Network Signals and Systems
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Lecture - 25
Graph Theory (Contd…)
Analysis of Resistive Networks Computer Aided Approach

Good evening friends. Today we shall be taking up analysis of resistive networks or computer aided approach using graph theory.

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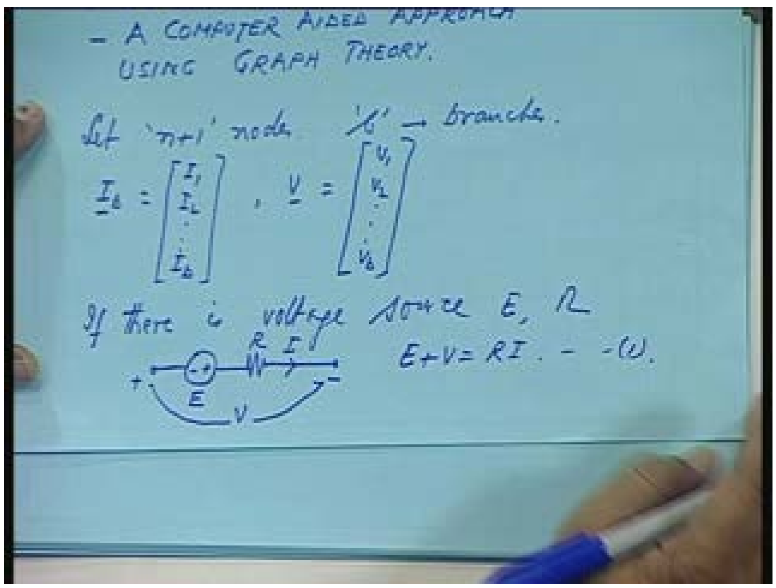

So we will be going back to the equations that we derived in our earlier lectures on graph theory. So let us have a system of n plus1 nodes and b number of branches then I can write the branch current vector as I_1, I_2 and so on I_b. Similarly, the voltage vector for different branch elements will be like this excuse me, if there is a voltage source, if there is a voltage source then we can write say voltage source with a resistance R then we can show each element like this, this is E, this is current I, this is R and this is V. The element voltage V then E plus V will be equal to R into I, if E is connected in the opposite direction then it will be minus E plus V is equal to RI, if E is connected like this.

So we get minus E plus V equal to RI thus we can write E_k plus V_k equal to R_k into I_k, k equal to1 to b this is generalized equation and we can write in a vector form E plus V equal to R into I where R will be the matrix R will be having only diagonal elements R_1, R_2, R_2, R_b rest of the elements are 0. Now this is a very simple equation using ohms law, now you will consider a loop, a loop l in a graph I will call it if we consider a loop then all the branch voltages in that loop if we consider then k equal to 1 to say b_l, b_l is the number of branches in that loop l, in loop l then sigma V_k, k varying from1 to b_l will be equal to R_k, I_k again k_1 to b_l.

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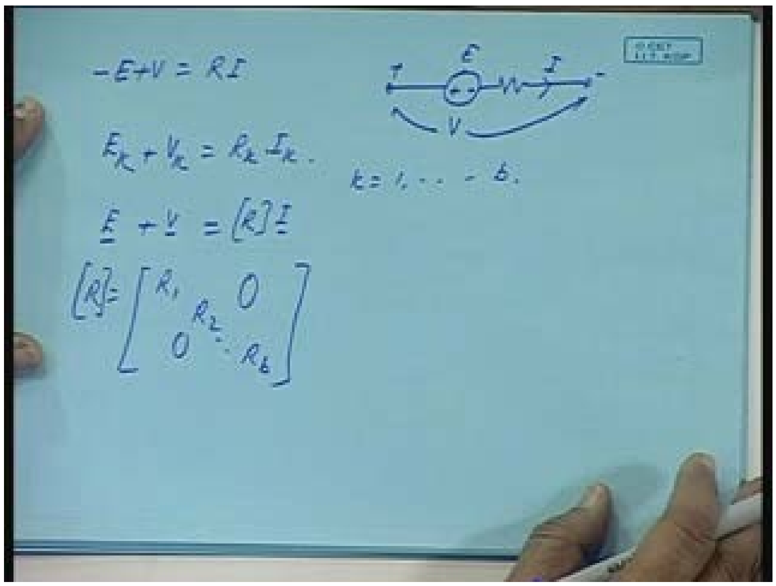

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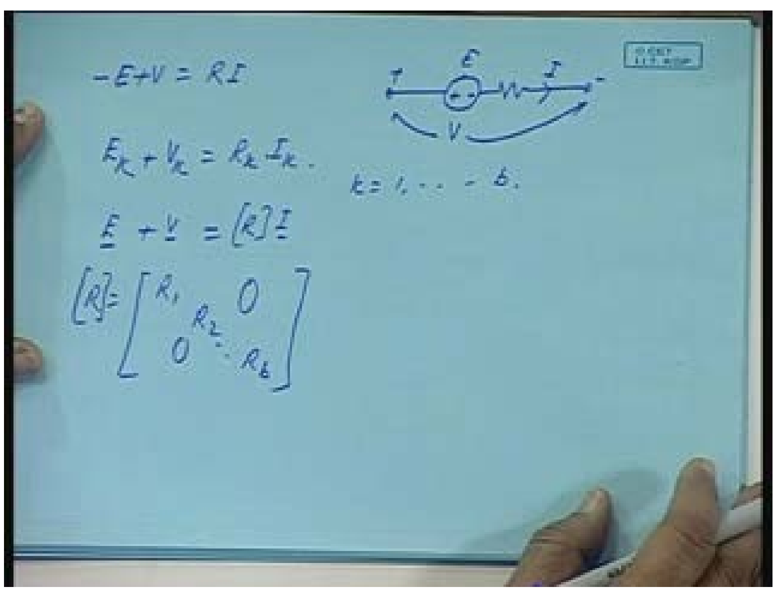

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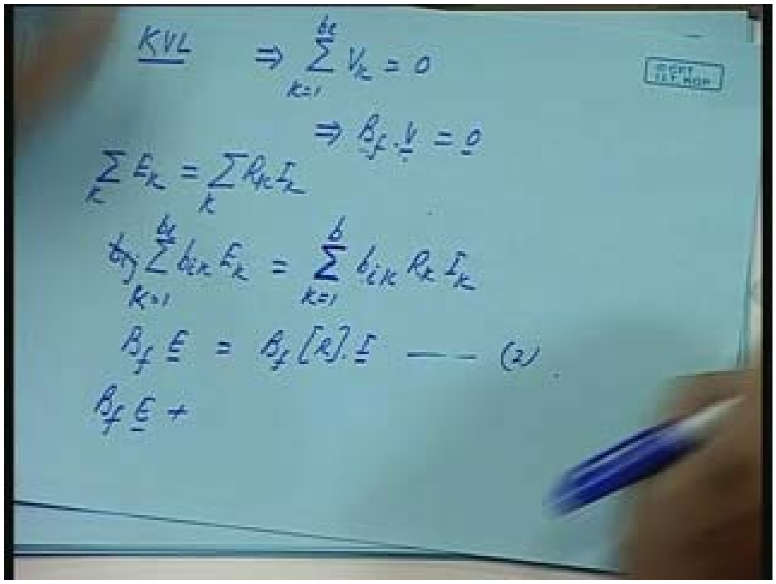

Now if you apply the Kirchhoff's voltage law states sigma V_k in a loop is equal to 0 agreed which means Bf into V equal to 0, V_k sigma V_k if it is for all the branches and if we consider all possible loops then I can take the loop matrix, fundamental loop matrix into V, V bar that will be equal to 0. Now using this if in the previous equation, we use this condition then we find E_k equal to sigma R_k, I_k submitted over k or you can write in terms of the elements after all these are the elements in the loop.

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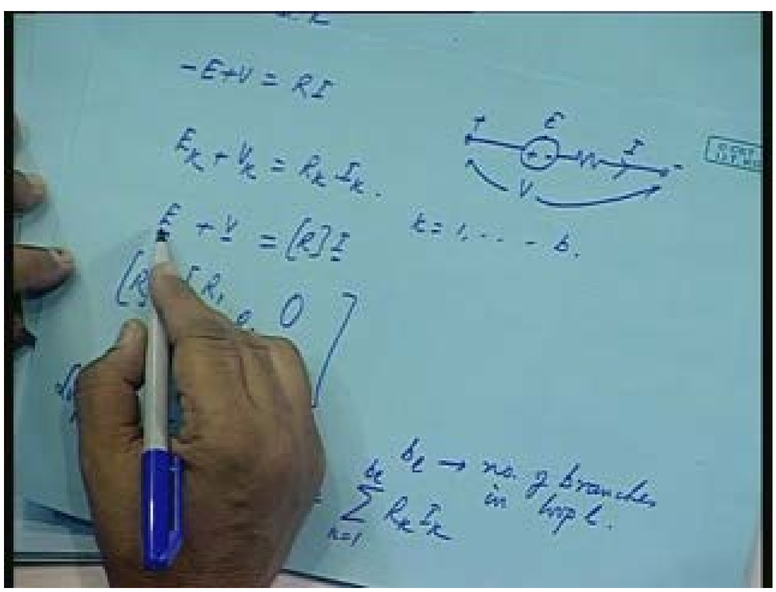

So we may write b_{ik} okay b_{ik}, E_k, k varying from 1 to b_l equal to sigma b_{ik}, R_k, I_k basically this refers to those elements which are falling in the loop and b_{ik} are the elements of that fundamental matrix. So that takes care of those elements in a loop or generalizing this bf, this is for all i's, so Bf into E will be equal to Bf into matrix R into I this is a very important equation that you get in terms of the loop matrix Bf. Now this we could have got from the other equation also Bf into E plus if would have multiplied this by Bf and Bf into V is equal to 0 that we have already seen we could have got the same relation.

