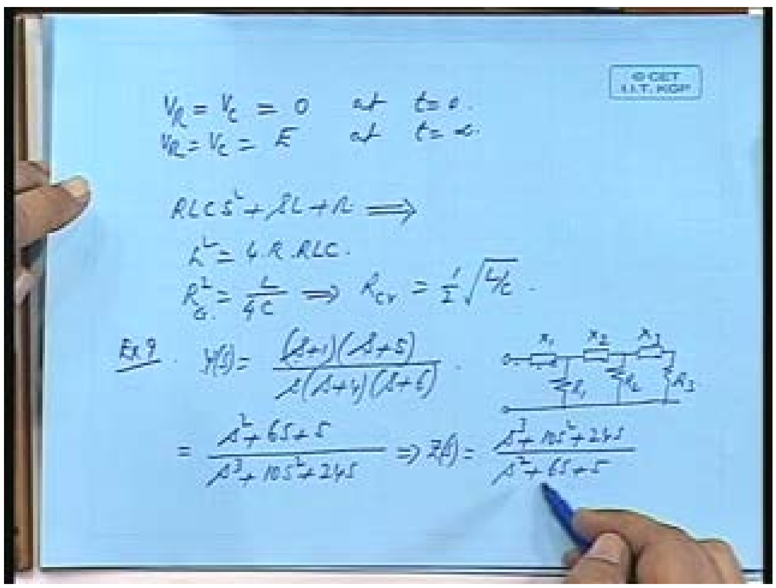

So V_R which is equal to V_C will be equal to 0 at t equal to 0, V_R equal to V_C equal to E, at t equal to infinity okay. For critical damping this quadratic we have got RLCS squared plus SL plus R, so equate this square is equal to 4 SC, so L squared is equal to 4 into R into RLC so that gives me R squared equal to L by C into 4 okay or R critical, R critical is 1 by 2 root L by C. So this will be the condition for critical tamping. Next we have another problem a network is given by Y(s) equal to s plus 1 into s plus 5 divided by s into s plus 4 into s plus 6 you are asked to show the unknown arms of this network in this form, what will be this unknown arms X_1, X_2 and X_3, it continues this is given as R_1, R_2, R_3 what to be the nature and the values of these X_1, X_2 and X_3.

Obviously, this is given in a ladder form okay so we try to find out we try to find out the ladder structure. So this is s squared plus 6s plus 5 divided by s cubed plus 4 plus 6, 10s squared plus 24s okay. This is Y(s) so correspondingly Z(s) you see the first element if it is present then it has to start with Z(s) which will be s cubed plus 10 s squared plus 24s divided by s squared plus 6s plus 5 okay Z(s) of this form, is it RL or RC network obviously, Y(s) having a pole closest to the origin closest to the imaginary axis is giving me RL, it is an RL network. So this will be an inductance resistance inductance resistance and so on.

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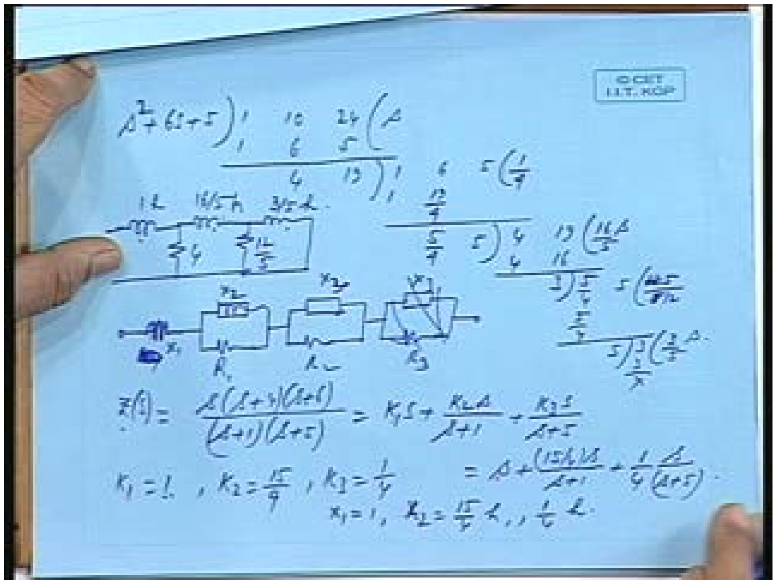

