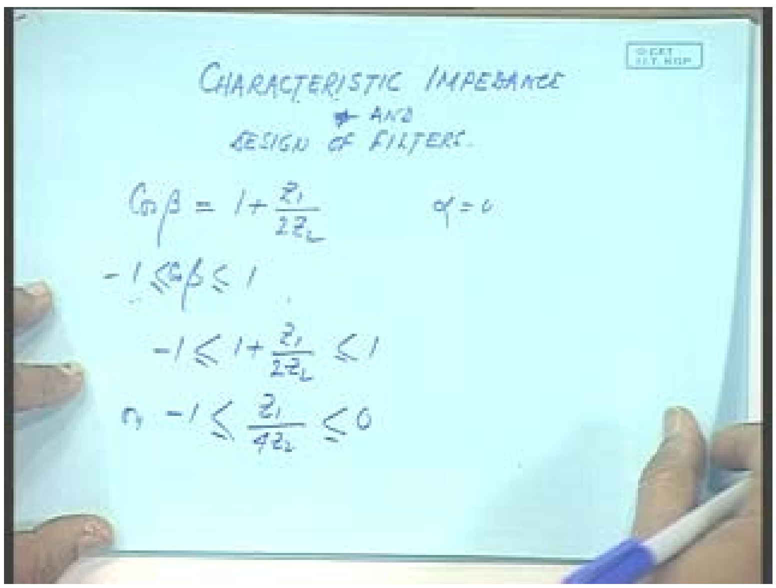
Networks Signals and Systems
Prof. T. K. Basu
Department of Electrical Engineering
Indian Institute of Technology, Kharagpur
Lecture - 24
Graph Theory (Contd…)
Characteristic Impedance and Design of Filters

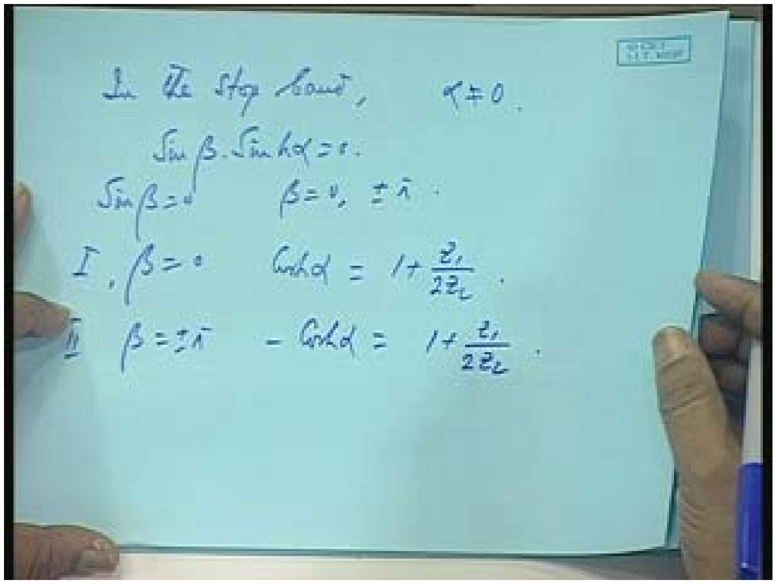
Okay, good morning friends last time we discussed about the properties of impedance, characteristic impedance and today we will continue that and discuss about design of filters.

(Refer Slide Time: 00:59)



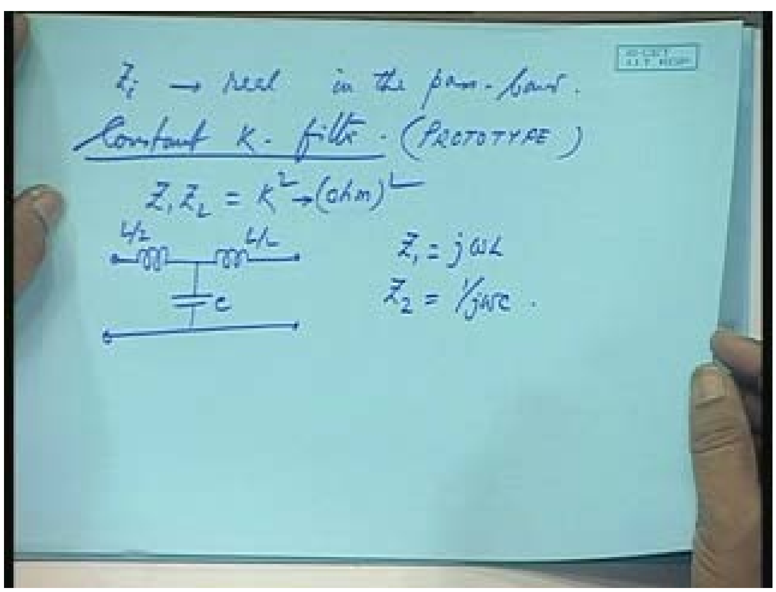
Now we saw last time cosine beta was 1 plus Z_1 by twice Z_2 when alpha equal to 0 that is in the pass band this means because beta is sorry cosine beta is between minus 1 and plus 1, so you can write minus 1, 1 plus Z_1 by twice Z_2 less than equal to 1, if I subtract 1 from all the 3 then it reduces to Z_1 by 4 Z_2 less than equal to 0 okay. Now in the stop band, in the stop band we have in the stop band where alpha is not equal to 0, we put the other condition sine beta, sine hyperbolic alpha equal to 0 okay that means there are 2 possibilities if sine beta equal to 0 then beta can be either 0 or plus minus phi if beta is 0 then cos h hyperbolic alpha that is cos h alpha is 1 plus Z_1 by twice Z_2 and if beta equal to plus minus phi then minus cos h alpha it will be 1 plus Z_1 by twice Z_2, this you saw last time.

(Refer Slide Time: 01:57)



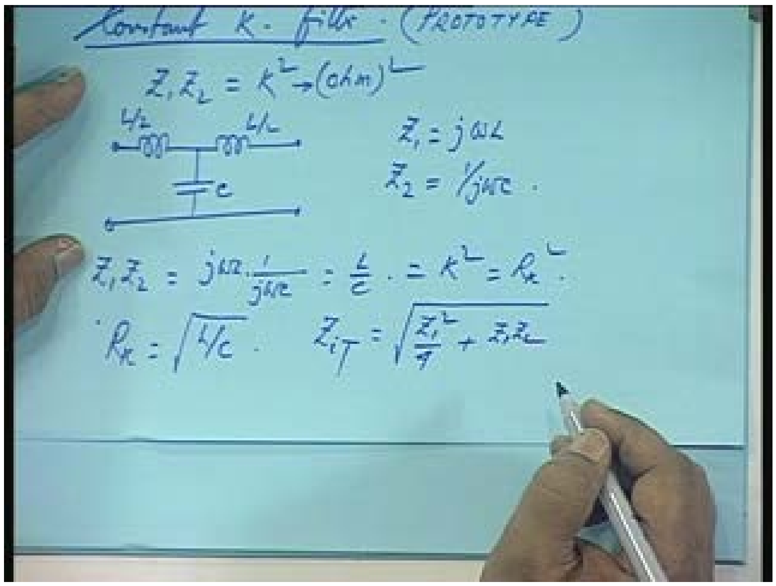
So from here we can find if I take Z_1 by 4 Z_2 along this x axis then between 0 and minus 1 of this this is a pass band and beta will be varying like this. So in the pass band beta varies like this when alpha is 0 and again here and here beta exists. Next this characteristics this characteristic impedance Z_i is real in the pass band, what does it mean? It means, it means that in the pass band power has to be transmitted if, if Z_i is not real if it is imaginary then no power will be transmitted.

(Refer Slide Time: 03:56)

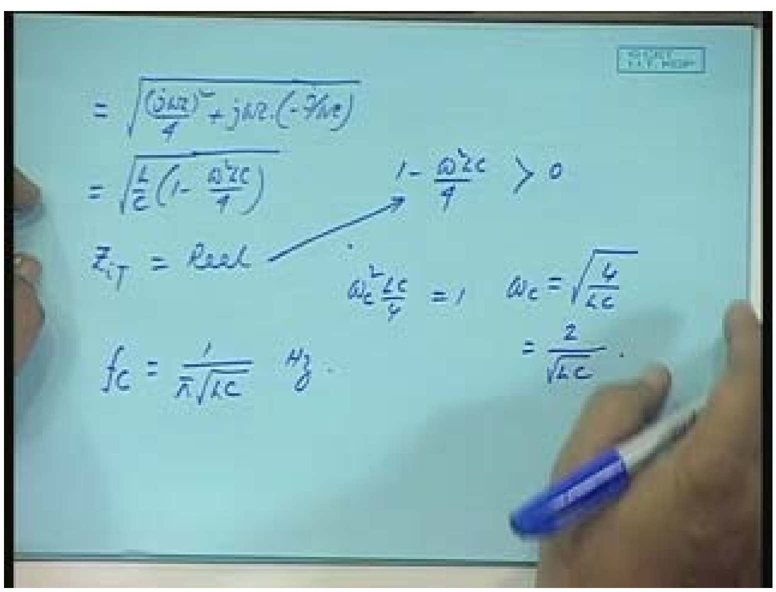


So in the stop band it may be imaginary but in the pass band it must be real there can be anything in between. Now we will design constant K filter, this is known as a prototype filter. The specifications are given like this suppose Z_1 and Z_2 are the 2 arms series and short terms then product of these 2 will be equal to some constant K squared it has the dimension of ohms squared okay.

(Refer Slide Time: 05:55)



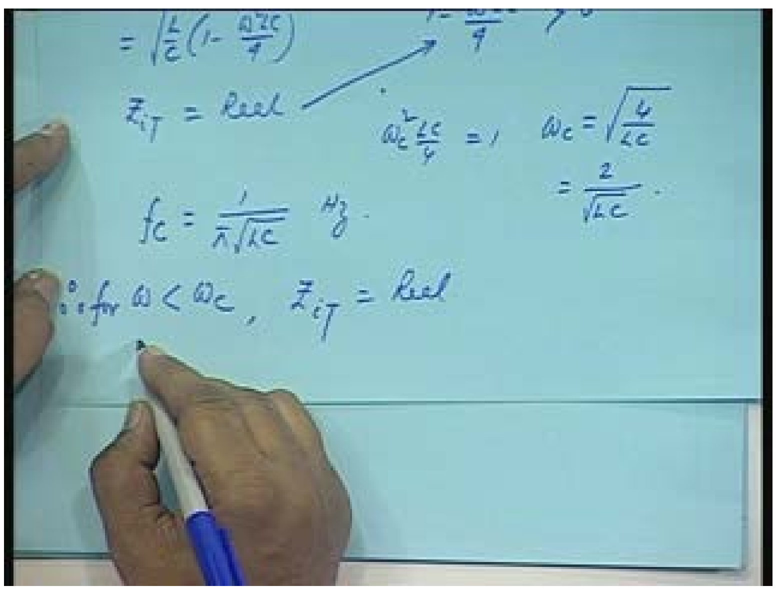
(Refer Slide Time: 07:02)



Now let us take a very simple example. Let us take this as an inductance L by 2 and L by 2 and a capacitance C that is Z_1 is equal to j omega L and Z_2 is equal to1 by j omega C okay then what will be Z_1, Z_2 it will be j omega L into1 by j omega C and that is equal to L by C and that is equal to K squared. So it is a resistive this product is a constant, so it is a resistive element there is no j involved so you can write K squared as some R_K squared all right where R_K is the resistance its value will be root over of L by C. For a t network we know the iterative impedance is root over of Z_1 squared by 4 plus Z_1, Z_2 okay.

