**Network Signals and Systems**

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**Lecture - 27**

# Parts of Network Functions

Good after noon friends, today we shall be discussing about parts of network functions. Now before we go to that I will just briefly summarize what we did yesterday in the Bott Duffin synthesis then we will go to parts of networks.

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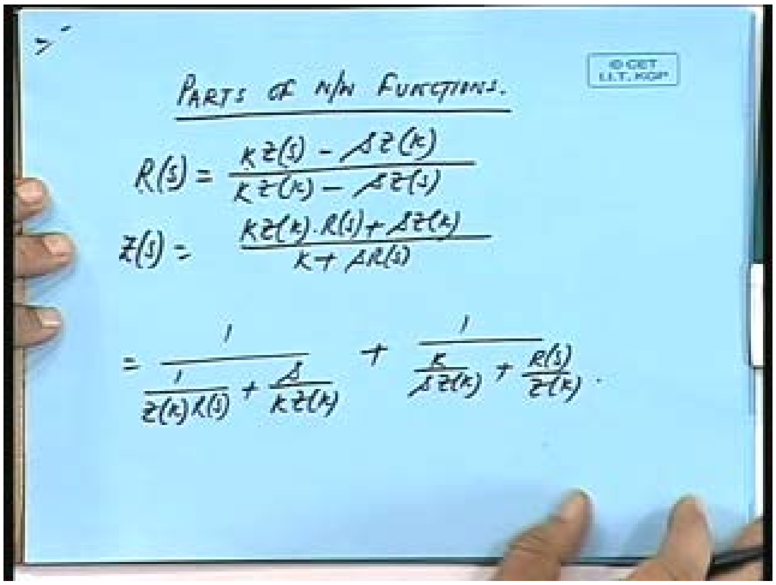


So the Bott Duffin synthesis we defined the Richard function which is a positive real function in terms of z(s) as kz(k) minus sz(s) if z(s) is a positive real function then all for all positive values of k, R(s) will also be a positive real function from there we wrote z(s) in terms of R(s) as kz(k), R(s) plus sz(k) divided by k plus sr(s) and this we broke up in this form if you remember in the form of 2 parallel elements put in series with anotheR(s)et of this type k by sz(k) plus r(s) by z(k) okay.

So this was shown to be like this, this is the admittance of a capacitor is that all right. So this is this admittance of a capacitor, so1 by kz(k) Farads will be the value of this capacitance this is say this is some $z_1$ then in parallel with this capacitor this impedance $z_1$ will come similarly, this one is the admittance of a reactor its value is z(k) by k, so many inductance in parallel with that we have $z_2$ which means z(k) by R(s) is that all right.

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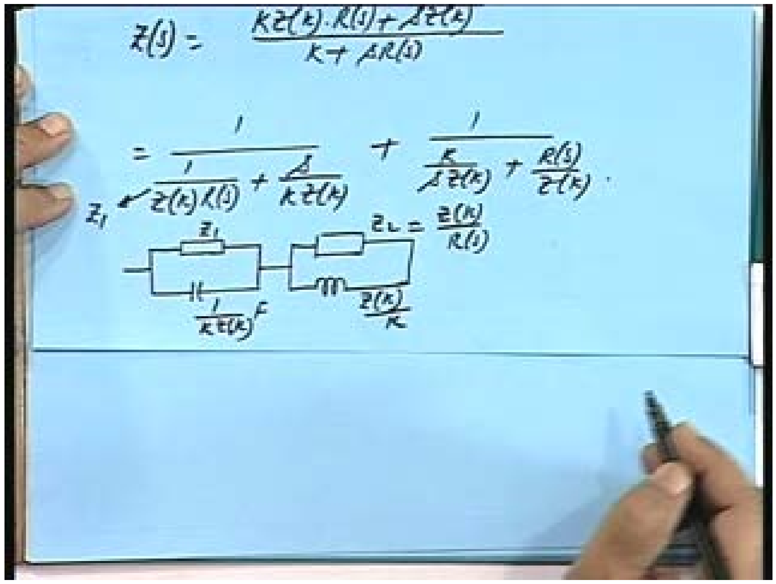


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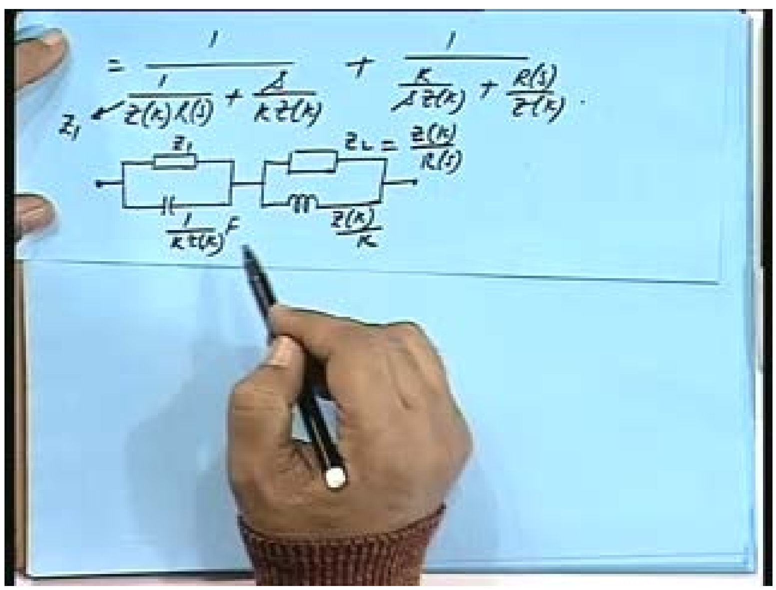


So this is what we wrote yesterday now $z_1$ to be calculated $z_1$ is z(k) into R(s), R(s) being a positive real function z(k) is having a constant value for a particular value of k, real value. So this is an impedance function realizably impedance function similarly, z(k) by R(s) is the other impedance function you can see one is z(k) into R(s) the other one is z(k) by R(s). We started of with this point if at a particular omega the real part is 0 real z j omega 1 equal to 0 then z j omega 1, z j omega 1 will be just some reactive element it may be plus j $x_1$ or minus j $x_1$.

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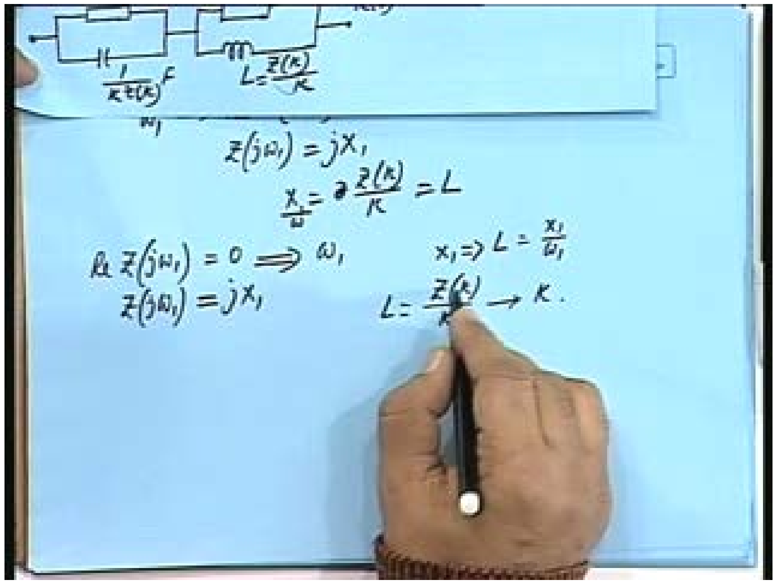

Suppose it is plus j $x_1$ if it is plus j $x_1$ it will be inductive, now I can have this at omega 1 suppose at omega 1, this is 0, this is 0 impedance that means at omega 1 z(k) into R(s) is 0 okay then the current will flow through this and then z by z(k) by R(s) if R(s) is having a 0 at omega 1 then 1 by R(s) will be having a pole. So if this is 0 impedance then this will be infinite impedance so the current will be made to pass through this short circuit 0 impedance and then going through this because this open circuit. So it will be appearing to be this inductance, so z(k) by k we started of with z(k) by k is equal to $x_1$ this was what a starting point.

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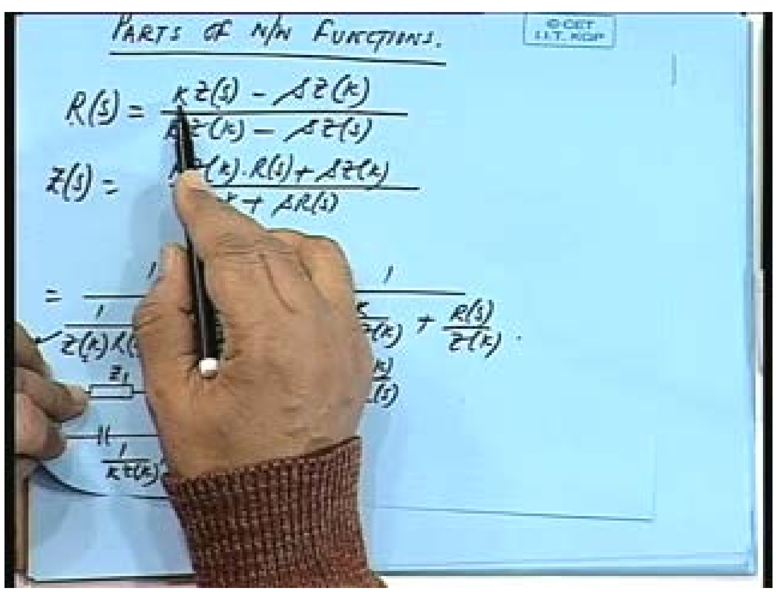

So $x_1$ has been evaluated for the given z(s) we found out the frequency we found out the frequency where this real part is 0 okay, we determine the frequency at that frequency what is this one we calculated this was j $x_1$. So this $x_1$ will be coming out of mind you $x_1$ represents the reactance, so $x_1$ by omega is the inductance that is equal to z(k) by k okay $x_1$ by omega we calculate that as this inductance l okay that is z(k) by k so from $x_1$ you calculate l which is $x_1$ by omega 1 and the that that is equated to z(k) by k.

