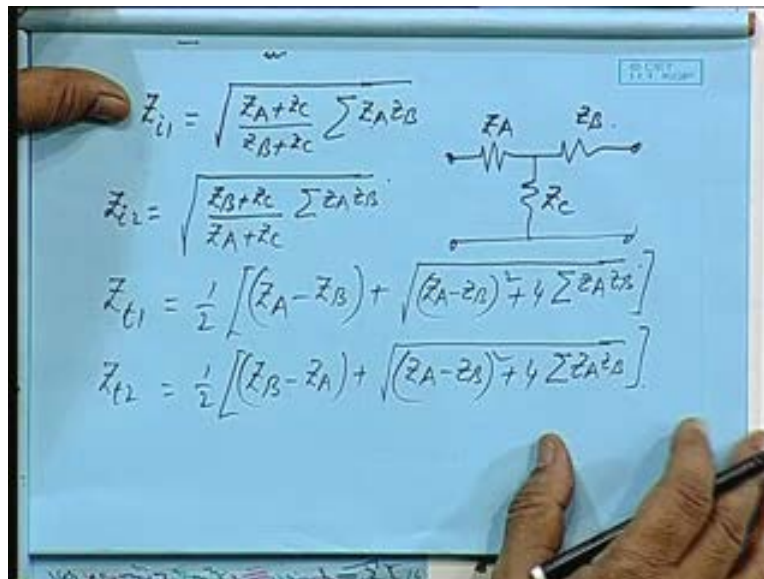


Network Signals and Systems
Prof. T. K. Basu
Department of Electrical Engineering
Indian Institute of Technology, Kharagpur
Lecture - 23
Graph Theory (Contd...)

Image Impedance, Iterative Impedance and Characteristic Impedance (Contd...)

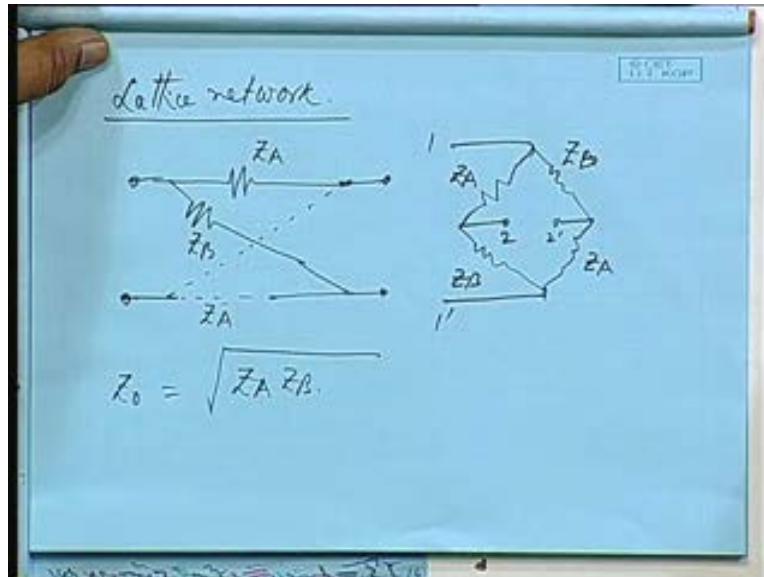
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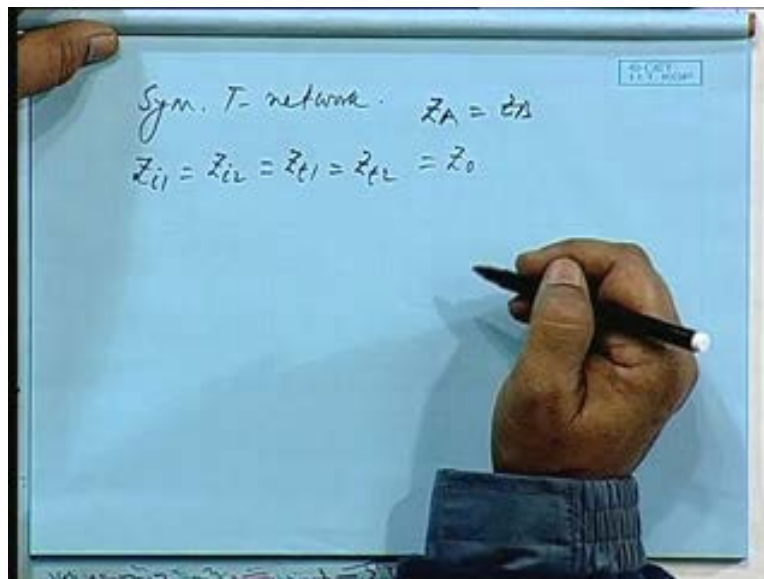
The image impedances and iterative impedances and we derived these relations for a t network that was Z_A plus Z_C by Z_B plus Z_C into sigma Z_A , Z_B , this means combination of 2 impedances at a time. So it was a structure like this Z_A , Z_B , Z_C and this was Z_B similarly, Z_{i2} we derived as Z_B plus Z_C divided by Z_A plus Z_C into sigma Z_A , Z_B , this we derived last time. We also derived for the iterative impedances Z_{t1} as half of Z_A minus Z_B plus under root Z_A minus Z_B whole squared plus 4 times that sigma term Z_A , Z_B . Similarly, Z_2 was Z_{t2} was half Z_B minus Z_A plus Z_A minus Z_B whole squared the same expression okay.

Then we discussed about a network that is a Lattice network where we considered Z_A , Z_B Z_A , this was the notation for the counter parts, this is Z_B and this is Z_A , this can also be seen as a bridge network okay like this, this was say Z_B , Z_A , Z_A , Z_B and this was the output terminals 2, 2 dash, 1, 1 dash and for this kind of a symmetric network we saw this Z not the characteristic impedance was Z_A and Z_B product okay also for the t network we saw if it is a symmetric t network, for a symmetric t network Z_{i1} is equal to Z_{i2} is equal Z_{t1} is equal to Z_{t2} by symmetric t network I mean Z_A is equal to Z_B and that was equal to Z not okay then that relation becomes very simple.

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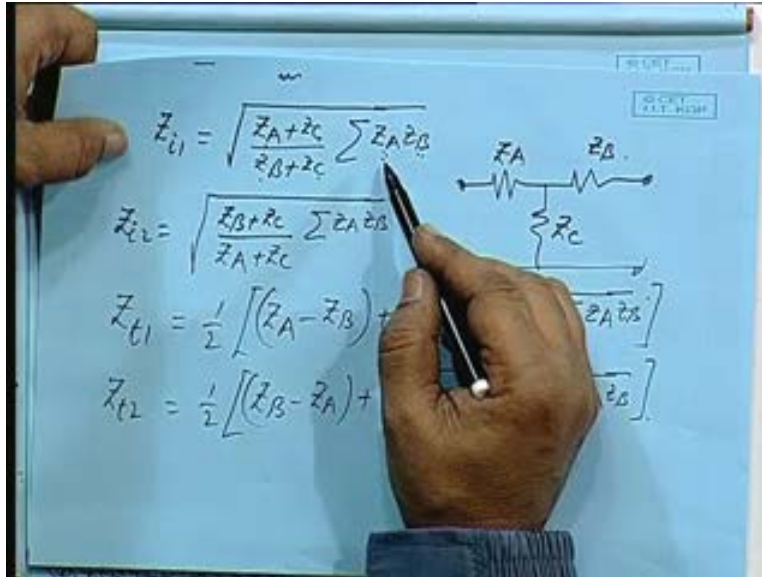
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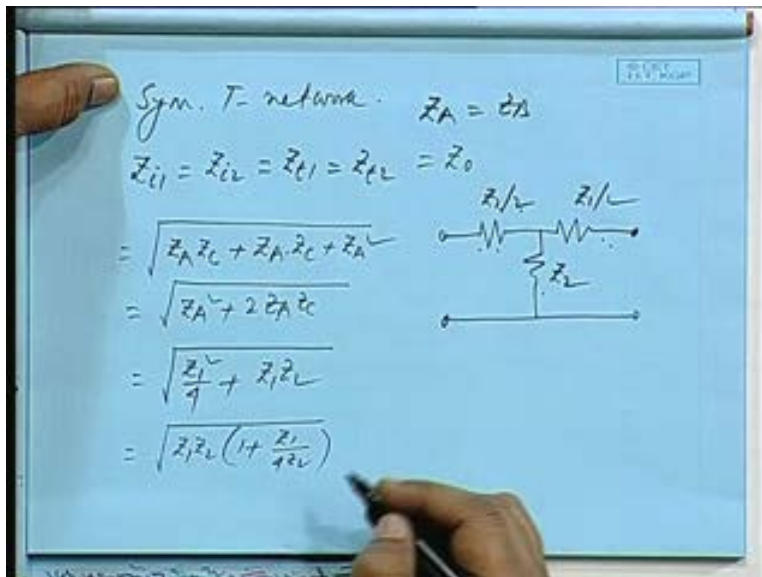
We can see from here itself Z_A plus Z_C , Z_B plus Z_C they will be getting cancelled so you will be left with left with only $Z_A Z_B$ product so that gives me the product as Z_A , Z_C plus Z_A , Z_C plus Z_A squared that is equal to Z_A squared plus twice Z_A , Z_C . Now if I have, if I have a network a balance network where the impedances are like this a symmetric network like this Z_1 by 2, Z_1 by 2 and Z_2 then Z_A is Z_1 by 2, this also Z_1 by 2. So this will become Z_1 squared by 4 plus 2 into Z_1 by 2 into Z_2 , so this will be Z_1 , Z_2 one may