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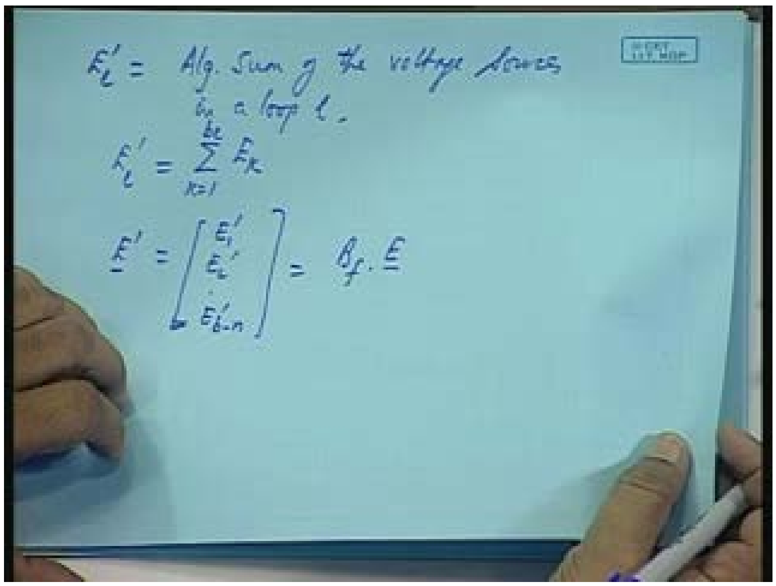

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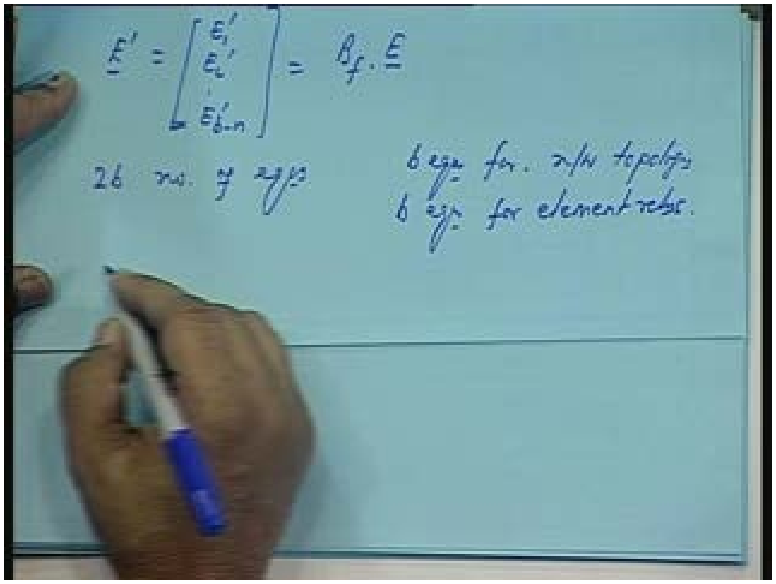

Now if you consider E_l dash as a algebraic sum of the voltage sources in a particular loop in a loop l then E_l dashed is nothing but summation E_k, k varying from 1 to b_l, b_l will be the number of elements in that loop. So I can write E vector, E dashed vector as E_1 dashed, E_2 dashed and like that E_b minus n dashed b minus n will be the number of independent loops and that is equal to all the fundamental loops that is equal to Bf into E all right because E_1 dash E_2 dash etcetera these are nothing but the total loop voltage.

So we need 2b number of equations that is b equations for the network topology, for the interconnections and b equations, b number of equations for element relations. To solve in a circuit therefore we require 2b number of equations. Now for a dc circuit, for a dc circuit we first of all solve the voltages and currents you are given the network connections, the resistances and value of independent source voltages, independent source voltages. These are the quantities given that is we know the interconnections resistances and independent source voltages.

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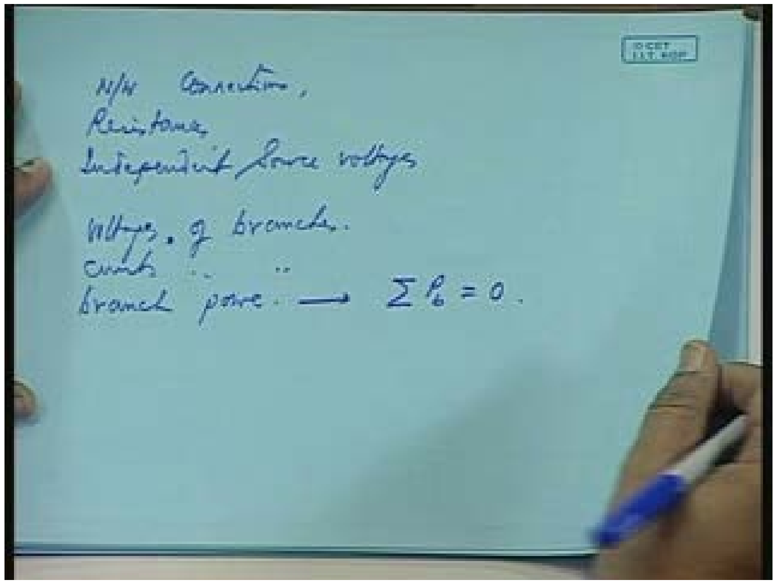

You are required to solve voltages of different branches, voltages of branches, currents of branches and then branch power and you have to check the summation of branch power should be equal to 0. This is deligens theorem which states that sum of the total branch power is equal to 0, so you will have to verify this relationship. So in the classical approach we solve n equations for n nodes applying KCL and that is sigma I_k equal to 0 and for b minus n number of loops, we apply KVL to compute the b branch voltages okay.

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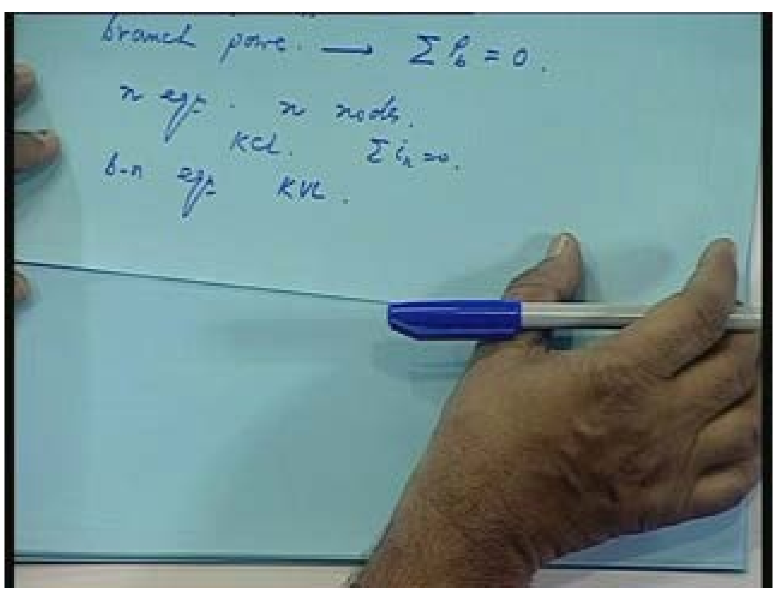

Now let us see how we can take care of current sources, current sources can be equivalently represented by voltage sources, so or voltage sources can be converted to a current source. Let us see their equivalents you all know that you must have done it in thevenin's equivalent and Norton's equivalent a similar treatment so it is just a repetition of that we repeat just for continuity.