So by continue division s cube s squared plus 6s plus 5, I will write only the coefficients 1, 10, 24, this will be s 1, 6, 5, 4 and19 , 1, 6, 5. So 1 by 4, 19 by 4 so this gives me 5 by 4 and 5, this is 4 and 19, 5 by 4, so that gives me 16 by 5 okay, no 3 into 5 by 12, 3 5s are 15 by 12 okay 5 by 4 then 5, 3, 3 by 5. So what you get this is s this is s, what you get is 1 Henry then this is an admittance of 4 ohms then this is 16 by 5 Henry and then 5 by 12. So 12 by 5 ohms and then 3 by 5 Henry.

So the unknown values are X_1 is1 Henry, X_2 is 16 by 5, X_3 is 3 by5 okay. The same function the same function you are also asked to evaluate the impedances X_1 take once again nature in the values of X_1, X_2, X_3 okay R_1, R_2, R_3 and this is R_4 what will be the values of X_1, X_2, X_3. So obviously this is a faster one realization, so Z(s) start with Z(s), so Z(s) is s into s plus 4 into s plus 6 by s plus 1 into s plus 5. So I break it up into K_1 into S since this is a fire plus K_2 by s plus 1 should I have K_2 by s plus1 or K_2 into s by s plus 1. I have already identified this is an RL network so for an RL network in faster one synthesis, you remember the Z synthesis will require K_2(s) by s plus 1 if you do not take K into s you are landing up into trouble you saw earlier also we will get negative values of K_2, K_3 etcetera. Some of them may come out to be negative.

So this one K_1 will be if I divide by s make a stand into infinity this becomes 1, K_2 divided by s multiplied by s plus 1 make s plus 1 equal to 0, so this is 3 into 5 by 4, 15 by 4, s plus 5, so this will be minus 1 plus 1 and 4, so 1 by 4. So these values are this is 1 okay, so if it is like this obviously you do not you do not require all of them. There may be sorry, I forgot to mention here this is X_1, this is X_2, X_3, this will not come because the order is only this much, so X_1, so

K_1 represents X_1 okay, this is s plus K_2 is 15 by 4 by s plus 1s plus 1 by 4s by s plus 5. So for this one this is 1 Henry, X_1 is 1 Henry, from here you can see if I make s stand into 0 stand into 0 this is 15 by 4.

So X_2 is 15 by 4 Henry and X_3 if I make s stand into infinity s stand into 0 this is 1 by 4, 1 by 4 Henry okay. So these are the 3 values 1, 15 by 4 and 1 by 4. S by 4 into 5 it should be 20 sorry, 4 into 5 if I can make s stand into 0, it is s by 4 into 5, so it is 1 by 20 Henry. Next example if you have a little more time. Determine the value of b, c and a for the impedance function Z(s) equal to 5 into s into s squared plus b divided by s squared plus c where, this network has been partly realized as a Henry and half a farad and 1 Henry that means a network has been given this is a structure 2 values are known, this is not yet known but it transfer function is of this form what will be b, c and a, this is the problem.

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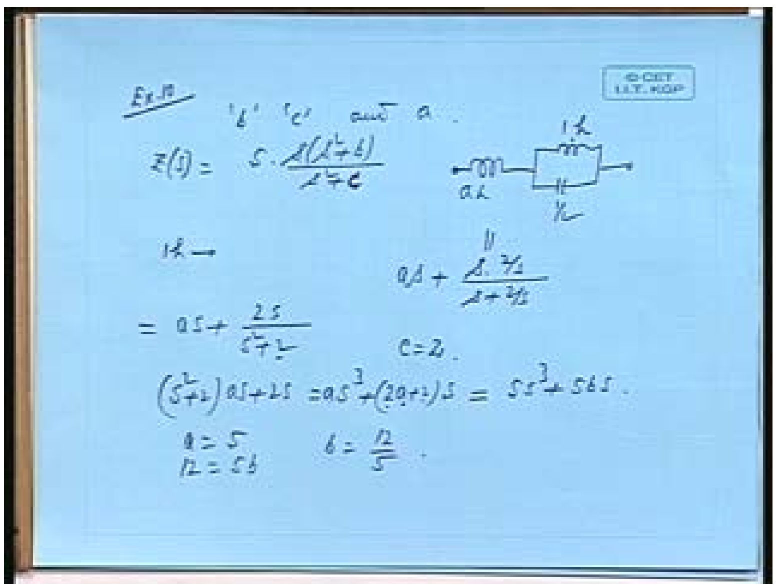