write this as root over of Z_1, Z_2 into 1 plus Z_1 by 4, Z_2 this you saw last time. For a phi network one may derive this for a phi network.

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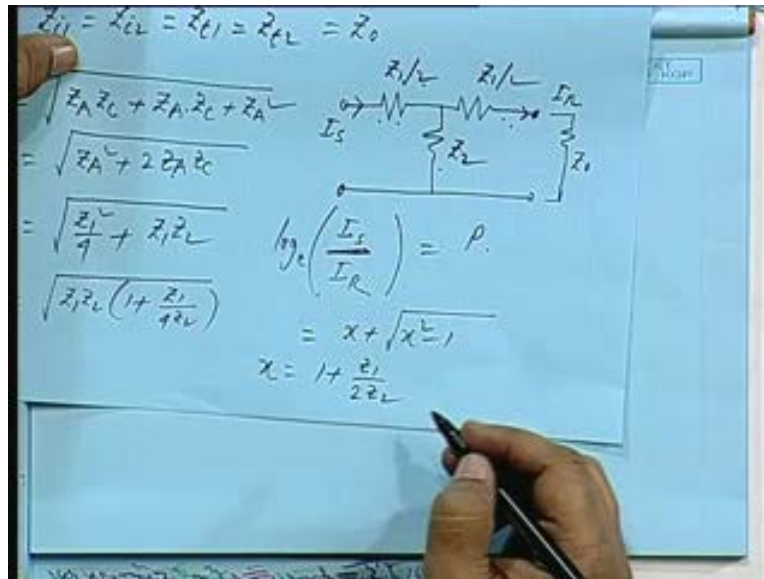
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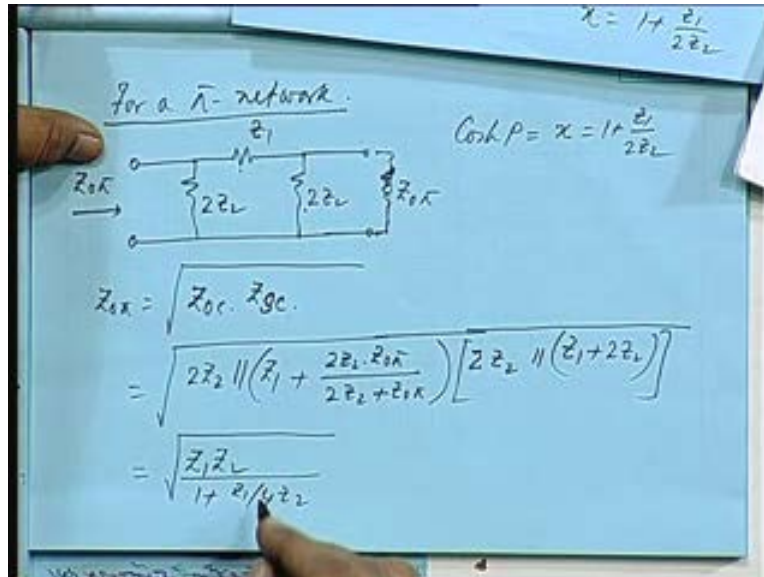


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Similarly well we have also seen if this side is loaded by the characteristic impedance Z_0 then if this is a sending in current this is a receiving in current we also established a relation between I_s by I_R in a logarithmic scale this was defined as a quantity P which is basically representing the ratio by which the current falls okay in a logarithmic scale and that was shown to be \log_e of I_s by I_R to the base p and this was x plus root over of x squared minus 1 this kind of especially got where x is 1 plus Z_1 by twice Z_2 okay.

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So this expression that we got last time give me cos of P is equal to x and that is equal to 1 plus Z_1 by twice Z_2 . Let us see for phi network what will be the situation for a phi network similarly, we can go by the short circuit and open circuit test, short circuit and open circuit impedances are evaluated and you can get the characteristic impedance, what will be the characteristic impedance if I call it $Z_{naught\ phi}$ this impedance $Z_{naught\ phi}$ the impedance in from this side is also $Z_{naught\ phi}$ what is that $Z_{naught\ phi}$.

So $Z_{naught\ phi}$ will be performed the short circuit and open circuit impedances, you evaluate from there and that will be z open circuit into z short circuit see if I short this it will be twice Z_2 in parallel with $Z_{naught\ phi}$ $Z_{naught\ phi}$ sorry, in series with Z_1 again the combination in parallel with this. So it is $2Z_2$, Z_2 till parallel with Z_1 plus parallel combination of twice Z_2 into $Z_{naught\ phi}$ by twice Z_2 plus $Z_{naught\ phi}$ okay. This is the short circuit impedance, open circuit impedance will be twice Z_2 in parallel with Z_1 plus twice Z_2 okay so twice Z_2 in parallel with Z_1 plus twice Z_2 .

So this is the open circuit parameter, open circuit impedance, short circuit impedance take the product and evaluate this $Z_{naught\ phi}$ you have to change sides and then make all kinds of manipulations you can get the result I leave it to you as an exercise finally you can verify for yourself this will boil down to it is a very interesting expression Z_1 , Z_2 divided by 1 plus Z_1 by 4 Z_2 interestingly. you see for a t network it is Z_1 , Z_2 into 1 plus Z_1 by 4 Z_2 under root here it is squared root of Z_1 , Z_2 divided by the same factor, you see this is dimension less so it is something little more than 1 you are multiplying once in the other case you are dividing.

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$$= \sqrt{Z_A Z_C + Z_A Z_C + Z_A}$$

$$= \sqrt{Z_A + 2 Z_A Z_C}$$

$$= \sqrt{\frac{Z_1}{4} + Z_1 Z_L}$$

$$= \sqrt{Z_1 Z_L \left(1 + \frac{Z_1}{4 Z_L}\right)}$$

$$\log_e \left(\frac{I_s}{I_R} \right) = \rho$$

$$= x + \sqrt{x^2 - 1}$$

$$x = 1 + \frac{Z_1}{2 Z_L}$$

$$= \sqrt{2 Z_2 \parallel \left(Z_1 + \frac{2 Z_1 Z_{0T}}{2 Z_2 + Z_{0T}} \right) \left[2 Z_2 \parallel (Z_1 + 2 Z_L) \right]}$$

$$= \sqrt{\frac{Z_1 Z_L}{1 + Z_1 / 4 Z_L}}$$

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For a π -network.

$$\text{Coeff } \rho = x = 1 + \frac{Z_1}{2 Z_L}$$

$$Z_{0T} = \sqrt{Z_{0T} Z_{0R}}$$

$$= \sqrt{2 Z_2 \parallel \left(Z_1 + \frac{2 Z_1 Z_{0T}}{2 Z_2 + Z_{0T}} \right) \left[2 Z_2 \parallel (Z_1 + 2 Z_L) \right]}$$

$$= \sqrt{\frac{Z_1 Z_L}{1 + Z_1 / 4 Z_L}} \quad Z_{0T} Z_{0R} = Z_1 Z_L$$

So if I take the product you can see Z_{0T} multiplied by Z_{0R} will be the product of these 2 it will be Z_1, Z_2 this is also very interesting result. Now let us see what will be the ratio that we saw last time the ratio of the current in a logarithmic scale what will be this value in case of a phi network. So once again in case of a phi network if I take this as the receiving end current, this as the sending end current, suppose this intermediate current is some I_1 then how much is I_R in terms of I_1 .