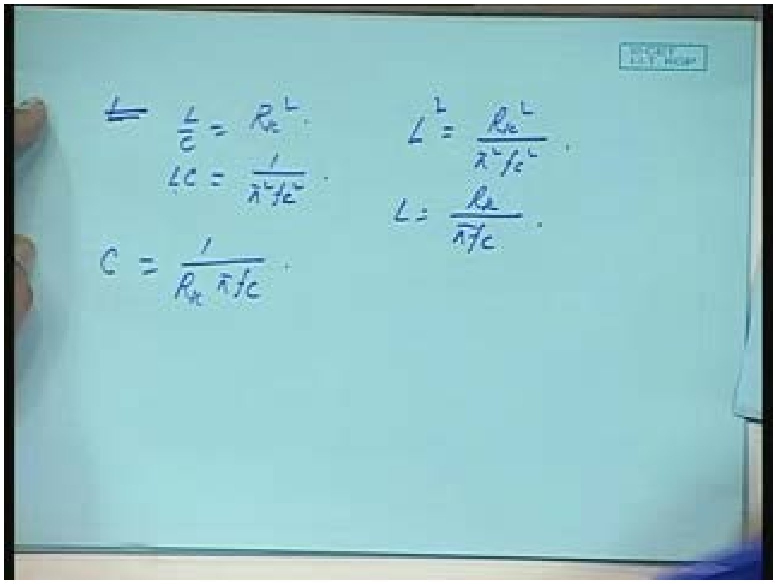
Let us compute this, this will be equal to root over of j omega L squared by 4 plus j omega L into minus j by omega C agreed so that comes to root over of L by C, 1 minus omega square LC by 4 okay. Now so long as this is greater than 1 omega square LC by 4 this quantity is greater than 0 that is this quantity is less than 1 then this is always positive. So Z_{iT} will be equal to real, this is positive means square root of that will be always real for this condition now at the critical value omega C this will be equal to 0.

(Refer Slide Time: 08:45)

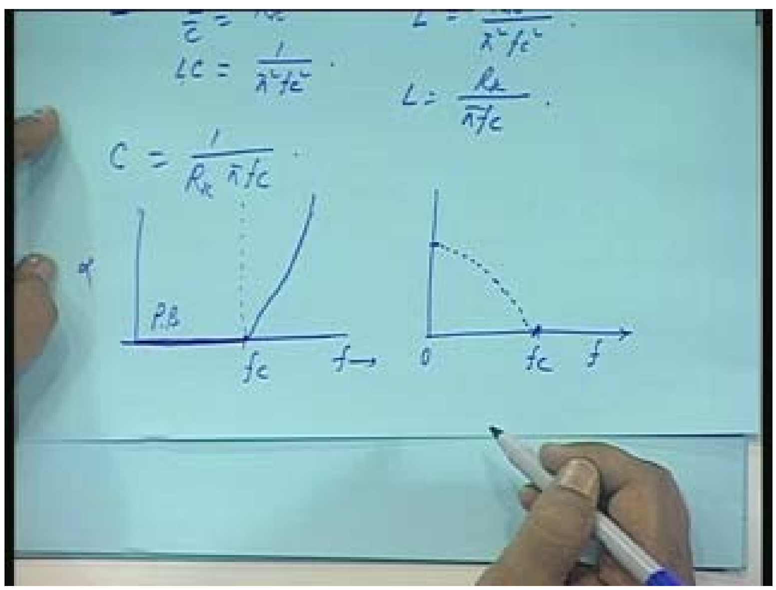


So this will just seems to be real when omega C squared LC by 4 is equal to 1 or omega C is under root 4 by LC that is equal to 2 by root LC. So corresponding frequency will be1 by phi root LC, so many hertz agreed. So you find that for omega less than omega c Z_{iT} is real okay and you can write therefore L you see we had L by C as R_K squared and here we are having LC from here okay LC equal to 1 by phi square f_c squared. See if I take the product L squared is equal to R_K squared by phi squared f_c squared or L is equal to R_K by phi f_c.

(Refer Slide Time: 09:07)



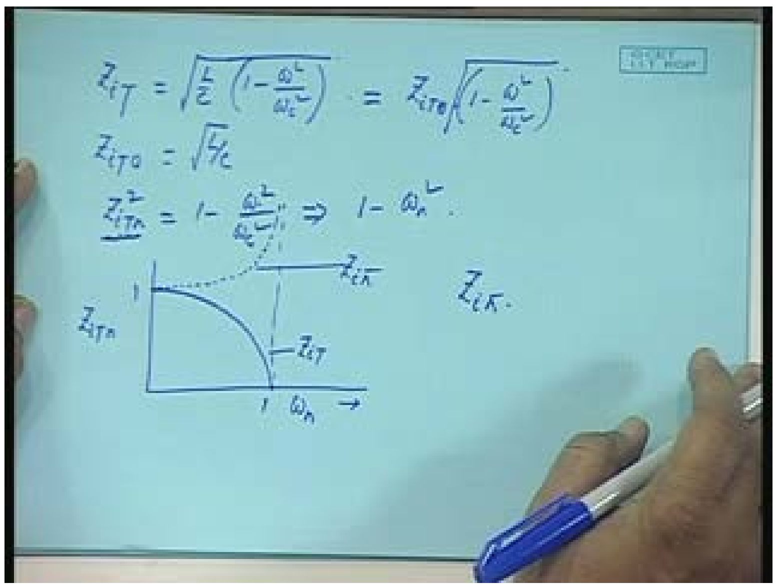
(Refer Slide Time: 10:17)



Similarly, C can be computed from either of them by substituting the value of L, R_K phi f_c okay what will be the variation of, what will be the variation of alpha and beta from the expressions that we had earlier. We can see alpha in the pass band will remain this is frequency f alpha will remain 0 in the pass band and it will be going up like this, it will be going up like this to infinity. Similarly, beta, beta will be in the pass band it will be falling to 0 like this.

We will get the expressions for alpha and beta very soon therefore Z_i for a t network is root over of L by C, 1 minus omega squared by omega C squared and Z_{iT} at omega equal to 0 will be just root L by C. Therefore, if I normalize this I can write this as Z_{iT0} into 1 minus omega square by omega C squared or Z_{iT} normalized I can write this as Z_{iT} normalized okay, Z_{iT} normalized squared if I am 1 minus is this all right omega squared by omega C squared Z_{iT} was root over of this and is this all right. So I can write this you can see if I normalize the frequency with respect to omega C then I can write this as 1 minus omega normalized square.

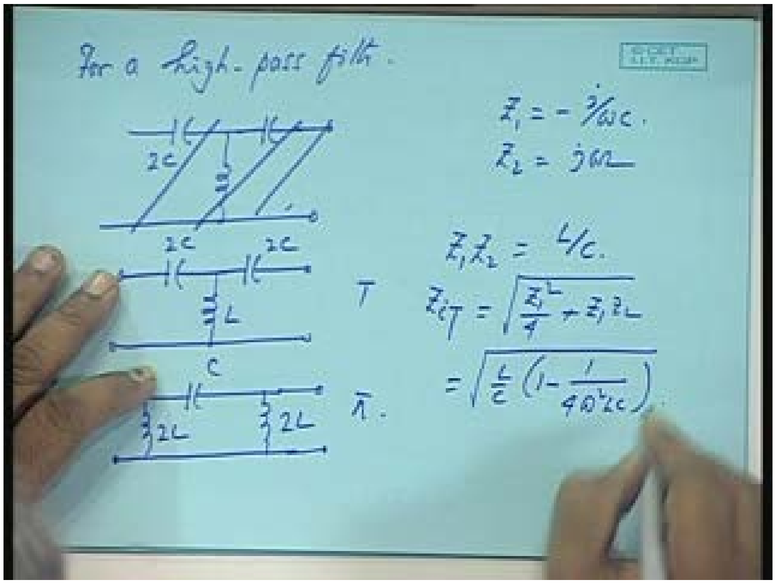
(Refer Slide Time: 11:23)



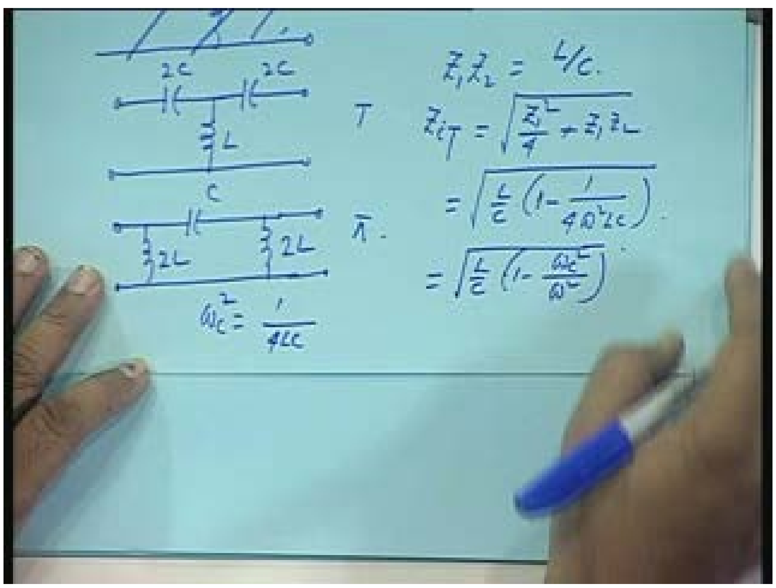
So this is equation of a circle if I write omega n and Z_{iTn} then, this is 1, this is 1, this is how Z_{iT} varies, one may take a phi network and you will find that for a phi network it will be going up like this. I leave it as an exercise 1 may derive the relation for Z_{iP} similarly, Z_i phi similarly, for a high pass filter let us see the relationship. For a high pass filter what we do we take a capacitance in series and an inductance in shunt? So if I have Z_1 by2 it will be Z_1 is minus j by omega C so Z_1 by 2 will give me 2 C and Z_2 is j omega L.

So here you are having sorry for a high pass filter in a t network we have 2C and 2C and an inductance L this is for a T and for a phi network we will have 2L and 2L like this okay. Now what will be the relationship for the different parameters and the frequency say Z_1 into Z_2 once again will be L by C with the same logic and then Z_{iT} for example, if I take will be root over of Z_1 squared by 4 plus Z_1, Z_2 substitute the values for Z_1 and Z_2 see you get L by C into 1 minus 1 by 4 omega squared LC.