So let us see these 2 combinations 1 Henry and a half farad will give me 1 Henry. I can write this as a s plus, this is s into 2 by s divided by s plus 2 by s. So that gives me a s plus 2s by s squared plus 2, correct if I wrong, is it all right and this is equal to this. So c has to be equal to 2 okay c has to be equal to 2, if I add these S squared plus 2 into a s plus,2 S that gives me S cubed plus twice a plus 2 into S and that is equal to 5 S cubed plus 5 S cubed plus 5 b s. So here a s cubed sorry a is equal to 5 and if I put equal to 5, 5 into 2, 10 plus 2, 12 is equal to 5 b therefore b is equal to 12 by 5 okay, it is a very simple example. Well, before we take up any other problem I think we will stop here today. We will continue in the next class because there is not much of time left.

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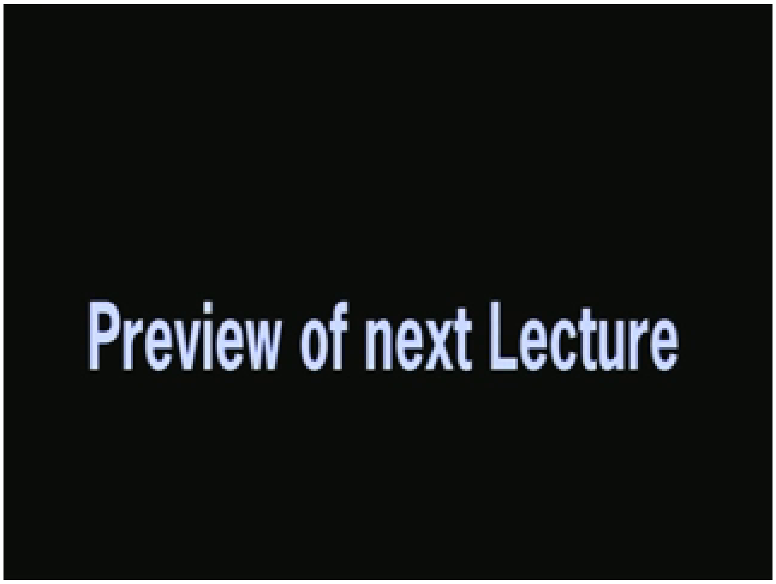

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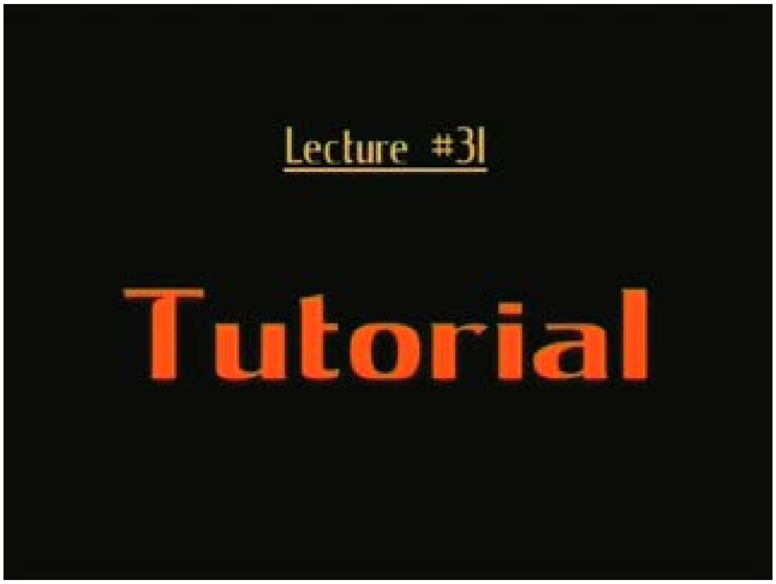

Okay, Good morning friends, today we will have another tutorial session. The first question is determine the Laplace inverse of factorial n divided by s into s plus 1 into s plus 2 into s plus n okay. So easiest method is to find out the partial fractions I will write A 0, A 1 A 2, S plus 2 and so on, A k by S plus k general term okay. So how much will be A 0, A 0 multiply by s this F(s) make s stand into 0, so that gives me factorial n divided by 1, 2, 3, 4 up to n that is equal to 1, A

1 multiply by s plus 1 make s plus 1 equal to 0. So what we get A 1 as 1 into 1, 2, 3, 4 up to n minus 1.

