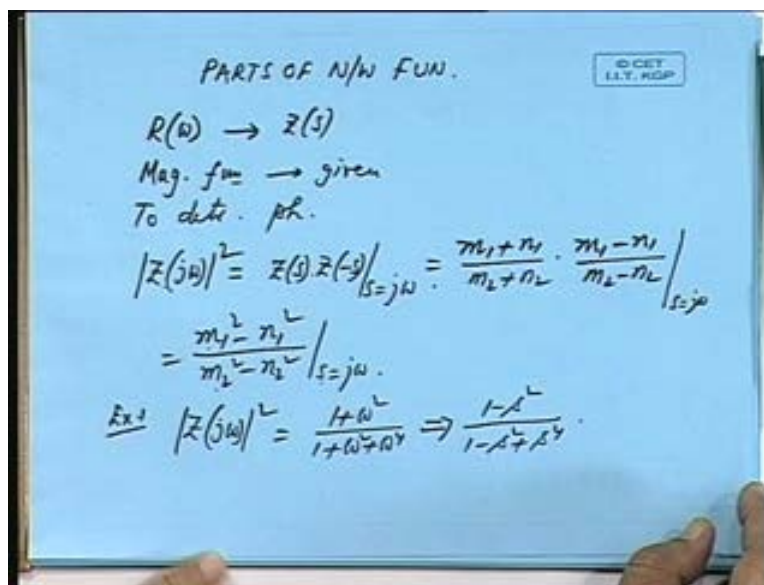


**Network Signals and Systems**  
**Prof. T. K. Basu**  
**Department of Electrical Engineering**  
**Indian Institute of Technology, Kharagpur**  
**Lecture - 28**  
**Parts of Network Functions (Contd...)**

Good morning friends we shall continue with the topic on parts of network functions.

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PARTS OF N/W FUN.

$R(\omega) \rightarrow Z(s)$   
 Mag. fun  $\rightarrow$  given  
 To det. ph.

$$|Z(j\omega)|^2 = Z(s)Z(-s)|_{s=j\omega} = \frac{m_1+n_1}{m_2+n_2} \cdot \frac{m_1-n_1}{m_2-n_2} \Big|_{s=j\omega}$$

$$= \frac{m_1^2 - n_1^2}{m_2^2 - n_2^2} \Big|_{s=j\omega}$$

Ex:  $|Z(j\omega)|^2 = \frac{1+\omega^2}{1+\omega^2+0.5\omega^4} \Rightarrow \frac{1-A^2}{1-A^2+B^4}$

Yesterday, we discussed about functions  $r(\omega)$  being given how to get  $z(s)$  okay so we desorb this by substituting  $\omega^2 = -s^2$ , we took the roots in the left of plane and finally from the coefficient of the numerator  $a_0, a_1$  and so on.

Now today we shall be discussing of about the magnitude function, the magnitude function, if the magnitude function is given how to calculate phase magnitude function is given to calculate to determine phase. Now the magnitude  $z^2$  can be written as  $z(s)$  into  $z(-s)$  at  $s = j\omega$  is that all right and what is  $z(s)$  if I write in terms of even and odd parts  $m_1$  plus  $n_1$  by  $m_2$  plus  $n_2$ , this is  $z(s)$  and what is  $z(-s)$ ,  $m_1$  minus  $n_1$  it is only the odd part which will be having a negative sign now divided by  $m_2$  minus  $n_2$  is that all right at  $s = j\omega$ . So what you are getting is  $m_1^2$  square minus  $n_1^2$  square by  $m_2^2$  square minus  $n_2^2$  square to  $s = j\omega$  is that all right.

So if you given  $z$  magnitude square it is basically  $m_1^2$  square minus  $n_1^2$  square by  $m_2^2$  square minus  $n_2^2$  square at  $s = j\omega$  see if I make reverse substitution I will get  $m_1^2$  square  $s$

minus  $n_1$  square  $s$  by  $m_2$  square minus  $n_2$  square  $s$  from here okay. So there if I factorize and drop out the roots in the right up plane I will get  $m_1$  plus  $n_1$  and correspondingly  $m_2$  plus  $n_2$  it is very simple. So let us take one example  $z$   $j$   $\omega$  square is equal to  $1$  plus  $\omega$  square by  $1$  plus  $\omega$  square plus  $\omega$  to the power  $4$ , so that gives me if I make reverse substitution it will be  $1$  minus  $s$  square by  $1$  minus  $s$  square plus  $s$  to the power  $4$  okay.

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Mag. fun  $\rightarrow$  given  
To det. pr.

$$|Z(j\omega)|^2 = Z(s)Z(-s) \Big|_{s=j\omega} = \frac{m_1+n_1}{m_2+n_2} \cdot \frac{m_1-n_1}{m_2-n_2} \Big|_{s=j\omega}$$

$$= \frac{m_1^2 - n_1^2}{m_2^2 - n_2^2} \Big|_{s=j\omega}$$

Ex:  $|Z(j\omega)|^2 = \frac{1+\omega^2}{1+\omega^2+\omega^4} \Rightarrow \frac{1-s^2}{1-s^2+s^4}$

$$= \frac{(1+s)(1-s)}{(s^2+\sqrt{3}s+1)(s^2-\sqrt{3}s+1)} \Rightarrow z(s) = \frac{1+s}{s^2+\sqrt{3}s+1}$$

So what are the roots of this  $1$  plus  $s$  into  $1$  minus  $s$  okay what are the roots of this  $s$  square, see  $1$  plus  $s$  to the power  $4$  plus twice  $s$  square minus  $3s$  square, so it will be  $1$  plus root  $3$   $s$  into  $s$  square minus root  $3s$  plus  $1$  all right. Obviously, the quadratic be the negative sign this is  $m_2$  minus  $n_2$ , this is  $m_2$  plus  $n_2$ , this  $m_1$  plus  $n_1$ , this  $m_1$  minus  $n_1$ . So that gives me  $z(s)$  equal to the left up plane roots the factor corresponding to that is  $1$  plus  $s$  divided by this  $1$   $s$  square plus root  $3$   $s$  plus  $1$  is that all right  $s_1$  plus  $s$  by  $s$  square plus root  $3$   $s$  plus  $1$  realization of this you can realize this by Bott Duffin or Brute synthesis okay we are not going to the realization part at this stage  $z(s)$  is this much.