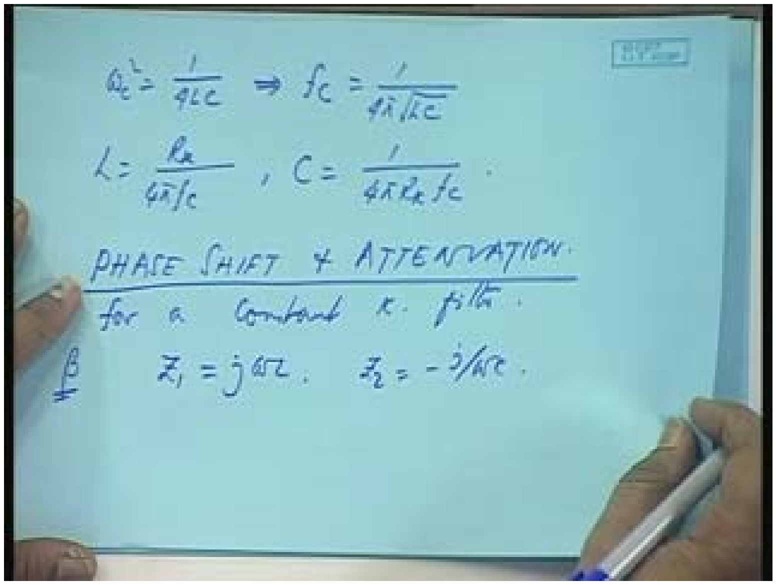
(Refer Slide Time: 14:19)



(Refer Slide Time: 16:49)

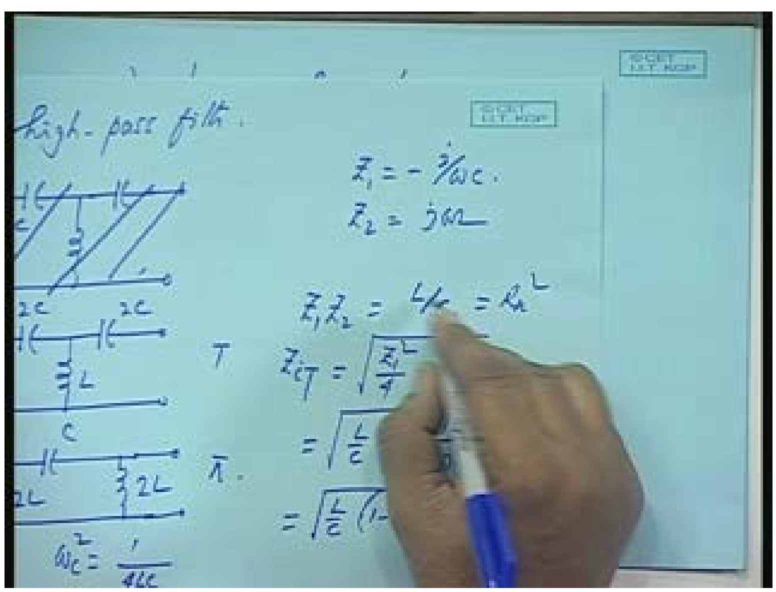


(Refer Slide Time: 17:20)



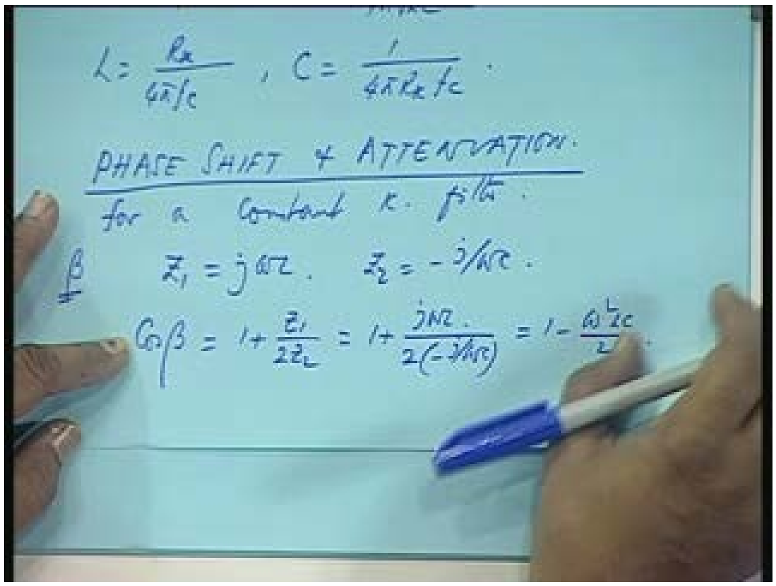
Once again when this becomes 0 that is at the critical frequency we get omega C as that frequency which will make the 0. So it will be omega C will be okay omega C squared will be 1 by 4 LC okay so if I substitute this here it will be root over of L by C into 1 minus omega C squared by omega squared agreed. So omega C squared is equal to 1 by 4 LC basically gives me a frequency f_c equal to 4 phi root LC there will be if I take squared root it will be 2 root LC and then 2 phi will make it 4 phi root LC.

(Refer Slide Time: 17:46)

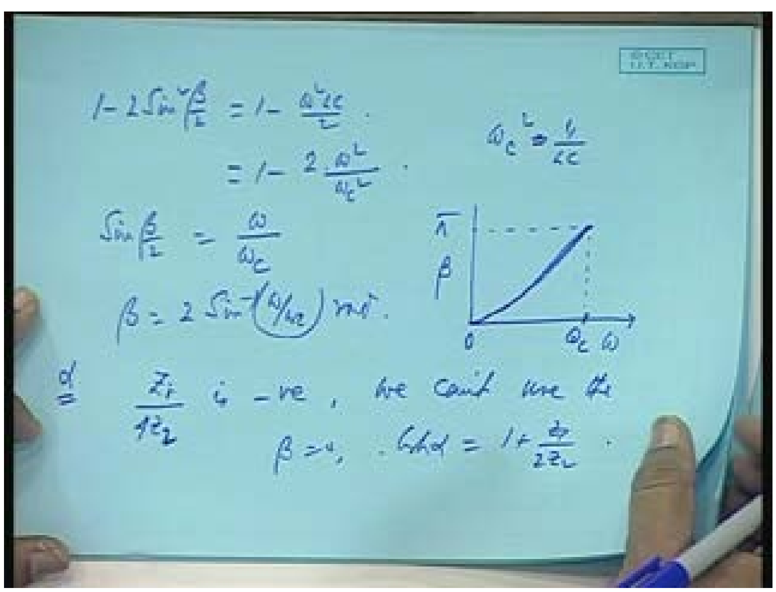


Now if I use once again the same relationship Z_1, Z_2 which is equal to K squared R_K squared is L by C and omega C is this. So L comes out to be R_K by 4 phi f_c and C as 1 by4 phi R_K fc okay, what will be the phase shift? What will be the phase shift and attenuation? Now let us see the phase shift and attenuation for a constant K filter, for a constant K filter. Now phase shift beta if you want to compute Z_1 is j omega L and Z_2 is minus j by omega C okay, you are taking the low pass filter then what will be cosine beta that is 1 plus Z_1 by twice Z_2.

(Refer Slide Time: 19:04)

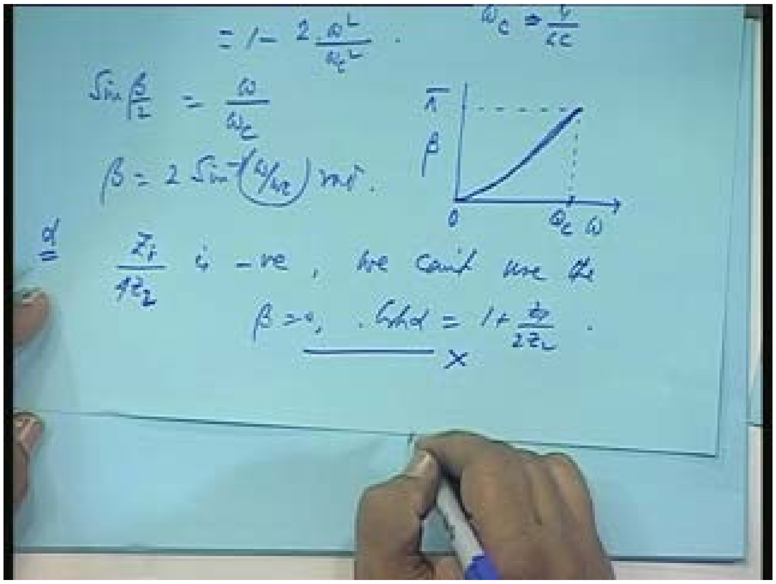


(Refer Slide Time: 19:38)

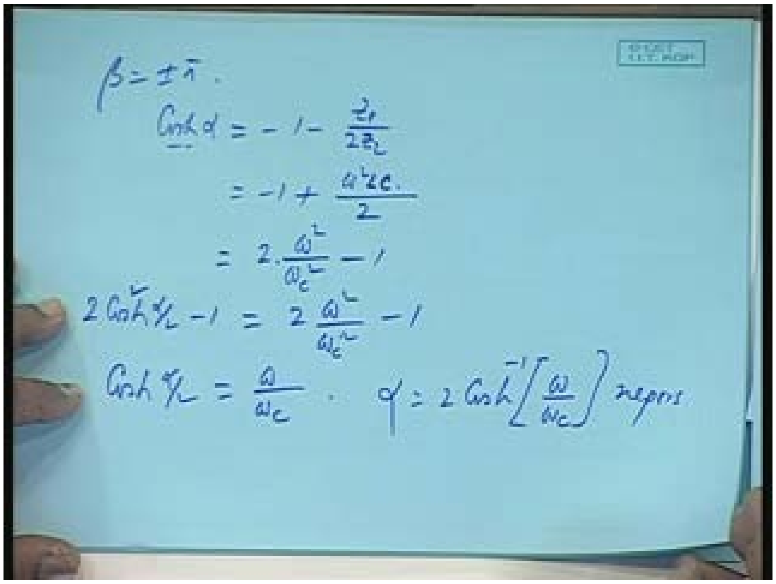


So that gives me 1 plus j omega L by 2 into minus j by omega C that gives me 1 minus omega squared LC by 2 okay cos beta is equal to1 minus LC omega squared LC by 2. So cosine beta we can write as 1 minus twice sine squared beta by 2 and that is equal to 1 minus omega squared LC by 2 and we also saw this to be equal to 2 into omega squared by omega c squared therefore because if you remember omega c squared was 4 by LC. So that gives me this so sine beta by 2 can be you can equate these 2 would be equal to omega by omega c therefore beta equal to 2 sine inverse omega by omega c is that all right.