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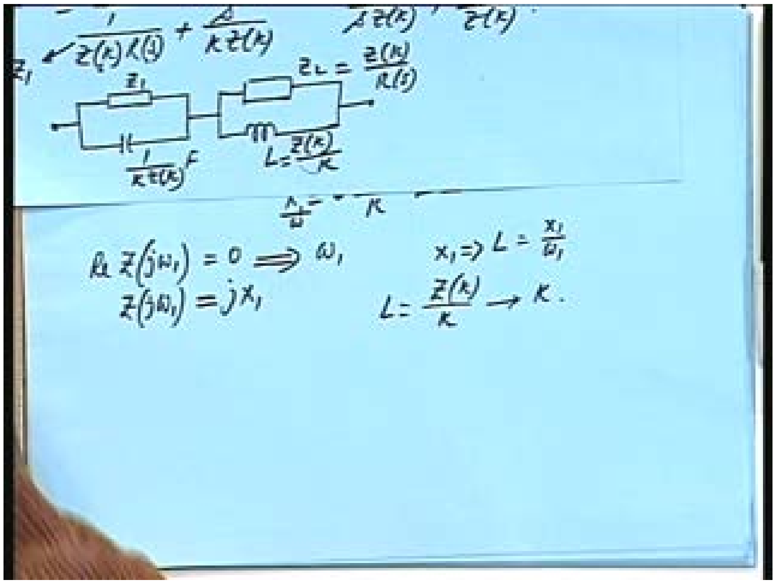

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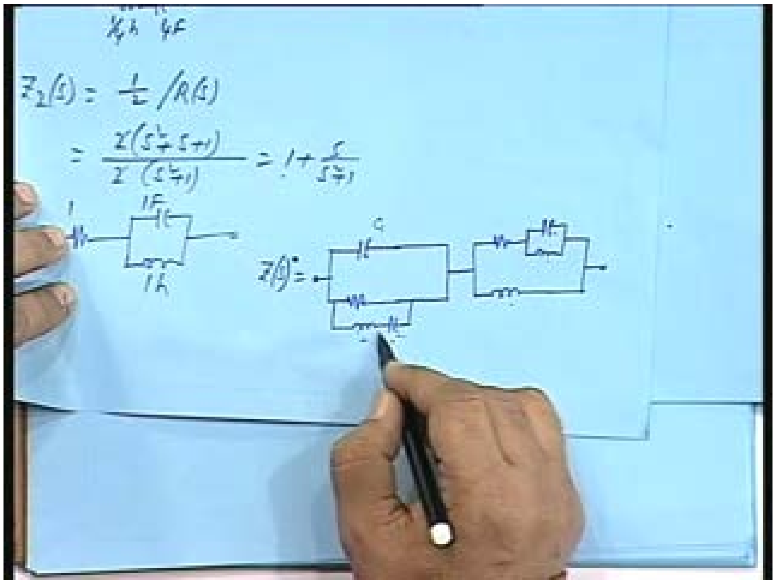

So calculate k okay once you know the value of k, l is known, k is known, so z(k) is also known, substitute here you get z(k) into R(s). See if I put the value of k here I get R(s) z(s) is given I get R(s) all right and we saw yesterday R(s) is having a 0 at omega 1 okay and $z_2$ was infinity whenever there is a pole, how to realize it make partial fractions. So you made a partial fraction and then we got $z_2$ this impedance similarly, we inverted $z_1$ and got the values of this impedances in the form of 2 admittances.

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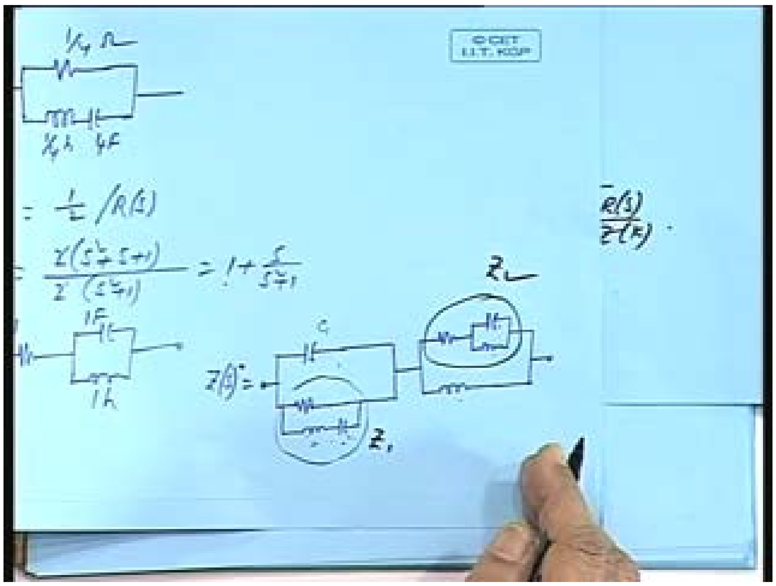


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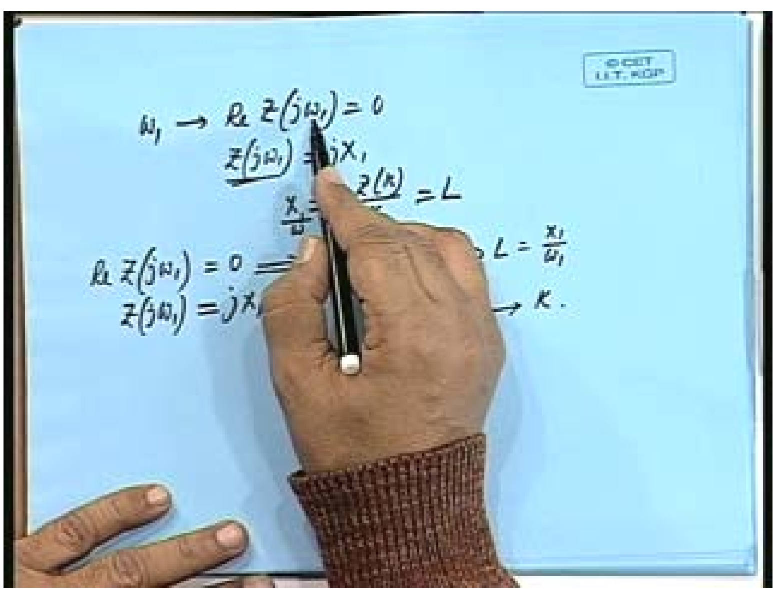

So the values that we got I will just show you the final values that we got yesterday. It was like this, the structure came like this there was a capacitor and you had r and l c in series combination of r and l c in this mode, 2 parallel elements in parallel with $c_1$. So this was equal to $z_1$, this is $z_1$ similarly, this was $z_2$ okay if you have instead of plus j $x_1$ at that particular frequency omega 1 when the real part vanishes suppose z is having a minus j exactly can be plus j x or minus j x.

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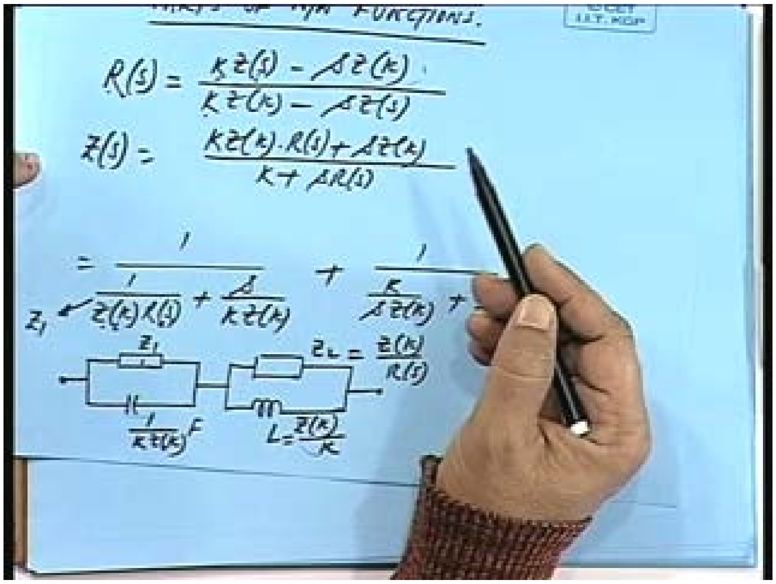


Suppose it is minus j $x_1$ then we would like to this is capacitive at that frequency so we would like to equity it to this capacitive impedance that means as if the current is made to flow through this so this is a open circuit all right. So $z_1$ which is z into R(s) should have a pole here at omega 1 and automatically this will have a 0. So that current will flow like this and then through this short circuited path okay. So z(k) into R(s) will be equal to 0 at that particular frequency omega 1, z(s), R(s) is having a 0 means you have to equate this to 0 at that frequency omega 1. So you can calculate the value of k okay, $z_1$ is infinity sorry, $z_1$ is infinity so the denominator should be 0, k into z(k) should be s into z(s) at s equal to j omega 1. So it is this denominator which has to be equated to 0 and then you determine the value of k rest of the thing is simple, straight forward.

Now we take up this topic parts of network function. We shall come back to some numerical problems at a later stage. Some times for this function, we may be given just the real part or the imaginary part the variation of this with respect to frequency is specified or variation of this with respect to frequency is specified. Can you realize z or sometimes it may be written z magnitude which is a frequency dependant quantity and e to the power j theta which is again frequency dependant either this magnitude or the phase will be given to you, can you find out z that means from the real part can you realize the imaginary part and hence the total function or from the magnitude can you estimate the phase and hence the overall function. You can realize it under certain conditions for example, suppose you are having a simple r l network okay now suppose you are given the imaginary part imaginary part is j omega l okay.