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Sym. T-network. $Z_A = Z_B$

$Z_{ii} = Z_{il} = Z_{li} = Z_{ll} = Z_0$

$= \sqrt{Z_A Z_L + Z_A Z_C + Z_A}$

$= \sqrt{Z_A + 2Z_A Z_C}$

$= \sqrt{\frac{Z_A}{1} + Z_1 Z_L}$

$= \sqrt{Z_1 Z_L \left(1 + \frac{Z_1}{Z_1 Z_L}\right)}$

$\log_e \left(\frac{I_S}{I} \right) = P$

$= x$

$x = 1 + \dots$

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For a $\bar{\pi}$ -network.

$\cosh P = x = 1 + \frac{Z_1}{2Z_L}$

$Z_{0x} = \sqrt{Z_{0c} Z_{0o}}$

$= \sqrt{2Z_2 \parallel \left[Z_1 \parallel (Z_2 + 2Z_L) \right]}$

$= \sqrt{\frac{Z_1 Z_L}{1 + \frac{Z_1}{2Z_L}}}$

$Z_{0x} = Z_1 Z_L$

Let us evaluate that I_R in terms of I_{one} it will be I_1 multiplied by twice Z_2 divided by the total impedance. So twice Z_2 divided by twice Z_2 plus Z naught phi is that all right and once again I_1 is what kind of share this current will have from I_S , it will be I_S multiplied by this impedance twice Z_2 divided by the total impedance which is twice Z_2 plus Z_1 plus parallel combination of this 2. So twice Z_2 plus Z_1 plus twice Z_2 , Z naught phi by twice Z_2 plus Z naught phi. So this fraction of I_S is I_1 then that gets multiplied by this factor to give you the total current is that okay.

So I_S by I_R if one evaluates I am not going to the details IS by IR if one evaluates and sorry and computes the logarithmic, in the logarithmic scale the propagation this will be equal to log of twice Z_2 plus after all reductions it will you have to substitute the value of Z naught phi which we have already go Z_1 , Z_2 by 1 plus Z_1 by 4, Z_2 under root. So you have substitute there and then after simplification it boils down to Z_1 , Z_2 into 1 plus how much Z_1 by twice Z_2 or 4 Z_2 you check up Z_1 by 4, Z_2 divided by Z_2 okay.

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Handwritten derivation on a whiteboard:

$$I_R = I_S \frac{2Z_2}{2Z_2 + Z_1}$$

$$= I_S \frac{2Z_2}{2Z_2 + Z_1 + \frac{2Z_2 \cdot 2Z_2}{2Z_2 + Z_1}} \cdot \frac{2Z_2}{2Z_2 + Z_1}$$

$$\log_e \left(\frac{I_S}{I_R} \right) = \log_e \left[1 + \frac{Z_1}{2Z_2} + \frac{\sqrt{Z_1 Z_2} \left(1 + \frac{Z_1}{2Z_2} \right)}{2Z_2} \right]$$

Below the main equation, there is a note: $\coth P = x = 1 + \frac{Z_1}{2Z_2}$

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Handwritten derivation on a whiteboard:

$$P = \log_e \left(\frac{I_S}{I_R} \right) \rightarrow \text{Napier Neper}$$

$$\frac{P_1}{P_2} = \left(\frac{I_S}{I_R} \right)^2 \quad \begin{array}{l} \text{(Voltage gain)} \\ \text{or} \\ \text{(Current gain) = Power gain} \end{array}$$

$$10 \log_{10} \left(\frac{I_S}{I_R} \right)^2 \rightarrow \text{dibel}$$

$$\log_{10} \left(\frac{P_1}{P_2} \right) = \log_{10} \left(\frac{I_S}{I_R} \right)^2 = 2 \log_{10} \left(\frac{I_S}{I_R} \right)$$

$$= 2 \times \log_e \left(\frac{I_S}{I_R} \right) \times \log_e 10$$

So this once again gives me an expression similar to if I assume this to be x plus x squared minus 1, so the expressions are identical whether it is a ϕ network or a T network and once again we get $\cos P$ if you take the exponential of that that gives me this is equal to P , so if you take e to the power P is so much all right, e to the power P is x plus root over of x squared minus 1 hence we get $\cos P$ is equal to x and x is this quantity $1 + Z_1$ by twice Z_2 okay. So so far we have derived the conditions, we have derived the relation between the 2 currents when the network is terminated by the characteristic impedance in both the cases that is a T network or a ϕ network of course symmetrical network.

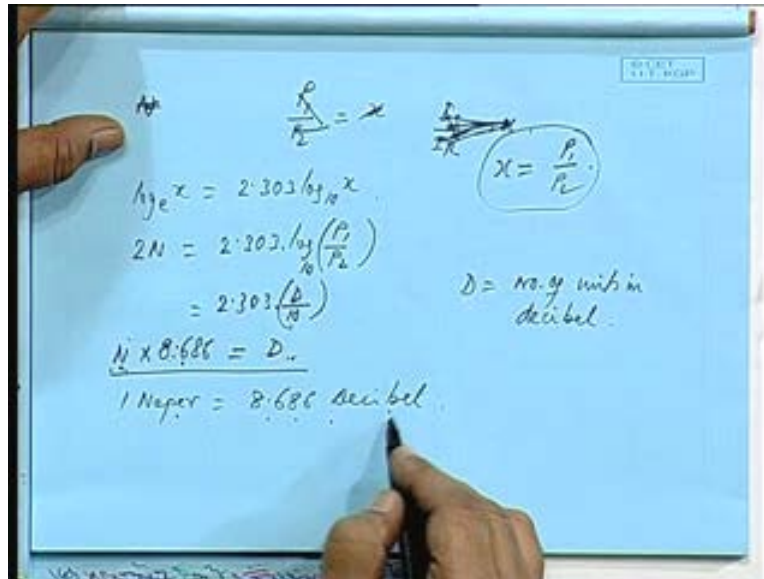
Now from this point we bring the concept of a filter before we go to that I will just explain what this quantity P means, this P was \log of I_S by I_R . Now this unit is after the name of the mathematician Napier, the name of the unit is Neper mind you this is a dimension less quantity it is just a logarithmic quantity it refers to the current gain or current loss we can say how current propagates, how it gets diminished or reflected by this ratio when the network is terminated by the characteristic impedance.

Now in any network if we take the ratio of the power P_1 is the power that is pumped into the circuit, P_2 is the power that is extracted at the receiving end then it will be equal to if it is compared with the same say, if the current is flowing through the same resistance. So 1ohm then it will be proportional to I_S by I_R squared okay that is voltage gain or current gain if you know then square of that will be the power gain. Okay mind you this is power P_1 , P_2 etcetera whereas this P refers to only the ratio, this logarithmic ratio.

Now we define we have already done in bode a plot when we talk about voltage gain or current gain I_S by I_R \log of that with the base 10 was the unit bel okay and 10 times that was decibel okay. So you take smaller units smaller denomination to define a unit, now what will be the relationship between decibel and Neper. Now see Neper is okay, let us see \log of P_1 by P_2 is \log of I_S by I_R **I am** I am extremely sorry. The ratio of power when you take the logarithm with the base 10 and you take a denomination of 10 then that becomes the unit of that becomes decibel all right the unit is decibel if this 10 is not there it is bel so bel is basically logarithm of the ratio power and logarithm of ratio of current or voltage, we are defining as Neper.

So \log of P_1 by P_2 to the base 10 is \log of I_S by I_R to the base 10 squared, so it is 2 \log of I_S by I_R to the base 10 and if one change as this to 2 into \log of I_S by I_R to the base e then this will be multiplied by \log of 10 to the base e okay. Suppose the number of units number of units, number of units in Neper for such a ratio is N then say the ratio P_1 by P_2 is say x okay.