(Refer Slide Time: 22:06)



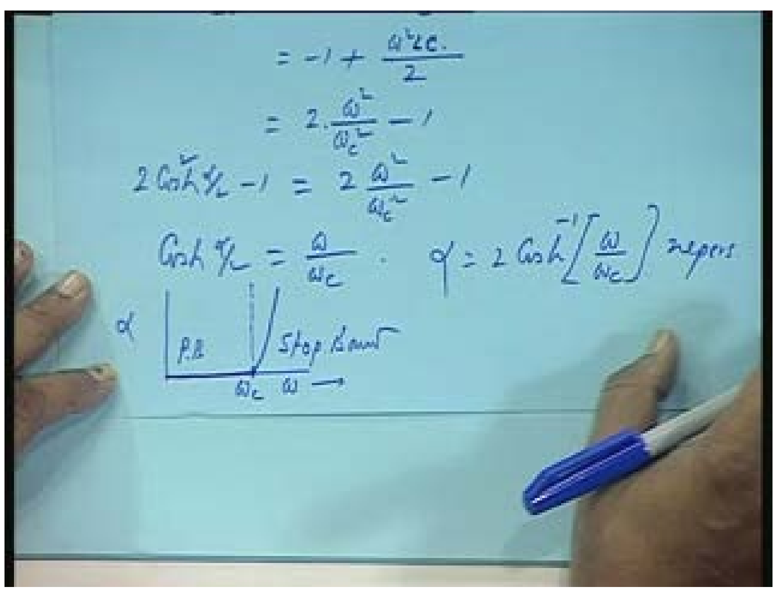
(Refer Slide Time: 22:23)



So many radians okay so we may show this in the pass band it comes from 0 it goes to phi. This somewhat like this, this is omega c. Now what will be the nature of alpha since Z_1 by 4 Z_2 is negative in this case we cannot use the first condition, is it not? We are taking 2 opposite elements inductance and capacitance so Z_1 by Z_2 is negative so you cannot use the first condition and we have to take the that is beta equal to 0, if beta equal to 0 cos h alpha we have seen is 1 plus Z_1 by twice Z_2 so you cannot take this because cos h alpha is always greater than 1 and if this is of opposite sign it will be canceling. So it will be less than 1 so we have to take the other condition so this is ruled out.

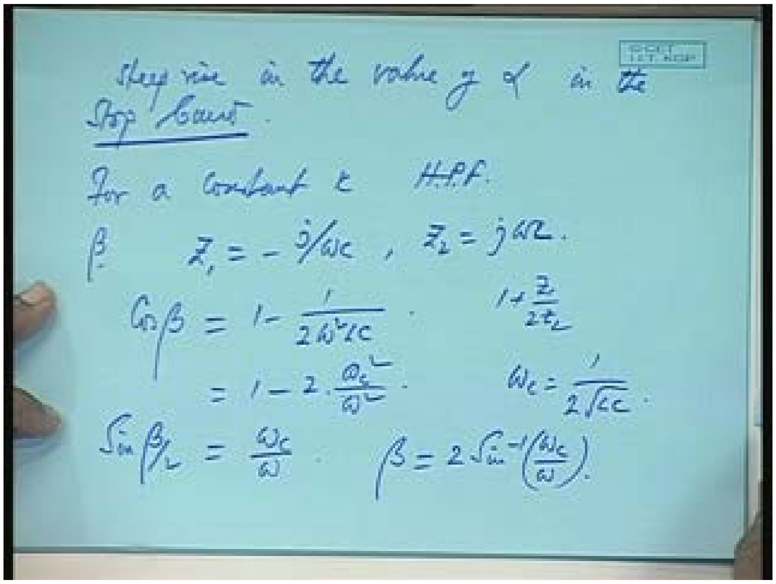
So the other condition is when beta is equal to plus minus phi, so that time cos h alpha is minus1 minus Z_1 by twice Z_2, this is a situation when Z_1 and Z_2 are of opposite sign. So this will be minus 1 plus omega squared LC by 2 if I substitute Z_1 and Z_2 I will get omega squared LC by 2 which is 2 into omega squared by omega c squared minus 1 and what is cos h alpha I can write this as twice cos h hyperbolic alpha by 2 minus 1 cos squared. So that is equal to twice omega squared by omega c squared minus 1 so equate the 2 that gives me cos h alpha by 2 as omega by omega c or alpha equal to twice cos h inverse omega by omega c, so many Nepers okay.

(Refer Slide Time: 24:29)



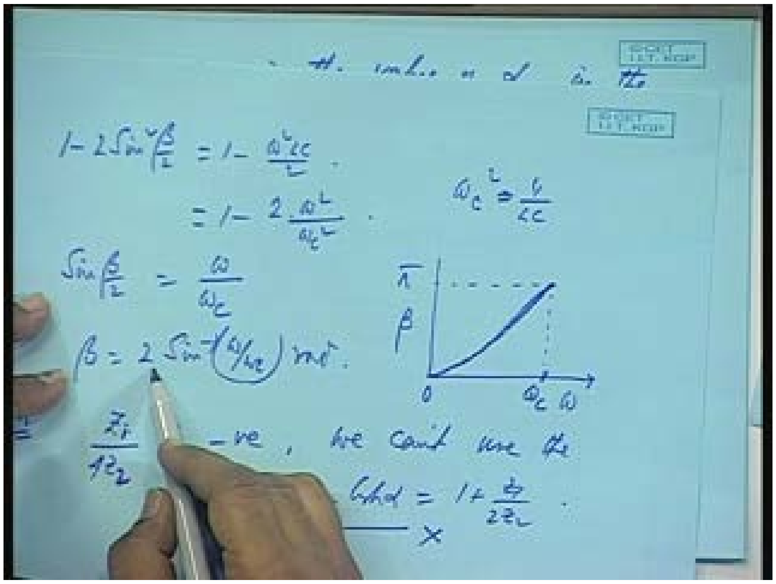
So alpha varies this is omega C alpha varies like this, it is in the pass band, it is 0 and then it goes on increasing but in the stop band, in the stop band the increase is not very rapid it is according to this function, see it will be gradually increasing. We want this rise to be very very sharp okay you want very good cut off characteristics. So we shall come back to that very soon, so our aim will be for a steep rise in the value of alpha in the stop band, will take it up very soon.

(Refer Slide Time: 24:31)



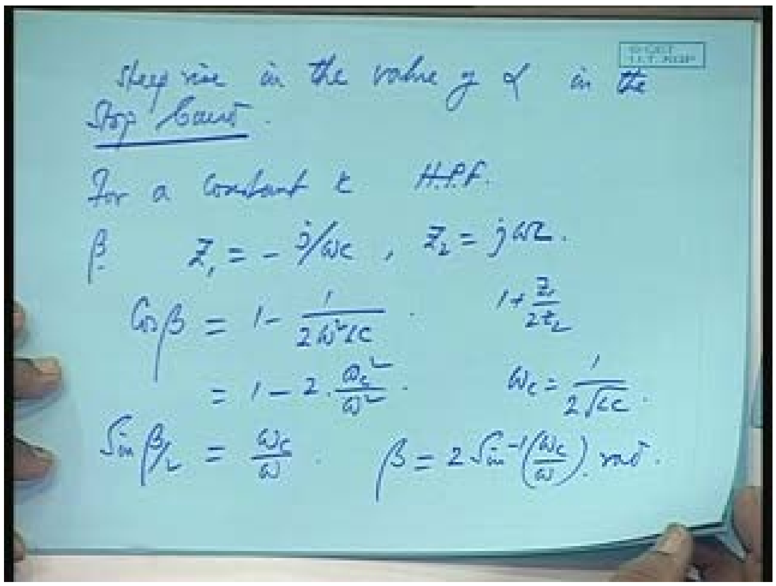
Let us examine this for a high pass filter now, for a constant K high pass filter what will be the variation of beta once again, for a high pass filter we have Z_1 and Z_2 like this, Z_2 is omega L into j Z_1 is minus j by omega C, so what will be cosine beta? It will be 1 minus 2 into omega squared LC if I substitute the values of Z_1 and Z_2 in this expression okay. So that is 1 minus 2 into omega c square by omega square if you remember the expression for omega c, omega c was 1 by 2 root LC so omega c squared I can get1 by 4 LC.

(Refer Slide Time: 26:42)

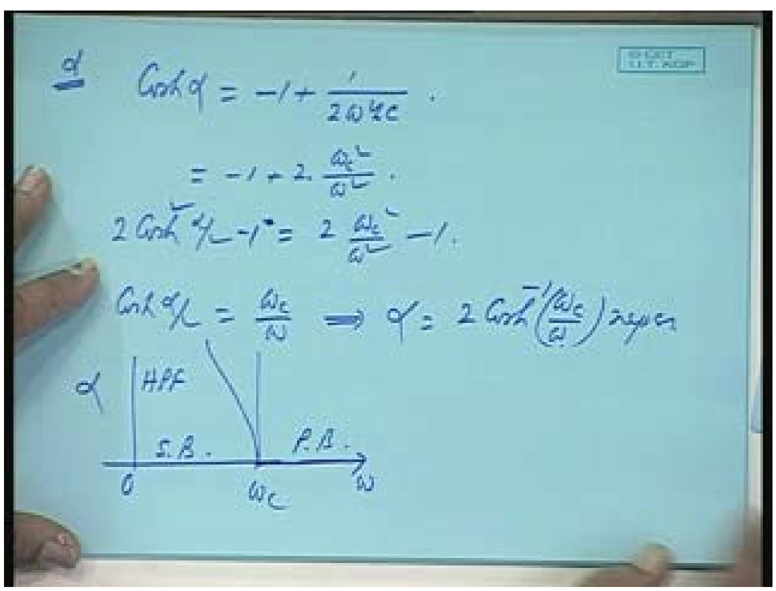


So that after manipulation gives me this so sine of beta by 2, in this case will be omega c by omega or beta will be twice sine inverse omega c by omega okay. If you look at the earlier expression for a low pass filter it is 2 sine inverse omega by omega c radians and now it is 2 sign inverse omega, it is just inverse of the other one, so many radians.

(Refer Slide Time: 26:49)

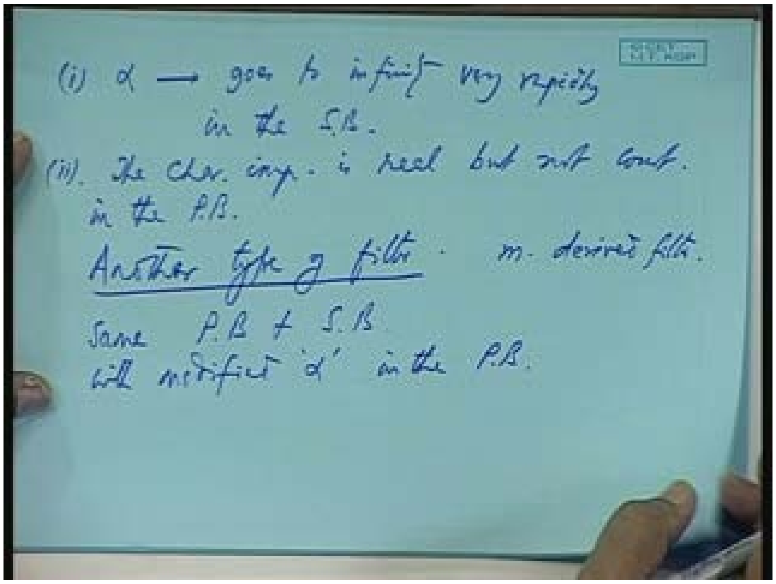


(Refer Slide Time: 26:59)