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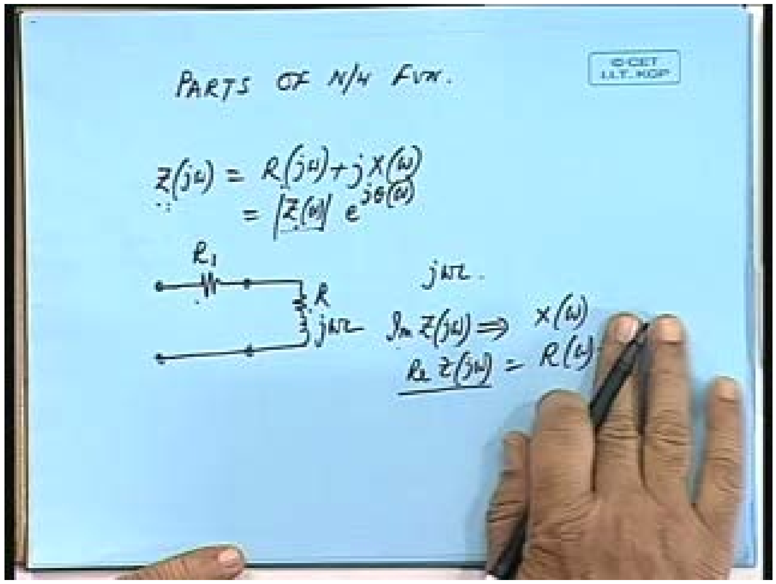


Now it can have a real part which can be either r or $r_1$ plus R, so from z if I am given only x omega that is imaginary part of z then the real part can be r omega or this plus r dash any additional resistance I can put but still because this is not specified, it can be anything.

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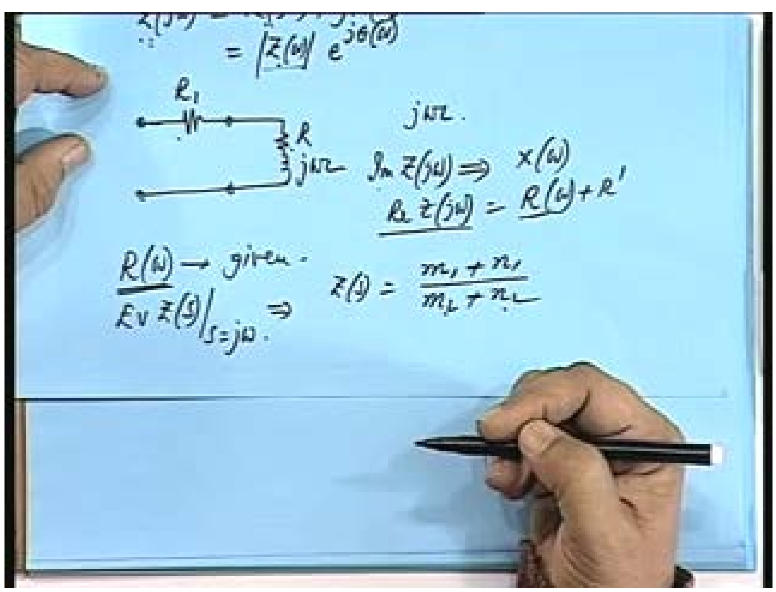

So I can realize this only when it is having a minimum value that is just what is required to realize z(s) whatever r omega is required okay. So it is not possible to determine r omega under any condition there can be 100s of solutions for a given x omega. If the imaginary part is specified from there I can find out only one unique value of r under certain conditions we say minimum resistance function similarly for the other one if r omega is specified you can find out x omega only under minimum inductance condition. Otherwise, I can keep on putting some more inductance till r omega will not be disturbed is it not so we shall be determining only those minimum reactance or minimum resistance required to realize that function. Now suppose you are given the a real part given, how do you realize z(s). Let us take a very simple case if r omega is given what will be z(s) r omega is basically even z(s) when I put s equal to j omega, is it not. If you remember even part of z(s), so z(s) can be written as, z(s) can be written as even part plus odd part, even part plus odd part, $m_1$, $m_2$ are the even parts, $n_1$, $n_2$ are the odd parts, this we discussed while testing the positive realness of the function if you remember.

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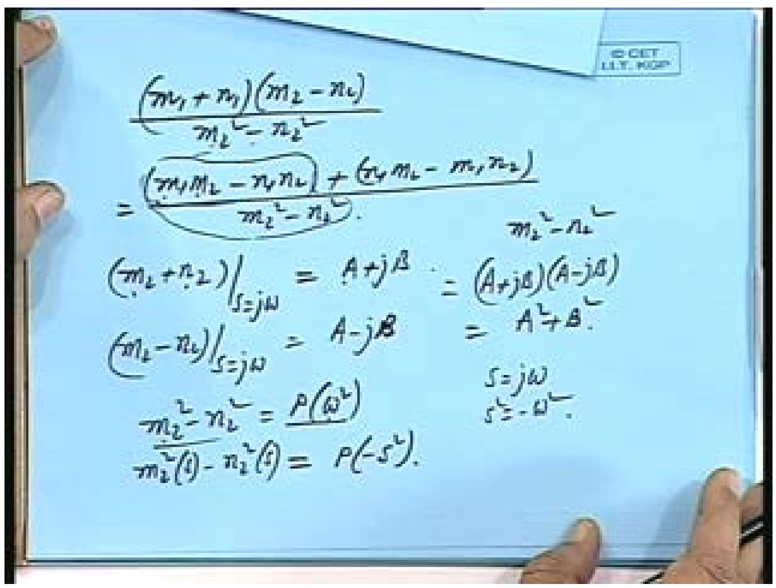


So this can be written if I try to find out the even part and odd part separately then it will be $m_1$ plus $n_1$ into $m_2$ minus $n_2$ divided by $m_2$ square minus $n_2$ square. I have just multiplied by sorry, $m_2$ minus $n_2$ both sides, so I have got $m_1$ in $n_1$ plus $m_2$ minus $n_2$, $m_2$ square minus $n_2$ square that gives me $m_1$, $m_2$ minus $n_1$, $n_2$ these are the even part plus $n_1$, $m_2$ minus $m_1$, $n_2$ odd part divided by $m_2$ square minus $n_2$ square. If you remember we discussed this point $m_2$ plus $n_2$ into $m_2$ minus $n_2$ will be all ways a mod, is it not.

Suppose $m_2$ plus $n_2$ at s equal to j omega suppose, this is a plus jB then what will be $m_2$ minus $n_2$ it will be a minus jB is it not even part $m_2$ plus $n_2$ see even part will given you real part odd part will give you imaginary part and it is a odd part to sign is changed, so it is going to minus jB if I multiply this you will get a plus jB into a minus jB all right that will be $m_2$ square minus $n_2$ square which is nothing but a square plus b square.

So this is a positive quantity so this is a positive quantity all ways for all values of s equal to j omega. Therefore, this is even into even this is odd into odd so this is even part and this is odd part all right. So our starting point will be this $m_2$ square minus $n_2$ square, so $m_2$ square minus $n_2$ square it is a function of omega square see odd function square will be even. So it will be something omega omega cube and so on if I square it you will get all omega score square omega4 all even parts of omega.

Similarly this one, so this will be something like a function of omega square s equal to j omega means s square is equal to minus omega square. So in this if I put omega square is equal to minus s square I get $m_2$ s minus $n_2$ square s, is it not will be p minus s square is that okay I will take an example and show you.

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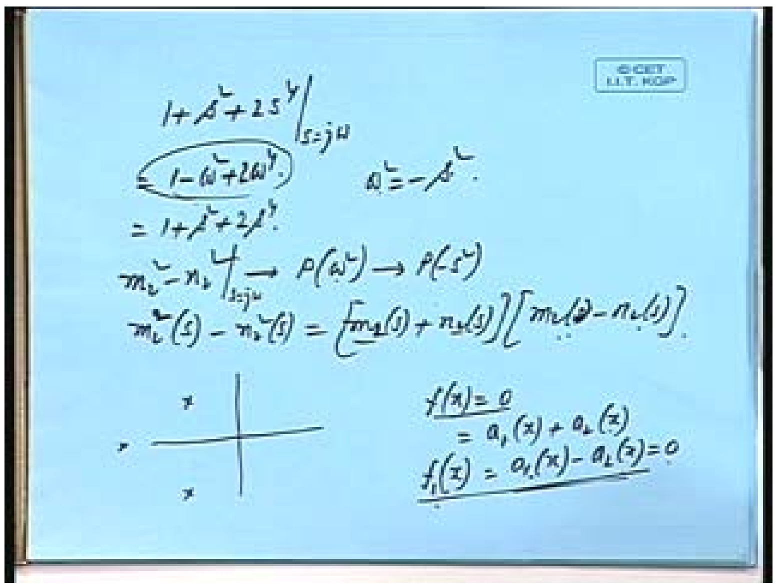


Suppose I have 1 plus s square plus 2 s to the power 4 if I put s equal to j omega what you get 1 minus omega square plus 2 into omega to the power 4. Suppose you are given these function how do I get back to this put omega square is equal to just reverse substitution omega square is equal to minus s square. So that gives me 1 plus s square plus twice s to the power 4, is it not. So that is what precisely I have done $m_2$ square minus $n_2$ square is given in terms of omega square. So make a substitution omega square equal to minus s square all right that will give you $m_2$ square s minus $m_2$ square s in the s domain all right and that is nothing but $m_1(s)$ plus sorry $m_2(s)$ plus $n_2(s)$ into $m_2(s)$ minus $n_2(s)$ okay.

Now you see $m_2(s)$ plus $n_2(s)$ suppose this is having a roots like this then what will be the roots of $m_2(s)$ minus $n_2(s)$, $m_2(s)$ minus $n_2(s)$ it is the odd part which is negated, odd of a polynomial which is negated. So if you are having a polynomial containing roots in the left of plane, roots in the left of plane then if I take just the negative sign of the odd terms and generate another polynomial then that will have roots in the right of plane, just the mirror images there is something very interesting. You have f(x) equal to 0 all right which is a function of say $a_1(x)$ even parts plus $a_2(x)$ odd parts then $f_1(x)$ which is given by $a_1(x)$ minus $a_2(x)$ equal to 0 this will have roots which will be mirror of this if this is having roots in the left of plane f(x) is having roots in the left of plane then $f_1(x)$ will have roots in the right of plane where $f_1(x)$ is having the even part as it is and odd part with a negative sign is that all right.