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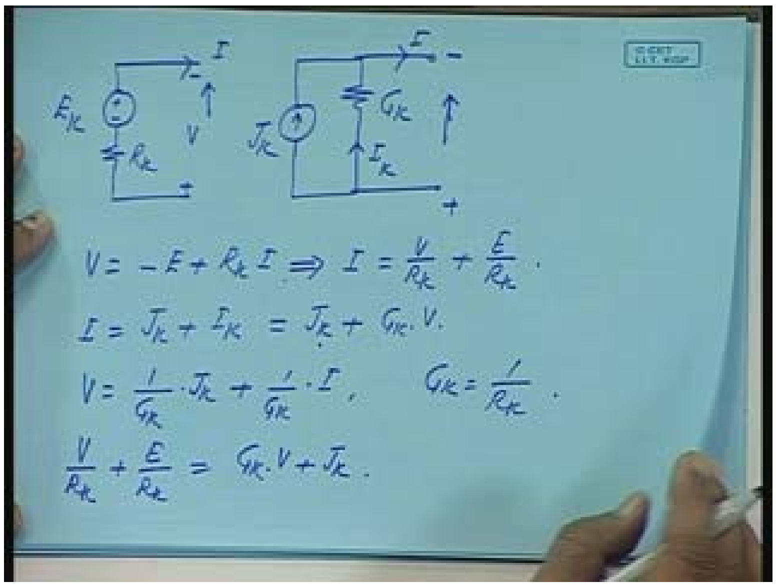

We take G_k as a conductance of the current source where E_k is the voltage source with a series resistance R_k and was this is minus, this is plus, this is voltage V, same voltage V. So this is I_k all right, so you have got V equal to minus E plus R_k into I which gives me I equal to V by R_k plus E by R_k. Again I equal to J_k plus I_k okay again J_k plus G_k into V, so if you equate you find 1 by R_k is G_k, so V can be written as also1 by G_k into J_k plus1 by G_k into I where G_k will be1 by R_k i can equate these G_k is equivalent to1 by R_k if I substitute here I can rewrite this at J_k commutating with the other other term.

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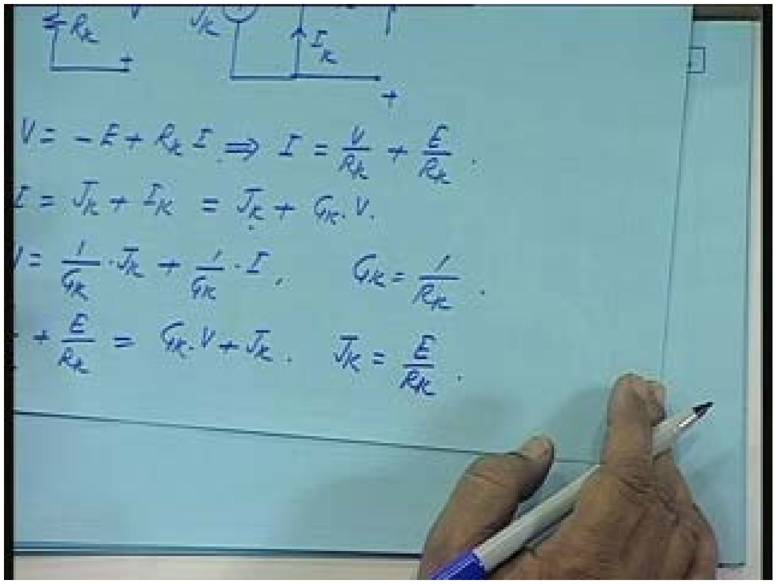

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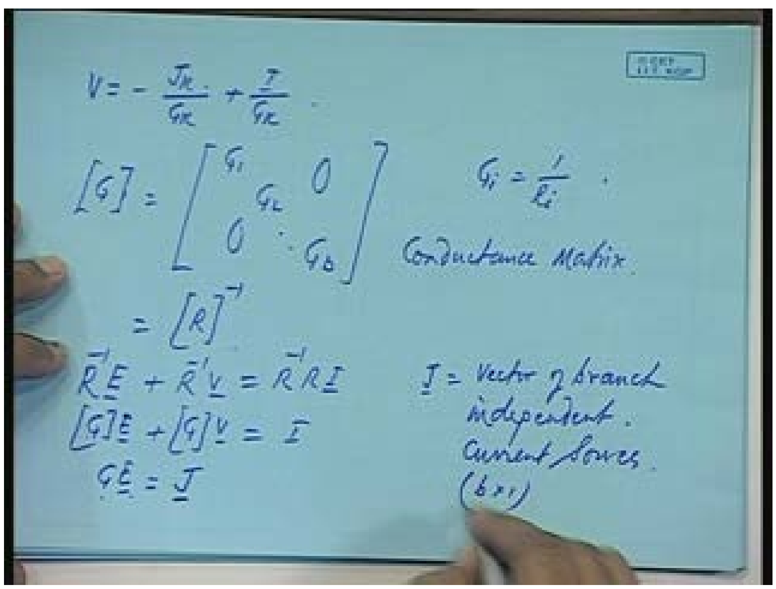

So V by R_k plus E by R_k can be written as G_k into V plus J_k that means J_k equal to E by R_k okay. So you can convert one to the other, so you have got V equal to minus J_k by G_k plus I by G_k. So we define a matrix G which will be G_1, G_2, G_b where G_i is1 by R_i, so this is a conductance matrix which is nothing but R inverse. Thus we can write R inverse E plus R inverse V is equal to R inverse R into I, I have just taken the first equation or even before that this equation the other this equation I have just multiplied by R inverse both sides so R inverse E is nothing but G into E they are all vectors plus G into V is equal to I okay or G into E is equal to J. We write this term that is taking the source voltages only the equivalent terms, the source current vector J, G into E equal to J okay J is the vector of branch independent current sources that will be having the dimension b by 1.

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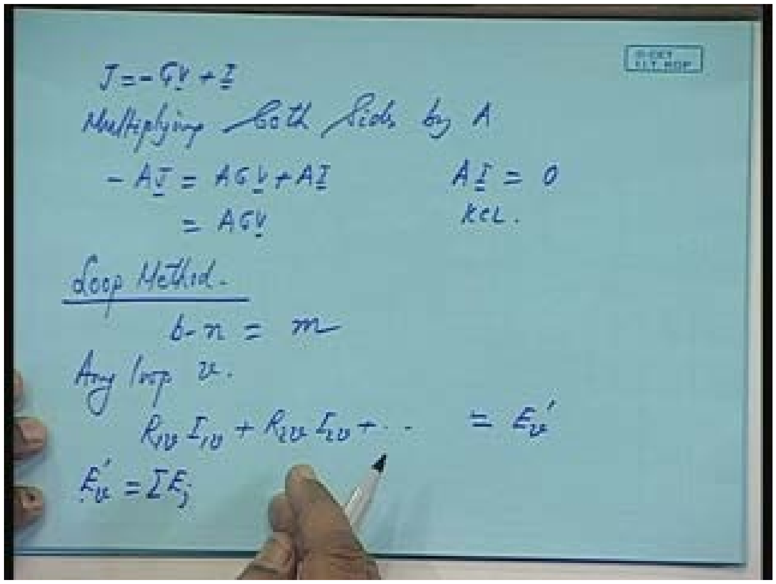

So you get J plus we can write J equal to sorry GV plus I, j equal to GV if I take to this side it will be minus GV plus I if I multiply both sides, multiplying both sides by matrix A we get minus A times J equal to AGV plus AI and what is AI, we have proved earlier it is equal to 0 by KCL, so it is AGV. Now you come to loop method, now let us consider let us consider the fundamental loops, let us consider the fundamental loops that is b minus n number of loops, let b minus n b be m okay. Now any loop if you consider any loop v, you can write the equation loop equation R_{1v}, I_{1v} plus R_{2v}, I_{2v} etcetera, number of depending on the number of elements in that particular loop is equal to E_v dashed where, E_v dashed is equal to summation of the source voltages in a particular loop summation of the source voltages in a particular loop that is if there are different branches having different sources, so in that loop you add up the source voltages so that is Ev dashed in that loop number V okay so I_{jv} we write I_{jv} equal to current in the resistor R_{jv} that is branch j in loop v, v is a particular loop.

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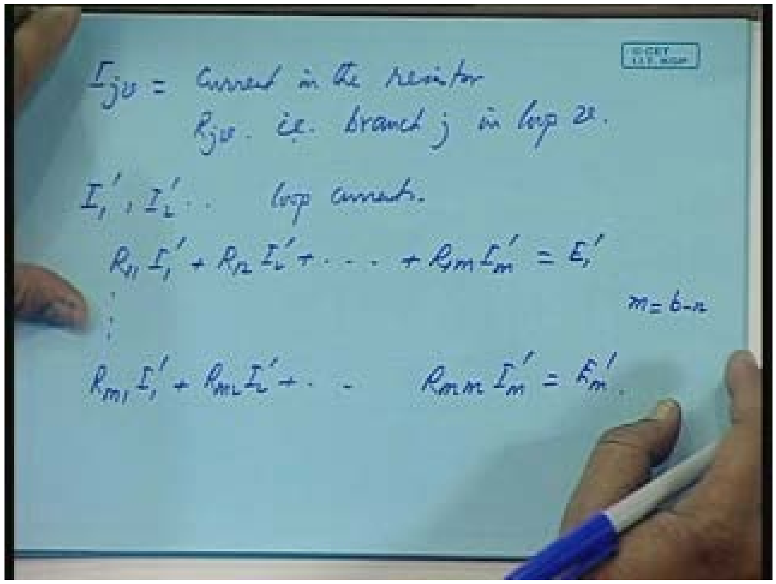

So we are considering any particular branch j and corresponding current as I_{jv} okay and if we write now I_1 dashed, I_2 dashed etcetera as the loop currents loop currents in loop number 1, loop number 2, loop number 3 and so on. We can write this equation R_{11}, I_1 dashed plus R_{12}, I_2 dashed and so on plus R_{1m}, I_m dashed is equal to E_1 dashed where there are m loops, m fundamental loops, m is equal to b minus n. So like this we will have R_{m1}, I_1 dashed plus R_{m2}, I_2 dashed and so on R_{mm}, I_m dashed equal to E_m dashed okay.