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Okay I_S by I_R is x , so if it is x then \log of x to the base e will be $2.303 \log$ of x to the base 10 okay and $2N$ is therefore $2.303 \log$ of P_1 by P_2 to the base 10 , should we take okay I have I am extremely sorry, I was starting with this ratio P_1 by P_2 so if P_1 by P_2 is x then it will be simply like this. So this will be 2.303 to the base this is in decibel, so D by 10 okay.

So N into if you take this 10 on this side 2.303 if you divide N into 8.686 will be the number of units in decibel okay where D is the number of units in decibel for that same quantity that means 1 Neper is equal to 8.686 decibel, mind you N is the number of units. So if the number of units is small okay then multiply by 8.686 you get number of units in decibel its like 1 rupee corresponds to 100 paise.

So paise is a smaller denomination but then number of units will be more for a, for a coin if we take say 101 paise coin, so number of units will be 100 the value of that coin is less 1 rupee it is just a single 1 coin that will be representing a higher denomination okay. So if the denomination is higher number of units will be less so N multiplied by 8.686 will be equal to D it is something like rupee multiplied by 100 you get 100 coins. So the value of D is very very small, so 1 paise if you multiply by 100 by this number then only you get 1 rupee it means that so 1 Neper is equal to 8.686 decibel I hope this point is clear.

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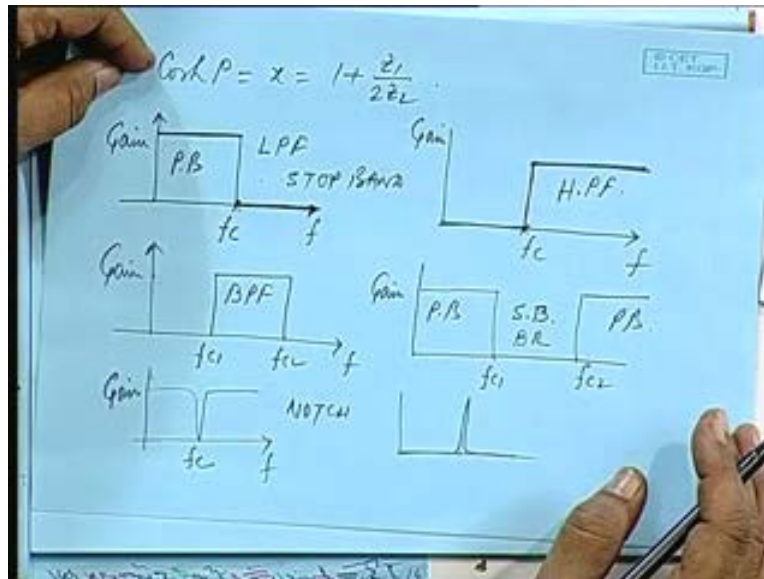
$$= I_s \cdot \frac{Z_L}{2Z_L + Z_1 + \frac{2Z_1 \cdot Z_0 \sqrt{x}}{2Z_1 + Z_0 \sqrt{x}}} \cdot \frac{2Z_L}{2Z_L + Z_0 \sqrt{x}}$$

$$\left(\frac{I_s}{I_L} \right) = \log_e \left[1 + \frac{Z_1}{2Z_L} + \frac{\sqrt{Z_1 Z_0} \left(1 + \frac{Z_1}{2Z_L} \right)}{Z_L} \right]$$

$$= \log_e \left[x + \sqrt{x^2 - 1} \right] = P$$

$$\text{Cos } P = x + \sqrt{x^2 - 1} \Rightarrow \text{Cos } P = x = 1 + \frac{Z_1}{2Z_L}$$

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Now again we will come back to the relationship that we have derived $\cos P$ is equal to x that was $1 + Z_1$ by twice Z_2 okay. So we will start from this point we will start from this point for going into the design of a filter. Now what does a filter do in a filter there are let us have a very basic elementary idea about the filter that we are going to discuss now filter is a network which will be allowing say this is a gain some of the frequencies present in the signal to pass through on attenuated there is no attenuation there is no change and rest of the frequencies are arrested they are not allowed to go this is the case

of an ideal filter, ideal low pass filter. An ideal filter will have a constant gain in the pass band, this is the pass band and it has a 0 gain in the stop band.

Similarly, a high pass filter, an ideal high pass filter will have a gain characteristics like this this frequency we call a cut off frequency, so beyond this cut off frequency the gain is uniform 1 and before that it is not allowed to pass through. So this is a high pass filter similarly we also have a band pass filter that is certain band is allowed f_{c1} , f_{c2} this is the band pass filter and we also have a band gap or band rigid filter that is frequencies below f_{c1} will be allowed to go will be allowed to pass through similarly, frequencies above f_{c2} will be allowed.

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$$P = \alpha + j\beta$$

$$\frac{I_s}{I_R} = e^{\alpha + j\beta} = e^{\alpha} \cdot e^{j\beta}$$

$$I_s = |e^{\alpha} / I_R| e^{j\beta} \quad \cosh P = x$$

$$\cosh(\alpha + j\beta) = x = 1 + \frac{z_1}{2z_2}$$

So this is a pass band, this is a pass band and this is a stop band okay. So this is band rigid filter or band stop filter. If in this band stop filter, if you have this gap very very narrow almost like this then it is a notch filter similarly, similarly if it is allowed only in this narrow band this is an inverted notch okay. Now we will see what this factor P means P may be a complex quantity when you are taking the ratio I_s by I_R in a complex network this may have a phase this may have a phase, so this is a complex network. So the logarithm of that also may be a complex quantity means the current magnitude changes its phase is also changing. So let that complex factor be alpha plus j beta that means I_s by I_R is basically e to the power alpha plus j beta which means e to the power alpha is the magnitude path and e to the power j beta what is it mean I_s is equal to e to the power alpha times I_R in magnitude.

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$$I_R = I_s \frac{2Z_L}{2Z_L + Z_0K}$$

$$= I_s \frac{2Z_L}{2Z_L + Z_0 + \frac{2Z_L Z_0K}{2Z_L + Z_0K}} \cdot \frac{2Z_L}{2Z_L + Z_0K}$$

$$\log_e \left(\frac{I_s}{I_R} \right) = \log_e \left[1 + \frac{Z_0}{2Z_L} + \frac{\sqrt{2Z_L \left(1 + \frac{Z_0}{Z_0K} \right)}}{Z_L} \right]$$

$$= \log_e \left[x + \sqrt{x^2 - 1} \right] = P$$

$$\Rightarrow \cosh P = x = 1 + \frac{Z_0}{2Z_L}$$

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$$P = \alpha + j\beta$$

$$\frac{I_s}{I_R} = e^{\alpha + j\beta} = e^\alpha \cdot e^{j\beta}$$

$$I_s = |e^\alpha / I_R| e^{j\beta} \quad \cosh P = x$$

$$\cosh(\alpha + j\beta) = x = 1 + \frac{Z_0}{2Z_L}$$

\times LPF
 HPF

So the receiving end current gets multiplied by e to the power α to give you the magnitude of I_s and with a phase shift of β so the receiving end current if you multiply a factor e to the power $j\beta$ if you give a phase shift then only you will get I_s the angle associated with I_s so P is equal to $\alpha + j\beta$ what is $\cosh(\alpha + j\beta)$ we have seen $\cosh P$ is equal to x this is $\cosh P$, this is $\cosh P$, so that is equal to x and that was $1 + Z_1$ by twice Z_2 . Now our filter design starts from this point onward.

Now from the discussion on what exactly the filter is suppose to do we have seen the filter is basically suppose to allow a certain band of frequency on attenuated. So for the low pass filter if you consider if I consider alpha when alpha is 0, this is 1 that means the ratio of the current is 1 current is not suffering any change in magnitude. So for alpha this value up to f_c should be 0 and then ideally speaking it should be going to 0 value there should not be any current received and hence I_S by I_R will tend to infinity ideally this should be the characteristics of low pass filter.