Let us take another example  $z$   $\omega$  square is equal to  $\omega$  square plus  $16$  by  $\omega$  square  $4$  plus  $10$   $\omega$  square plus  $9$ . So I can write  $z(s)$  into  $z$  minus  $s$  as  $16$  minus  $s$  square divided by  $s$  to the power  $4$  minus  $10$   $s$  square plus  $9$  which gives me  $4$  plus  $s$  into  $4$  minus  $s$ , this gives me  $s$  square minus  $9$  into  $s$  square minus  $1$  which gives me finally  $4$  plus  $s$  into  $4$  minus  $s$  by  $s$  plus  $3$  into  $s$  plus  $1$  into  $s$  minus  $3$  into  $s$  minus  $1$ . So what will be  $z(s)$   $4$  plus  $s$  by  $s$  plus  $3$  into  $s$  plus  $1$  okay is this all right. Like this you can see for any function mind you  $z$  magnitude square will be a function of  $\omega$  square only there is nothing like  $\omega$ ,  $\omega$  to the power  $3$  and so on, this will be an even function of  $\omega$ , this will also be an even function of  $\omega$ . Next we go to phase to magnitude, phase to magnitude, phase to magnitude okay.

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$$\frac{z(s)z(-s)}{z(s)} = \frac{16 - s^2}{s^4 - 10s^2 + 9}$$

$$z(s)z(-s) = \frac{16 - s^2}{s^4 - 10s^2 + 9} = \frac{(4+s)(4-s)}{(s^2-9)(s^2-1)}$$

$$= \frac{(4+s)(4-s)}{(s+3)(s+1)(s-3)(s-1)}$$

$$z(s) = \frac{4+s}{(s+3)(s+1)}$$

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$$z(s)z(-s) = \frac{16 - s^2}{s^4 - 10s^2 + 9} = \frac{(4+s)(4-s)}{(s^2-9)(s^2-1)}$$

$$= \frac{(4+s)(4-s)}{(s+3)(s+1)(s-3)(s-1)}$$

$$z(s) = \frac{4+s}{(s+3)(s+1)}$$

hate to say.  $z(s) = \text{Ev } z(s) + \text{Od } z(s)$

$$z(jw) = \text{Re } z(jw) + j \text{Im } z(jw)$$

$$= \text{Ev } z(s) \Big|_{s=jw} + j \text{Od } z(s) \Big|_{s=jw}$$

Now if I see the angle  $z(s)$  if I write as even  $z(s)$  plus odd  $z(s)$  then  $z(j\omega)$  this will correspond to a real part this is nothing but even  $z(s)$  at  $s$  equal to  $j\omega$  and this gives me odd  $z(s)$  okay at  $s$  equal to  $j\omega$   $j$  will come out of this, is that okay. So what is  $\tan \theta$  you would give an say the angle  $\tan \theta$  is odd  $z(s)$  at  $s$  equal to  $j\omega$  but odd  $z(s)$  will generate  $j$  term okay mind you divided by even  $z(s)$  all right. So this is nothing but if I write  $j \tan \theta$ ,  $j \tan \theta$  you will get  $n_1, m_2$  minus  $m_1, n_2$  divided by  $m_1, m_2$  minus  $n_1, n_2$  okay at  $s$  equal to  $j\omega$  all right this is nothing but okay.

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$$Z(s) = \frac{N(s)}{D(s)}$$

$$Z(j\omega) = \text{Re } Z(j\omega) + j \text{Im } Z(j\omega)$$

$$= \text{Ev } Z(s) \Big|_{s=j\omega} + j \text{Od } Z(s) \Big|_{s=j\omega}$$

$$\tan \theta = \frac{\text{Od } Z(s) \Big|_{s=j\omega}}{\text{Ev } Z(s) \Big|_{s=j\omega}}$$

$$j \tan \theta = \frac{n_1 m_2 - m_1 n_2}{m_1 m_2 - n_1 n_2} \Big|_{s=j\omega}$$

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$$\tan \theta = \frac{\text{Od } Z(s) \Big|_{s=j\omega}}{\text{Ev } Z(s) \Big|_{s=j\omega}}$$

$$j \tan \theta = \frac{n_1 m_2 - m_1 n_2}{m_1 m_2 - n_1 n_2} \Big|_{s=j\omega} = \dots$$

$$\frac{N}{D}$$

$$H(s) + N(s) \Rightarrow (n_1 m_2 - m_1 n_2) + m_1 m_2 - n_1 n_2$$

$$= (m_1 + n_1)(m_2 - n_2)$$

Ex.  $\tan \theta = -\omega^5$        $\omega = \frac{s}{j}$

$$j \tan \theta = -j\omega^5$$

$$= -s^5$$

Suppose this is equal to this 1 even  $z(s)$  plus odd  $z(s)$  is say after division I am making a reverse substitution suppose we get  $m$  by  $n$  okay  $m$  by  $n$ . So  $m$  plus  $n(s)$  if the ratio is  $m(s)$  by  $n(s)$  if it is  $m(s)$  by  $n(s)$  then what is  $m(s)$  plus  $n(s)$  this plus this okay which will be  $n_1, m_2$  minus  $m_1, n_2$  plus  $m_1, m_2$  minus  $n_1, n_2$  which means  $m_1$  plus  $n_1$  into  $m_2$  minus  $n_2$  okay. So the 1 with  $m_1$  plus  $n_1$  term that is roots in the left up plane will be numerator and roots in the right up plane will if you shift those roots in the left up plane that is  $m_2$  minus  $n_2$  you get so you just change the sing of  $n_2$  that will give you the denominator all right. So let us take an example it will be clear.

Suppose tan theta is given as minus omega to the power 5 okay. So you substitute omega is equal to s by j so j tan theta you are computing j tan theta that is minus j omega to the power 5 and that gives me minus s to the power 5 all right. So this you are calling as m by n, so s to the power 5 by 1, is it not this will be s to the power 5 by 1 with a negative sign.

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$$j \tan \theta = \frac{M_1 - N_1 + j\omega}{N_1 M_2 - N_2 M_1} \Big|_{s=j\omega} = \frac{M}{N}$$

$$M(s) + N(s) \Rightarrow (\pi_1 M_2 - N_1 N_2) + N_1 M_2 - \pi_1 N_2$$

$$= (M_1 + N_1)(M_2 - N_2)$$

Ex.  $\tan \theta = -\omega^5$        $\omega = \frac{s}{j}$

$$j \tan \theta = -j \omega^5$$

$$= -\frac{s^5}{1}$$


$$M + N = 1 - s^5$$

$$= (1 - s)$$

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$$1 - s^5 = 0 \quad s^5 = 1$$