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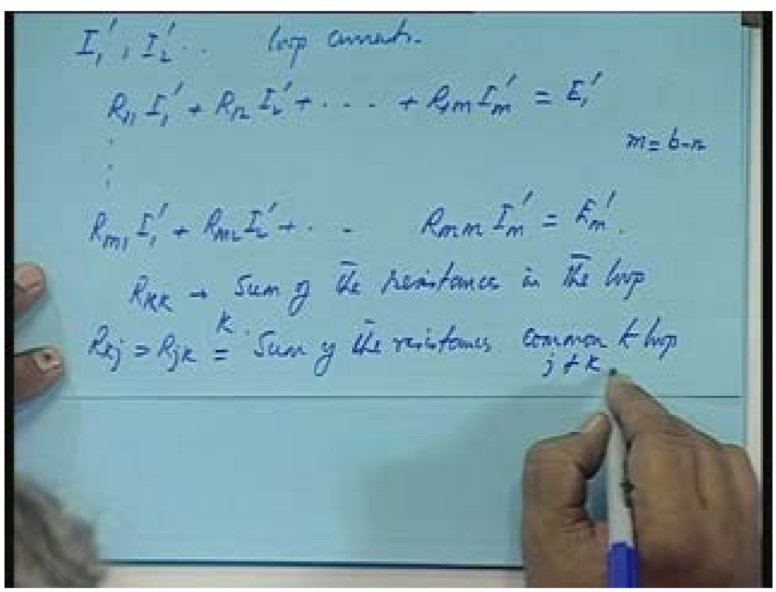

Here $R_{11,}$ R_{22} etcetera that is R_{kk} will be the sum of the resistances in the loop k and R_{kj} is equal to R_{jk} is sum of the resistances common to loop j and k, common to both loop and j and k will be the element R_{jk} it is same as R_{kj} all right. So there are m equations and m unknowns that is the loop currents, so you can solve you can write in the matrix form and then we can solve by taking the inverse of the matrix.

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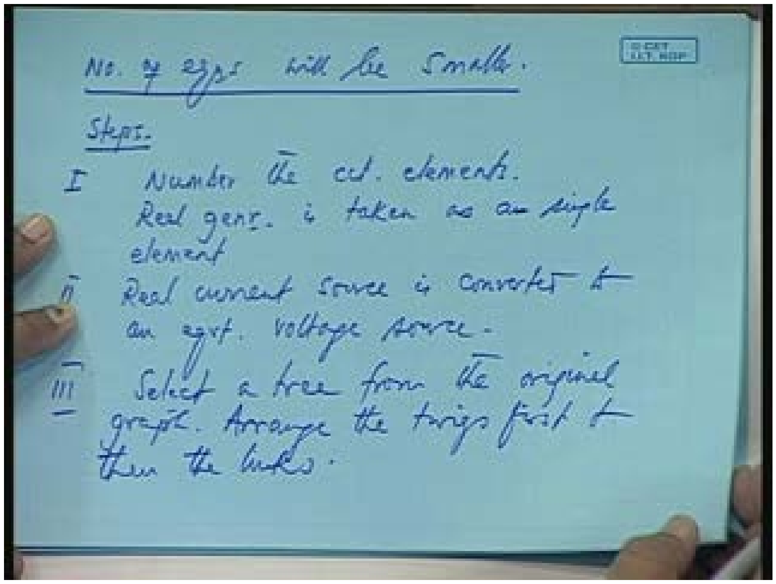

The number of equations will be smaller; this is the most important advantage of this method. So we follow the following steps 1 number the circuit elements take a real generator, real generator means a voltage source with a resistance, a real generator is taken as an element as an single branch, as a single element by element we mean the branch.

Next step real current source is converted to an equivalent voltage source, an equivalent voltage source that means wherever there are current sources you convert them to voltage sources and a put it in the branch and then select a tree from the original graph we have already seen earlier how to select the branches of a possible tree by those transformation of matrices and selecting a particular set of nodes and elements. So we select a tree from the original graph this may not be a unique one and then we arrange the twigs first then the links, so arrange the twigs first and then the links.

So we find out the fundamental loop matrix, find the fundamental loop matrix Bf and number the twigs that is Bf you write in a partitioned form bf which one should come first you find out twigs and links we did write like this, twigs and links then draw on the circuit, draw on the circuit the branch currents in the direction of the graph. Normally, we choose the direction of the graph as the direction of the current through a particular element. In the direction of the graph branches then you write the system equation solve for I_1 dashed, I_2 dashed, I_3 dashed and so on up to I_m dashed.

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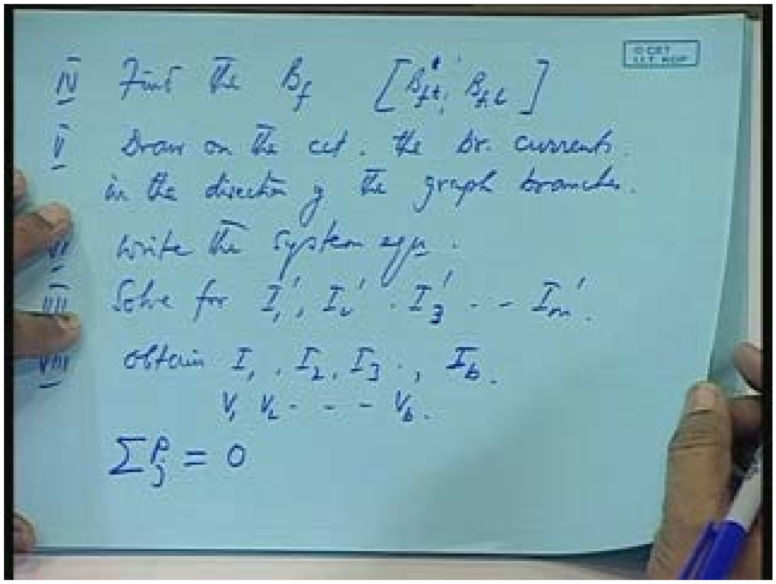

Once you have got these currents obtain I_1, I_2, I_3 the branch currents and the branch voltages V_1, V_2 etcetera up to V_b then calculate power in each branch and then check whether this condition is satisfied, this is the diligens theorem that has to be satisfied. Let us take up an example but then it will be clear.

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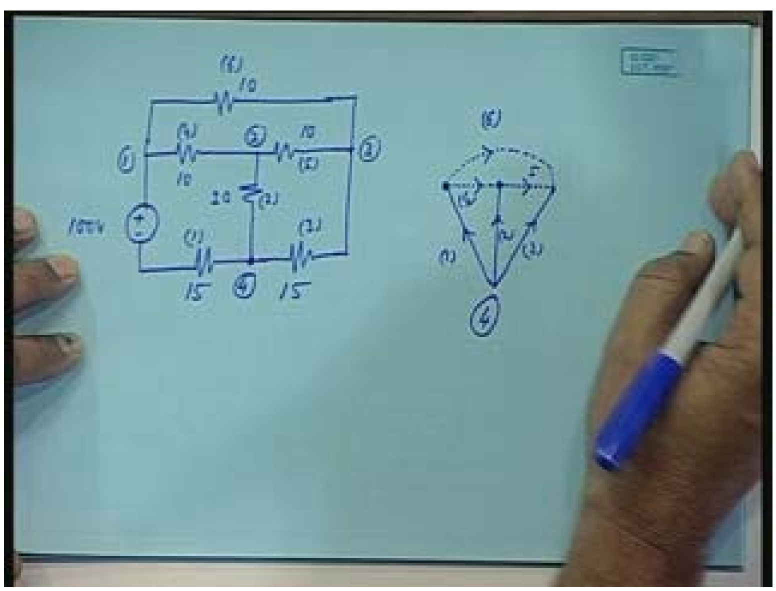

Let us have a circuit like this, this is 15 ohms, this is 15 ohms, this is element number 1 this is 100 volts, this is 10 ohms, this is element number 4, element number 6, this is also

10 ohms, this is also 10 ohms, this is element number 5, this is element number 2 of value 20 ohms and this is element number 3, this is node 3, this is node 1, this is node 2 and this is reference node 4.

So we draw a tree of graph we draw, we can take number 4 as the reference node and I can take this as the tree, these are the 3 different directions, this is element 1, element 2 element 3 and then I will show the other branches of the graph that is the links by dotted lines and this is element number 6, this is element number 4, this is element number 5 and I have taken the directions like this.