Similarly, for the high pass filter for the high pass filter, we should have alpha very large up to f_c and then it should be 0 okay. So this is high pass filter, this will not be allowed this is the forbidden Z_1 this is the pass band you can draw similar characteristics for band pass and band rigid filter.

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Handwritten notes on a blueboard:

$$\cosh(\alpha + j\beta) = 1 + \frac{Z_1}{2Z_2}$$

$$\cosh \alpha \cos \beta + j \sinh \alpha \sin \beta = 1 + \frac{Z_1}{2Z_2}$$

Assume Z_1, Z_2 reactive elements

$$1 + \frac{Z_1}{2Z_2} \rightarrow \text{Real}$$

$$\therefore \sinh \alpha \sin \beta = 0$$

Case I $\sinh \alpha = 0 \quad \alpha = 0$
 This corresponds to the Pass band
 $\cosh \alpha = 1$
 $\cos \beta = 1 + \frac{Z_1}{2Z_2}$

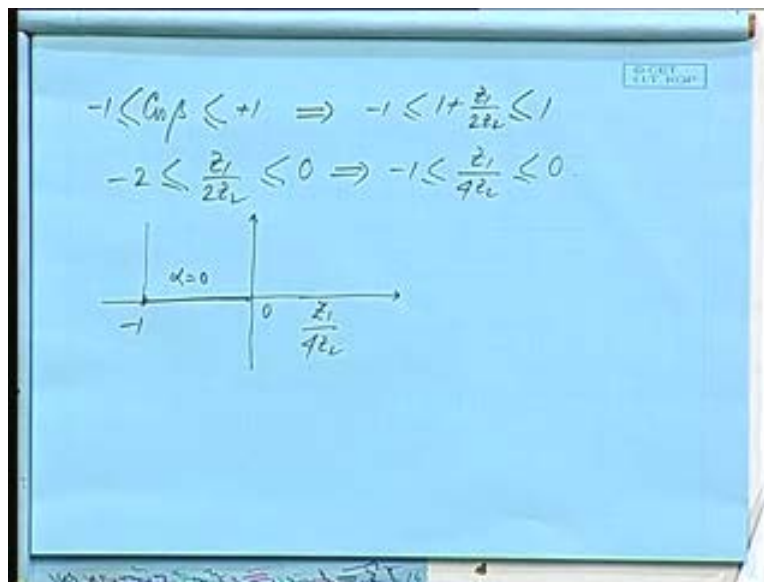
So let us come to the expression for the cos h P term this is 1 plus Z_1 by twice Z_2 , this means cos h hyperbolic alpha, cos beta plus j times sin h alpha, sin beta is equal to 1 plus Z_1 by twice Z_2 . Now let us assume that Z_1 and Z_2 are loss less that means when we are transmitting a signal when we are transmitting a signal through the network there is no loss in the network all right, power is directly given to the load. So Z_1 and Z_2 are loss less reactive element, reactive elements okay.

So this will be there may be of the same type, there may be of the opposite type we will study if Z_1 by twice Z_2 this is both of them are reactive then this will be a real quantity. So this whole expression is real, so the imaginary part is 0 therefore 1 plus Z_1 by twice Z_2 is real therefore sin h alpha multiplied by sin beta is 0. So there are 2 possibilities either this is 0 or this is 0 okay so we will take up we will take up one at a time case 1

when $\sin h \alpha$ is 0 that means α is equal to 0. So this is the pass band you have just now seen when α is equal to 0 when α is equal to 0 it is a pass band here α is 0, so this is a pass band this is the stop band.

So this corresponds to the pass band okay. Now for $\sin h \alpha$ to be 0 that is α to be 0 what are the conditions on Z_1, Z_2 so $\sin h \alpha$ is 0, so $\cos h \alpha$ is 1, so here $\cos h \alpha$ into $\cos \beta$ $\cos h \alpha$ is 1, so $\cos \beta$ is equal to $1 + Z_1$ by twice Z_2 . Therefore, $\cos \beta$ is equal to $1 + Z_1$ by twice Z_2 okay $\cos \beta$ in magnitude cannot be more than 1 that means we know $\cos \beta$ must lie between plus 1 and minus 1, is it not?

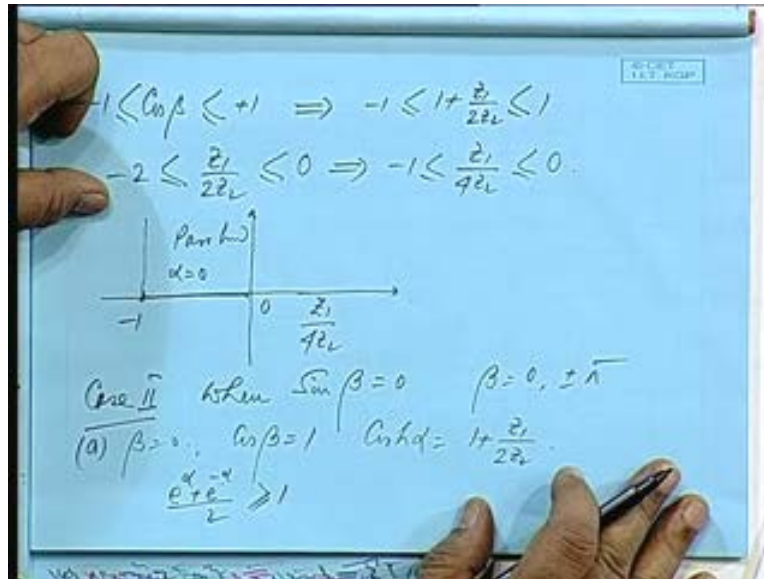
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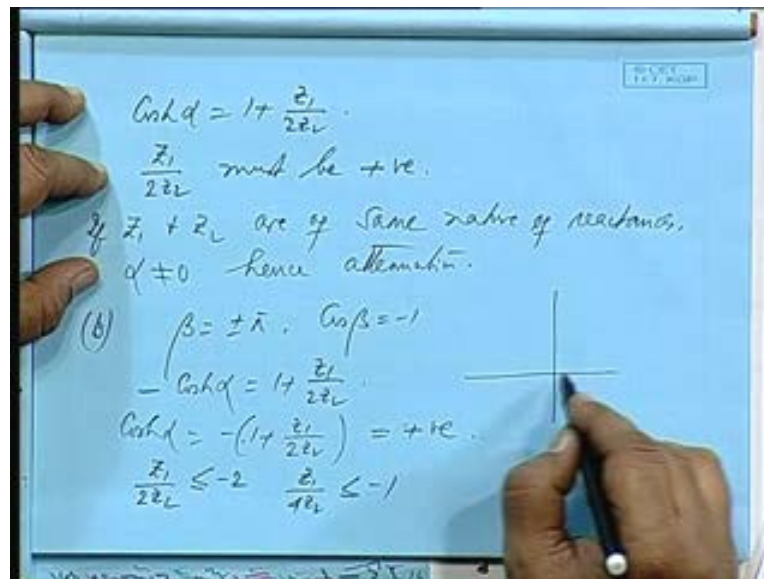
So that means $1 + Z_1$ by twice Z_2 , $1 + Z_1$ by twice Z_2 should lie between minus 1 and plus 1. So minus 1, $1 + Z_1$ by twice Z_2 should be 1 subtract 1 from all of them that gives me minus $2 Z_1$ by twice Z_2 less than equal to 0 which means if I further divide by 1 minus 1, Z_1 by $4 Z_2$, 0 what does it mean? It means from 0 if I take Z_1 by $4 Z_2$ this ratio then it should lie between 0 and minus 1, Z_1 by $4 Z_2$ should lie between 0 and minus 1 then this is the pass band here only α is equal to 0, okay.

Let us take the second case, when sine beta is equal to 0, so this is the pass band when sine beta is equal to 0 there are 2 possibilities beta can be 0 or plus minus phi. Now case, first case if beta is 0 that is sine beta is 0 then $\cos \beta$ is equal to 1 therefore $\cos h \alpha$ becomes $1 + Z_1$ by twice Z_2 , the $\cos h \alpha$ is all ways a positive quantity, is it not? e to the power α plus e to the power minus α by 2 you put any value of α fraction, integer anything it will be always positive more than 1, this is also more than 1.