$$s = e^{j2\pi n/5} = e^{j2\pi n}$$

$$= 1, e^{j72^\circ}, e^{j144^\circ}, e^{j216^\circ}, e^{j288^\circ}$$


$$\pi_1 + \pi_2 = (s - e^{j144^\circ})(s - e^{j216^\circ})$$

$$= (s^2 + 2 \cos 36^\circ s + 1) (s^2 + 2 \cos 72^\circ s + 1)$$

$$= (s^2 + 1.618s + 1) (s^2 + 0.618s + 1)$$

$$\pi_2 + \pi_3 = (s^2 + 2 \cos 72^\circ s + 1)$$

$$= (s^2 + 0.618s + 1)$$

$$Z(s) = \frac{s^2 + 1.618s + 1}{(s^2 + 0.618s + 1)}$$

So what will be  $m$  plus  $n$ ,  $1 - s$  to the power 5 is that all right. So  $1 - s$  to the power 5 what are the factors  $1 - s$ , now let me work it out here,  $s$  to the power 5, no you do it this way  $1 - s$  to the power 5 equal to 0 you want to find out the roots say  $s$  to the power 5 is equal to 1 so on the unit circle, so that is equal to  $e$  to the power  $j2\pi n$ . So what will be  $s$  to the power  $j2\pi n$  by 5 put  $n$  equal to 0 1 to so if I put  $n$  equal to 0 this is 1 root then  $2\pi$  by 5, 72 degrees, this is 72 degrees  $2\pi$  by 5 then 2 into  $2\pi$  by 5, 144 degrees okay then 3 into 72 so 216 degrees, so the values are  $1 e$  to the power  $j72$  degrees,  $e$  to the power  $j144$  degrees,  $e$  to the power  $j216$  degrees,  $e$  to the power  $j288$  degrees, these are the 5 roots is that all right, **that equal to be 1**. Obviously 1 into  $e$  to the power so 1 to the power 1 by 5, so that will be 1 they will not be on the unit circle.

So what are the factors corresponding to  $m_1$  plus  $n_1$ , it will be this this is giving me  $m_2$  minus  $n_2$  so  $m_2$  plus  $n_2$  will be just images of this. So  $m_1$  plus  $n_1$  will be  $s$  minus this  $a$  to the power  $j144$  degrees into  $s$  minus  $a$  to the power  $j$  this one 216 degrees all right. So that gives me  $\cos 144$  degrees okay that means this one how much is this angle 180 minus 144, 36 degrees 36 degrees, so  $\cos 36$  degrees so  $s$  plus  $\cos 36$  degrees and this will also give me  $\cos 36$  degrees so 2 into  $\cos 36$  degrees. So  $s$  square plus 2 into  $\cos 36$  degrees into  $s$  plus the imaginary parts  $\alpha$  square plus  $\beta$  square is 1 it will be 1 is that all right. Any quantity coming out of complex pair of roots on the unit circle will be  $s$  square plus 2 into the real part which is  $\cos 36$  degrees into  $s$ ,  $s$  square plus 2 into  $\alpha$  into  $s$  plus 1  $\alpha$  square plus  $\beta$  square is 1. So it is  $s$  square plus  $\cos 36$  degrees approximately .8, so  $1.6 s$  plus 1, 36.8 degrees is .8,  $m_2$  minus  $n_2$  you know so  $m_2$  plus  $n_2$  will be this is 72 degrees.

So **s** square plus 2 into  $\cos 72$  degrees into  $s$  plus 1 straight away images of these will be on this side all right. So the real part of this will be 72 degrees,  $\cos 72$  degrees  $s$  square  $\cos 72$  degrees is approximately .38 degrees approximately .3, so 2 into .3, so  $0.6 s$  plus 1. So  $z(s)$  is this one will be the numerator  $s$  square plus  $1.6 s$  plus 1 divided by  $s$  square plus  $0.6 s$  plus 1 approximately this will be the function, is that okay. So we have seen how to calculate the function  $z(s)$  from the phase.

Now from here you can calculate the magnitude, once you know the actual  $z(s)$  you can calculate the magnitude **sure**, yes please, **that kind of problems when when we can be in practical use**, no in the practical system sometimes we measure only the phase we are not in a position to estimate say the gain then what will you do okay. Sometimes you try to track both magnitude and phase all right and try to find out  $z(s)$  it is a cross check, it is a cross check.

Suppose the magnitude function is known we take only the real part or the only the total magnitude with frequency then you approximate it by even polynomial ratio, ratio of even polynomials all right and then only from there you can find out just previous to this you have found out  $z(s)$  into  $z$  minus  $s$ , is it not. So factorize it find out the right up plane roots left up plane roots all right take only the left up plane roots and that gives you the function  $z(s)$  you are tracking only the phase then from the phase if you can estimate again phase function in the form of polynomials then you can estimate  $z(s)$ . In the laboratory depending on your facilities available the accuracy for measurements of phase and magnitude may be different. So you try to

find out  $z(s)$ , approximate  $z(s)$  from both sides another example  $\tan \theta = \frac{\omega^3 - 3\omega}{1 - 3\omega^2}$  all right.

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$$\tan \theta = \frac{\omega^3 - 3\omega}{1 - 3\omega^2}$$

$$j \tan \theta = \frac{j\omega^3 - 3\omega}{1 - 3\omega^2}$$

$$= \frac{-s^3 - 3s}{1 + 3s^2} = \frac{-s(s^2 + 3)}{1 + 3s^2}$$

$$M+N = -s^3 + 3s^2 - 3s + 1$$

$$= -(s-1)^2$$

$$\frac{m_1+n_1}{m_2+n_2} = \frac{1}{(1+s)^2} \rightarrow \text{Not a dr. pt. immittance fun.}$$

$$= \text{Tran. imp.}$$

So how much is  $j \tan \theta$  whenever the phase function is given in the form of  $\tan \theta$  we write  $j \tan \theta$  that will be  $j \omega^3 - 3\omega$  by  $1 - 3\omega^2$  make the reverse substitution  $s = j\omega$ . So that gives you  $j \omega^3$  means  $-s^3$  minus  $3s$  okay divided by  $1 - 3\omega^2$  becomes  $1 + 3s^2$  all right. So this will give me  $-s^3 - 3s$  divided by  $1 + 3s^2$  all right. So what is  $m + n$ , what is  $m + n$  minus  $s^3$  just add these  $2 + 3s^2 - 3s + 1$ , so that gives me  $s^3 - 3s^2 + 3s - 1$  minus  $s^3$  whole cube is that all right,  $1 - s^3$  whole cube.

So where are the roots in all on the right up plane at  $s = 1$ , so what will be  $m_1 + n_1$  by  $m_2 + n_2$  this corresponds to  $m_2 + n_2$  this does not give any finite roots that is  $0s$ , so  $1 + s^2$  whole cube is that all right. So this is not a driving point impedance, it is an impedance function, it is a transfer impedance function, in case of a transfer impedance function you may have difference in the degree more than 1 but in case of a driving point impedance or admittance function, we know the difference in the degree should be restricted to 1.

So here you can have more than 1, so this is not a driving point function **sir why do you sir de difference is 1 and the difference is greater than 1**, not greater than 1 it may or may not be 1. So transfer function is a ratio between 2 voltages or voltage and current, so the transfer function can be anything it can have only thing the roots must be in the left of plane that is poles must be in the left of plane yes, this is an insist able system any stable system may have a phase function like this but this is not an impedance function, driving point impedance function not a driving point impedance or admittance function. I will call it immittance function all right, so this is

basically a transfer impedance about the properties of transfer impedance we will discuss later on when you go for 2 port synthesis.