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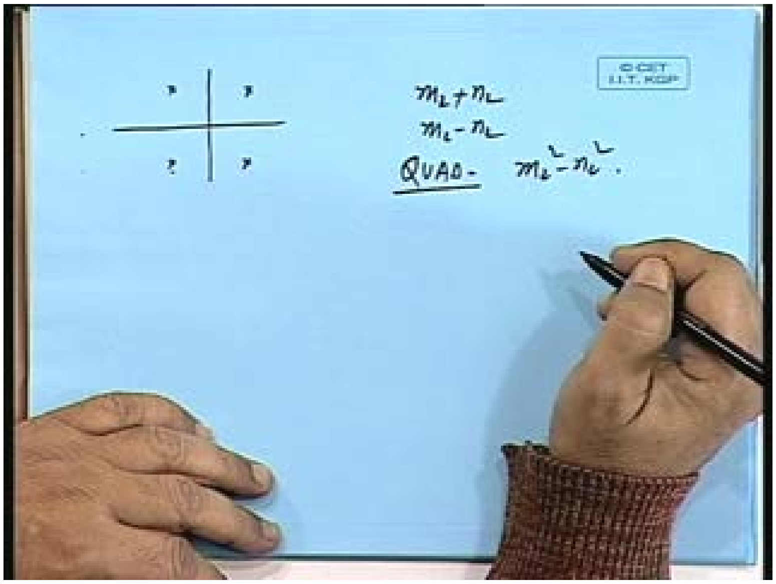


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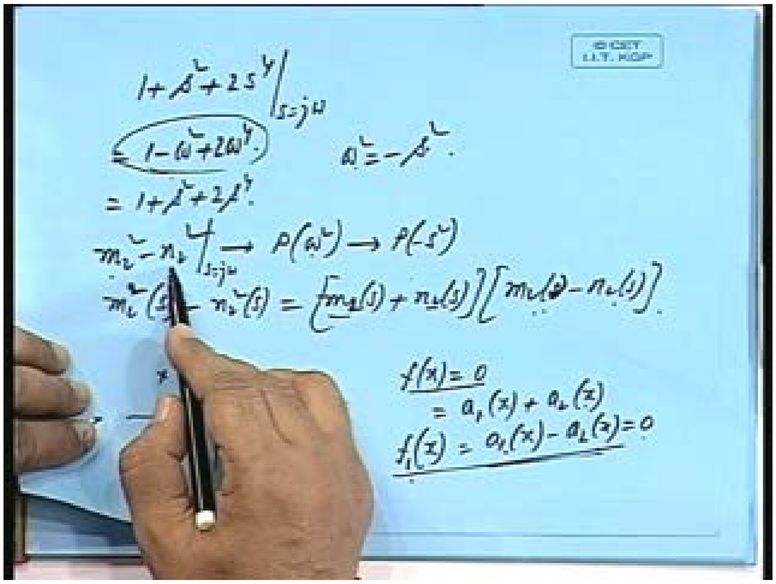

So for any complex pair of roots for example if you have a complex pair like this for $m_1$ plus sorry $m_2$ plus $n_2$, one pair of roots is like this then for $m_2$ minus $n_2$ correspondingly you have root here mirror images. So you get quad of points for each complex root set you get actually quad of points for the function into quad minus $m_2$ square okay.

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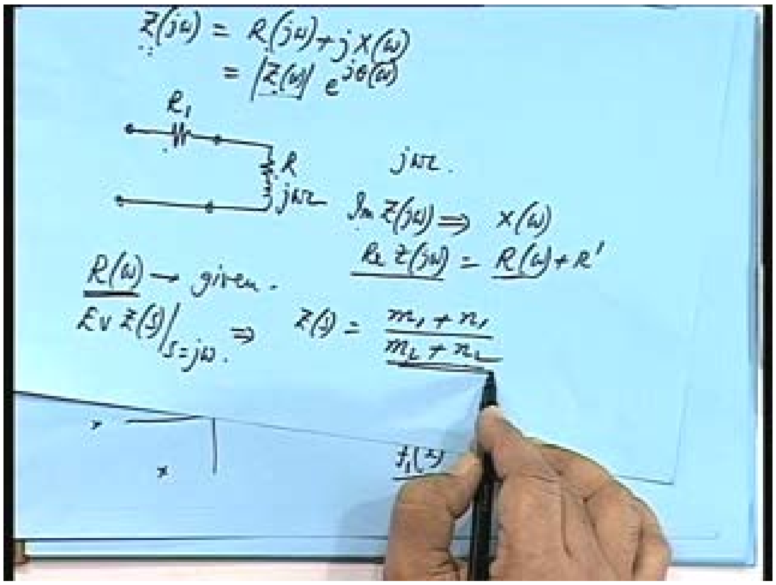


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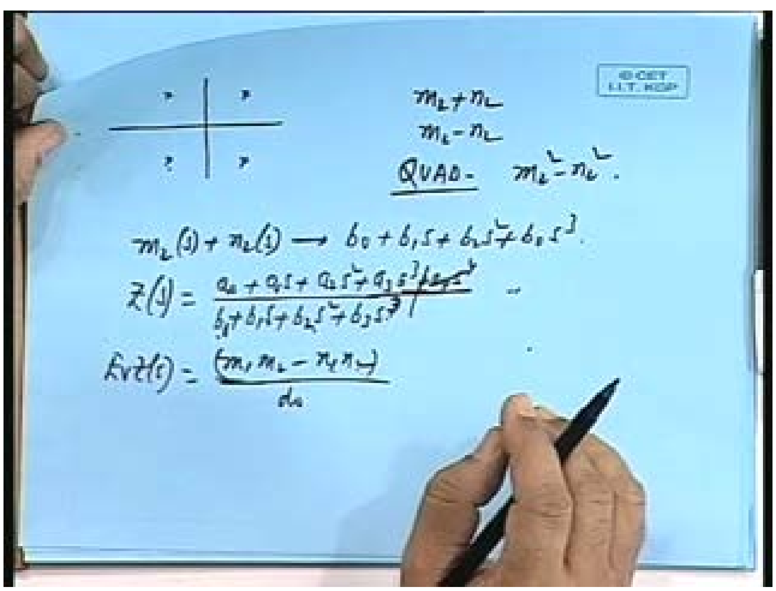


So problem is simplified you are given $m_2$ square minus $n_2$ square in terms of omega, so make s substitution omega square is equal to minus s square then factorize it then chose the factors corresponding to the left of plain roots that will give you $m_2(s)$ plus $n_2(s)$ why left of plain roots because original function z(s) is a positive real function which should satisfy Rowther hortz criterion that means all the roots of the numerator and the denominator must lie in the left of plain. So $m_2(s)$ plus $n_2(s)$ corresponds to the function z(s) it is the denominator of z(s), is it not which is a realizable function. So the roots are here are lying in the left of plain, so you select out the roots corresponding to the left up plain.

Once you have done that then $m_2(s)$ plus $n_2(s)$ is defined $m_2(s)$ plus $n_2(s)$ is selected. Now you chose z(s) equal to suppose this is equal to some $b_0$ plus $b_1(s)$ plus $b_2(s)$ square. Suppose it is like this may be $b_0(s)$ to the power 3 $b_0$ plus $b_1(s)$ plus $b_2(s)$ square plus $b_3(s)$ to the power 3 then what will be the numerator like it can have $a_0$ plus $a_1(s)$ plus $a_2(s)$ square plus $a_3(s)$ cube okay. Any question? The condition for a positive real function is the power of s should differ at the most by 1 can I take $a_4(s)$ to the power 4, see by division there will be a free s term that means some reactance element additive reactance element will come it can be anything that becomes indeterminate, is it not.

As I was telling you right in the beginning I can identify from that only x corresponding to the minimum value any additional x will also give me the same real part, real part remains unchanged if you keep on adding reactances. So we will consider only that reactance function which is just just required to realize set us. So we will not go further beyond this s to the power 4 by s to the power 3 will give me by division 1 free s term something like an inductor, free inductor and that can have any co efficient $a_4$, $a_5$, $a_6$, it can have anything. So we will take this as z(s) and then what will be a real part of z(s), what is the even part of z(s) from here you take $m_1$, $m_2$ minus $n_1$, $n_2$ okay.

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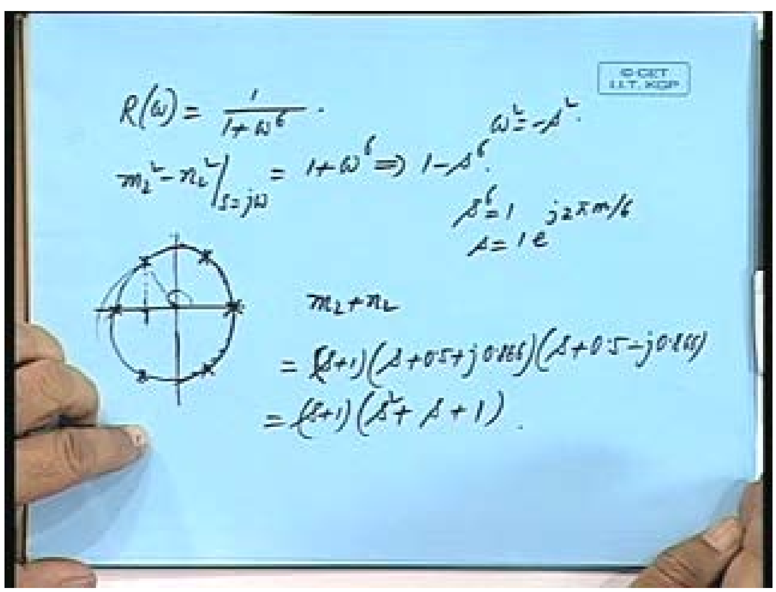

So that means $a_0$ plus $a_2$ s square into $b_0$ plus $b_2$ s square minus the odd part product put s equal to j omega that should correspond to the numerator of the given real function, we have started of with the denominator $m_2$ square minus $n_2$ square the denominator. Now you equate this to the numerator given let us take an example, it will be clear okay.