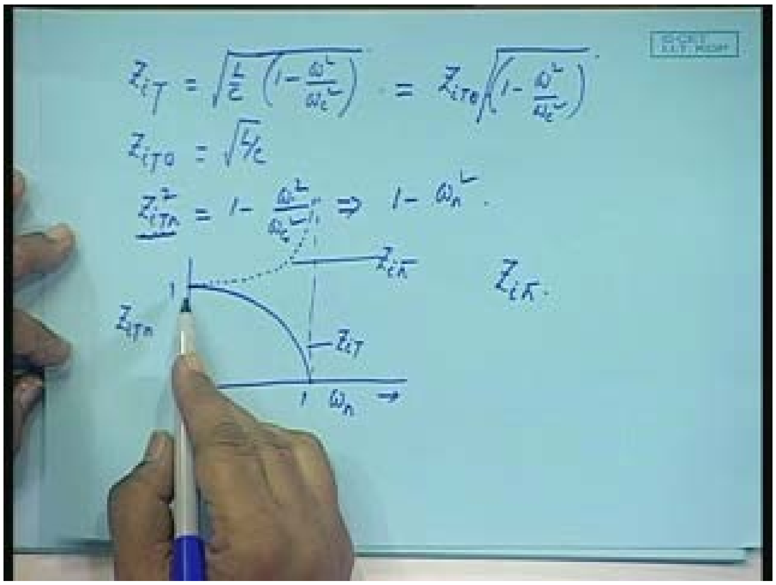


Let us see the expression for alpha cos h alpha is minus 1 plus 1 by twice omega squared LC okay I have just put the conditions for Z_1 and Z_2 etcetera those values, so you get this that is minus 1 plus 2 into omega c squared by omega squared. So this is nothing but twice cos h squared alpha by 2 minus minus 1 that is equal to twice omega c squared by omega squared minus 1 or cos alpha by 2 is omega c by omega. Hence, we get alpha equal to twice cos h inverse omega c by omega so many Napers and what you got earlier was twice cos h inverse omega by omega c so here also it is just reverse omega c by omega is obtained from the previous one just by inverting it.

(Refer Slide Time: 29:02)

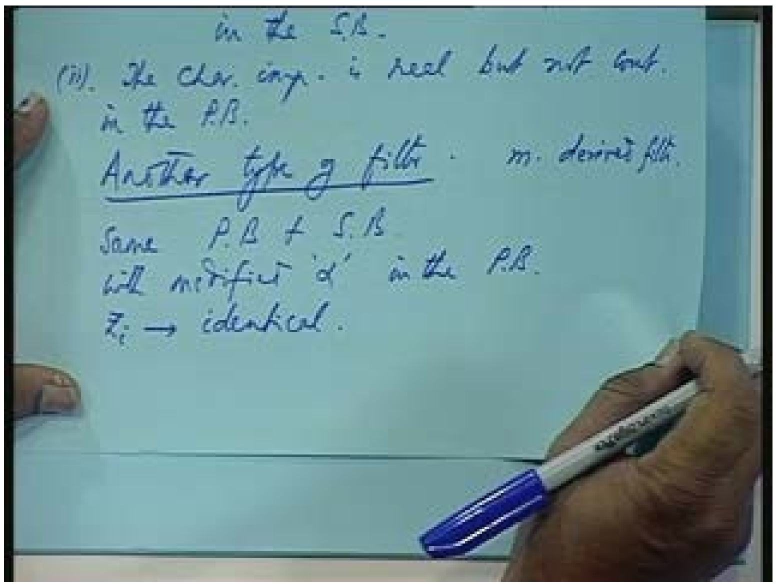


(Refer Slide Time: 30:05)

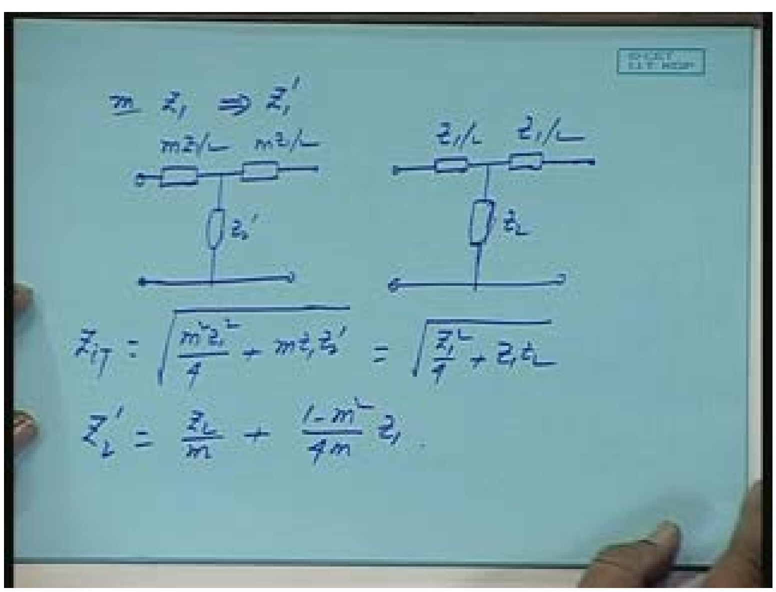


So alpha in this case, we will start from here and then this is a pass band it will be 0 and this for a high pass filter okay. So this is a stop band all right, so once again we want this to be very very sharp so what should we do about it we want alpha goes to infinity, goes to infinity very rapidly in the stop band. The characteristic impedance is real but not constant in the pass band it varies as we have seen earlier the characteristic impedance was gradually changing the normalized value was changing for 1 to 0. So in the pass band the characteristic impedance does not remain constant, we want that also to be substantially constant.

(Refer Slide Time: 31:14)



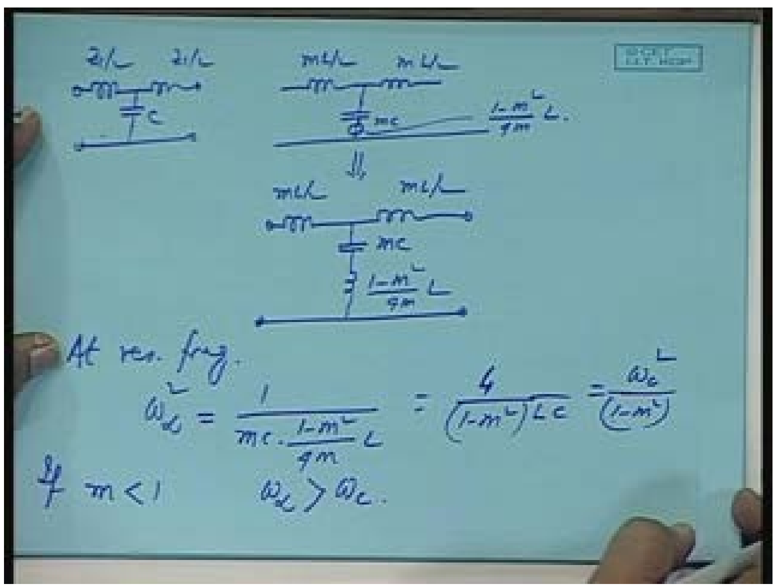
(Refer Slide Time: 31:28)



So we derive another type of filter from the prototype we call it m derived filter so we have the same pass band and stop band same pass band and stop band for this filter but with with a modified value of alpha, with modified alpha characteristics in the pass band and it is characteristic impedance should be identical. Then, let us see what it will be like derive suppose we we take a t network and we multiply Z_1 by m, we are modifying Z_1 by m and that is Z_1 dash all right. So if we consider m Z_1 here, m Z_1 here, m Z_1 by 2 sorry then what will be this Z_2 dashed which will have an identical characteristic impedance, the 2 networks should have identical characteristic impedance.

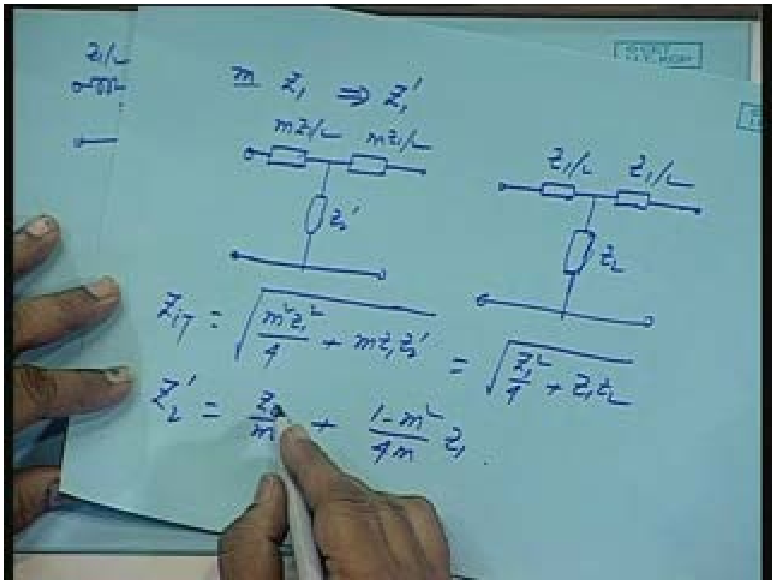
So what will be Z_2 dashed now Z_i the characteristic impedance for this t network will be root over of m square Z_1 squared by 4 plus m Z_1 and Z_2 dash which is here to be determined and that must be equated to root over of Z_1 squared by 4 plus Z_1, Z_2 which means if I squared them equate them Z_2 dashed will come out to be Z_2 by m plus 1 minus m squared by 4 m Z_1 that means if I have a low pass filter with an inductance like this and a capacitance like this, this was Z_1 by 2, Z_1 by 2 and this was C, we can have the m derived m derived filter as m Z_1 by 2.

(Refer Slide Time: 33:29)

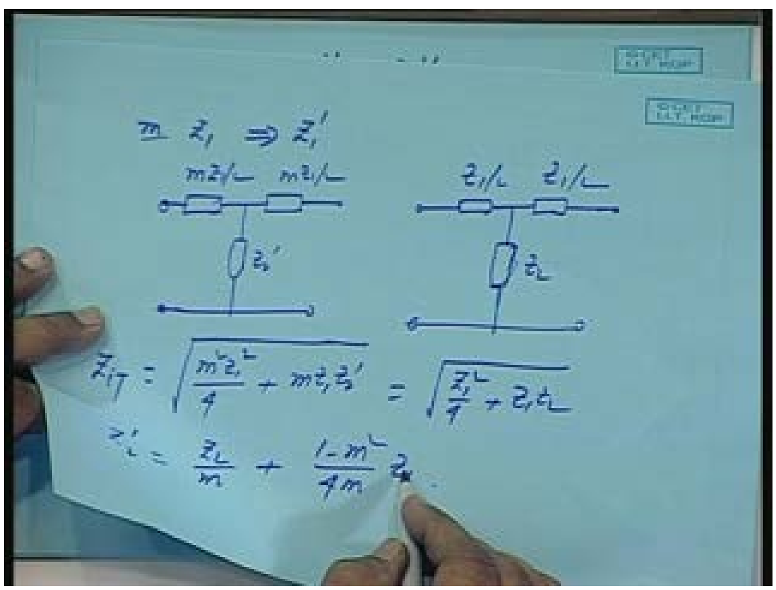


So it will be m L by 2, m L by 2, is it not m Z_1 by 2, m Z_1 by 2, if I take an LC combination like this then it will be this one and in the shunt element it is Z_2 by m, Z_2 was a capacitance so it will be m into c plus another element which is given here as 1 minus m squared by 4 m into Z_1. So since Z_1 is an inductive element, so this additional element will be 1 minus m squared by 4 m into L. So there is an inductance here sorry redraw it m L by 2, m L by 2 and a capacitance and an additional inductance. Now you see there is a change in the structure there is an additional inductance that comes into picture 1 minus m squared by 4 m into L. Now what does it do, it gives me a resonating element, this resonating element, this resonance frequency can be so adjusted that it becomes a short at that resonating frequency okay that frequency can be adjusted and at that time the entire signal gets shorted there is no transmission.