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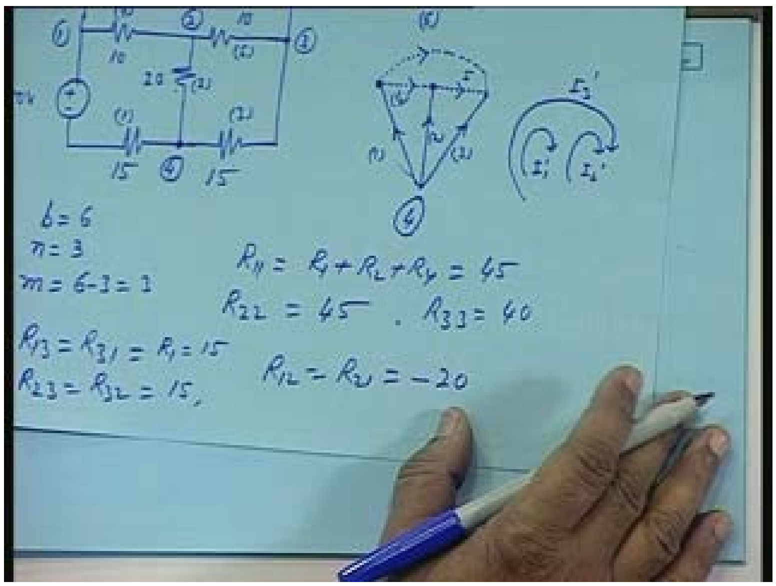

I will show the loops in this diagram here which will be referring to the loop will be sorry, this is with the first link loop 1 that is the loop current as I_1 dashed then I take I_2 dashed for loop 2 and the outer one, I_3 dashed which will be taking number 6 they are corresponding to the links number 4, 5 and 6, I have shown it separately because it will bit it will be quite clumsy here. So I wanted to be shown and this to be shown separately, now number of branches is equal to 6, n is equal to n plus 1 is 4.

So n is equal to 3, so number of fundamental loops will be 6 minus 3 that is 3 and there are 1, 2, 3 loops here, these are the3 fundamental loops we have chosen. R_{11} will be R_1 plus R_2 plus R_4, in the first loop R_{14} and R_2 these are the 3 resistances earlier we had mentioned that the diagonal elements will correspond to the elements in a particular loop, the resistive elements in a particular loop the sum of that. Similarly, now okay that comes to the values are given here 15,10 and 20, so that comes to 45 ohms I am attracting ohms every time the unit is same a standard one. R_{22} similarly, in loop 2 it will be R_2, R_5 and R_3 so 2, 5, 3 again 15, 10 and 20, 45 ohms. R_{33} will be the third loop that is 1, 3 and 6, 15, 15 and 10 that is 40. Now R_{33} is 40, what will be R_{13} that will be same as R_{31} if there is any element common between loop 1 and loop 3 that is this element that is 15 ohms only R_1 the element that is common between 2 and 3 between loop 2 and loop 3 is element 3 okay element 3 is15 ohms and R_{12} is same as R_{21} is the element common between 1 and 2.

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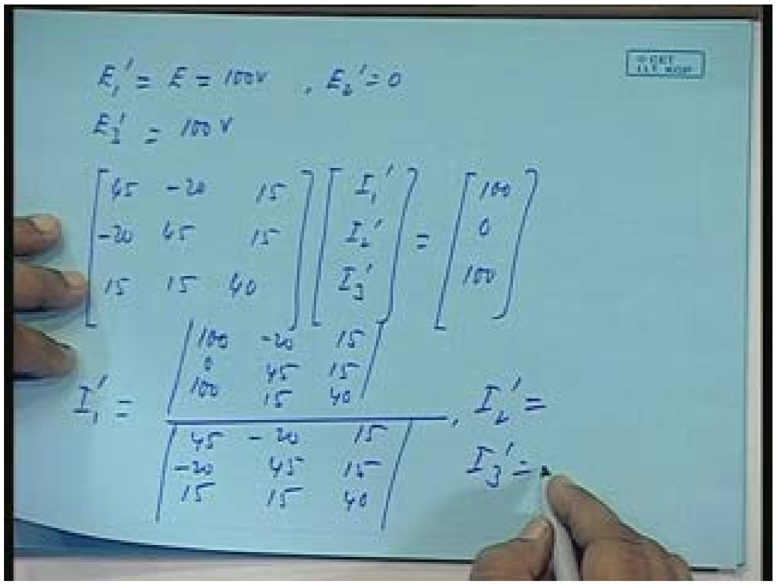

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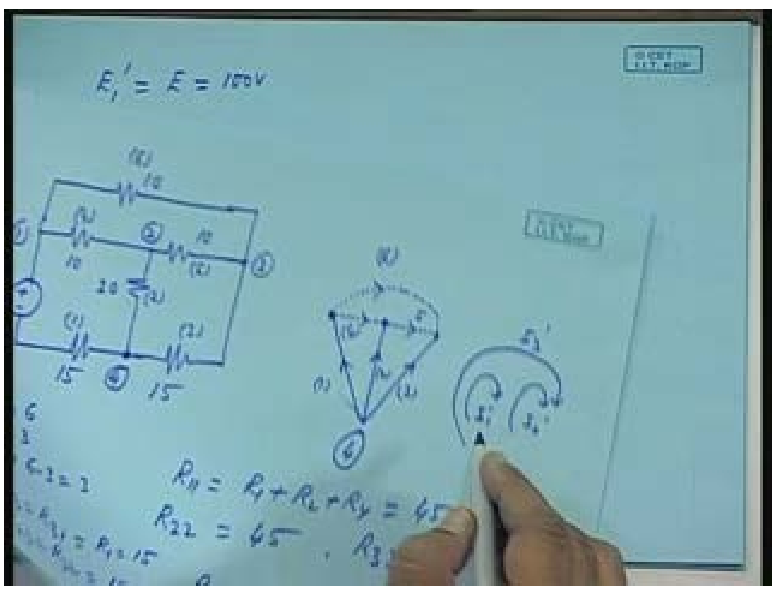

Now the directions are opposite, so these 2 elements will be negative minus 20 because this loop current and this loop current they are opposing here the, in the common element

2. So there value will be minus 20, so let us now write with this the equations again the voltages E_1 dashed is equal to E itself in the element 1 in the, see in loop 1 there is only one source and the sum of the voltage sources will be just 100 volts okay.

So E_1 dashed is equal to E itself that is 100 volts in loop 2 there is no source voltage so E_2 dashed is 0 in loop 3 again E_3 dashed outer loop will contain this voltage see E_3 dashed is also 100 volts. So you get the equation like this if you permit me to write the this values in the R matrix it will be 45, 45 and 40, R_{11} R_{22} and R_{33} then minus 20, minus 20, 15, 15, 15, 15, this is that R matrix into I_1 dashed, I_2 dashed, I_3 dashed, you get the voltage vector E dashed which is 100, 0, 100.

So by applying Cramer's rule you can solve for I_1 dashed, I_2 dashed and I_3 dashed for example I_1 dashed will be the first column replace by this, so 100, 0, 100 then minus 20, 45, 15, 15, 15, 40 determinate this divided by 45, minus 20, 15, minus 20, 45, 15, 15, 15, 40. So this is I_1 dashed similarly we can evaluate I_2 dashed and I_3 dashed. So after you have evaluated I_1 dashed, I_2 dashed and I_3 dashed we will find out the branch currents we know the relationship between the branch currents and the loop currents. In this case the branch current I_1 if we refer to the diagram here.

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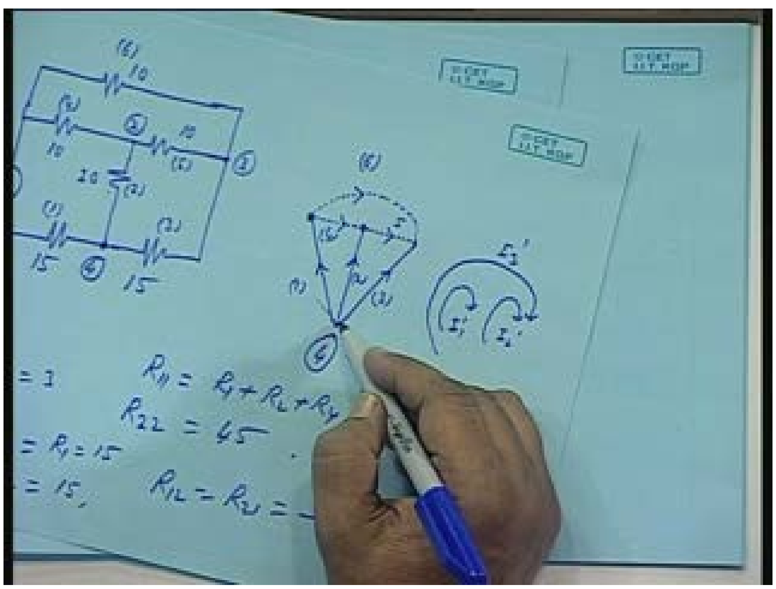

This branch current will be the loop current I_1 dashed plus the loop current I_3 dashed they are in the same sense, so both the loop currents will add up together to give the branch current I_1.