A simple example we shall take say, suppose the real part r omega is given as 1 by 1 plus omega to the power 6, 1 plus omega to the power 6 then $m_2$ square minus $n_2$ square that is equal to j omega that is given as 1 plus omega to the power 6. So you make a substitution omega square is equal to minus s square, so how much will it be 1 minus s to the power 6 okay 1 minus s to the power 6 if I factorize s to the power 6 equal to 1 okay then s will be 1 e to the power j, 2 pie n by 6 okay m is any integer.

So the values are m is equal to 0 means this one, so is a unity circle this is 1 root m is equal to 2 means, 2 pie by 6, m is equal to 3 this one 4,5, 6 with a spacing of 60 degrees and these are the 3 roots lying in the left of plane. So these 3 will continue to $m_2$ plus $n_2$ okay, so $m_2$ plus $n_2$ straight away you can write how much is it this is 1, this 120 degrees so s plus 1 factor corresponding to this this is plus 1 factor corresponding to this will be s, this is minus .5 cos 60, so s plus .5 plus this is j .866 minus j .866 so one is plus j .866 and s plus minus j .866 is that all right.

So that gives me s plus 1 into s plus is alpha plus j Beta and alpha minus j Beta if I multiply it will be alpha square plus beta square which is 1. So s square plus 1 plus 2 into real part which is .5, 2 into .5 is 1 into s is that all right. If these are alpha and j Beta then when you form a polynomial corresponding to this pair you get s square plus 2 into alpha s plus alpha square plus beta square. This will be the factor corresponding to one pair of roots and if the roots are on the unity circle then alpha square plus beta square is 1 okay this is alpha, this is beta.

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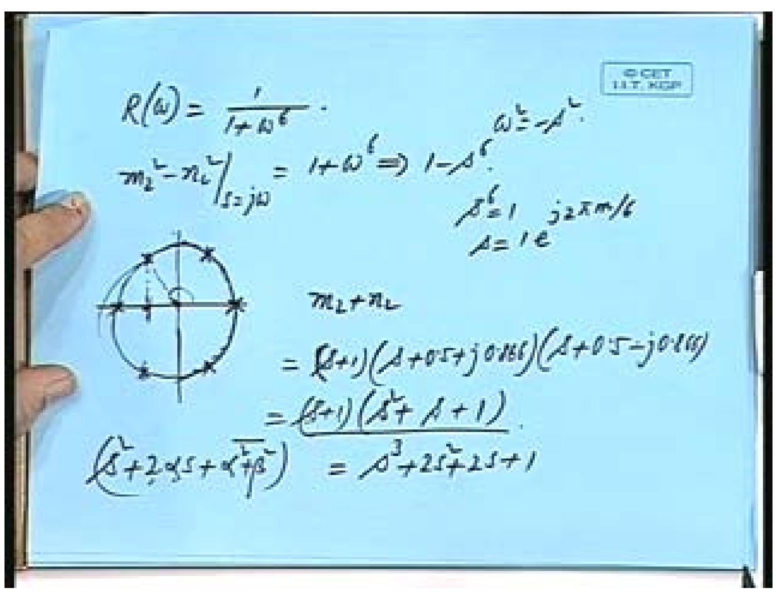

So 2 into alpha here is .5 so 2 into .5 that gives me 1 into s plus 1 so this is my denominator polynomial $m_2$ plus $n_2$ is that all right. So let us write this as s to the power 3 is that okay, s to the power 3 plus s square plus s square twice s square plus 2 s plus 1 so and what is the numerator of this one okay. So we shall write z(s) as a polynomial some $a_0$ plus $a_1$(s) plus $a_2$(s) square plus $a_3$(s) cube divided by 1 plus 2 s plus 2 s square plus s cube I just put in reverse order it can I mean that is immaterial you can put in any order okay.

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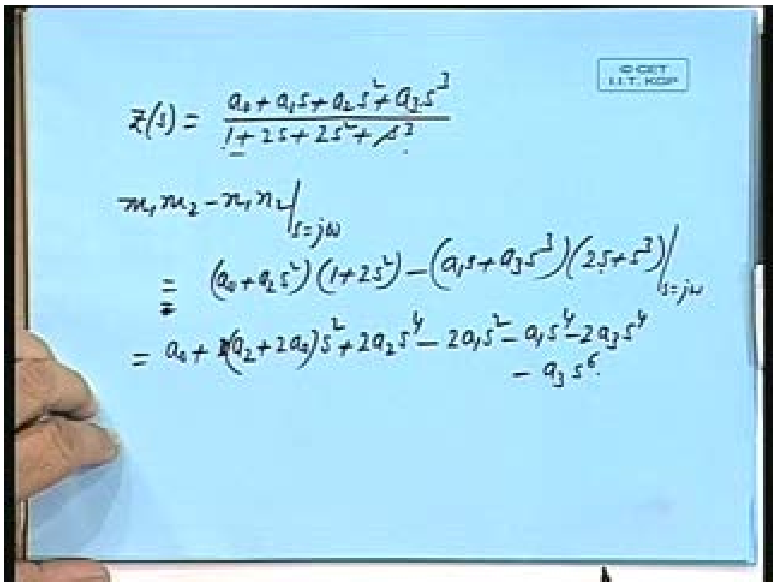


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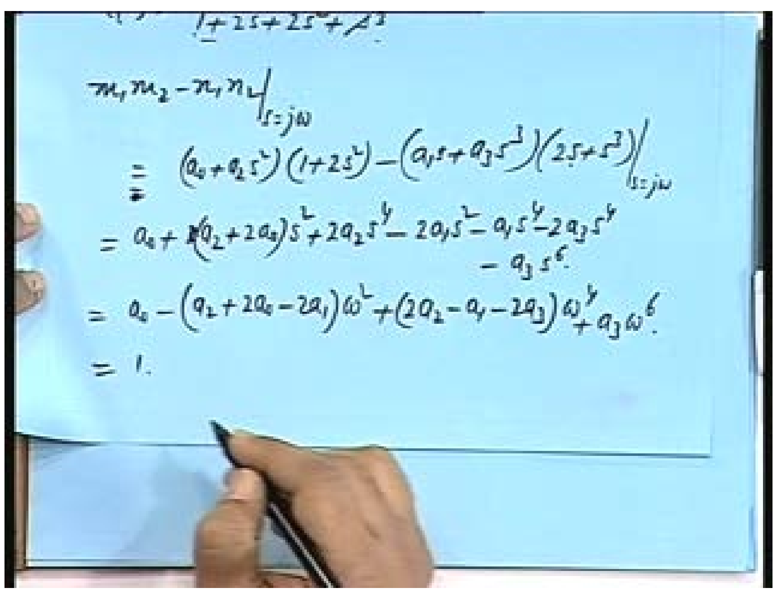

So this is the denominator once you have determined the denominator, you assume a polynomial for the numerator then what will be $m_1$, $m_2$ minus $n_1$, $n_2$ at s equal to j omega it will be the even part $m_1$ is $a_0$ plus $a_2$(s) square into 1 plus 2s square minus $a_1$(s) plus $a_3$(s) cube is that all right into 2s plus s cube at s equal to j omega is that okay. So that gives me let us simplify this $a_0$ plus twice $a_2$ sorry plus $a_2$ plus twice $a_0$ into s square plus twice $a_2$(s) to the power 4 minus twice $a_1$(s) square okay minus $a_1$(s) to the power 4 minus twice $a_3$(s) to the power 4 minus $a_3$(s) to the power 6 okay, you may put s equal to j omega.

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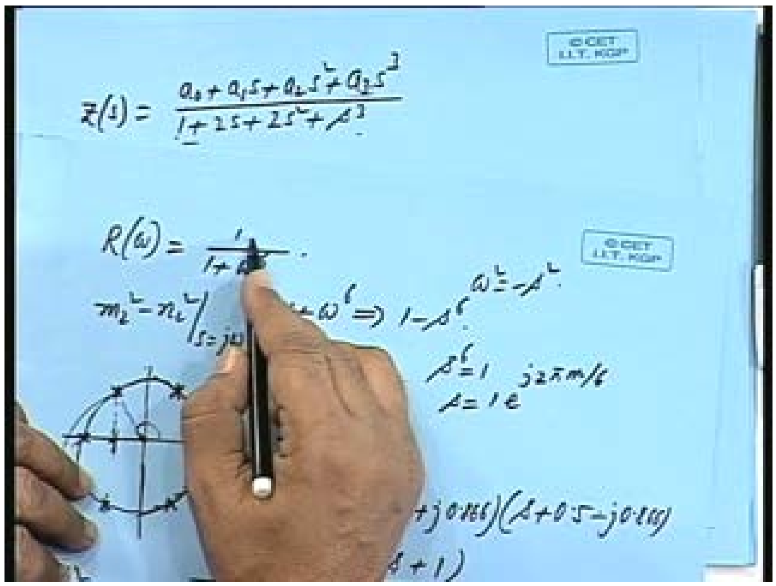


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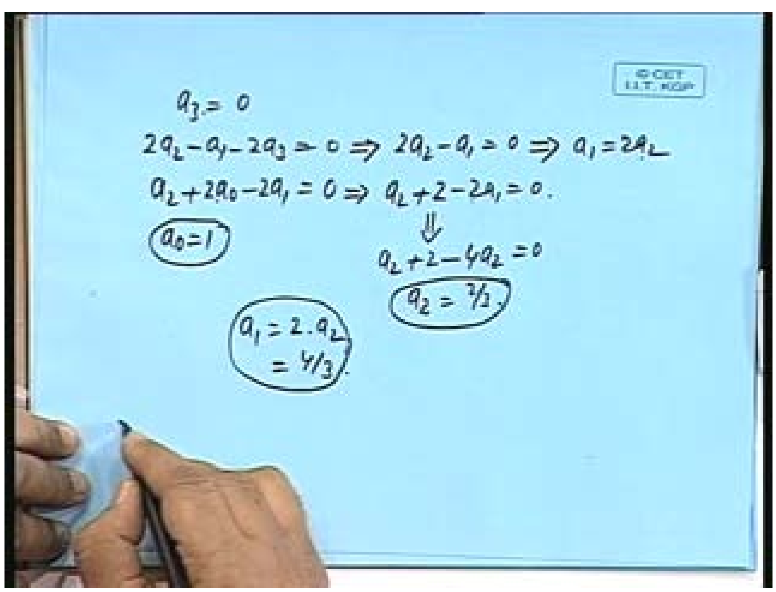

So that gives me $a_0$ plus minus $a_2$ plus twice $a_0$ minus twice $a_1$ into omega square plus twice $a_2$ s to the power 4 minus $a_1$ minus twice $a_3$ omega ratio of 4 minus and minus plus $a_3$ omega to the power 6, is that okay and that was equal to 1, numerator was given as 1 omega square co efficient is 0 omega to the power 4 is 0, omega to the power 6 is 0. So will equate each of this co efficient, so co efficient wise if we go last one is omega to the power 6 $a_3$, what is the co-efficient here is 0 all right, so $a_3$ is 0 then twice $a_2$ minus $a_1$ minus twice $a_3$ is 0 which means $a_3$ is already 0. So twice $a_2$ minus $a_1$ is 0, next $a_2$ plus twice $a_0$ minus twice $a_1$ is 0, co efficient of omega square that is also 0 okay and then lastly $a_0$ and that is equal to1 $a_0$ is equal to1.