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$$\begin{aligned}
 \operatorname{Re} z(j\omega) &= \frac{1+\omega^L}{1+\omega^L+\omega^4} \\
 &\Rightarrow \frac{1-s^L}{1-s^L+s^4} \\
 s^4 - s^L + 1 &\Rightarrow (s^2 + 2s^2 + 1) - 3s^2 \\
 &= (s^2 + 1)^2 - (\sqrt{3}s)^2 \\
 m_2 + n_2 &\Rightarrow (s^2 + \sqrt{3}s + 1) \\
 \operatorname{Re} z(s) &\Rightarrow \frac{a_0 + a_1 s + a_2 s^2}{s^2 + \sqrt{3}s + 1} \Rightarrow \operatorname{fv.} \frac{(a_0 + a_2 s^2)(s^2 + 1) - \sqrt{3} a_1 s}{m_2 - n_2} \Big|_{s=j\omega} \\
 &= \frac{a_2 \omega^4 + (\sqrt{3} a_1 - a_1 - a_2) \omega^2 + a_0}{-a_2}
 \end{aligned}$$

Now we shall take up 1 or 2 more examples, it is all about realization of networks from given parts. Let us take another example real part of  $z(j\omega)$  okay  $1 + \omega^2$  by  $1 + \omega^2 + \omega^4$  what will be  $z(s)$ . So yesterday we had handled this kind of problems, so what will be  $1 + \omega^2 + \omega^4$  did, we discuss about this particular problem yesterday,  $1 + \omega^2$  by did I take this very example then otherwise we will skip to another problem, no I do not think so.

We took up another example,  $1 + \omega^2$  by  $\omega^4$ , no no not this example okay, I do not think we did it okay. I have chosen this because we want to see 2 types of functions given by the same expressions. So what we do here make a substitution  $s^2$  equal to  $-\omega^2$  so that will give me  $1 - s^2$  by  $1 - s^2 + s^4$ . So what are the roots of this  $s^4 - s^2 + 1$  gives me  $s^2$  to the power 4 plus twice  $s^2$  plus 1 minus  $3s^2$ . So it is  $s^2 + 1$  whole square minus root 3  $s$  square.

So the roots corresponding to  $m_2 + n_2$ ,  $m_2 + n_2$  will be having the left up, left up plane roots or this. So  $s^2 + \sqrt{3}s + 1$  this is the denominator, so numerator let us assume  $a_0(s) = a_0 + a_1(s) + a_2(s)^2$  divided by  $s^2 + \sqrt{3}s + 1$  okay. So even part is how much  $a_0 + a_2(s)^2$  into  $s^2 + 1$  minus root 3  $a_1(s)^2$  divided by  $m_2^2 - n_2^2$  square which is this. See if I put  $s$  equal to  $j\omega$  here I get real  $z(j\omega)$  okay and that gives me how much  $a_0$  sorry  $a_2 \omega^4 + a_0(s)^2$  all right plus  $a_2(s)^2$  minus root 3  $a_1$  is that okay. So  $a_0$ , so plus root 3  $a_1$  minus  $a_0$  minus  $a_2$  into  $\omega^2 + a_0$ , this is the numerator is that okay equate the coefficients.



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$$\begin{aligned} a_2 &= 0 \\ \sqrt{3}a_1 - a_0 - a_2 &= 1 \Rightarrow \sqrt{3}a_1 - 1 = 0 \\ a_0 &= 1 & a_1 &= \frac{1}{\sqrt{3}} \\ \therefore z(s) &= \frac{1 + \frac{1}{\sqrt{3}}s}{s^2 + \sqrt{3}s + 1} \\ &= \frac{s + \sqrt{3}}{\sqrt{3}(s^2 + \sqrt{3}s + 1)} \\ z(s) &= \frac{s(s^2 + \alpha)(s + A)}{(s^2 + \gamma)(s + \delta)} \rightarrow LC \end{aligned}$$

So  $a_2$  is 0 next  $\sqrt{3} a_1$  minus  $a_0$  plus  $a_2$   $\sqrt{3} a_1$  minus  $a_0$  minus  $a_2$  is equal to 1 co-efficient of  $\omega^2$  and  $a_0$  is equal to 1 substitute here  $a_2$  is 0 so  $\sqrt{3} a_1$  and  $a_0$  is 1 minus 1 is 0, so  $a_1$  is 1 by  $\sqrt{3}$  therefore  $z(s)$  will be  $a_0$  that is 1 plus  $a_1(s)$ , so 1 by  $\sqrt{3} s$   $a_2$  is 0 divided by a square plus  $\sqrt{3} s$  plus 1. So that gives me  $s$  plus  $\sqrt{3}$  divided by  $\sqrt{3}$  into  $s^2$  plus  $\sqrt{3} s$  plus 1 this is  $z(s)$ , is that all right.

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$$Rx = \frac{s+4}{(s+1)(s+3)}$$

That is an interesting point to be noted in case of synthesis that if you have a 2 element synthesis if you have a 2 element synthesis from the network function, you can identify whether it will be a<sub>2</sub> element synthesis or a<sub>3</sub> element synthesis. For example z(s) if you are having functions like s square plus alpha s square plus beta and so on s square plus gamma, s square plus delta functions of this type and if the poles and 0s are interlaced they are coming alternately then it is an r l c function, is it not l c. You can have Foster1, Foster2, Cauer1, Cauer2 realizations and in those realizations you have minimum number of elements they are canonic forms if z(s) is having real roots s plus alpha, s plus beta and so on and if they are interlaced then they can be either a r l or r c depending on whether for z(s), whether the pole is closest to the origin. The first one starts with a pole or not then it will be an r c with the first root is a 0 nearest to the origin is a 0 then it will be an r l function and we can realize it again by 4 canonic forms.

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Handwritten mathematical derivation on a blue sheet of paper:

$$\text{Ex 2 } z(s) = \frac{s^2 + 16}{s^4 + 10s^2 + 9}$$

$$z(s)z(-s) = \frac{16 - s^4}{s^4 + 10s^2 + 9} = \frac{(4+s)(4-s)}{(s^2+9)(s^2+1)}$$

$$= \frac{(4+s)(4-s)}{(s+3)(s+1)(s-3)(s-1)}$$

$$z(s) = \frac{4+s}{(s+3)(s+1)}$$

Partial fraction decomposition:

$$z(s) = \frac{Kv z(s)}{s+3} + \frac{Kd z(s)}{s+1}$$

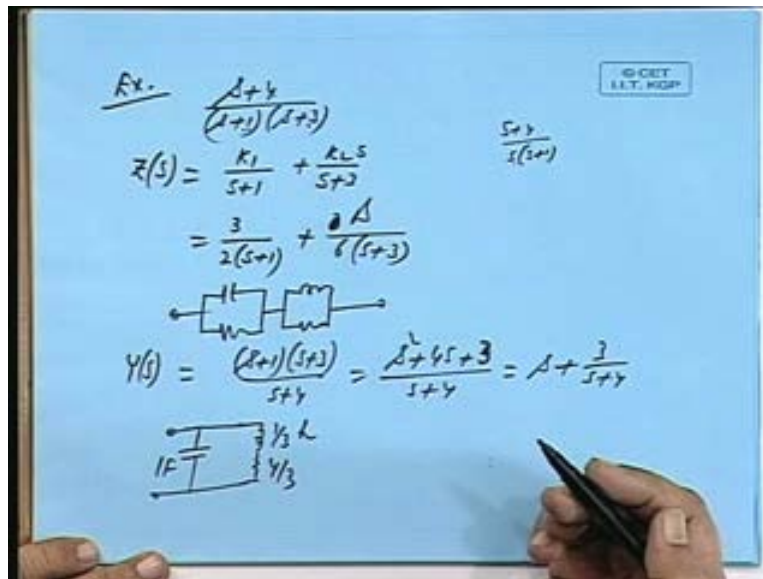
$$z(j\omega) = \text{Re } z(j\omega) + j \text{Im } z(j\omega)$$

$$= \text{Ev } z(s) \Big|_{s=j\omega} + j \text{Od } z(s) \Big|_{s=j\omega}$$

Suppose it is a positive real function but roots are not alternately coming then it is neither r l nor r c, it will be an r l c network. This is also what we saw earlier, if it is tested for positive realness, if it is tested for positive realness and the conditions are satisfied then it is realizable but if the poles and 0s are not interlaced then it will not be an r l or r c then it will be r l c and if it is an r l c network then we will find now you cannot have necessarily a Foster 1 or Foster 2 realization you have to go for Bott Duffin or Brune's synthesis, well not that you cannot factorize it I will give an example, you cannot factorize it so easily. Let us see, I will take a simple example, let me see if I have that okay.

Let us take a function s plus 4 by s plus 1 into s plus 3 okay. This is a function which we got just sometime back if you remember the first example but I worked out yes it is the 1 s plus 4 by s plus 3 into s plus 1 okay. So how to realize this, it is a positive real function though poles and 0s are not coming alternately there are 2 poles first then there 0.

(Refer Slide Time: 35:50)



So it is neither r l nor r c, so I thought of trying to realize it  $z(s)$  suppose it is  $k_1$  by  $s$  plus 1 plus  $k_2$  by  $s$  plus 3 obviously, one of them will be negative because poles and 0s are not coming alternately. Let us see how much is  $k_1$ , how much is  $k_1$  multiplied by  $s$  plus 1 put  $s$  plus 1 equal to 0 so 0, 3 by 2, 3 by 2 into  $s$  plus 1, so this will give me an r c combination.

Now here it is  $k_2 s$  plus 3 here 0 if I put this will be 1 this will be 3 minus 3, so minus 1 third, so it will not work. Suppose let us put it as  $s$  into this then how much is  $k_2$  multiplied by  $s$  plus 3 divide by  $s$ , so it will be  $s$  plus 4 by  $s$  into  $s$  plus 1 and then make  $s$  plus 3 equal to 0. So this will be 3 divided by sorry, 1 divided by 3 into 2 minus 3 into minus 2, so minus 6 plus 6, 1 by 6 into  $s$  plus 3 into  $s$ , what is this an r c combination, what is this an r l combination. So if someone goes by Foster 1 realization it is possible for such a simple function it is possible to get this, problem comes when you are having quadratics and not so easily factorizable, for in a factorizable form then you have to go for Brune's synthesis or Bott Duffin synthesis, not necessarily all ways you will be having that kind of synthesis even by Bott Duffin synthesis also you may be landing up in similar functions all right. Let us see what canonic means let us this is Foster 1 synthesis all right means we are putting series elements  $z_1, z_2, z_3$ , let us try  $y$  s Foster 2 then it is  $s$  plus 1 into  $s$  plus 3 divided by  $s$  plus 4 and that is equal to  $s$  square plus 4  $s$  plus 3 by  $s$  plus 4 okay  $s$  square plus 4  $s$  by  $s$  plus 4 is  $s$ ,  $s$  into  $s$  plus 4  $s$  plus 3 by  $s$  plus 4.

So this is an, this is a capacitor in parallel with and  $s$  plus 4 by 3 is the impedance. So it is 1 third 4 by 3 ohm resistance and 1 third Henry inductor is 1 Farad capacitor. Now you see here you are having 3 elements, here you are having 4 elements. So  $y(s)$  and  $z(s)$  you see the difference all right if someone goes for Cauer synthesis, ladder synthesis, let us see what it gives,  $z(s)$  equal to  $s$  plus 4 by  $s$  square plus 4  $s$  plus 3 is it not I can write this as 1 by  $s$  square plus 4  $s$  plus 3 by  $s$  plus 4. So let us carry out the division  $s$  square plus 4  $s$  plus 3  $s$  into  $s$  square plus 4  $s$  here so 3  $s$  plus 4,  $s$  by 3  $s$  4, 3 3 by 4, 3 okay.

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Handwritten mathematical derivation on a blue background:

$$z(s) = \frac{s+2}{s^2+4s+3}$$

$$= \frac{1}{\frac{s^2+4s+3}{s+2}}$$

$$= \frac{1}{s + \frac{1}{\frac{3}{2} + \frac{4}{3}}}$$

Partial fraction decomposition:

$$\frac{s+2}{s^2+4s+3} = \frac{1}{s} + \frac{\frac{2}{3}}{s+3}$$

Circuit diagram showing a series combination of a capacitor with admittance  $\frac{1}{11F}$  and a parallel combination of a resistor with impedance  $\frac{1}{3}$  and a branch with impedance  $\frac{4}{3}$ .

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Handwritten mathematical derivation on a blue background:

$$z(s) = \frac{4+s}{3+4s+s^2}$$

$$= \frac{1}{\frac{3+4s+s^2}{4+s}}$$

Partial fraction decomposition:

$$\frac{4+s}{3+4s+s^2} = \frac{13}{4} \frac{1}{s+5} + \frac{16}{13} \frac{1}{s+3} - \frac{2}{13} \frac{1}{s}$$

Circuit diagram showing a parallel combination of a capacitor with admittance  $\frac{1}{11F}$  and a branch with impedance  $\frac{1}{3}$ .

So it starts with 1 by s plus 1 by s by 3 plus 1 by 3 by 4, so 4 by 3 so what does it give me admittance of capacitor then an impedance of 1 third Henry and 4 by 3 this is Cauer 1 okay, Cauer 1 synthesis is same as Foster 2 the same thing 1 Farad, is it not, it is coming as Foster 2, Cauer 1 and Foster 2, Cauer 2 you reverse the order and then see z(s) equal to 4 plus s divided by 3 plus 4 s plus s square equal to 1 by 3 plus 4 s plus s square divided by 4 plus s. So let us divide 4 plus s 3 plus 4 s plus s square so this will be 3 by 4, 3 plus 3 by 4 s, so 13 by 4 s plus s square 4

plus s how much is it, 16 by 13 is that all right that gives me 4, 16 by 13 s in the denominator all right.