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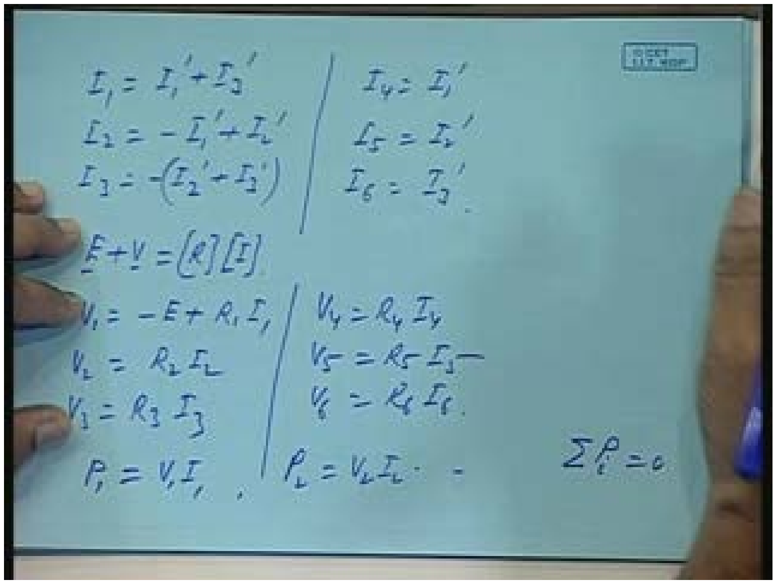

So it will be I_1 dashed plus I_3 dashed similarly, I_2 will be minus I_1 dashed plus I_2 dashed this current, is this loop current minus this loop current similarly, I_3 will be this current which is opposite. So minus I_3 dashed and plus I_2, so I_2 dashed I_2 is minus I_1 plus I_2 dashed I_3 will be I_2 dashed sorry, I_3 will be I_2 dashed and I_3 dashed both will be opposite to this so minus of I_2 dashed plus I_3 dashed I_4 is I_1 dashed I_4 is same as I_1 dashed I_6 is I_3 dashed and I_5 is I_2 dashed.

So after we have evaluated these branch currents, we apply the relationship that is V_1 will be minus E plus R_1I_1, V_2 since there is no source connected it will be just R_2I_2, V_3 will be R_3I_3, V_4 is equal to R_4I_4, V_5 is equal to R_5I_5, V_6 is equal to R_6I_6 there is no voltage source connected in this. So we have computed the voltages the currents and hence calculate P_1 P_2 etcetera P_1 is V_1I_1, P_2 is V_2I_2 and so on and then check sigma P_i. Now if you solve this values of I_1, I_2, I_3 and check this you will get this is equal to 0 the algorithm is very quick but how do you explain this to the computer. So computer has to be given a direction in a very systematic way.

So that it can handle a very large data set when the network is having too many nodes and too many branches then you have to give a systematic order to the computer to compute these matrices and arrange these equations such that the solution can be obtained very quickly and we can use the some of the relationships that we have already established in a simple way, we will see that now.

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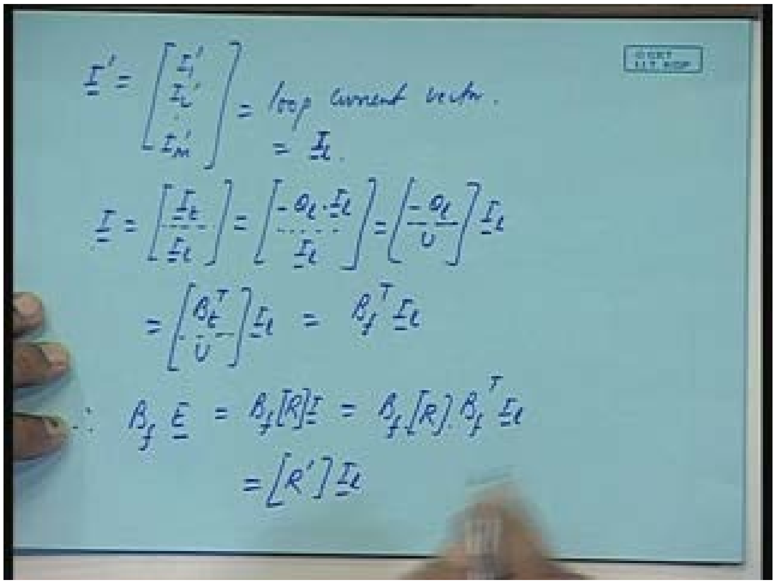

Let us consider I_1 dashed vector, I dashed is equal to I_1 dashed, I_2 dashed and so on, there are m loops so this is loop current vector which is same as I_l we studied earlier. Now if you remember okay I will just quickly mention the relationship that we are going to use here and you have already deduced it in our earlier lectures if you remember I can be broken up in to 2 components corresponding to the twigs and links and corresponding to the twigs it was shown, it is Q_l into I_l and this is I_l itself, so I_l can be taken out.

So it is minus Q_l and matrix, identity matrix into vector I_l which is minus Q_l is again if you remember B_t transpose and U into I_l and what is this nothing but b fundamental transpose the total B matrix into I_l. So I is related to I_l the link currents by this therefore B_f into E, I can write as B_f into R into I okay E is equal to R into I which is B_f into R actually I should write matrix R to be more specific into now I am writing this B_f transpose I_l okay and this you are defining as R dashed okay.

So B_f R B_f transpose I am writing as R dashed, so this will be R dashed into I_l and what is I_l is basically I dashed itself, is it not I dashed is I_l so I might self - write I_l or I dashed. This is the matrix loop equation, matrix loop equation R dashed is known as loop resistance matrix m by m okay. It is a symmetric matrix that one can prove mind you this is not same as R, it is different from R, R matrix will have only R_1 or I can write R_{11} doesn't matter R_2, R_b these are all 0s whereas in R dashed they are not 0s.

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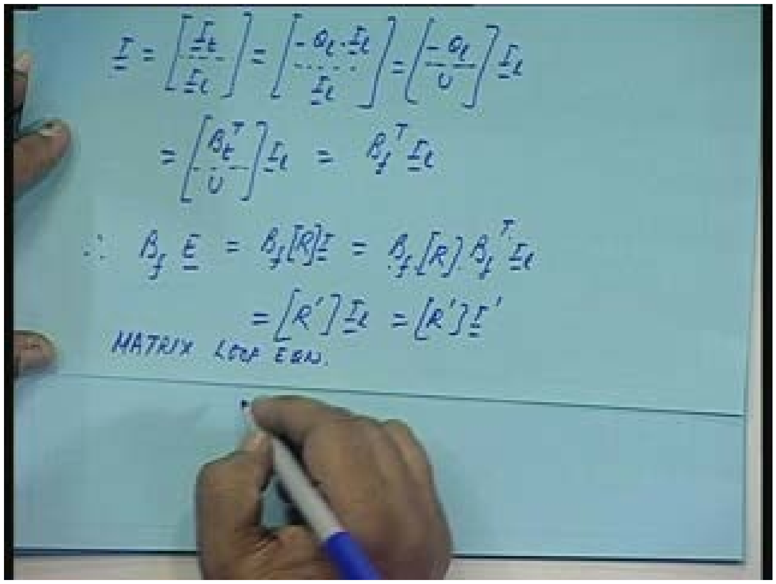

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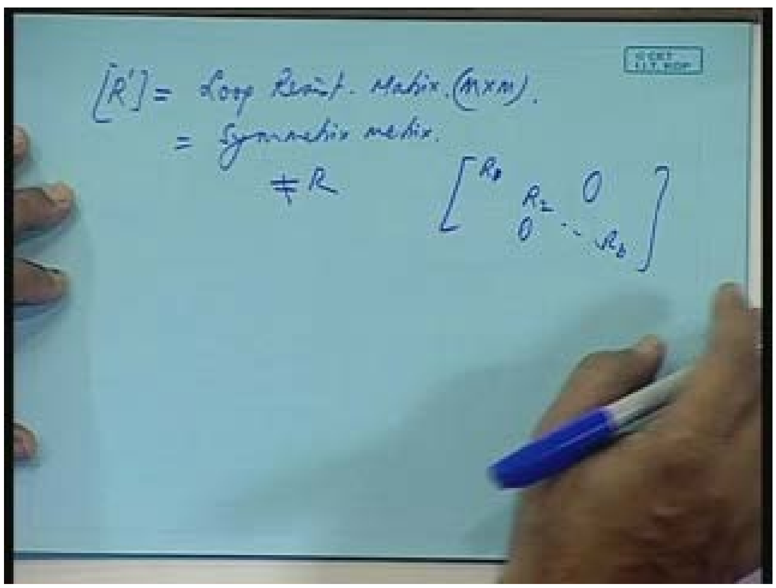

So the steps are very simple specify the circuit elements in triplets, specify in triplets the circuit elements that is k, i, j is element number starting node and finishing node and each element will also be specified by type and value. So these 3 nodes then type that is whether RLC etcetera and then value will see how to take LC etcetera into account. Formulate matrix A then obtain a tree then rearrange the columns of A, rearrange A exactly what we did earlier okay that is rearrange the columns first the twigs and then the links rearrange A that is A twigs and A links.