(Refer Slide Time: 34:16)

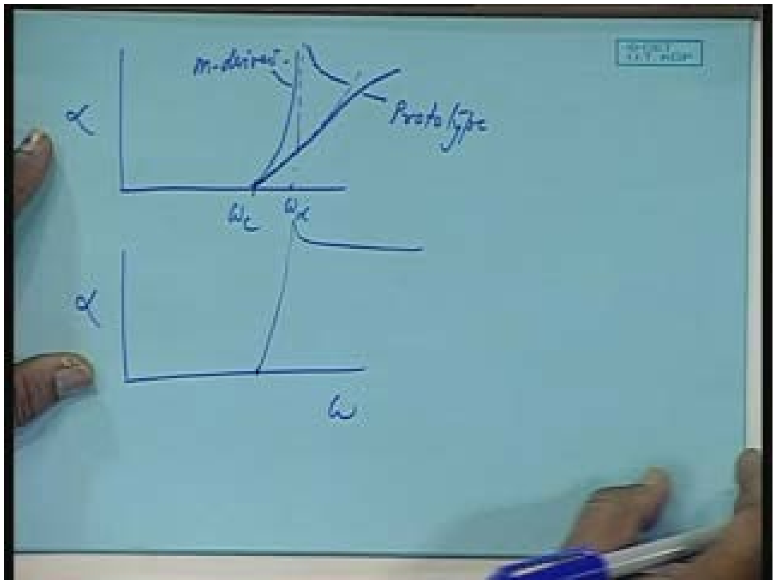


(Refer Slide Time: 34:35)



So it becomes alpha equal to 0, there is no transmission of signal on this side. So let us see alpha becomes infinity sorry, alpha becomes infinity. So at resonance frequency of this shunt term that is omega infinity squared will be LC, MC into 1 minus m squared by 4 m into L which means 4 by 1 minus m squared m will get cancelled into LC, this is a frequency at which this will resonate. If m is less than 1 then this is positive, this is positive, so omega infinity will be a frequency above omega c see 4 by LC is nothing but omega c squared.

(Refer Slide Time: 37:29)

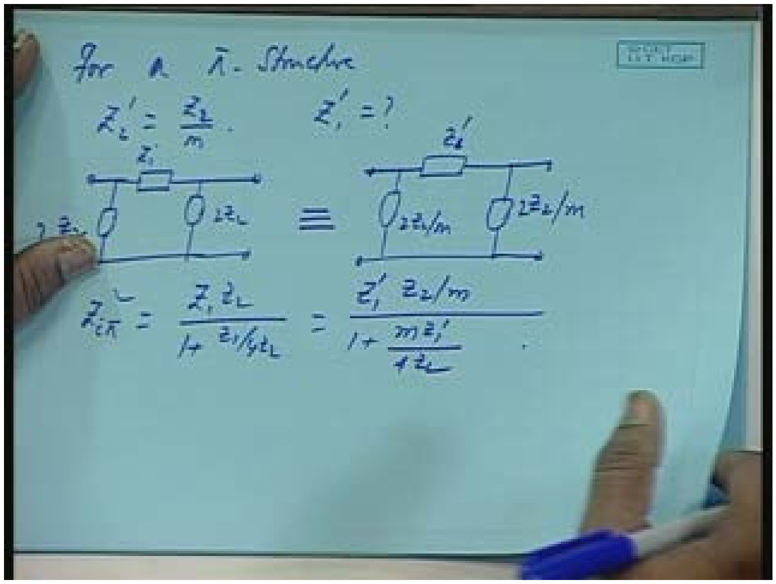


So since m is a fraction so this will be 1 minus something this will be positive but less than 1 so it will be a frequency higher than omega c, so omega infinity is greater than omega c. Now you can adjust the value of m to get the desired value of omega infinity what happens to alpha this is our omega c, so at some frequency omega infinity alpha was going on increasing like this now it will be shooting up to infinity at this point okay.

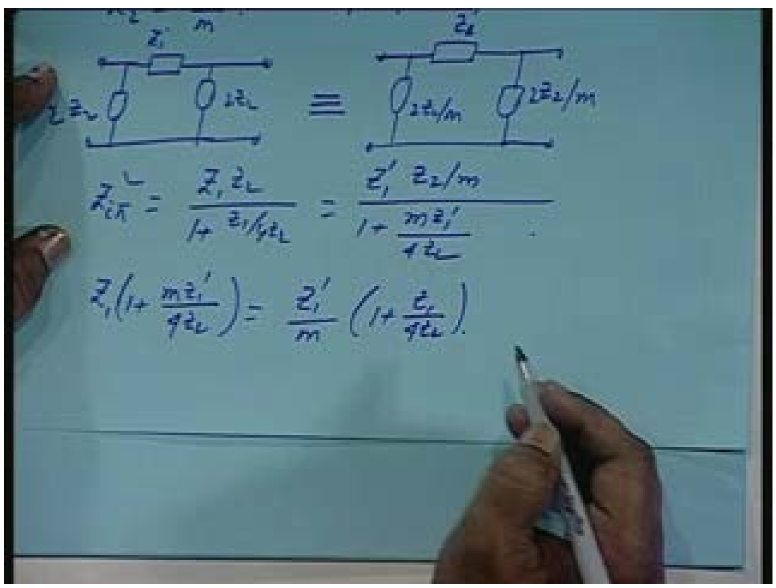
So this may be the gain of a prototype filter sorry not gain the value of alpha and this is for the m derived filter. If I put them in cascade this will be m derived after resonance again it will fall, so if I put them in cascade resultant alpha will be at omega C it will be like this then it will be practically going to infinity and then again fall like this. So we get a very sharp rise in the value of alpha in the stop band okay. For a phi structure we take Z_2 dashed to be equal to Z_2 by m then what will be Z_1 dashed that is we have Z_1 twice, Z_2 twice, Z_2 and we want to derive an m derived filter having the same iterative impedance, characteristic impedance what should be these value Z_2 dashed, Z_1 dashed this we have taken as Z_2 by m. So it will be twice Z_2 by m twice Z_2 by m.

So once again we equate the 2 characteristic impedance, so Z_i phi squared the expression is Z_1, Z_2 by 1 plus Z_1 by 4 Z_2 and on this side you will have Z_1 dashed, Z_2 dashed is Z_2 by m_1 plus m Z_1 dashed by 4 Z_2 okay. If you multiply cross multiply Z_1 into 1 plus Z will get cancelled so Z_1 into 1 plus m Z_1 dashed by 4 Z_2 equal to Z_1 dashed by m into 1 plus Z_1 by 4 Z_2 okay. From here after simplification we get this as] sorry Z_1, Z_2 divided by Z_2 by m plus Z_1 by 4 into1 by m minus m which after simplification gives me I can multiply by m. So m Z_1 into Z_2 into 4 m by 1 minus m squared one may simplify this you can see for yourself Z_2 by m plus Z_1 by 4 m into 1 minus m square. So I have just divided by 1 minus m square here.

(Refer Slide Time: 38:45)

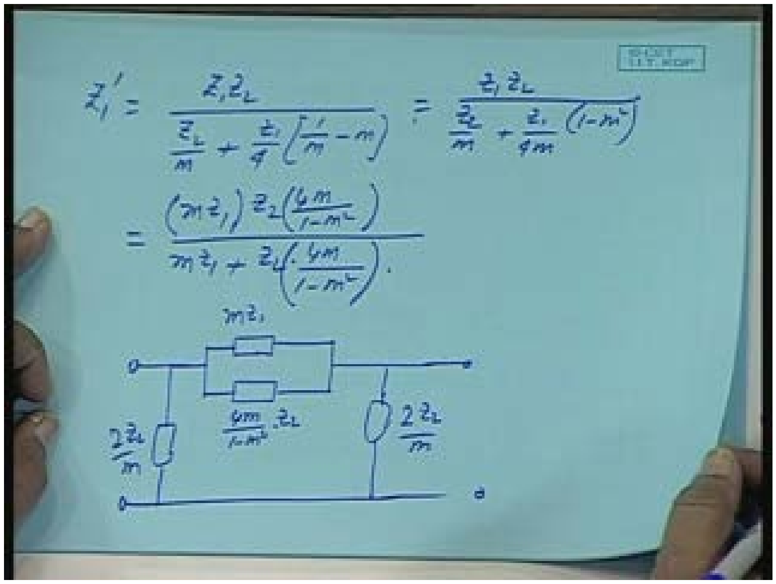


(Refer Slide Time: 40:40)

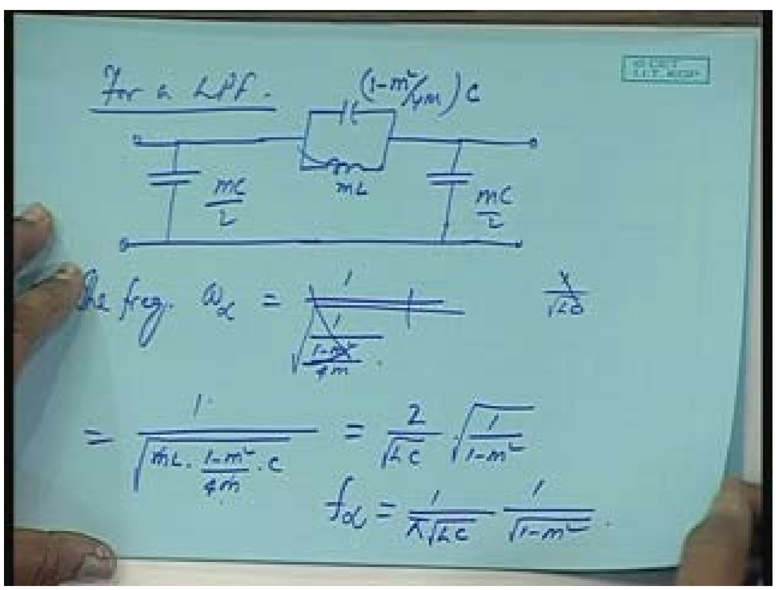


So divided by m Z_1 plus Z_2 into 4 m by 1 minus m squared okay. This has been just after adjustment of terms are written like this as if it is Z_1, Z_2 by Z_1 plus Z_2, so it is a parallel combination okay. So Z_1 will be basically a parallel combination of m Z_1 and 4 m by 1 minus m squared into Z_2 in parallel and then the shunt elements are twice Z_2 by m, twice Z_2 by m okay. So you can have the elements of L and C for a phi network whether it is a high pass filter or a low pass filter. So for a low pass filter for example, we can have mc by 2 that will give me twice Z by m, this will be 1 minus m squared by 4 m into c this one will be m into L because it is m into Z_1 if it is a low pass filter, then m into Z_1 will correspond to m into L and similarly, here mc by 2. So this is the m derived filter.