So that gives me 4 plus 16 by 13 s now that gives me minus 3 by 13 s. So since there is a negative sign it is not possible to realize the Cauer 2, Cauer 2 does not exist that means you cannot have something like resistance and then capacitance and so on then an inductance or resistance, it will feel.

(Refer Slide Time: 40:22)

$$z(j\omega) = \frac{1+\omega^2}{1+\omega^2+\omega^4}$$

$$z(s)z(-s) = \frac{1-s^2}{1-s^2+s^4} = \frac{(1+s)(1-s)}{(s^2+\sqrt{3}s+1)(s^2-\sqrt{3}s+1)}$$

$$z(s) = \frac{s+1}{s^2+\sqrt{3}s+1}$$

$$\text{Realizable } z(s) = \frac{s^2+2s+16}{s^2+2s+16}$$

So canonic forms of that kind Cauer 1, Cauer 2, Foster 1, Foster 2 all may not be applicable here in 3 element networks, some of them may succeed, some of them may not not obviously, that you have seen just now I have tried all the 4 methods and number of elements also differ in some case we have got 3 element, in some case we got 4 elements in Bott Duffin synthesis we have seen we can avoid the use of that Brune's transformer with 100 per cent coupling but number of elements will be more all right, number of elements will be more.

Let us take 1 or 2 examples on  $z(s)$  with 3 elements that is Brune's and Bott Duffin's synthesis. Some of the interesting networks okay sorry before we go to that there is another function suppose real  $z$   $j$   $\omega$  equal to  $1 + \omega^2$  by  $1 + \omega^2 + \omega^4$  already it was sorry, real  $z$  was given Suppose this is magnitude square suppose this is equal to magnitude square then what will you do have you understood if  $z(s)$  square is there then you make substitution  $z(s)$   $z$  minus  $s$  will be  $1 - s^2$   $1 - s^2$  divided by  $1 - s^2 + s^4$  have I have I done it sorry, just now, it was the real part, no I did the other problem that was the real part. So this will be  $1 + s$  into  $1 - s$  okay and how much is this  $s^2 + \sqrt{3}s + 1$  into  $s^2 - \sqrt{3}s + 1$  okay so how much is  $z(s)$  choose only the left up plane roots. So  $s + 1$  by  $s^2 + \sqrt{3}s + 1$  is that all right. Earlier while computing for the when it was given as real  $z$  equal to this we, we got

the same denominator but the numerator was remember 1 plus s by root 3 something like that know I will just **yes 1 plus 1 by root 3 s**, 1 plus s by root 3 that was the function.

So you can see the difference yes, 1 plus s by root 3 was that so that is the only difference in the numerator denominator remains same okay. Now I will take general r l c synthesis by both Brune's method and Bott Duffin's method this is just a devising what we have learnt earlier. So realize  $z(s)$  equal to  $s^2 + 2s + 16$  divided by  $s^2 + 2s + 4$  okay.

(Refer Slide Time: 43:36)

The image shows handwritten mathematical derivations on a blue background. The equations are as follows:

$$z(s)z(-s) = \frac{1-s^2}{1-s^2+s^4} = \frac{(1+s)(1-s)}{(s^2+\sqrt{2}s+1)(s^2-\sqrt{2}s+1)}$$

$$z(s) = \frac{s+1}{s^2+\sqrt{2}s+1}$$

Then, the real part of  $z(s)$  is derived as:

$$\text{Re } z(s) = \frac{s^2+2s+16}{s^2+2s+4}$$

Next, the real part of  $z(j\omega)$  is calculated by substituting  $s = j\omega$ :

$$\text{Re } z(j\omega) = \frac{(s^2+16)(s^2+4) - 4s^2}{(s^2+4)^2 - 4s^2} \Big|_{s=j\omega}$$

$$= \frac{s^4+16s^2+64}{s^4+4s^2+16} \Big|_{s=j\omega} = \frac{\omega^4-16\omega^2+64}{\omega^4-4\omega^2+16} = \frac{(\omega^2-8)^2}{-4\omega^2}$$

Let us try by Brune's synthesis so what was the starting point a real  $z(j\omega)$  we compute that is  $m_1, m_2$  minus  $n_1, n_2$  by  $m_2$  square minus  $n_2$  square so  $s^2 + 16$  even part into  $s^2 + 4$  minus  $2s$  into  $2s$ ,  $4s^2$  divided by  $s^2 + 4$  whole square  $m_2$  square minus  $4s^2$  square  $n_2$  square  $s$  equal to  $j\omega$  is that all right.

So that gives me  $s$  to the power 4 plus 16 plus 4, 20 minus 4, so  $16s^2 + 64$  divided by  $s$  to the power 4 plus 8  $s^2$  minus 4  $s^2$  plus 16 is that all right,  $s$  equal to  $j\omega$ . See if I put  $s$  equal to  $j\omega$   $\omega$  to the power 4 minus 16  $\omega^2$  plus 64 divided by  $\omega$  to the power 4 minus 4  $\omega^2$  plus 16. Now this you can see is  $\omega^2$  minus 8 whole square okay,  $\omega^2$  minus 8 whole square. So this function becomes 0 at  $\omega^2$  is equal to 8 we have to first of all track that frequency at which the real part vanishes, is it not.

(Refer Slide Time: 45:17)

$\text{Re } z(j\omega_1) = 0 \quad \text{at } \omega_1 = \sqrt{8} = 2\sqrt{2}$   

$$z(j\omega_1) = \frac{16 - 8 + 2j \cdot 2\sqrt{2}}{-8 + 4 + 2j \cdot 2\sqrt{2}}$$

$$= \frac{8 + 4j\sqrt{2}}{-4 + 4j\sqrt{2}}$$

$$= \frac{\sqrt{96} \angle \tan^{-1}(1/\sqrt{2})}{\sqrt{48} \angle \tan^{-1}(\sqrt{2})} = \sqrt{2} \angle -90^\circ = -\sqrt{2}j$$

$$X_1 = -\sqrt{2}$$

$$L_1 = \frac{X_1}{\omega_1} = \frac{-\sqrt{2}}{2\sqrt{2}} = -\frac{1}{2} \text{ H}$$

The diagram shows a right-angled triangle with the angle  $\theta$  between the hypotenuse and the positive x-axis. Below it, a circuit diagram shows an inductor with value  $-\frac{1}{2} \text{ H}$  connected to a ground symbol.

So the real part  $z(j\omega_1)$  equal to 0 at  $\omega_1$  equal to  $\sqrt{8}$  that is  $2\sqrt{2}$ . Once you have traced that frequency  $\omega_1$  calculate  $z(j\omega_1)$  that will be the imaginary part is it not that will be the imaginary part. So if I put  $\omega$  equal to  $2\sqrt{2}$  in this how much is it  $16 - s^2$  is  $16 - 8$ , correct me if I am wrong, okay plus  $2$  into  $s$  into  $j$  into  $2\sqrt{2}$  divided by  $s^2 + 4$ . So  $8 + 4j\sqrt{2}$  divided by  $-4 + 4j\sqrt{2}$  is that okay.