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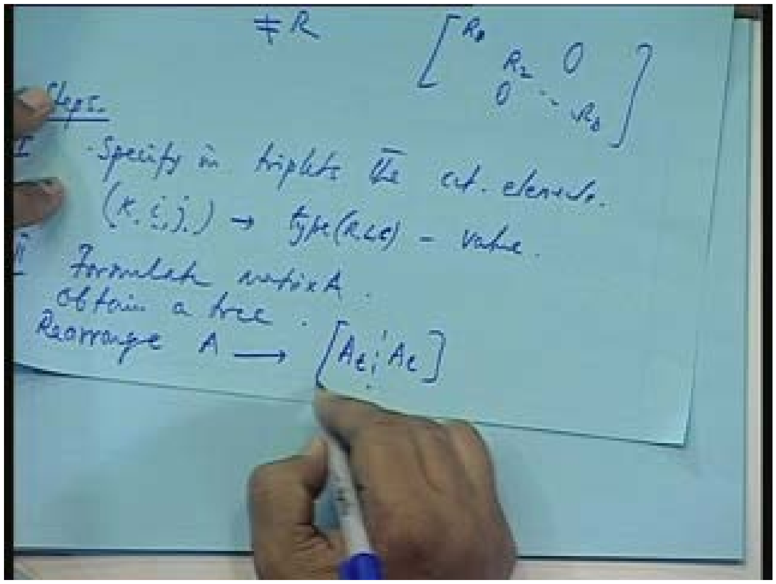

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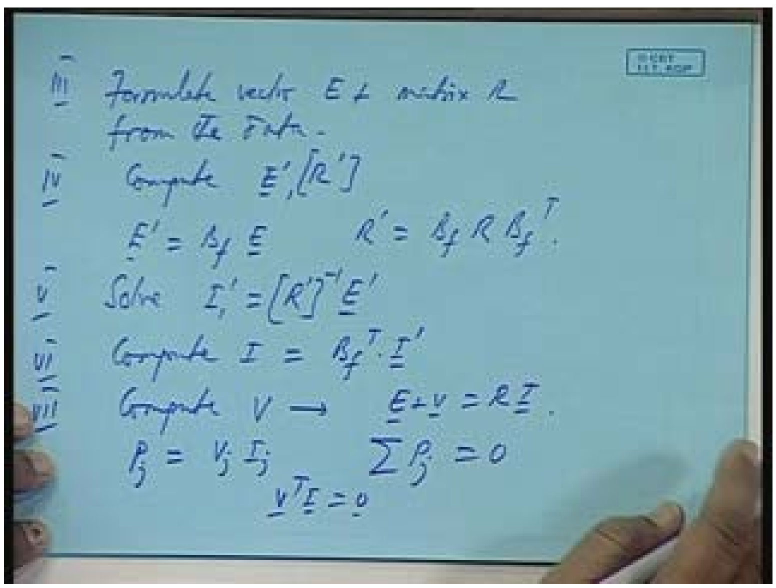

Then formulate vector E and matrix R from the data that we have already entered and in the previous step after rearranging the matrix A and then compute E dashed vector and R dashed matrix, E dashed is equal to B_f E and R dashed equal to B_f R, B_f transpose. So solve next step, solve I_1 dashed equal to R dashed inverse E dashed then compute I from this B_f transpose into I dashed. It is so systematic and then compute V from this relation E plus V is equal to R into I and then compute P_j equal to V_j into I_j and then summation P_j check that is same as V transpose I should be equal to 0, V transpose I is basically this sum. So for the last example that we took a matrix A 1, 2, 3, the elements are 1, 2, 3, these 3 elements form a tree so we have just taken as it is in this is a relatively simple example.

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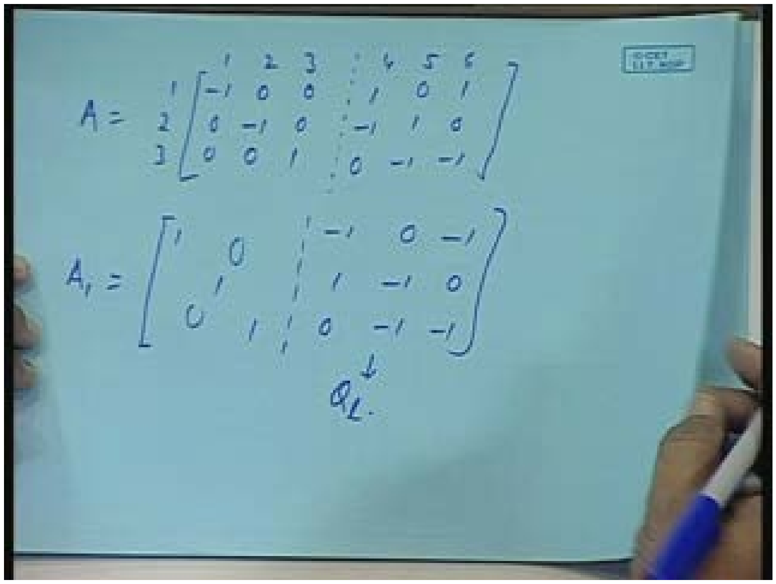

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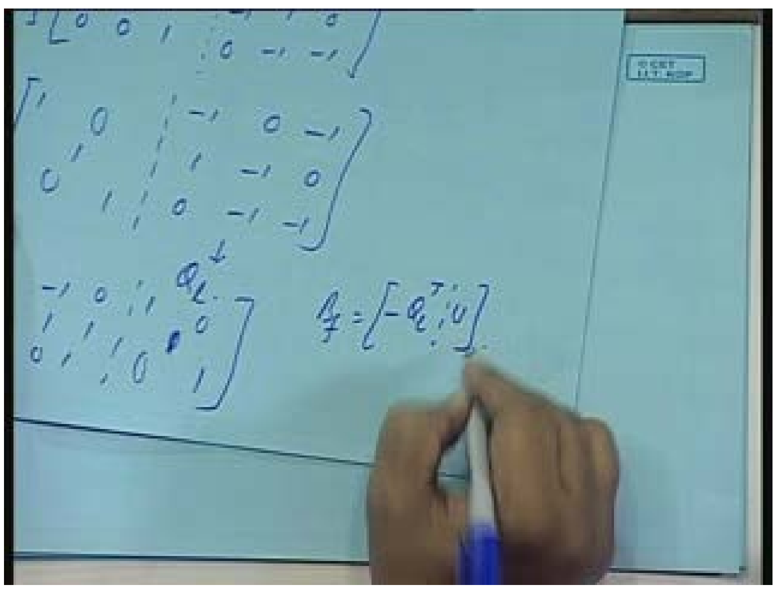

So there connections will be 1, 0, 0 if we refer to the diagram let me see the network that we took any way if you check the values 0, minus 1, 0, 0, 0, 1 then 1, minus 1, 0, 0 1, minus 1 and 1, 0, minus 1. So with the transformation so that we get unity matrix here, we get after transformation 1, 1, 1, minus 1, 0, minus 1, 1, minus 1, 0, 0, minus 1, minus 1. So this is nothing but Q_l if you remember we are trying to find out matrix Q from a matrix by transformations. So this is Q_l, so if you know Q_l you know B_t and hence B_f, so B_f if you remember it will be transpose of this that will be coming first and followed by a unity matrix. So it will be 1, minus 1, 0, then 0, 1, minus 1, 0 just put opposite signs and then transpose you will get this matrix.

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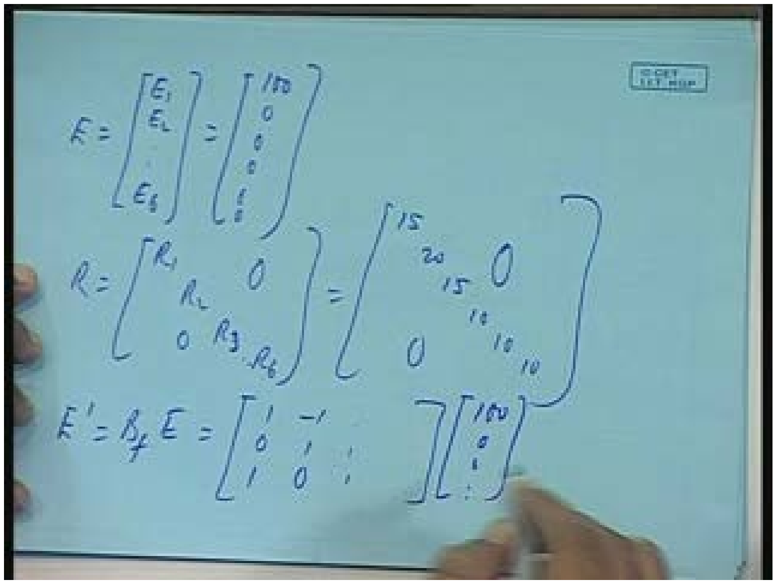