So if I substitute here I will get $a_2$ plus 2 minus twice $a_1$ equal to 0 okay. From here I get $a_1$ equal to twice $a_2$ is it not $a_1$ is equal to twice $a_2$ substitute in this one that gives me $a_2$ plus 2 minus twice $a_1$ means minus 4 $a_2$ equal to 0 or $a_2$ is equal to 2 by 3, substitute here $a_1$ is 2 into $a_2$ so 4 by 3. So the co-efficient are therefore numerator I will straight away substitute these values $a_0$ is this $a_2$ is this and $a_1$ is this.

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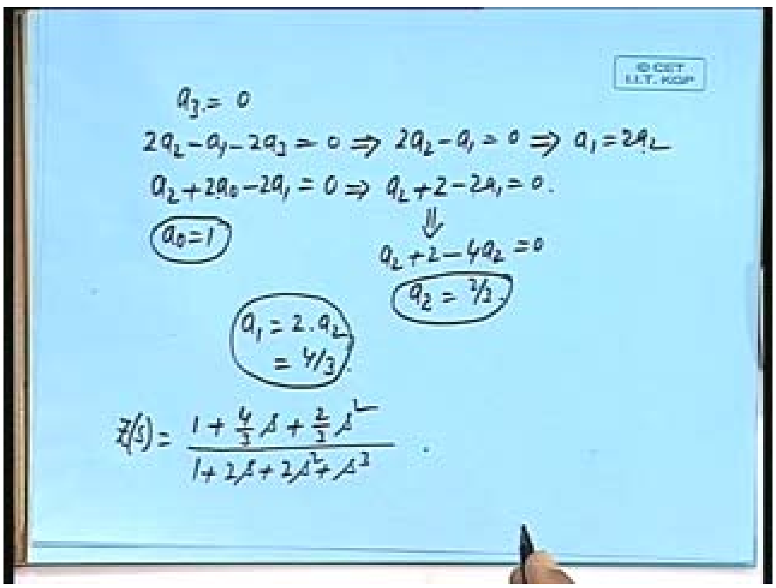


So how much is z(s), z(s) will be therefore $a_0$, 1 plus $a_1$(s) that is 4 by 3s plus $a_2$(s) square 2 by 3 s square divided by the numerator is this and the denominator is 1 plus 2 s plus 2 s square plus s cube, is this all right. So we started of with the denominator polynomial made a substitution of omega square is equal to minus s square got the roots of the total set that was $m_2$ square minus $n_2$ square then find out all the roots take the roots of left of plane from the polynomials, write in the polynomial form and then assume a numerator of maximum power of the denominator okay then find out the numerator of the even part from this relation $m_1$, $m_2$ minus $n_1$, $n_2$ put s equal to j omega equate with a given function r omega.

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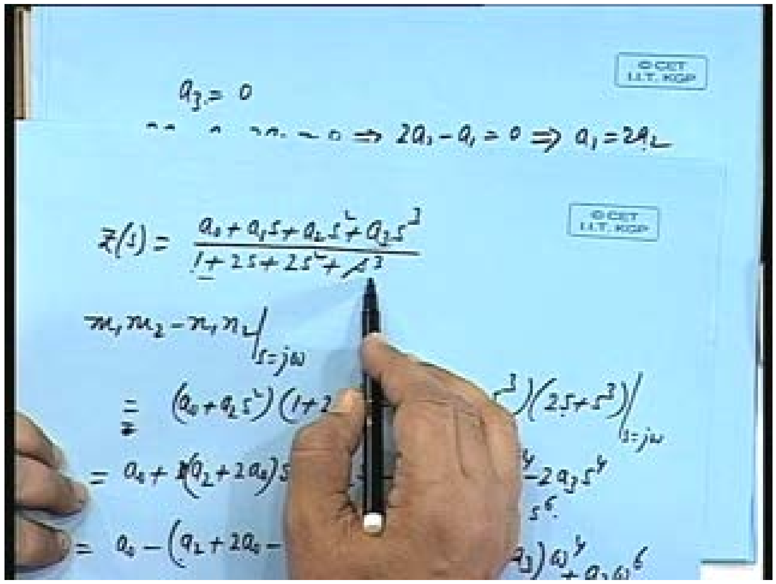


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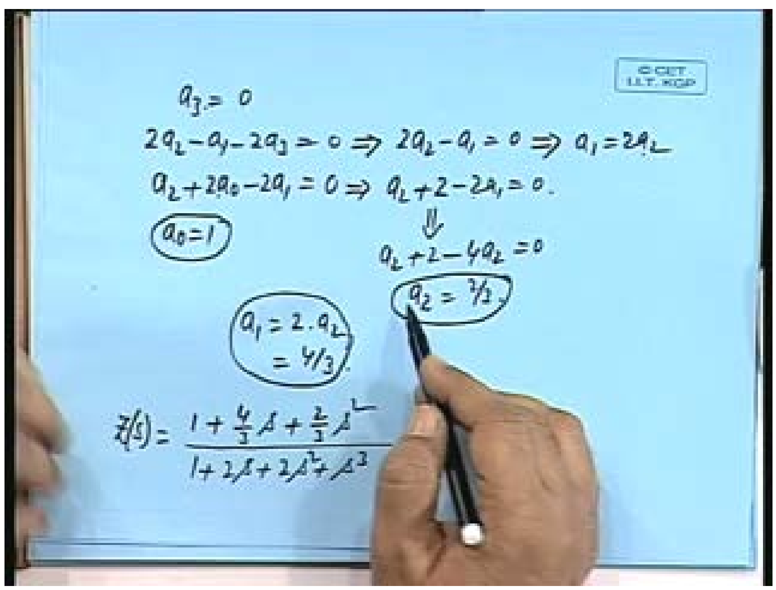


So that should be equated to 1, so if you equate now the co-efficients you get all these values okay coefficients of omega square if we get then you get simple linear relations in terms of this constants $a_0$, $a_1$, $a_2$, so after that you fed the polynomial. So this is z(s) sir $a_3$, $a_3$ was found to be 0 is this all right. Had I,  had 1 plus omega square.

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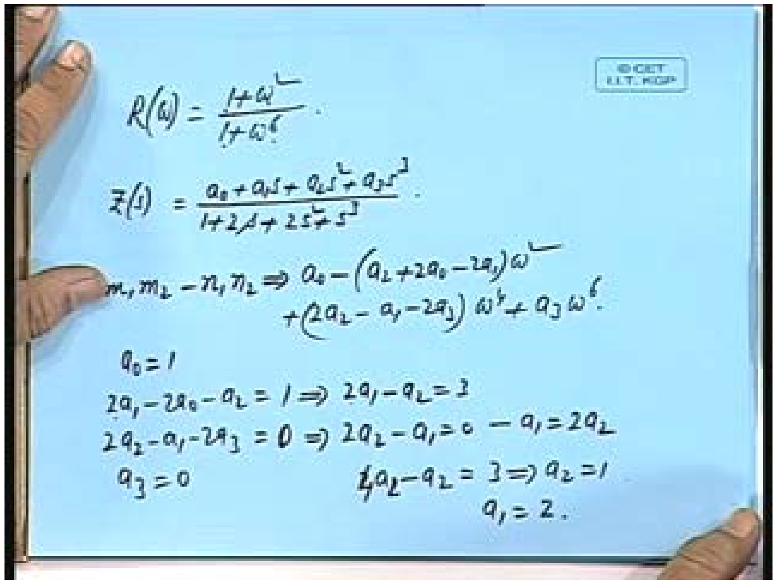


Let us take the same example, let us see whether it is possible whether it is possible. You have r omega equal to 1 plus omega square by 1 plus omega to the power 6, is it possible to have this function and if it is so what will be okay. Let us, let us have this 1 plus omega square then this will give me the same polynomial all right. So I will write z(s) equal to 1 plus 2s plus 2s square plus s cubed we know if I have a factor of 1 plus omega to the power 6 as $m_2$ square minus $n_2$ square corresponding $m_2$ plus $n_2$ polynomial is this taking only the left of plane roots okay and the numerator let us assume $a_0$ plus $a_1$(s) plus $a_2$(s) square plus $a_3$(s) to the power 3. Now after all these we got $m_1$, $m_2$ minus $n_1$, $n_2$ as this, so we shall be using this $a_0$ minus $a_2$ plus twice $a_0$ minus twice $a_1$ omega square plus twice $a_2$ minus $a_1$ minus twice $a_3$ into omega to the power 4 plus $a_3$ into omega to the power 6 okay. This was our numerator, is it not $m_1$ $m_2$ minus $n_1$ $n_2$ starting with this. Now that has to be equated to 1 plus omega square now.