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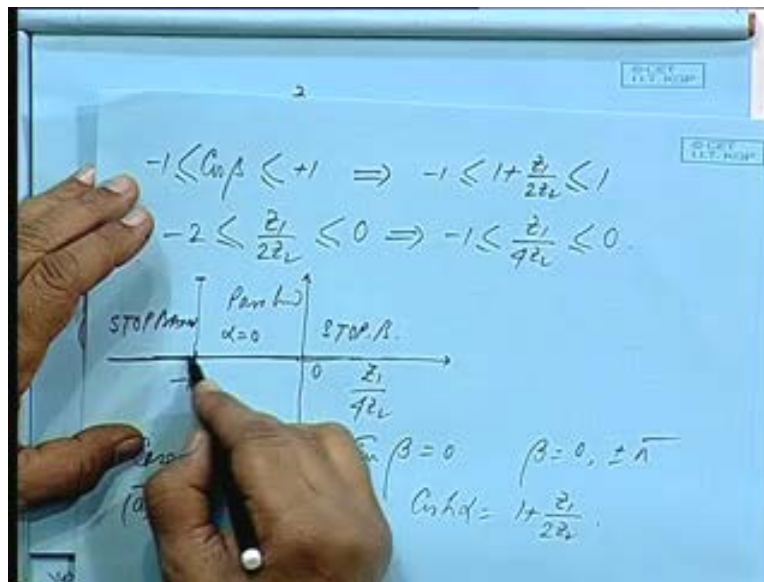
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So added together divided by 2 is also greater than 1 is always greater than equal to 1 when it is equal to 1, when alpha is equal to 0. So cos h alpha which is greater than 1 should be 1 plus you have got cos h alpha is equal to 1 plus Z_1 by twice Z_2 . So Z_1 by twice Z_2 must be positive which means Z_1 and Z_2 are of the same type if there of the same type of reactances then they will be definitely some value of alpha other than 0 that means there will be attenuation okay. See, if we have Z_1 and Z_2 , if Z_1 and Z_2 are of same nature that is both are inductive, are both are capacitive are of same nature of

reactances then alpha is not equal to 0. Hence, there will be attenuation. So it will be in the stop band an ideal filter will have infinite attenuation if there is some attenuation it will not be the pass band. Now in the second case, we have beta is equal to plus minus phi so cos beta is minus 1 therefore minus cos h alpha is equal to 1 plus Z₁ by twice Z₂, now this quantity you can see from here cos h alpha is therefore minus of 1 plus Z₁ by twice Z₂. This can be realized only when this quantity is highly negative, so that this plus 1 is swamped out and the overall quantity becomes plus okay.

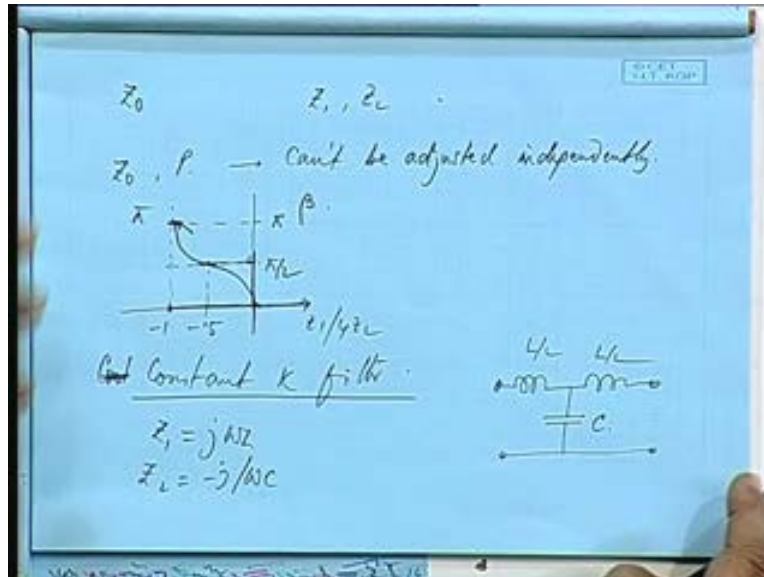
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So this needs since this is always positive so Z₁ by twice Z₂, Z₁ by twice Z₂ you can see for yourself must be less than minus 2 that means Z₁ by 4 Z₂ must be less than equal to minus 1. So that will be giving me alpha not equal to 0 and beta will have a value of plus minus phi. So Z₁ by 4 Z₂ less than minus 1 this is the region where beta will be plus minus phi and this is the stop band, this is the forbidden region and here also when both of them are positive, both of them are of the nature, the ratio is positive this is also a stop band okay.

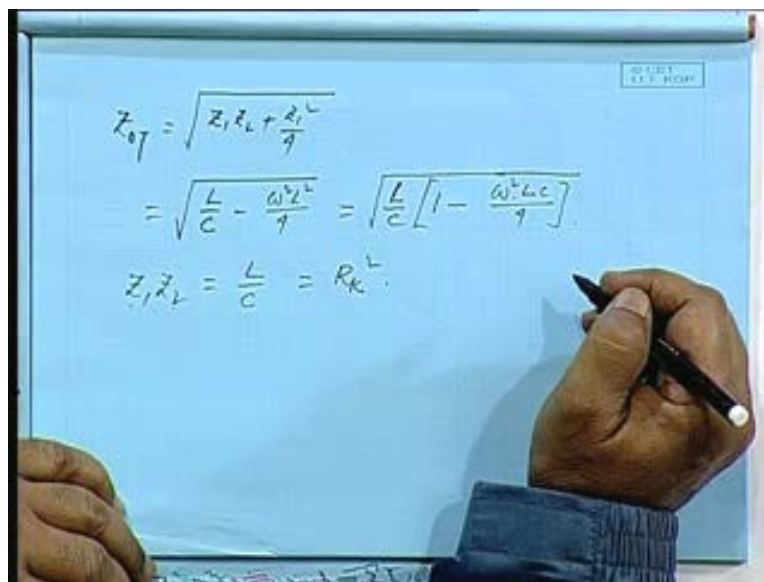
So this is what we have got now. Now what about Z₀, how will that change? Now the characteristic impedance of the filter will be either real or imaginary it cannot be in between it cannot be just a complex number either in the pass band, it is alpha equal to 0 or in the stop band it will be very large but beta will be having a phase shift of either 0 or plus minus phi, in the 2 regions, in the 2 stop bands. In the case of purely reactive elements say Z_A or Z₁ and Z₂ purely reactive then you do not get a situation when this will be complex X that is, this is the ideal situation.

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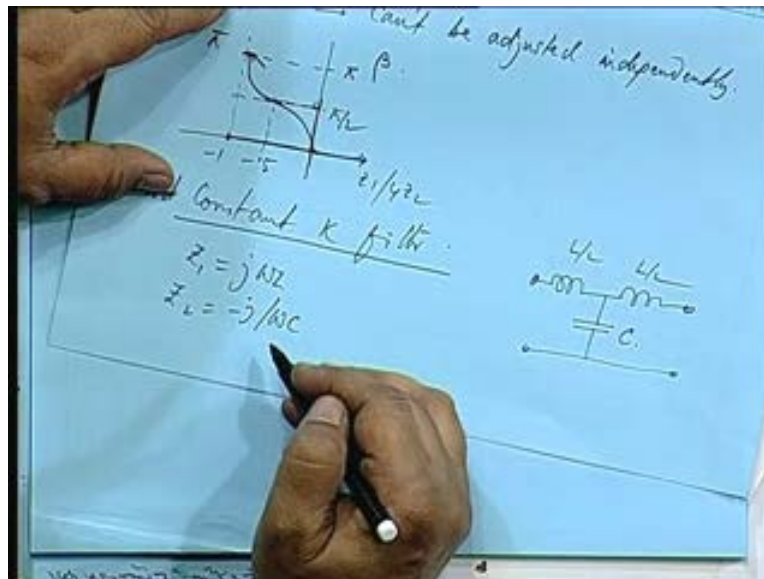


So Z naught let us see the characteristic impedance Z naught and P propagation constant this cannot be adjusted independently. You cannot adjust them independently one is dependent on the other, cannot be adjusted independently. This is a very important point. Now let us see how beta varies, beta was 0 here this is minus 1, this is Z_1 by 4 Z_2 and here sorry alpha was 0, here beta was 0 here, some are here it was say phi it will be we will see later on how this characteristics will change it will be something like this at minus .5 beta will be phi by 2, this is phi okay. So this is beta.