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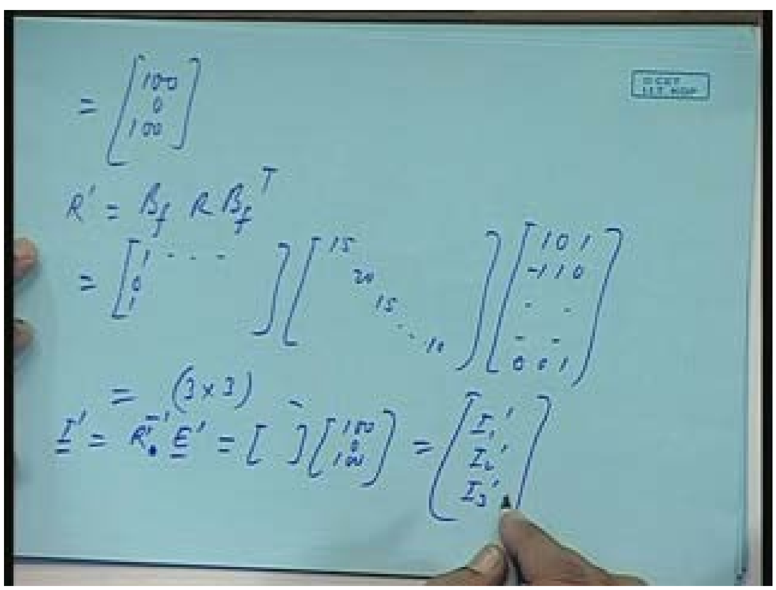

So 0, 1, 1, 0, 1, 1 then 1, 0, 1, 1, 0, 1 and then unity matrix 1 sorry 1, 1, 1 and then 0s, this is matrix B_f because B_f is nothing but minus q link transpose partitioned U. Therefore E in our case it is E_1, E_2 up to E_6, it is 100 and rest are all 0s, R is equal to R_1, R_2, R_3 up to R_6 the values are given rest are all 0. So it is like this 15, 20, 15, 1, 0, 1, 0, 1, 0, E dashed is equal to B_f into E which will be B_f is this matrix multiplied by 100. So I will not write all the terms it will be 1, 0, 1, minus 1, 1, 0 and so on you can complete B_f multiplied by 100, 0, 0 etcetera okay.

So that gives me 100, 0, 100 okay and R dashed will be B_f, R, B_f transpose, so you can compute this B_f you know 1, 0, 1 etcetera then you write R, 15, 20, 15 and so on the 6 diagonal terms multiplied by 1, 0, 1, minus 1, 1, 0 and so on 0, 0, 1 is B_f transpose. So it is a 3 by 3 matrix okay so I want the sorry, I dashed will be R_1 sorry, R dashed inverse the inverse of 3 by 3 matrix into E dashed okay where, E dash you have already computed E dashed as 100, 0, 100.

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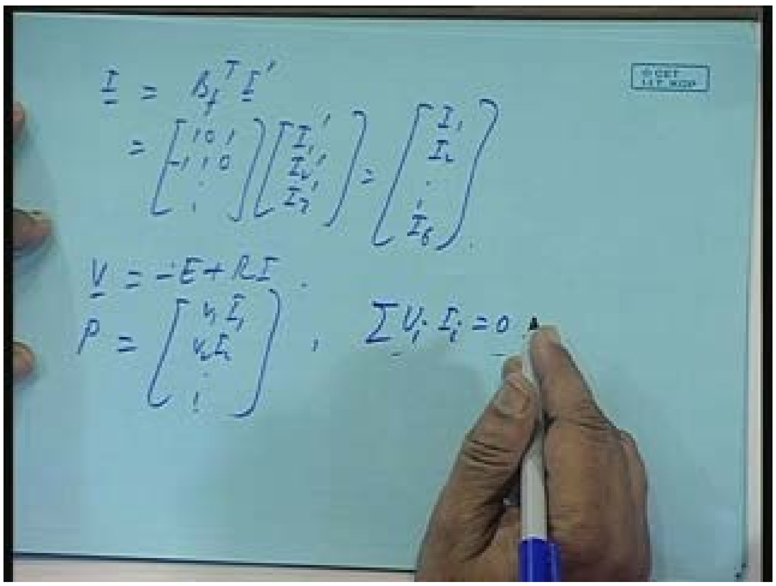

So whatever is the inverse of this matrix this is the 3 by 3 matrix that we get that multiplied by 100, 0, 100. So that gives me I_1 dashed, I_2 dashed, I_3 dashed, once you have got this compute vector I that is B_f transpose into I dashed and B_f transpose is known 1, 0, 1, minus 1, 1, 0 and so on and that multiplied by I_1 dashed, I_2 dashed, I_3 dashed. So you get 6 values I_1, I_2 and so on up to I_6 okay.

So V you can compute minus E plus RI and hence P will be V_1, I_1, V_2, I_2 and so on that will give you the branch or total sum of that will be equal to 0 okay. So sigma V_I I_i will be equal to 0, so this you can check now I have left the problem unfinished I have just given you the hints how to solve it you can try it yourself, you will get the result, thank you very much.

Preview of Next Lecture

Lecture - 26

R-L-C 2-Terminal Network

Good afternoon friends. Today we shall be discussing about synthesis of RLC network, when you say synthesis you are still restrict to our self ourselves to 2 terminal network that is 1 port network.

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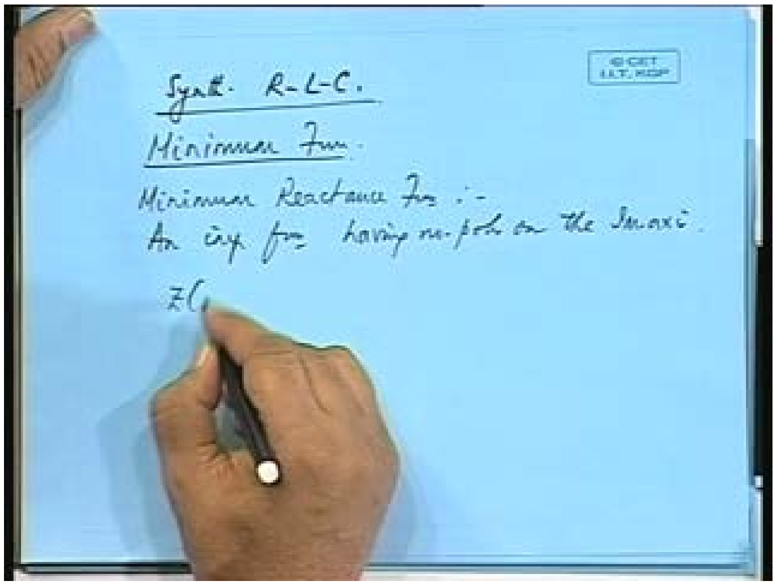

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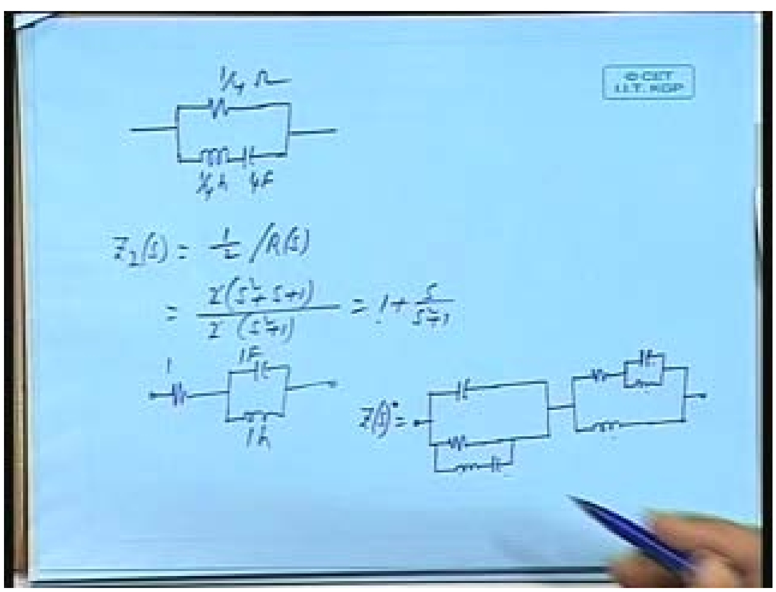

Now before we go to the synthesis we will define some functions, special type of functions, minimum functions, what is a minimum reactance function? First, we define a minimum reactance function, a minimum reactance function we get when an impedance function has got no poles on the imaginary axis that is an impedance function having no poles on the imaginary axis that means in Z(s) was in parallel with the capacitor okay.

So you have final Z(s) has a capacitor in parallel with that $Z_1(s)$ that is one forth ohm resistance and network like this and then an inductor and this one is resistance. So this is the structure of Z(s), the number of elements required here will be more you see it is no more canonic in that sense because we are getting negative inductor in case of Brooney's synthesis we coupled it with a transformer. The number of elements there we got much less here it will be 1, 2, 3, 4, 5, 6, 7, 8 never the less, this is easier to implement there is no coupling involved okay. So thank you very much, we shall continue with this with a few more examples in the next class.