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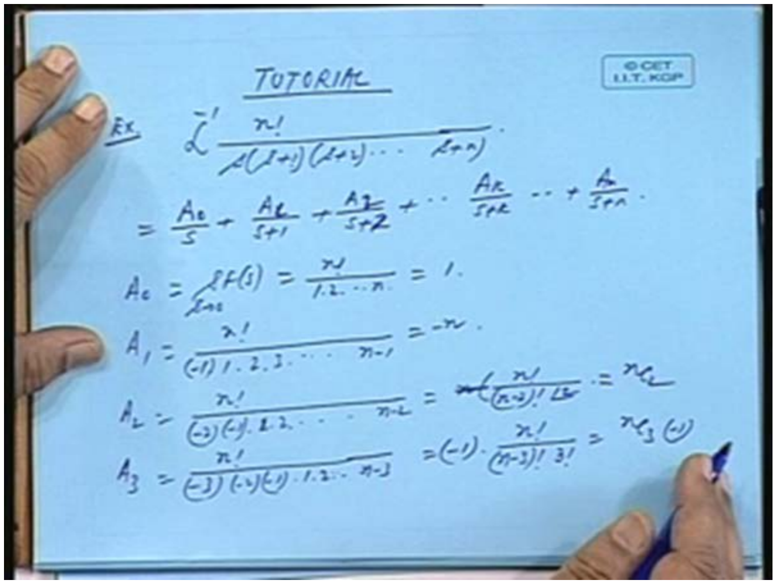

So that gives me n with a negative sign because the first one will be minus 1, A 2 will be factorial n divided by multiply by s plus 2 then make s plus 2 equal to 0 should be minus 2 minus 1 minus 2 minus 1 then 1, 2, 3 up to n minus 2 and that will give me plus n into see I get minus 1, minus 2, n minus 2, so factorial n divided by factorial n minus 2 into factorial 2, A 3 and what is this you can identify this as nc 2, A 3 similarly will be factorial n divided by minus 3, minus 2, minus 1 then 1, 2, 3 up to n minus 3. So I get minus 1 into factorial n by n minus 3 factorial into factorial 3 that is nc3 with a minus 1.

Network function y (s) is shown like this for y (s), some y (s), this is minus 30 degree, these are all 20 degree per decade slopes, this is 5, 10, 20, 40, this is 0.2 these are not to the scale exactly, this is 0.2, minus 30, we have to be the value of y (s) and the next question is, is it deriving a driving point impedence by admittance function, is it a driving point admittance function, if not justify, if yes state that is yes and then realize one in faster 2 form, so if yes then realize in faster two form and if not justify, why it is not a driving point admittance function, let us write y as, V 2 by I 1, I 1 by V 2, I 1 by I 2 and so on.

That means you may be given the specification either in terms of Z 12, Y 12, Y 22 and so on and Z 11, so specification can be given in terms of Z 11, Z 22 or Y 12, Y 22 or V 2 by V1 and so on. You have to determine a possible network so, so far we have studied driving point synthesis now we shall going for 2 port synthesis with transfer synthesis, okay some of the basic condition for these impedence and admittance function that is transfer impendences and transfer admittances a very interesting and so the transfer function also, we will find the numerator and denominator polynomials N (s) by D (s) they need not have all the properties that are listed in the driving point synthesis that is the difference in degree can be more than 1, difference in degree can be more than 1, can be may or may not be but D (s) must be Hurwitz polynomial, roots must be always in the left of plane, so D (s) must be Hurwitz, there can be multiple roots, multiple zeros on N (s), there can be all possible combinations. So we shall study in details, what are the conditions to be satisfied for 2 port synthesis, what are the conditions for Y 12, Y 12 or Z 11, Z 12, these care of admittance or impedence function. Thank you very much.