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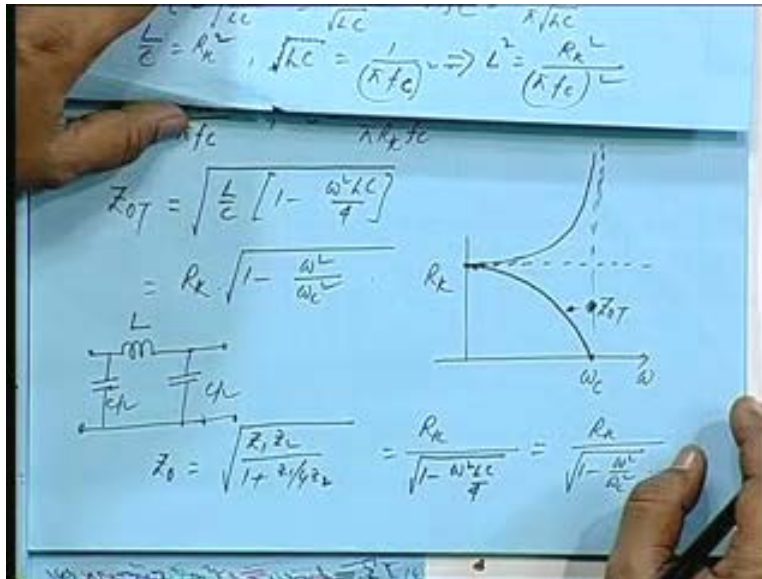
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Now we will go for the design of a constant k filter, constant k filter. Let us take 2 elements say Z_1 is $j \omega L$ just an inductance that means we have L by 2 and then L by 2 and a capacitance C here, Z_2 is equal to minus j by ωC okay, what will be the characteristic impedance for this? We are considering for the time being only a T network we can also see for the pi network. So Z_{OT} is root over of Z_1, Z_2 plus Z_1 squared by 4 and Z_1, Z_2 if you take the product Z_1 into Z_2 it is ωL multiplied by 1 by ωc so L by c okay, root over of L by c plus Z_1 squared by 4 will be giving me minus ω squared L squared by 4 okay. Now if the product Z_1, Z_2 is taken Z_1, Z_2 is L by C we define this as a constant, constant k filter means it is totally resistive. So this constant is having a ohm value R_K is some defined in terms of ohms, so this is known as a constant k filter design when we take Z_1, Z_2 as the product R_K and we call this a prototype design there can be other derived filters from this prototype.

So L by C so this gives me root over of R_K square or okay I can take L by C outside then 1 minus ω square L, L by C is R_K, R_K squared, so this will be LC by 4 okay. When ω squared LC by 4 equal to 1 this becomes 0 okay. The reactants become 0 and this frequency we call ωC is therefore 1 by 4 by LC under root which is 2 by root LC corresponding radian frequency to hertz if we convert this will become 1 by ϕ root LC so many hertz. You have got 2 equations L by C is R_K squared and root LC is 1 by ϕf_c . So you may be given a specification in terms of the cut off frequency f_c and the constant R_K if you are given these 2 values as a filter specification then we can calculate L and C just take the product that gives me L if you take LC . So LC it will be ϕf_c squared then if you take the product it will become L squared is equal to R_K squared by ϕf_c squared.

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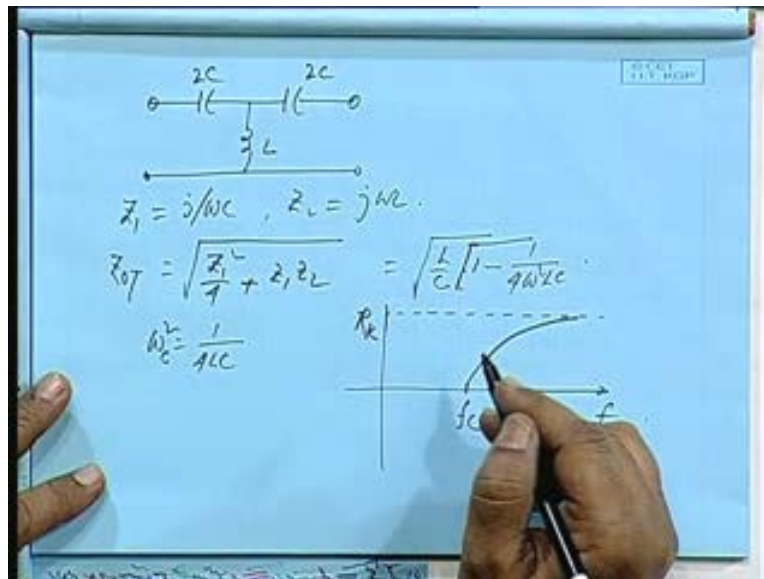


Hence, L will become R_K by πf_c okay and similarly, by eliminating LC can be determined from any of the equations into f_c . So normally the specifications are given in terms of this constant impedance and f_c , so Z_{OT} therefore comes out as which was given as L by C $1 - \omega^2 LC$ by 4 I can write this as L by C is Z_1, Z_2 or R_K squared, so this is R_K into $1 - \omega^2 LC$ by 4 . If you look at it LC by 4 was 1 by ω_c^2 , so squared so with respect to frequency it will start when $\omega = 0$ it is R_K from this value and it will gradually fall like this okay.

So the characteristic equation does not remain constant as such it varies with frequency in this manner this is for T network Z_{OT} one may derive similarly for a π network, the situation is like this you have C by $2, C$ by 2 okay. So for a π network what will be the values of Z naught, Z naught you know is root over of Z_1, Z_2 divided by $1 + Z_1$ by $4 Z_2$.

So substitute for this Z_1, Z_2 is the same product okay which will be R_K and $1 + Z_1$ by $4 Z_2$ once again by the same logic if you substitute for Z_1 $j \omega L$ for $Z_2, 1$ by $j \omega C$ minus j by ωC okay that gives me $1 - \omega^2 LC$ by 4 which is again written as $1 - \omega^2$ by ω_c^2 . So when ω is very very small this is R_K , when ω is very very large up to ω_c , when ω is ω_c this becomes 1 , so $1 - 1$ is 0 , so this becomes infinity. So it will be shooting up at this frequency, it will go to infinity, it will be approaching infinity mind you when you go beyond ω_c it becomes imaginary both the cases, it becomes imaginary.