So $a_0$ is 1 then this one, this is minus so if I divide this twice $a_1$ minus twice $a_0$ minus $a_2$ is equal to 1 co efficient of omega square okay co efficient of omega square is 1. So I am equating this to 1 then this one is 0 twice $a_2$ minus $a_1$ minus twice $a_3$ equal to 1 and $a_3$ is 0 is that all right. So if I put $a_0$ equal to 1 what do I get here omega 4 sorry, it is 0, yes it is 0. So $a_3$ is also 0 so from here if I substitute here $a_0$ what do I get twice $a_1$ minus2 minus 2 minus $a_2$ minus 2 if I bring to this side that become equal to 3 okay and twice $a_3$ is 0 so twice $a_2$ minus $a_1$ is 0 okay that means $a_1$ is equal to twice $a_2$, $a_1$ is equal to twice $a_2$ if I substitute here twice $a_1$ is 4 $a_2$ minus $a_2$ is equal to 3 or $a_2$ is equal to 1 is that all right, $a_2$ is equal to 1 and $a_1$ is equal to 2 into $a_2$, so 2.

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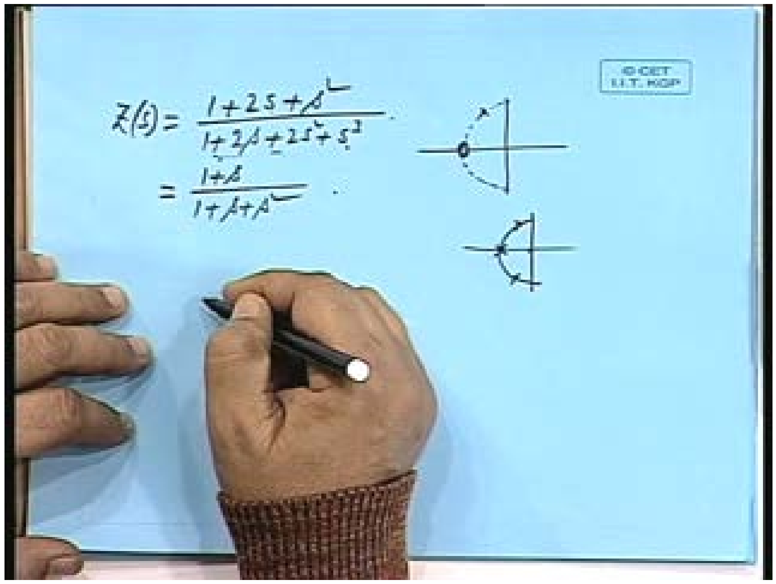


So what will be the polynomial now it will be z(s) will be 1 plus twice s plus $a_2$ is 1s square divided by 1 plus 2s plus 2s square plus s cube is it all right, parts of network functions I have not yet tested, I have not yet tested whether this is a positive real function or not, what you have checked is given the real part construct the total function all right, given the real part from the total function. So this is basically 1 plus s whole square is it not 1 plus t $a_2$(s) plus s square 1 plus s whole square divided by this 3 roots. So you you having a pole 0 distribution these are the 2, 0s.

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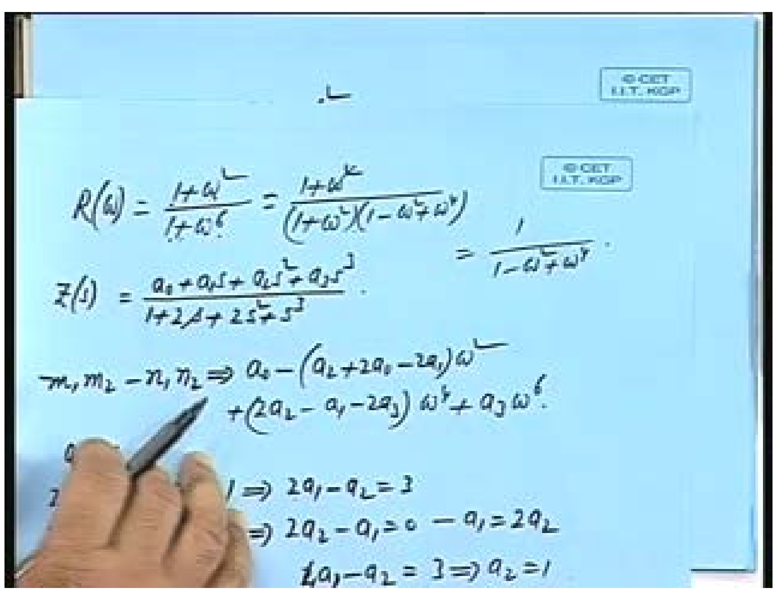

Now see something very interesting can you tell me what this one plus 2s and this is also 1 plus s is common know, this what 1 plus s if I remember these are this polynomial was coming out of this 3 roots, is it not. So s plus 1 into s square plus s plus 1 and then poles you can selected basically it is 1 plus s by 1 plus s plus s square will you get this 1 plus omega square by basically 1 plus omega square is a factor here know 1 plus x cube 1 plus x into 1 minus x plus is square, is it not. So this will go, so basically what I give you is okay so even if you have forgotten to cancel it here, it is reflected in this 1 plus s is the common factor pole 0 cancellation basically this is the function and if you take the real part will get this it is same as this one.

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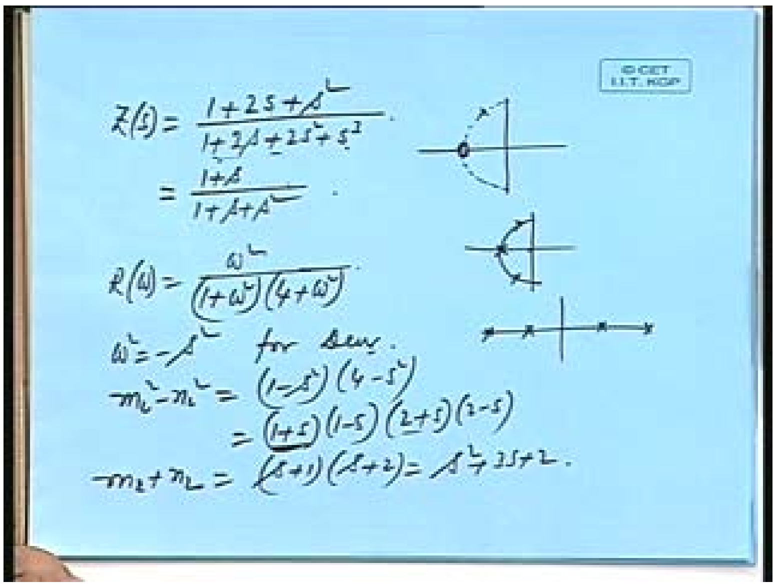


If I have there is a function given real omega is equal to omega square by 1 plus omega square into4 plus omega square 1 plus omega square into 4 plus omega square. So what we have do put omega square is equal to minus s square all right for the denominator then that will give you $m_2$ square minus $n_2$ square, so 1 minus s square into 4 minus s square all right. So this is 1 plus s into 1 minus s and this is 2 plus s into 2 minus s is it not so there are roots here at minus 1, minus 2 plus 1, plus 2. So $m_2$ plus $n_2$ I will select as the left of plane roots 1 plus s and 2 plus s, is it not. So that is s square plus 3s plus 2 is that all right.

So what will be the function z(s) like the general term will be $a_0$ plus $a_1$(s) plus $a_2$(s) square divided by 2 plus 3s plus s square all right. So what will be the real part $m_1$, $m_2$ minus $n_1$, $n_2$ s square plus 2 into $a_2$(s) square plus a not minus 3 into $a_1$ into s square that gives me $a_2$ into s to the power 4 plus twice $a_2$ plus a not minus thrice $a_1$ into s square plus twice a not. If I put s equal to j omega, so $m_1$, $m_2$ minus $n_1$, $n_2$ at s equal to j omega will be $a_2$ omega to the power 4 plus sorry minus twice $a_2$ plus $a_0$ minus thrice $a_1$ into omega square plus twice $a_0$ and that has to be equated to omega square. So $a_2$ is what is the co-efficient of omega to the power 4, 0 so $a_2$ is 0 okay and with a negative sign this is equal to 1, so if I put the negative sign inside thrice $a_1$

minus $a_0$ minus twice $a_2$ is equal to 1 and $a_2$ is 0. So that gives me thrice $a_1$ minus $a_0$ equal to 1 all right and then next twice $a_0$ is how much twice $a_0$ is 0.

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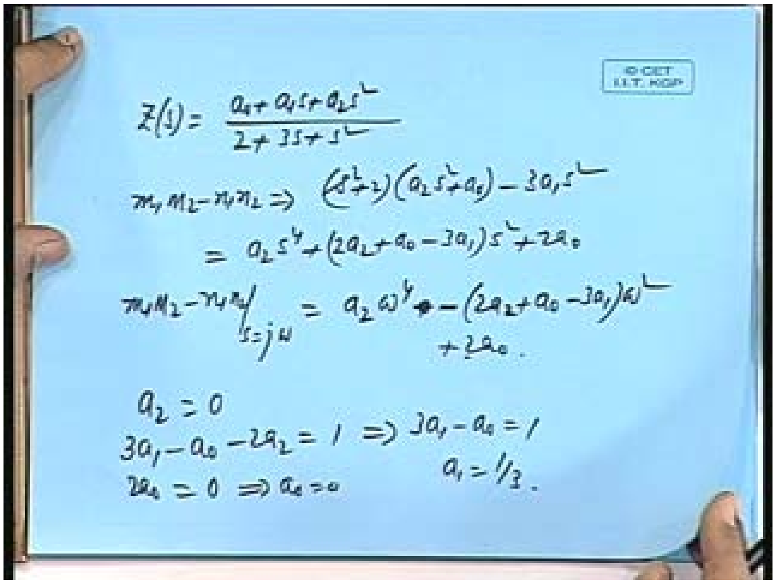