(Refer Slide Time: 41:17)

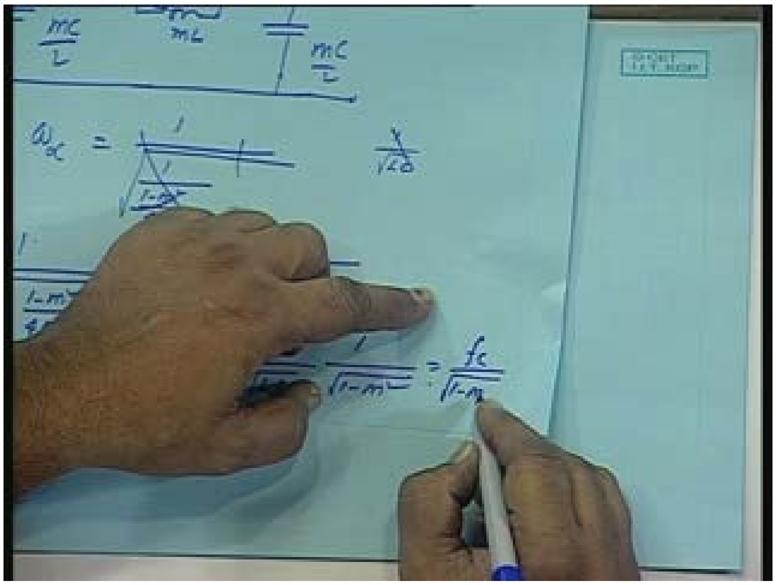


(Refer Slide Time: 43:42)

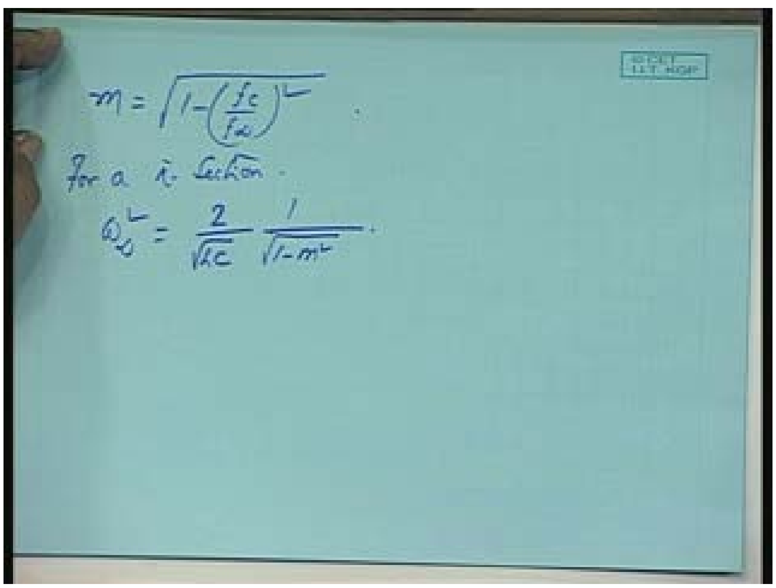


The frequency now, the frequency omega infinity at which the attenuation will be infinity that is alpha will be shooting up to infinity will be corresponding to an anti-resonance frequency here. At resonance of a parallel circuit this; this impedance will be infinite so there is no transmission of signal it is virtually cut off. In the earlier case it was shorting of the shunt element the other way of getting it is where the getting the same situation that is the transmission is stopped will be when the series element is cut off, so at that frequency you will have omega infinity equal to the elements are here. So1 by 1 minus m square by 4 m sorry it is 1 by root LC, is it not. So we put that value 1 by it is m into L into 1 minus m squared by 4 m into C so that gives me, is this all right, 2 by root LC into root over of 1 by 1 minus m square this m will get cancelled 4 will come here as 2, so 2 by root LC 1 minus m square and from here f infinity will be therefore 1 by phi root LC_1 by 1 minus m squared okay.

(Refer Slide Time: 47:02)

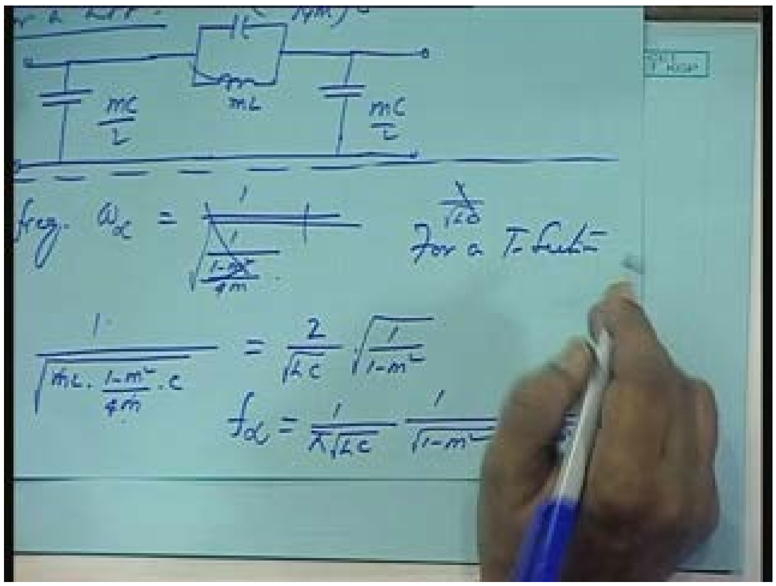


(Refer Slide Time: 47:14)

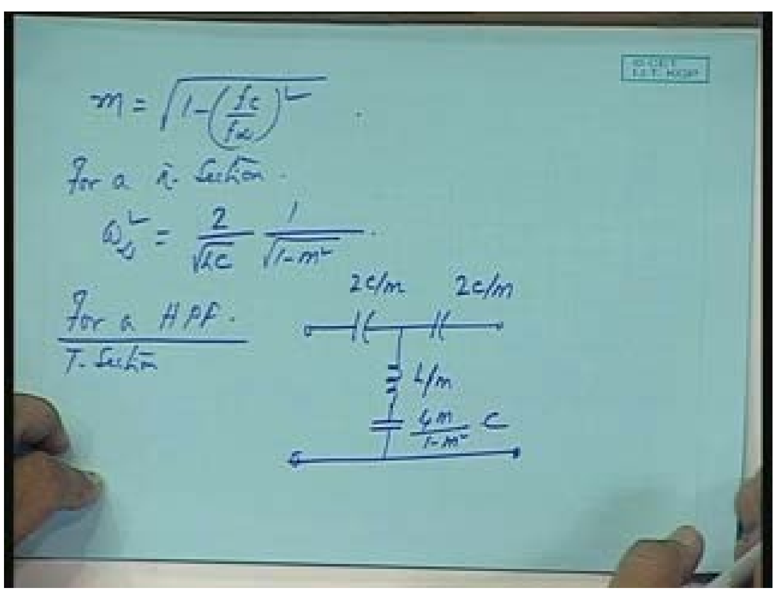


So from here this gives me f infinity also f_c also, so this is f_c by root over of 1 minus m square. So m will be1 minus f_c by f infinity squared for a phi section therefore omega infinity squared is 2 by root LC_1 by 1 minus m squared this was for a t section, this was for a t section and this is for a phi section okay. For a high pass filter therefore for t section we have 2 C by m, 2 C by m because it is m Z 1 by 2.

(Refer Slide Time: 48:01)

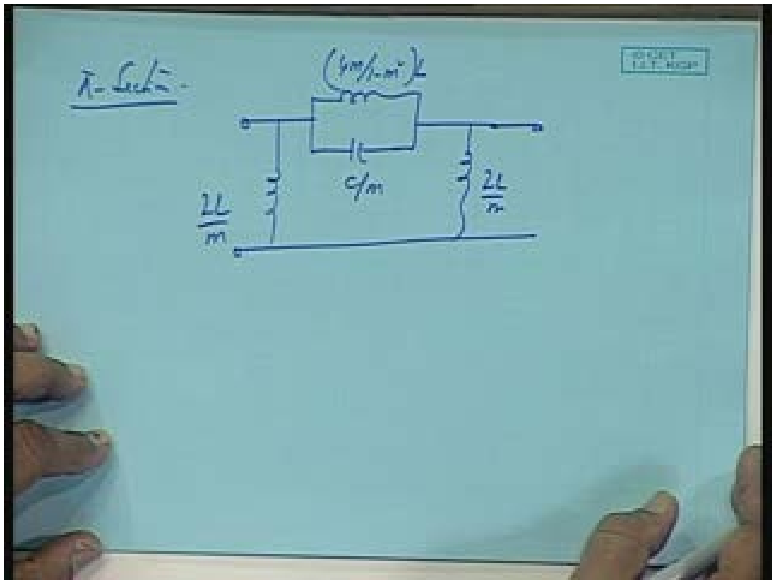


(Refer Slide Time: 48:15)

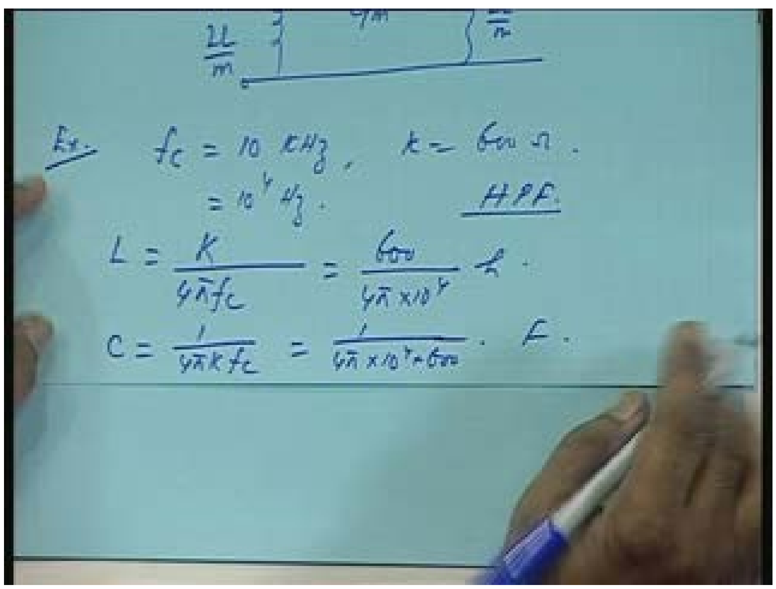


So corresponding reactance the elements will be capacitive elements of these values and this will be L by m and the capacitance of value 4 m by 1 minus m square C. For a high pass filter these element elements will be capacitive, this will be normally inductive so I have an additional element C when we take m derived filter and the phi section, the phi section for high pass will be similarly 4 m by 1 minus m square L and this will be C by m, this is 2L by m, 2 L by m, okay.