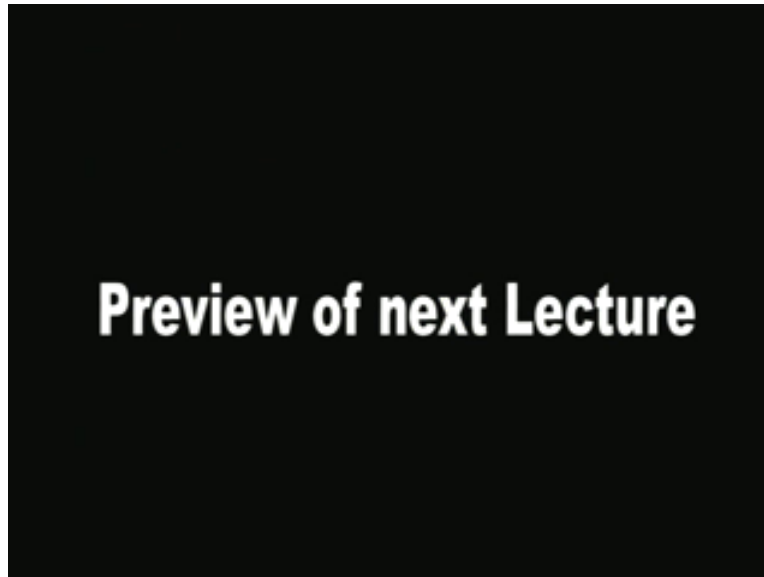
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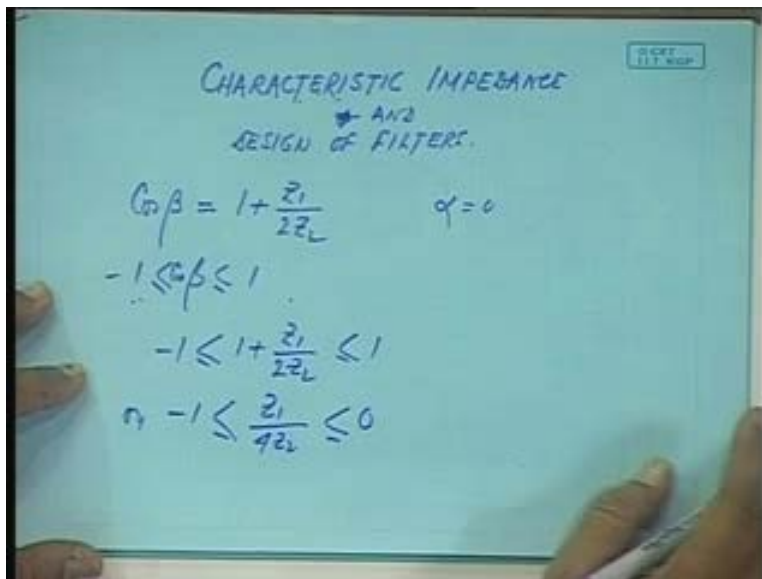
So the characteristic impedance so which is over from a real quantity to an imaginary quantity okay that means you cannot transmit power imaginary means the load impedance is imaginary, so you cannot transmit any power as simple as that. Now if you have instead of inductance if you have capacitance in the series and inductance in parallel this arrangement then what you get, Z_1 and Z_2 they have interchange their elements type. So this becomes ωC , Z_2 becomes $j\omega L$, so Z_{OT} I will just write the expression for the T network which is Z_1 squared by 4 plus $Z_1 Z_2$ okay that gives me root over of L by C into if you permit me to simplify 4ω squared LC okay.

So when 4ω squared LC this becomes equal to 1 that is ω squared is 1 by $4LC$ corresponding frequency we call it ωC then this becomes how much when ω square C ωC squared is equal to 4 , 1 by $4LC$ that is this whole quantity is 1 , so 1 minus 1 is 0 , so Z_{OT} is 0 . So here at this frequency it is 0 and when ω is very very large this becomes all most 0 , it is root L by C . So it will be tending towards R_k root L by C is R_k , so from f_c onward you can write f_c or ωC either you take f or ω axis, so it will become, it will tend to R_k we will stop here for today we will continue with this in the next class, thank you very much.

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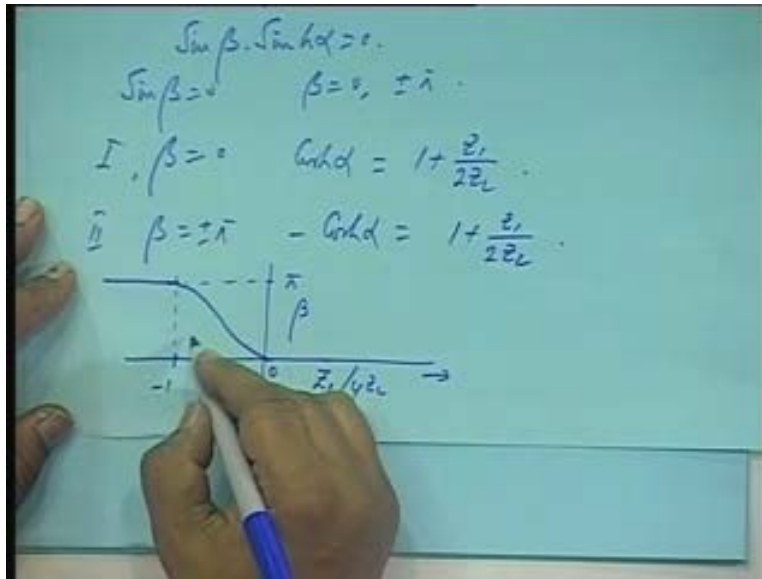


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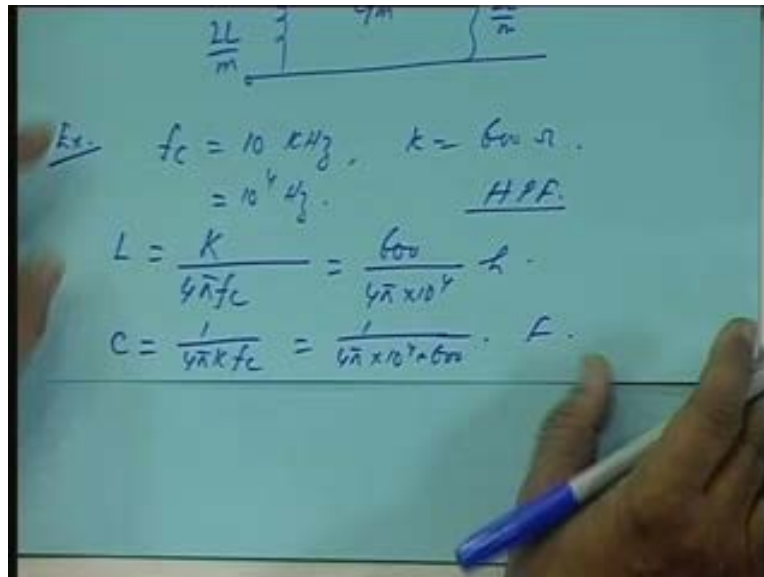


Okay, good morning friends. Last time we discussed about the properties of impedance characteristic impedance and today we will continue that and discuss about design of filters. Now we saw last time cosine beta was 1 plus Z_1 by twice Z_2 when alpha equal to 0 that is in the pass band this means because beta is sorry cosine beta is between minus 1 and plus 1. So you can write minus 1, 1 plus Z_1 by twice Z_2 less than equal to 1 if I subtract 1 from all the 3 then it reduces to Z_1 by 2 Z_2 less than equal to 0 okay.

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Now in the stop band, in the stop band, we have in the stop band where alpha is not equal to 0, we put the other condition sine beta, sine hyperbolic alpha equal to 0 okay that means there are 2 possibilities, if sine beta equal to 0 then beta can be either 0 or plus minus phi, if beta is 0 then cos h hyperbolic alpha that is cos h alpha is 1 plus Z₁ by twice Z₂ and if beta equal to plus minus phi then minus cos h alpha it will be 1 plus Z₁ by twice Z₂ this you saw last time. So from here we can find if I take Z₁ by 4 Z₂ along this x axis then between 0 and minus 1 of this, this is a pass band and beta will be varying like this.

So in the pass band that R_K okay and f_c this was the relation so it will be 600 by 4 phi into 10 to the power 4 so many Henrys okay. Similarly, capacitance would be 1 by 4 phi K into f_c that is 1 by 4 phi into 10 to the power 4 into 600 so many Farads okay.

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Ex 2 T-section L.P.F.
 freq f , $\alpha = 10 \text{ dB}$
 $\alpha = 10 \text{ dB} = \frac{10}{8.696} \text{ neper}$
 $= 1.15 \text{ neper}$
 $\alpha = 2 \cosh^{-1}\left(\frac{f}{f_c}\right)$
 $1.15 = 2 \cosh^{-1}\left(\frac{f}{f_c}\right)$
 $f = f_c \left[\cosh^{-1}\left(\frac{1.15}{2}\right) \right]$

Suppose you are asked to calculate, let us take another example you are asked to calculate for a T section low pass filter. Determine the frequency for which alpha frequency f for which alpha is equal to 10 dB, f in terms of f_c , frequency f in terms of f_c so alpha is given as 10 dB that is 10 by 8.696 if you remember we discussed earlier the relationship between dB and Neper. So that comes to approximately 1.15 Neper, so alpha is equal to twice cos h inverse f by f_c that is 1.15 is twice cos h inverse f by f_c okay. So from here you can see f will be f_c into cos h inverse 1.15 by 2, 1.15 by 2. So this is a relationship between f and f_c at this frequency, at this value of alpha okay.

So we will stop here for today and next time will be taking up some more basic elements of graph theory, how to solve networks, network problems with the help of graph theory. Thank you very much.