Now you can see for yourself  $8 + 4\sqrt{2}$ ,  $8 + 4\sqrt{2}$  somewhere here  $8 + 4\sqrt{2}$  how much is the magnitude  $8^2 + 32$ ,  $96$ , square root of  $96$  okay  $\tan^{-1} 1/\sqrt{2}$  and this is  $4$   $4$ 's are you see some where here  $4$   $4$ 's are  $16 + 32$ . So  $\sqrt{48} \tan^{-1} \sqrt{2}$  this is  $\tan^{-1} 1/\sqrt{2}$  okay and this is  $\tan^{-1} \sqrt{2}$  but it is more than  $90$  degrees. So how much is this angle, how much is this angle the difference is  $90$  degree, it is the denominator angle which is more.

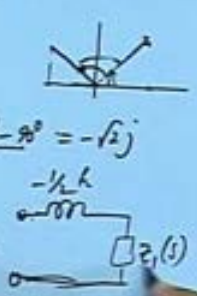
So if this is  $\theta$  this is  $90 + \theta$  is that all right magnitude is  $\sqrt{2}$  and denominator is having an angle of  $90$  degree that means  $-90$  in the numerator. So  $90$  degree in the numerator, so  $-\sqrt{2}j$  is that all right, it is just a simple competition. Otherwise also by rationalizing you will get  $-\sqrt{2}j$  I thought it is better to write in the polar form and you can imagine the angles very clearly okay one is  $\alpha$  the other one is  $1/\alpha$   $\tan^{-1} \alpha$  and  $\tan^{-1} 1/\alpha$ , so find out the difference in angle is that all right. So  $z$  is known so  $X$  is  $-\sqrt{2}$  I call it  $X_1$  so in Brune's synthesis if you get a negative reactance to start with what will that be it represented as  $L_1$  is equal to  $X_1/\omega_1$  which is  $-\sqrt{2}/2\sqrt{2}$ ,  $2\sqrt{2}$  is the frequency, thank you all right.

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$$\begin{aligned}
 &= \frac{s^2 + 2s + 16}{s^2 + 2s + 4} + \frac{1}{2}s \\
 &= \frac{2s^2 + 4s + 32 + s^3 + 2s^2 + 4s}{2(s^2 + 2s + 4)} \\
 &= \frac{s^3 + 4s^2 + 8s + 32}{2(s^2 + 2s + 4)} \\
 &= \frac{(s+8)(s+4)}{2(s^2 + 2s + 4)}
 \end{aligned}$$

So this will be minus half Henry so to start with you will have a minus half Henry inductor, do not get frightened in the realization in a transformer you will find that minus can be accommodated. So once you have identified this then you go for the next impedance  $z_1$  what is  $z_1(s)$ ,  $z_1(s)$  is  $z(s)$  minus  $l_1(s)$  which is nothing but what was  $z(s)$  sorry,  $s$  square plus  $2s$  plus  $16$  divided by  $s$  square plus  $2s$  plus  $4$  minus  $l_1(s)$ ,  $l_1$  is minus half so minus and minus will make it plus half  $s$  is that all right. So add these what do you get by adding this, so  $s$  cube  $s$  square plus  $2s$  plus  $4$  plus  $16$  by  $s$  square plus  $2s$  plus  $4$  plus half  $s$  is that all right.

(Refer Slide Time: 52:14)

$$\begin{aligned}
 z(j\omega_1) &= 0 \quad \text{at } \omega_1 = \sqrt{8} = 2\sqrt{2} \\
 z_1(j\omega_1) &= \frac{16 - 8 + 2j \cdot 2\sqrt{2}}{-8 + 4 + 2j \cdot 2\sqrt{2}} \\
 &= \frac{8 + 4\sqrt{2}j}{-4 + 4\sqrt{2}j} \\
 &= \frac{\sqrt{96} \angle \tan^{-1}(1/2)}{\sqrt{48} \angle \tan^{-1}(\sqrt{2})} = \sqrt{2} \angle -90^\circ = -\sqrt{2}j \\
 \frac{x_1}{\omega_1} &= \frac{\sqrt{2}}{2\sqrt{2}} = -\frac{1}{2} \text{ H}
 \end{aligned}$$




So you get 2 into s square plus 2 s plus 4 and here it is twice s square plus 4 s plus 32 plus s cubed plus twice s square plus 4 s, correct me if I am wrong. So that gives me s to the power 3 plus 4 s square plus 8 s plus 32 divided by 2 into s square plus 2 s plus 4 is that all right. This you can write as s cube plus 8 and s see s square plus 8 s square plus 8 into s plus 4, 4 into s square plus 8 s into s square plus 8 divided by 2 into s square plus 2 s plus 4. By the way, why did I take s square plus 8 as a **fact** factor, if you remember, if you remember in the network this function z(s) is having a value minus half sorry root 2 j at that frequency all right and it is also the value of this that we have equated.

(Refer Slide Time: 52:43)

$$\begin{aligned}
 z_1(s) &= z(s) - Ls \\
 &= \frac{s^2 + 2s + 4}{s^2 + 2s + 4} + \frac{1}{2} s \\
 &= \frac{2s^2 + 4s + 32 + s^3 + 2s^2 + 4s}{2(s^2 + 2s + 4)} \\
 &= \frac{s^3 + 4s^2 + 8s + 32}{2(s^2 + 2s + 4)} \\
 &= \frac{(s+4)(s+4)}{2(s^2 + 2s + 4)}
 \end{aligned}$$

(s+4)

So this will be  $a_0$  at that frequency if you remember this element  $z_1(s)$  must be 0 at that particular frequency  $\omega = 1$  okay. So this  $z_1(s)$  at  $\omega = 1$  that is  $2\sqrt{2}$  that is  $s^2 + \omega^2$  will be a factor in the numerator and  $\omega^2 = 8$ . So  $s^2 + 8$  will be a factor that we know because what we try to ensure is this impedance at that frequency is equal to the reactance at that frequency of this coil. So what will be the impedance of this it has to be 0 because this impedance is equal to this impedance so this has to be 0, so  $z_1(s)$  must have  $a_0$  at that frequency, so  $s^2 + 8$  must be having a numerator factor, must be a numerator factor of  $z(s)$  this  $z_1(s)$  allowed obviously, we are making it equal to that inductor okay, thank you very much we will stop here for today. We will continue with this in the next class.

**Preview of Next Lecture**  
**Lecture - 29**  
**Tutorial**

Good morning friends, we will continue with the numerical problem that we are discussing in the last class.