(Refer Slide Time: 50:09)

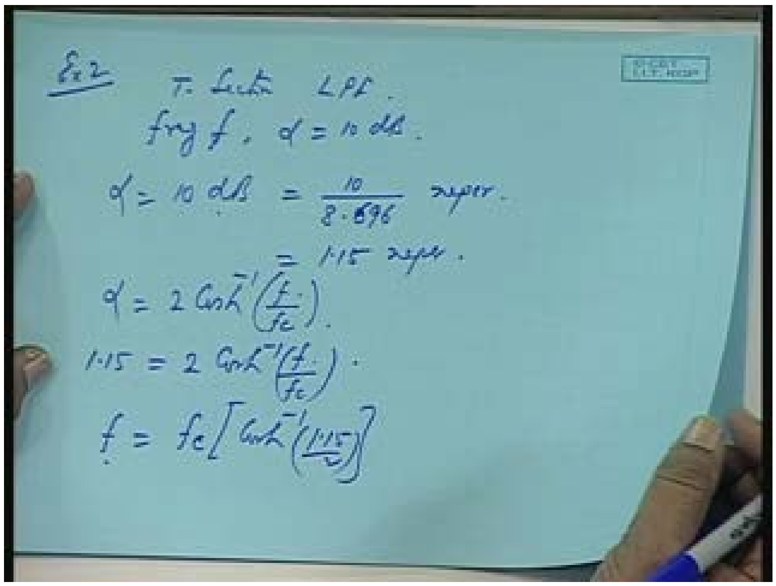


(Refer Slide Time: 50:14)



Let us take some examples suppose you are given fc equal to 10 kilo hertz, K equal to 600 ohms that is10 kilo hertz means 10 to the power 4 hertz. So what will be the value of L and C for a high pass filter you are asked to design a high pass filter, what will be the value of L and C for a high pass filter we saw n terms of K, K means that R_K okay and f_c this was the relation, so it will be 600 by 4 phi into 10 to the power 4. So many Henrys okay similarly, capacitance would be 1 by 4 phi K into f_c that is 1 by 4 phi into 10 to the power 4 into 600, so many farads okay.

(Refer Slide Time: 51:29)



Suppose you are asked to calculate let us take another example, you are asked to calculate for a t section low pass filter determine the frequency for which alpha frequency f, for which alpha is equal to 10 dB f in terms of f_c, frequency f in terms of f_c so alpha is given as10 dB that is 10 by 8.696 if you remember we discussed earlier the relationship between dB and Neper. So that comes to approximately 1.15 Neper, so alpha is equal to twice cos h inverse f by f_c that is 1.15 is twice cos h inverse f by f_c okay. So from here you can see f will be f_c into cos h inverse 1.15 by 2, 1.15 by 2.

So this is a relationship between f and f_c at this frequency, at this value of alpha okay. So we will stop here for today and next time we will be taking up some more basic elements of graph theory, how to solve networks, network problems with the help of graph theory, thank you very much.

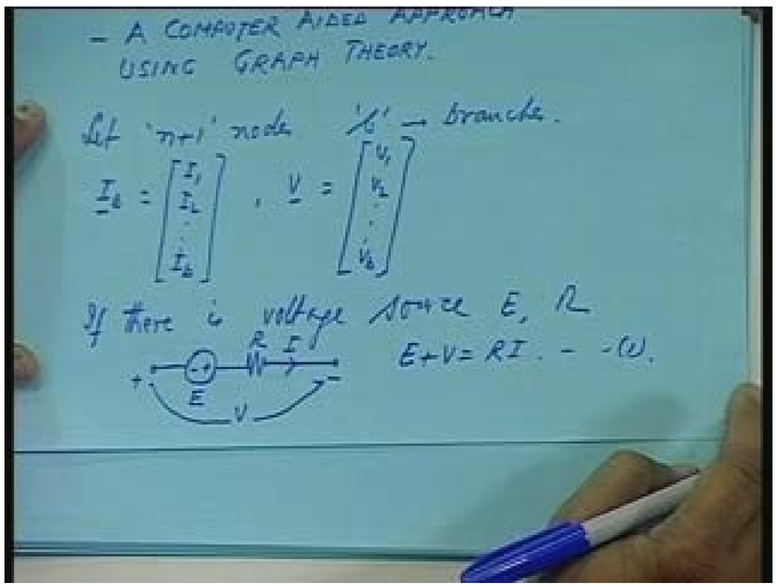
Preview of Next Lecture

Lecture - 25

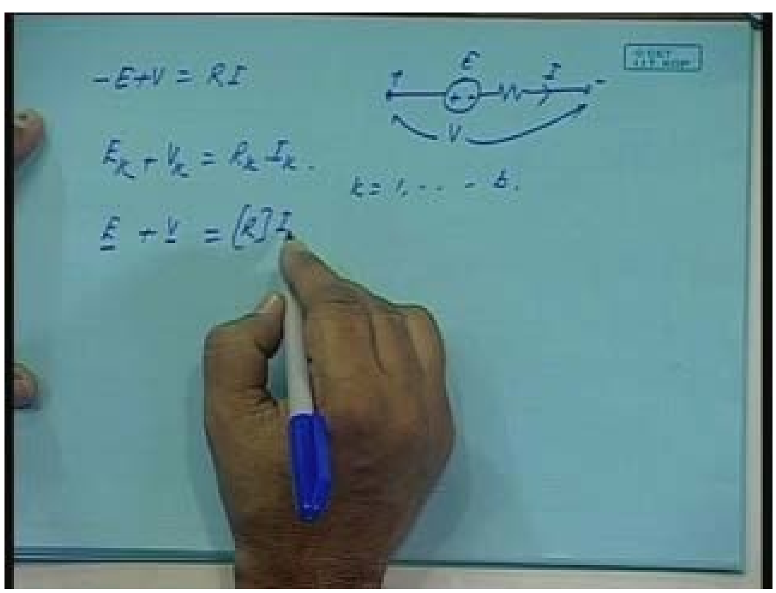
Analysis of Resistive Network Computer Aided Approach

Good evening friends, today we shall be taking up analysis of resistive networks or computer aided approach using graph theory.

(Refer Slide Time: 53:58)

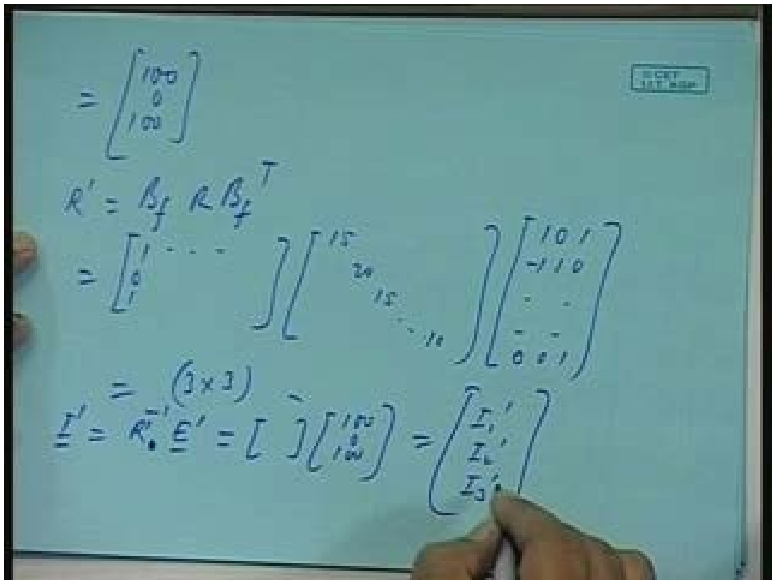


(Refer Slide Time: 55:38)

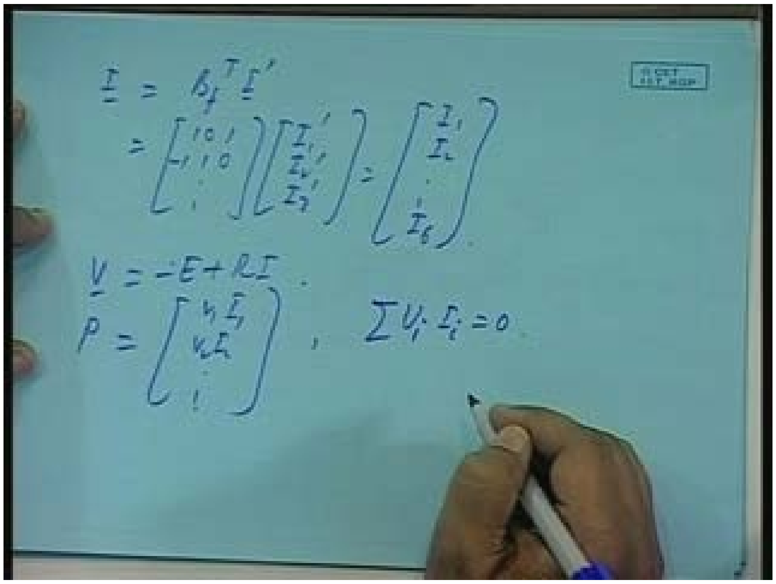


So we will be going back to the equations that we derived in our earlier lecture on graph theory. So let us have a system of n plus1 nodes and b number of branches then I can write the branch current vector as I_1, I_2 and so on I_B. Similarly, the voltage vector for different branch elements will be like this, excuse me, if there is a voltage source, if there is a voltage source then we can write say voltage source with a resistance R then we can show each element like this, this is E, this is current I, this is R and this is V, the element voltage V, then E plus V will be equal to R into I. If E is connected in the opposite direction then it will be minus E plus V is equal to RI, if E is connected like this.

(Refer Slide Time: 56:37)



(Refer Slide Time: 57:55)



So we get minus E plus V equal to RI thus we can write E_k plus V_k equal to R_K into I_k, k equal to1 to b this is generalized equation and we can write in a vector form E plus V equal to R into 1, 0, 1 etcetera then you write R, 15, 20, 15 and so on, the 6 diagonal terms multiplied by 1, 0, 1, minus 1, 1, 0 and so on 0, 0, 1 is Bf transpose.

So it is a 3 by3 matrix okay, so I want that sorry, I dashed will be R_1 sorry, R dashed inverse the inverse of 3 by3 matrix into E dashed okay where, E dashed you have already computed E dashed as 100, 0, 100. So whatever is the inverse of this matrix this is the 3 by 3 matrix that we get that multiplied by 100, 0,100. So that gives me I_1 dashed, I_2 dashed, I_3 dashed once you have got this compute reactor I that is Bf transpose into I dashed and Bf transpose is known 1, 0, 1, minus 1, 1, 0 and so on and that multiplied by I_1 dashed, I_2 dashed, I_3 dashed. So you get 6 values I_1 I_2 and so on up to I_6 okay.

So V you can compute minus E plus RI and hence P will be v_1, I_1, v_2, I_2 and so on that will give you the branch or total sum of that will be equal to 0 okay. So sigma V_i I_i will be kept to 0. So this you can check now I have left the problem unfinished I have just given you the hints how to solve it, you can try it yourself, you will get the result. Thank you very much.