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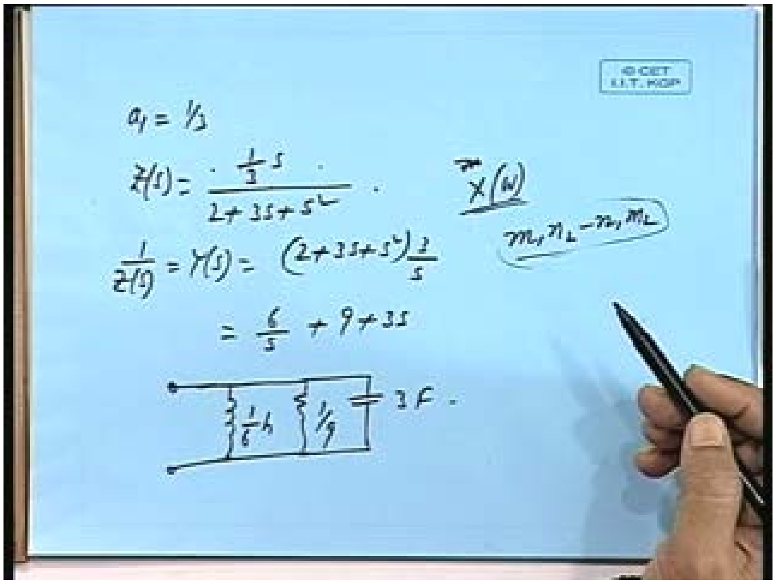


So $a_0$ is 0 so what does it mean $a_1$ is one third okay, $a_1$ is one third its very interesting only 1 of them is present. Now let us see what kind of function it is $a_1$ is equal to one third, so z(s), z(s) is one third s divided by 2 plus 3 s plus s square, can you realize it, is it a realizable function? There is nothing here nothing there only 1 term and here it is a quadrative what is 1 by z(s) that is y(s) if you invert it, it will be 2 plus 3s plus s square into 3 by s is not, so 6 by s plus 9 plus 3s. So this is an admittance of what 16th Henry this is 1 by 9 Farad, 1 by 9 ohms and this is by 3 Farads. So this is basically the network, is it not, so do not get frightened even if there is a single term here and there are 3 terms here because it is in the centre. So if I just divide I will get basically r l c.

I will get z(s) is that okay so given the real part of the function. We can calculate the overall function z(s) i mean that possessive already calculated x also because once you know z(s) all these co-efficeints then what will be x omega if I ask you to calculate the imaginary part, how did do? directly put s equal to j omega take out the imaginary part or otherwise $m_1$, $n_2$ the odd part minus $n_1$, $m_2$ okay this part divided by $m_2$ square minus $n_2$ square $m_2$ square minus $n_2$ square is already known that is 1 plus omega to the power 6 and so on like that. So it is this part which has to be computed okay from that $a_0$, $a_1$ all right in terms of $a_0$, $a_1$, $a_2$ we can calculate this and substitute once you have calculate this co-efficient substitute there we will get that all part.

Otherwise in z(s) put s equal to j omega take out the real part and the imaginary part that will give you x okay. So we will stop here for today, we will take up a few more problems in the next class along with problems of r l c synthesis, thank you very much.

**Preview of Next Lecture**

**Lecture - 28**

**Parts of Network Functions**

Good morning friends, we shall continue with the topic on parts of network functions.

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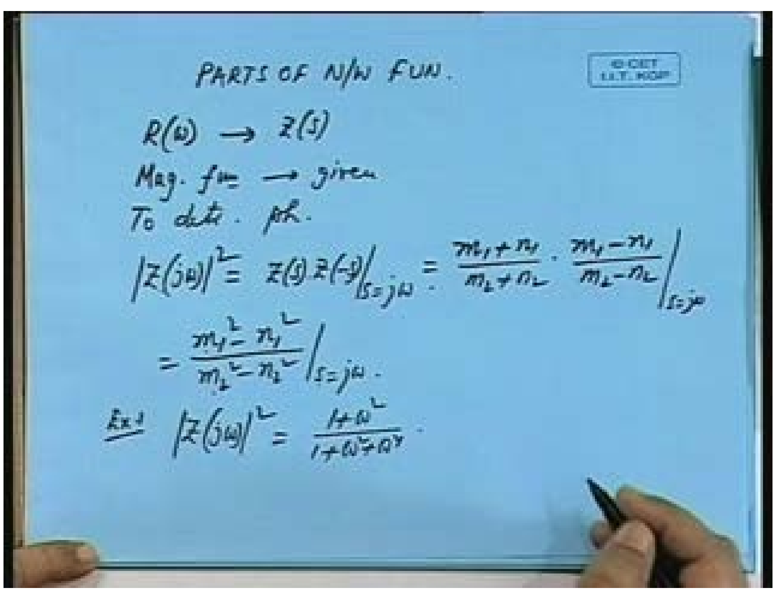

Yesterday, we discussed about functions r omega being given how to get z(s) okay. So we desorb this by substituting omega square equal to minus s square we took the roots in the left of plane and finally from the co-efficient of the numerator $a_0$, $a_1$ and so on. Now today we shall be discussing of about the magnitude function the magnitude function, if the magnitude function is given how to calculate phase, magnitude function is given to calculate, to determine phase.

Now the magnitude z square can be written as z(s) into z minus s at s equal to j omega is that all right and what is z(s)? If I write in terms of even and odd parts m1 plus n1 by m2 plus $n_2$, this is z(s) and what is z minus s? $m_1$ minus $n_1$ it is only the odd part which will be having a negative sign now divided by $m_2$ minus $n_2$ is that all right at s equal to j omega. So what you are getting is m1 square minus $n_1$ square by $m_2$ square minus $n_2$ square to s equal to j omega is that all right.

So if you given z magnitude square it is basically m1 square minus $n_1$ square by $m_2$ square minus $n_2$ square at s equal to j omega see if i make reverse substitution I will get $m_1$ square s minus $n_1$ square s by m2 square minus $n_2$ square s from here okay. So there if I factorize and drop out the roots in the right of plane I will get $m_1$ plus $n_1$ and correspondingly $m_2$ plus $n_2$, it is very simple. So let us take one example z j omega square is equal to1 plus omega square by 1 plus omega square plus omega to the power 4.

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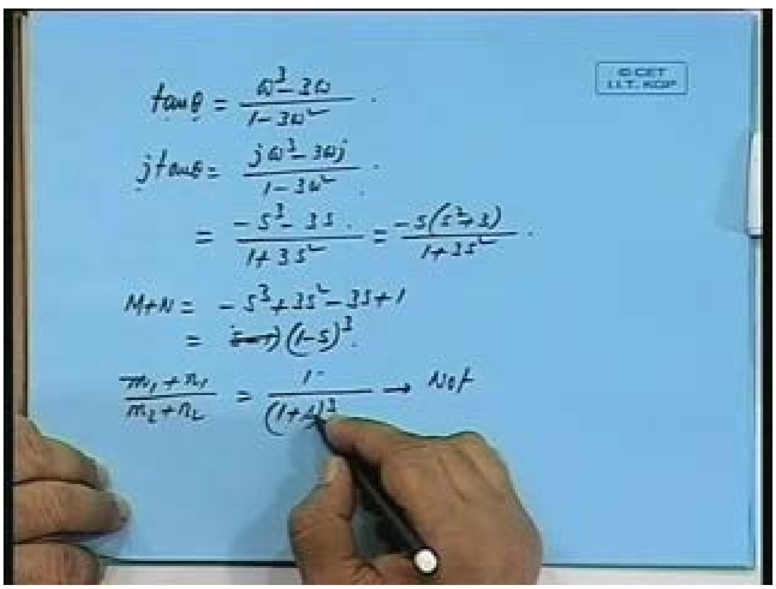


When you finite roots that is 0s so 1 by 1 plus s whole cube is that all right. So this is not a driving point impedance, it is an impedance function, it is a transfer impedance function, in case of a transfer impedance function you may have difference in the degree more than 1. But in case of a driving point impedance or admittance function, we know the difference in the degree should be restricted to 1. So here you can have more than 1, so this is not a driving point function sir why do you sir de difference is 1 and the difference is greater than 1, not greater than 1 it may or may not be 1.

So transfer function is a ratio between 2 voltages or voltage and current. So the transfer function can be anything it can have only thing the roots must be in the left of plane that is poles must be in the left of plane. Yes, this is an insist able system four into s square plus 8s into s square plus 8 divided by 2 into s square plus 2s plus 4 by the way why did I take s square plus 8 as a fact factor if you remember, if you remember in the network this function z(s) is having a value minus half sorry root 2 j at that frequency all right and it is also the value of this that we have equated.

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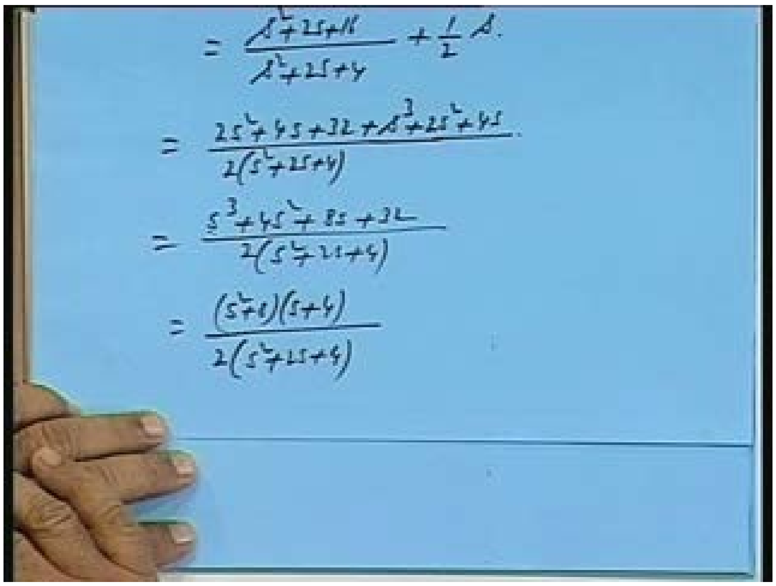


So this will be a 0 at that frequency if you remember this element $z_1(s)$ must be 0 at that particular frequency omega 1 okay so this $z_1(s)$ at omega 1 that is 2 root 2 that is s square plus omega 1 square will be a factor in the numerator and omega 1 square is 8. So s square plus 8 will be a factor that we know because what we try to ensure is this impedance at that frequency is equal to the reactance at that frequency of this coil. So what will be the impedance of this, it has to be 0 because this impedance is equal to this impedance, so this has to be 0. So $z_1$ s must have a0 at that frequency so s square plus 8 must be having a numerator factor must be a numerator factor of z(s) this $z_1(s)$. Obviously, we are making it equal to that inductor okay, thank you very much we will stop here for today. We will continue with this in the next class.