(Refer Slide Time: 54:07)

The image shows a blueboard with handwritten mathematical work. At the top, the transfer function is given as  $z(s) = \frac{s^2 + 2s + 16}{s^2 + 2s + 4}$ . Below this, it is noted that  $\omega_1 = \sqrt{8}$  and  $\alpha = -\frac{1}{2}$ , leading to  $s = -4s$ . A small circuit diagram shows a voltage source  $-\frac{1}{2}k$  in series with a component, with the resulting admittance  $z_1(s) = z(s) - 4s$ . This is simplified to  $\frac{(s^2 + 8)(s + 4)}{2(s^2 + 2s + 4)}$ . The partial fraction decomposition is shown as  $Y_1(s) = \frac{2(s^2 + 2s + 4)}{(s^2 + 8)(s + 4)} = \frac{k_1 s}{s^2 + 8} + \frac{k_2}{s + 4}$ . Finally, the coefficient  $k_1$  is calculated as  $k_1 = \frac{2(-4 + 25)}{s^2 + 4s} = \frac{-8 + 45}{-8 + 4s} = 1$ .

We took at function  $z(s)$  equal to  $s$  square plus  $2s$  plus  $16$  divided by  $s$  square plus  $2s$  plus  $4$ . We started of with this function and we found that at  $\omega_1 = 1$  is equal to root  $8$  the real part vanishes and we also obtained the value of  $\alpha_1$  as half  $s$  whether corresponding impedance this was  $l_1(s)$  all right and this was minus half  $s$  minus half Henry, this is  $z_1(s)$  which will be equal to  $z(s)$  minus  $l_1 s$  and that give me  $l_1$  is having a negative sign, so that give me a plus sign. So we got  $s$  square plus  $8$  into  $s$  plus  $4$  divided by  $2$  into  $s$  square plus  $2s$  plus  $4$  okay.

To realize a pole to realize a pole we know we can make partial fraction, so this  $0$  is to be converted to a pole so we can very easily realize  $z_1(s)$  in terms of admittance function which will be  $2$  into  $s$  square plus  $2s$  plus  $4$  divided by  $s$  square plus  $8$  into  $s$  plus  $4$ , you write as  $k_1(s)$  by  $s$  square plus  $8$  we know whenever there are roots  $1$  the imaginary axis it will be realized in terms of an  $l$  c network  $k_1(s)$  by  $s$  square plus  $8$  plus  $k_2$  by  $s$  plus  $4$ , this is to be seen later.

(Refer Slide Time: 57:12)

$$Z(s) = \frac{s^2 + 5s + 6}{s^2 + 5s + 4} \quad (s^2 + 5s + 4) = (s+2)(s+4)$$

$$A Z(s) = \frac{A(s+4) + B(s+2)}{s^2 + 5s + 4} = \frac{A s^2 + 4A + B s + 2B}{s^2 + 5s + 4}$$

$$A \equiv x \Rightarrow \frac{x^2 + 15x + 24}{x^2 + 17x + 16} = f(x)$$

$$f'(x) = 0 \Rightarrow (2x+15) \cdot (x^2 + 17x + 16) - (x^2 + 15x + 24) \cdot (2x + 17) = 0$$

$$\Rightarrow 17x^2 + 32x + 240 - (2x^2 + 17x + 16)(2x + 17) = 0$$

$$\Rightarrow 17x^2 + 32x + 240 - (4x^2 + 34x + 32x + 272) = 0$$

$$\Rightarrow 17x^2 + 32x + 240 - 4x^2 - 68x - 272 = 0$$

$$\Rightarrow 13x^2 - 36x - 32 = 0$$

$$x = \frac{36 \pm \sqrt{36^2 + 4 \cdot 13 \cdot 32}}{2 \cdot 13}$$

(Refer Slide Time: 58:02)

$$Z(s) = \frac{s^2 + 5s + 6}{s^2 + 5s + 4} \quad (s^2 + 5s + 4) = (s+2)(s+4)$$

$$A Z(s) = \frac{A(s+4) + B(s+2)}{s^2 + 5s + 4} = \frac{A s^2 + 4A + B s + 2B}{s^2 + 5s + 4}$$

$$A \equiv x \Rightarrow \frac{x^2 + 15x + 24}{x^2 + 17x + 16} = f(x)$$

$$f'(x) = 0 \Rightarrow (2x+15) \cdot (x^2 + 17x + 16) - (x^2 + 15x + 24) \cdot (2x + 17) = 0$$

$$\Rightarrow 17x^2 + 32x + 240 - (2x^2 + 17x + 16)(2x + 17) = 0$$

$$\Rightarrow 17x^2 + 32x + 240 - (4x^2 + 34x + 32x + 272) = 0$$

$$\Rightarrow 17x^2 + 32x + 240 - 4x^2 - 68x - 272 = 0$$

$$\Rightarrow 13x^2 - 36x - 32 = 0$$

$$x = -4 \pm \sqrt{16}$$

Let us see what this means how much is  $k_1$  multiply by  $s$  square plus 8 divide by  $s$  square plus 8 equal to 0. So this will give me 2 into **minus** 4 minus 8. So minus 4 plus 2  $s$  divided by  $s$  into  $s$  square plus 8, so  $s$  into  $s$  plus 4 so  $s$  square plus 4  $s$  which gives me this is minus 8 plus 4  $s$  divided by minus 8 plus 4  $s$ , so that is equal to 1, there must be some frequency where it is minimum omega square is equal to  $x$  square plus 15  $x$  plus 24  $x$  square plus 17  $x$  plus 16, I hope this is all right okay then 2  $x$  plus 15 into  $x$  square plus 17  $x$  plus 16, 2  $x$  plus 17 into  $x$  square

plus 15 x plus 24. So 2 x cube 2 x cube they get cancelled 2 x into okay. Let me rewrite it any way what I wanted to stress is you will get a real value of x that is equal to omega square.

(Refer Slide Time: 58:34)

$$F(x) = \frac{x^2 + 15x + 24}{x^2 + 17x + 16} = F(x)$$

$$F'(x) = 0 \Rightarrow (2x + 15)(x^2 + 17x + 16) - (x^2 + 17x + 16)(2x + 15) = 0$$

$$\Rightarrow 19x^3 + (32 + 25T)x + 240 = 0$$

$$= 42x^2 + (42 + 25T)x + 172x$$

$$\Rightarrow 28x^2 + 16x + 16 = 0$$

$$7x^2 + 4x + 4 = 0 \quad x = \frac{-4 \pm \sqrt{16 - 112}}{14}$$

So calculate **omega** omega and substitute that omega here in the real part that will give you the minimum value that is when the real part, real part varies like this it is this minimum value and after computing that r minimum subtract it from z(s), subtract it from z(s) whatever is left over you start realizing that z(s), the remainder z(s) there may be a small slip some where here we will discuss it in the next class, if time permits. Otherwise, you work it out your self and since there is not much of time okay, thank you very much. We will continue with this in the next class.